The F-Test by Hand Calculator

Where possible, one-way analysis of variance summaries should be obtained using a statistical computer package. Hand-calculation is a tedious and error-prone process, especially with large data sets. This section gives formulae for calculating the F-test statistic and introduces tables of the F-distribution for those situations in which computers with statistical software are not easily available.

We take each group in turn, calculating and recording their mean and standard deviation, as in the table below.[1] We then use the following formulae directly.[2] The degrees of freedom are $df_1 = k - 1$ and $df_2 = n_{tot} - k$.

$$\overline{x}_{..} = \frac{n_1\overline{x}_{1.} + n_2\overline{x}_{2.} + \ldots + n_k\overline{x}_{k.}}{n_1 + n_2 + \ldots + n_k}.$$

$$s_B^2 = \frac{n_1(\overline{x}_{1.} - \overline{x}_{..})^2 + n_2(\overline{x}_{2.} - \overline{x}_{..})^2 + \ldots + n_k(\overline{x}_{k.} - \overline{x}_{..})^2}{k-1},$$

$$s_W^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \ldots + (n_k - 1)s_k^2}{n_{\text{tot}} - k}, \quad \text{and}$$

$$f_0 = \frac{s_B^2}{s_W^2}.$$

[Substantial simplifications can be made when all group sizes are equal – see below.]

Example 1 For the reading-methods data of Example 10.3.1 in the text we have $k = 4$ groups with summary statistics given in Table 1.

Table 1 : Summary Statistics For the Reading Methods Data

1.	Both:	$n_1 = 22$	$\overline{x}_{1.} = 1.4590909$	$s_1 = 1.543545$
2.	Map Only:	$n_2 = 12$	$\overline{x}_{2.} = 1.2333333$	$s_2 = 1.441170$
3.	Scan Only:	$n_3 = 7$	$\overline{x}_{3.} = 0.9142857$	$s_3 = 1.301830$
4.	Neither:	$n_4 = 9$	$\overline{x}_{4.} = -0.5555556$	$s_4 = 1.134803$

Here, $n_{\text{tot}} = 22+12+7+9 = 50$ and the degrees of freedom are $df_1 = k-1 = 3$

[1]This subsection assumes the availability of a calculator which automatically calculates means and standard deviations.

[2]Recall that the grand mean, $\overline{x}_{..}$, is the average of all n_{tot} observations (regardless of group).

and $df_2 = n_{\text{tot}} - k = 46$.

$$\begin{aligned}
\overline{x}_{..} &= \frac{22 \times 1.4590909 + 12 \times 1.2333333 + 7 \times 1.301830 + 9 \times -0.5555556}{50} = 0.966 \\
s_B^2 &= \{22 \times (1.4590909 - 0.966)^2 + 12 \times (1.2333333 - 0.966)^2 \\
&\quad +7 \times (0.9142857 - 0.966)^2 + 9 \times (-0.5555556 - 0.966)^2\}/3 = 9.02052 \\
s_W^2 &= \frac{21 \times 1.543545^2 + 11 \times 1.441170^2 + 6 \times 1.301830^2 + 8 \times 1.134803^2}{46} = 2.029361 \\
f_0 &= 9.02052/2.029361 = 4.445.
\end{aligned}$$

This agrees with the computer generated value of f_0 given in Fig 10.3.2 in the text.

Simplifications for equal sample sizes

In many examples, all of the individual sample sizes n_i are the same, i.e. $n_1 = n_2 = \ldots = n_k (= n$, say). Here, we can obtain s_B^2 as follows. Enter the individual sample means $\overline{x}_{1.}, \overline{x}_{2.}, \ldots, \overline{x}_{k.}$ as k numbers into the calculator and obtain their sample standard deviation – we will write the result as $s_{\overline{x}}$. Calculate $s_B^2 = n\, s_{\overline{x}}^2$. Obtain s_W^2 as the sample mean of the k numbers $s_1^2, s_2^2, \ldots, s_k^2$ (note that it is the *squares* of the standard deviations which are being averaged).

Example The summary statistics in Table 2 relate to the cell ratio data described in Review Exercises 10, problem 12 and given in Table 10 there. There are $k = 6$ treatment groups and $n = 50$ observations in each group. The degrees of freedom are $df_1 = k - 1 = 5$ and $df_2 = n_{\text{tot}} - k = 6 \times 50 - 6 = 294$.

Table 2 : Summary Statistics For the Cell Ratio Data

1. Control:	$n_1 = 50$	$\overline{x}_1 = 0.2366$	$s_1 = 0.1124243$
2. Choral hydrate:	$n_2 = 50$	$\overline{x}_2 = 0.2686$	$s_2 = 0.1406111$
3. Hydroquinone:	$n_3 = 50$	$\overline{x}_3 = 0.2812$	$s_3 = 0.1204692$
4. Diazepam:	$n_4 = 50$	$\overline{x}_4 = 0.3116$	$s_4 = 0.1761535$
5. Econidazole:	$n_5 = 50$	$\overline{x}_5 = 0.2646$	$s_5 = 0.1258475$
6. Colchicine:	$n_6 = 50$	$\overline{x}_6 = 0.4482$	$s_6 = 0.1755189$

The 6 numbers in the means column have sample standard deviation $s_{\overline{x}} = 0.07575022$. Thus, $s_B^2 = n s_{\overline{x}}^2 = 50 \times 0.07575022^2 = 0.2869048$. The sample mean of *the squares of* the 6 numbers in the standard deviation column is[3] 0.02076634. Thus, $s_W^2 = 0.02076634$. Finally, $f_0 = s_B^2/s_W^2 = 0.2869048/0.02076634 = 13.81586$.

[3] i.e. the sample mean of $0.1124243^2, 0.1406111^2, \ldots,$ and 0.1755189^2 is 0.02076634.

Use of F-distribution Tables

Having obtained the F-test statistic using a hand-calculator, we need tables of the F-distribution in order to obtain the corresponding *P-values* . The F-distribution is very similar in shape to the Chi-square distribution.[4] However, since the F-distribution depends upon two "degrees of freedom" parameters, we need a complete page of tables for each upper tail area of the distribution. The "10%" table in Appendix 1 at the end of this module gives us the value f such that $\text{pr}(F \geq f) = 0.10$. For example, suppose that we look up the entry in the column of the 10% table defined by $df_1 = 4$ and the row defined by $df_2 = 8$ we find that it is 2.81. Thus, for $df_1 = 4$ and $df_2 = 8$, $\text{pr}(F \geq 2.81) = 0.10$. The 5% table in Appendix 2 and the 1% table in Appendix 3 work in the same way. From the $df_1 = 4$ column and $df_2 = 8$ row of each of the latter tables we find that $\text{pr}(F \geq 3.84) = 0.05$ (5% table) and $\text{pr}(F \geq 7.01) = 0.01$ (1% table). Thus, when $F \sim \text{F}(df_1 = 4, df_2 = 8)$, we have

$$\text{pr}(F \geq 2.81) = 0.10, \ \ \text{pr}(F \geq 3.84) = 0.05, \ \text{and } \text{pr}(F \geq 7.01) = 0.01.$$

Let us now use this information to bracket a *P-value* . Suppose that $f_0 = 3.12$. Since f_0 lies between 2.81 and 3.84 we have that the *P-value* lies between 0.10 and 0.05, and it is closer to 0.10 than it is to 0.05.

Suppose now that $df_1 = 13$, $df_2 = 25$ and the F-test statistic is $f_0 = 2.5$. When we look up the tables, say Appendix 1, we find that we have $df_1 = 12$ with $f = 1.82$ and $df_1 = 15$ with $f = 1.77$, but there is no entry for $df_1 = 13$. Without access to more detailed tables we have to choose one of these two entries. Which one? Since we always act conservatively, we choose the tabulated df_1 with the bigger f-value (which makes it harder to get significance), namely $df_1 = 12$. We therefore use the tabulated value of df_1 immediately *less* than the value required. What happens if df_2 is not tabulated? Inspecting the tables we see that we do the same thing with df_2 as with df_1. For example, if $df_2 = 33$ we enter the table with $df_2 = 30$.

Exercises

1. For the F-distribution, obtain upper and lower values that the *P-value* lies between in the following cases:
  - **(a)** $f_0 = 2.13$, $df_1 = 10$, $df_2 = 6$
  - **(b)** $f_0 = 2.41$, $df_1 = 5$, $df_2 = 25$
  - **(c)** $f_0 = 3.83$, $df_1 = 8$, $df_2 = 18$
  - **(d)** $f_0 = 3.41$, $df_1 = 4$, $df_2 = 30$
  - **(e)** $f_0 = 2.98$, $df_1 = 6$, $df_2 = 45$

2. If $0.01 <$ *P-value* < 0.05 for an F-test with $df_1 = 5$ and $df_2 = 23$, between what two values did f_0 lie?

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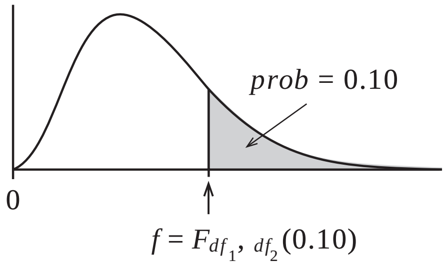
[4] In fact, if $F \sim \text{F}(df_1, df_2 = \infty)$ then $df_1 F \sim \text{Chi-square}(df_1)$.

3. Recompute the analysis of variance table for the variable DISPERSION in Exercises 10.3 in the text with the outlier in the Black group omitted. (The F-ratio is 17.08.) Were your intuitions about the effect of the outlier confirmed?

Appendix 1 *F*-distribution, 10% Table

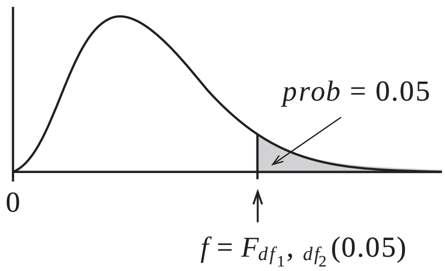
For fixed df_1, df_2, the tabulated value is the number $f = F_{df_1,df_2}(0.10)$ such that for $F \sim F(df_1, df_2)$, $\text{pr}(F \geq f) = 0.10$.



df_2 \ df_1	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	1000
1	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.7	61.2	61.7	62.3	62.8	63.3
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.46	9.47	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.15	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.82	3.79	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.17	3.14	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.80	2.76	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.56	2.51	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.38	2.34	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.25	2.21	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.16	2.11	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.08	2.03	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.01	1.96	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.96	1.90	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.91	1.86	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.87	1.82	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.84	1.78	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.81	1.75	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.78	1.72	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.76	1.70	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.74	1.68	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.72	1.66	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.70	1.64	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.69	1.62	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.67	1.61	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.66	1.59	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.65	1.58	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.64	1.57	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.63	1.56	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.62	1.55	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.61	1.54	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.54	1.47	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.48	1.40	1.29
80	2.77	2.37	2.15	2.02	1.92	1.85	1.79	1.75	1.71	1.68	1.63	1.57	1.51	1.44	1.36	1.24
100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66	1.61	1.56	1.49	1.42	1.34	1.21
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.41	1.32	1.19
1000	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.34	1.24	1.00

Appendix 2 *F*-distribution, 5% Table

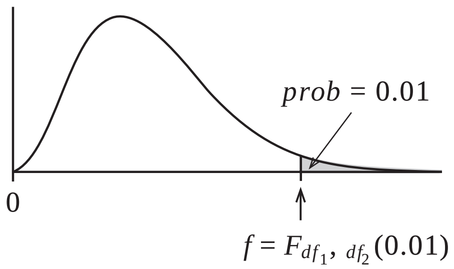
For fixed df_1, df_2 the tabulated value is the number $f = F_{df_1,df_2}(0.05)$ such that for $F \sim F(df_1, df_2)$, $\text{pr}(F \geq f) = 0.05$.



df_2 \ df_1	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	1000
1	161	199	215	224	230	233	236	238	240	241	243	245	248	250	252	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.62	8.57	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.69	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50	4.43	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81	3.74	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.38	3.30	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08	3.01	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86	2.79	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.70	2.62	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.57	2.49	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47	2.38	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.38	2.30	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.31	2.22	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25	2.16	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.19	2.11	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.15	2.06	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.11	2.02	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.07	1.98	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04	1.95	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.01	1.92	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	1.98	1.89	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	1.96	1.86	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.94	1.84	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.92	1.82	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.90	1.80	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.88	1.79	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.87	1.77	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.85	1.75	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84	1.74	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.74	1.64	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.65	1.53	1.39
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.79	1.70	1.60	1.48	1.32
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.77	1.68	1.57	1.45	1.28
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.55	1.43	1.25
1000	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.46	1.32	1.00

Appendix 3 *F*-distribution, 1% Table

For fixed df_1, df_2 the tabulated value is the number $f = F_{df_1,df_2}(0.01)$ such that for $F \sim F(df_1, df_2)$, $\text{pr}(F \geq f) = 0.01$.



df_2	df_1 = 1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	1000
1	4052	4999	5403	5624	5763	5858	5928	5981	6022	6055	6106	6157	6208	6260	6313	6365
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.5	26.3	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.8	13.7	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.38	9.20	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.23	7.06	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	5.99	5.82	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.20	5.03	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.65	4.48	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.25	4.08	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	3.94	3.78	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.70	3.54	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.51	3.34	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.35	3.18	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.21	3.05	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.10	2.93	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.00	2.83	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	2.92	2.75	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.84	2.67	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.78	2.61	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.72	2.55	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.67	2.50	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.62	2.45	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.58	2.40	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.54	2.36	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.50	2.33	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.47	2.29	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.44	2.26	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.41	2.23	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.39	2.21	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.20	2.02	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.03	1.84	1.60
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.27	2.12	1.94	1.75	1.49
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.22	2.07	1.89	1.69	1.43
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.86	1.66	1.38
1000	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.70	1.47	1.00