HOMEWORK 14   Due: March 26

1. A sample of size n is chosen randomly from a population that can be described by a Normal model with mean μ, and standard deviation σ.

   What is the sampling distribution model of sample means? Describe the shape, center, and spread.

   Shape: will be normal since the population is normal.

   Center: is the population mean, μ.

   Spread: the spread will be less than the population standard deviation: $\frac{\sigma}{\sqrt{n}}$

2. Place an X by any of the following statements that are NOT true according to the Central Limit Theorem:

   ___ An increase in sample size from $n = 16$ to $n = 25$ will produce a sampling distribution with a smaller standard deviation.

   _X_ The mean of a sampling distribution of sample means is equal to the population mean divided by the square root of the sample size.

   _X_ The larger the sample size, the more the sampling distribution of sample means resembles the shape of the population distribution.

   ___ The mean of the sampling distribution of sample means for samples of size $n = 15$ will be the same as the mean of the sampling distribution for samples of size $n = 100$.

   ___ The larger the sample size, the more the sampling distribution of sample means will resemble a normal distribution, regardless of the shape of the population distribution .

   ___ If the shape of the population distribution is itself normal, then the sampling distribution of sample means will resemble a normal distribution for *any* sample size.

3. The height of American adult women is distributed almost exactly as a normal distribution. The mean height of adult American women is 63.5 inches with a standard deviation of 2.5 inches. Imagine that all possible random samples of size 25 ($n = 25$) are taken from the population of American adult women's heights, and then the means from each sample are graphed to form the sampling distribution of sample means.

   a. Using the Central Limit Theorem, draw and label this sampling distribution. Show the mean and standard deviation on your drawing.

   The shape will be normal since the distribution of the height of women is almost exactly normal.
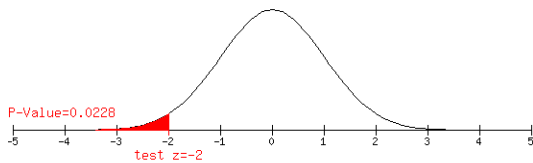   The mean will be 63.5 inches.
   The standard deviation will be $\frac{2.5}{\sqrt{25}} = 0.5$
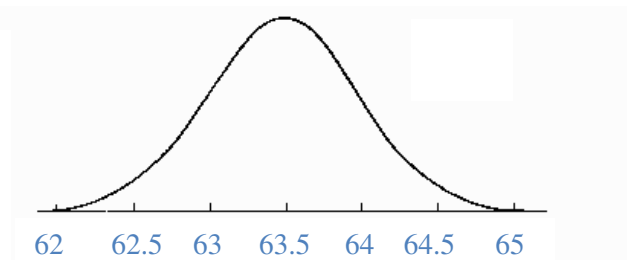
b. What is the probability that the mean height of a random sample of 25 women is less than 62.5 inches?

Since 62.5 is two standard deviations below the mean, you can use the Empirical Rule to find the approximate probability. Recall, 95% of the data is between two standard deviations below and above the mean. Thus, we have approximately 2.5% below 62.5 inches.
But if you don't remember this, you can find the probability using your z-table or calculator. The z-score is -2 (since 62.5 is two standard deviations below the mean). Thus, using the table, you get 2.28%.
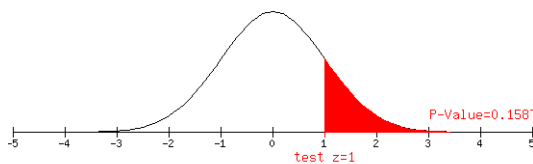
P-Value=0.0228

test z=-2

The probability that the mean height of a random sample of 25 women is less than 62.5 inches is about 2.3%.

c. What is the probability that the mean height of a random sample of 25 women is more than 64 inches?

Since 64 is one standard deviation above the mean, you can use the Empirical Rule again to find the approximate probability. Recall, 68% of the data is between one standard deviation below and above the mean. Thus, we have approximately 16% above 64 inches.
But if you don't remember this, you can find the probability using your z-table or calculator. The z-score is 1 (since 64 is one standard deviation above the mean). Thus, using the table, you get 15.87%.

P-Value=0.1587

test z=1

The probability that the mean height of a random sample of 25 women is more than 64 inches is about 16%.

4. The amount of time it takes to complete an exam has a skewed-to-left distribution with a mean of 65 minutes and a standard deviation of 8 minutes. A sample of 64 students is selected at random. Describe and draw a picture of the sampling distribution of the sample mean ($\overline{x}$) for samples of size n = 64.

The shape of the POPULATION distribution is left-skewed.
However, the shape of the SAMPLING DISTRIBUTION of the sample mean will be approximately normal since the sample size is large, 64.
The mean of the sampling distribution will be 65 minutes.
The standard deviation of the sampling
Distribution will be $\frac{8}{\sqrt{64}} = 1$

62   63   64   65   66   67   68
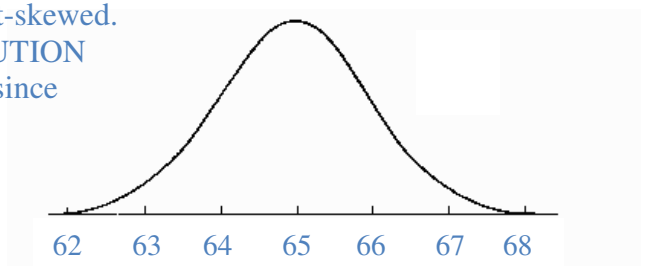
5. A brake pad manufacturer claims its brake pads will last for 38,000 miles, on average. Assume that the lifespans of the brake pads are normally distributed. Past analyses indicate that $\sigma = 5000$ miles. You work for a consumer protection agency and you are testing this manufacturer's brake pads using a random sample of 30 brake pads. In your tests, the mean lifespan of the brake pads you sample is 35,700 miles.

a. Would it be unusual to have an *individual* brake pad last for 35,700 miles? Why, or why not?

The z-score would be $\frac{35700-38000}{5000} = -0.46$. Since this z-score is between -2 and 2, it is a usual value. Thus, it wouldn't be unusual to have an individual brake pad last for 35,700 miles.

b. Assuming the manufacturer's claim is correct, what is the probability that the mean lifespan of the sample is as low as 35,700 miles? (That is, 35,700 miles or lower.)

The mean of the sampling distribution is still 38,000 miles, but the standard deviation is $\frac{5000}{\sqrt{30}} = 912.9$

The z-score would be $\frac{35700-38000}{912.9} = -2.52$.

Using the table, the probability is 0.0059.

Thus, assuming that the manufacturer's claim is correct, the probability that the mean lifespan of the sample of 30 brake pads is as low as 35,700 or lower is about 0.59%.

c. Using your answer from (b), what do you think of the manufacturer's claim?

Hard to believe that the manufacturer's claim is correct. The probability of getting a sample mean of 35,700 miles or lower in a random sample of 30 just by chance, assuming that the manufacturer's claim is correct, would be less than 1%.

d. Use your answers to parts a, b & c to structure a hypothesis test of the manufacturer's claim that $\mu = 38,000$ for his brake pads. Give:

(i)   the null hypothesis, in words:

The mean distance all the brake pads will last is 38,000 miles.

(ii) the null hypothesis, in symbols: $\mu = 38,000$ miles

(iii) the alternative hypothesis, in words:

The mean distance all the brake pads will last is less than 38,000 miles.

(iv) the alternative hypothesis, in symbols: $\mu \leq 38,000$

(choose >, < or ≠ to put here)

(v) the p-value if n=9: _____

The mean of the sampling distribution for sample sizes of 9 would be still 38,000 miles. The standard deviation, however would be $\frac{5000}{\sqrt{9}} = 1666.7$.

The z-score will be $\frac{35700-38000}{1666.7} = -1.38$.

And the lower tail probability (from the table) is 0.0838.

Thus, the p-value is about 8.4%.

(vi) your conclusion if n=9, based on how small the p-value is:

The p-value is not very low. That is, the probability of getting a sample mean of 35,700 miles or lower in a random sample of 9 just by chance, assuming that the manufacturer's claim is correct, would be about 8.4%. So this could happen quite often.
Based on this, we don't have enough evidence to reject the manufacturer's claim.

(vii) the p-value if n=36: _____

The mean of the sampling distribution for sample sizes of 36 would be still 38,000 miles. The standard deviation, however would be $\frac{5000}{\sqrt{36}} = 833.3$.

The z-score will be $\frac{35700-38000}{833.3} = -2.76$.

And the lower tail probability (from the table) is 0.0029.

Thus, the p-value is about 0.3%.

(viii) your conclusion if n=36, based on how small the p-value is:

The p-value is very low. That is, the probability of getting a sample mean of 35,700 miles or lower in a random sample of 36 just by chance, assuming that the manufacturer's claim is correct, would be about 0.3%. So this would not happen often just by chance.
Based on this, we have enough evidence to reject the manufacturer's claim.