

Ratio Equations

You cannot teach a man anything.

You can only help him discover it
within himself.

—Galileo.

Exhibit 2-1

OBJECTIVES

Upon completion of this chapter the clinician should be able to:

1. Identify and use ratios
2. Explain why intuitive operations should be practiced with equations
3. Identify and use integer factors
4. Identify and use non-integer factors
5. Explain and use correct order of operations in solving equations
6. Solve simple algebraic equations for x

KEY TERMS

denominator
division
equation
factor
fraction
integer
multiplication
numerator

order of operations
product
proportion
ratio
reciprocal
unknown
whole number

RATIOS

Why use ratios when there is a common sense answer to your question that I can do in my head? **Ratios** are important to setting up **equations**. The clinician may be able and strongly tempted to solve these simple problems early in the text without using the ratio/equation set-up process. Go ahead and figure out the common sense answer. Then go back and set up the equation. When word problems get more complex, the skills learned by practicing with the simple ratios here will help the clinician set up equations for these more complicated operations. So it is wise to resist going intuitively to the answer and skipping the set-up process. Later the answers will not be intuitive, and if you skip the set-up process now, you will have lost a tool you need to solve more complex problems. The purpose of these problems is to get you to begin to recognize the set-up process. Calculation of dosages is fairly simple and straightforward. It will become confusing when conversions between units such as volume to mass are given over periods of time or calculations are based on patient weight. As you become more experienced, you will learn to take shortcuts, and that is one advantage of experience.

A ratio is a constant relationship between two values. We deal with ratios every day. These include prices, such as a ratio of money to gallons of milk, or time, such as the number of hours per work shift. Ratios are used to calculate related values, such as costs or amounts of medication to administer.

Ratios may be written with a colon: 8 hours:1 shift

or as a fraction: $\frac{8 \text{ hours}}{1 \text{ shift}}$

Exhibit 2-2 Ratio Equations

When the expression contains a colon, it is read as 8 hours *to* 1 shift. When it is written with a fraction line, it is read as 8 hours *per* 1 shift. The form using a fraction line is preferred for dosage calculations. Some familiar ratios appear in Exhibit 2-3.

60 minutes/hour	60 min:1 hour
7 days/week	7 days:1 week
12 eggs/dozen	12 eggs:1 dozen
2 shoes/pair	2 shoes:1 pair

Exhibit 2-3 Common Ratios

A ratio is a constant relation **proportion**. *Constant* means the relation between the two values does not change. For each 1 dozen eggs there are 12 eggs, and for each hour of the day there are 60 minutes. It also means that for every part of 1 dozen eggs there is an exact (related) same part of 12 eggs. Similarly, it means that for every part of an hour there is an exact (related) same part of 60 minutes. One half of 1 dozen eggs is equal to one half of 12 eggs because 1 dozen eggs is related to 12 eggs.

If: 1 dozen eggs is equal to 12 eggs
 Then: $\frac{1}{2}$ (1 dozen eggs) is equal to $\frac{1}{2}$ (12 eggs)

Exhibit 2-4 Example Ratio Problem

If asked how many eggs are in one half dozen, a person can quickly answer, “There are 6.” This problem is easily solved intuitively without using paper and pencil. However, ratios are used to set up mathematical expressions called equations. Using an equation provides the clinician an organized approach to solving problems.

This is useful for problems more complex than “How many eggs are in one half dozen?”

Setting up a problem as an equation gives the clinician an organized mathematical approach to problem solving.

Exhibit 2-5 Setting Up Equations

This approach will work with all dosage calculation problems. During this course of study, intuitively solve those problems that seem simple. Then set up and solve them in a ratio equation. This will provide practice with equations. Solving problems intuitively without learning the skills of using equations will leave the clinician unprepared when difficult problems are encountered.

An equation is a mathematical expression that contains an equal sign (=) and two parts, one on each side of the equal sign. Each side of an equation is equal to the other.

The advantage of using an equation is that if one side is known, then the other side (which is not known) can be calculated. A clinician can manipulate an equation by adding, subtracting, multiplying, or dividing both sides without changing its value or the correct answer.

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A clinician can add, subtract, multiply, or divide both sides of an equation without changing its value or the correct answer.

Exhibit 2-6 Solving Equations

In Exhibit 2-7, both sides are divided by 3 to solve the equation and get all known and **integer** information on the one side and all the **unknown** on the other side.

Problem: $3x = 6$

Objective: Get x alone on one side of the equation so that the other side is equal to x .

Solution: To get $3x$ alone (to be $1x$), it is divided by the integer 3.

The same operation must be performed on both sides, or the equation will have been incorrectly changed.

$$3x = 6$$

$$3x \div 3 = 6 \div 3$$

$$1x = 2$$

Exhibit 2-7 Example Equation

It doesn't matter which side has the known information. A clinician may work left to right or vice versa. In Exhibit 2-7, an arbitrary choice was made to work from left to right. The equation could have been solved in the same manner from right to left, as in Exhibit 2-8.

Problem: $6 = 3x$

$$6 = 3x$$

$$6 \div 3 = 3x \div 3$$

$$2 = 1x$$

Exhibit 2-8 Example Equation

In Exhibit 2-9, both sides are multiplied by 2 to solve the equation.

Problem: $\frac{1}{2}x = 3$

Objective: Get x alone on one side of the equation so that the other side is equal to x .

Solution: To get $\frac{1}{2}x$ alone (to be $1x$), it is multiplied by the integer 2.

The same operation must be performed on both sides, or the equation will have been incorrectly changed.

$$\frac{1}{2}x = 3$$

$$\frac{1}{2}x * 2 = 3 * 2$$

$$1x = 6$$

Exhibit 2-9 Example Equation

In Exhibit 2-10, both sides have 2 added to solve the equation.

Problem: $x - 2 = 4$

Remember that x is the same as $1x$.

Objective: Get x alone on one side of the equation so that the other side is equal to x (or $1x$).

Solution: To get $x-2$ alone (to be $1x$), the integer 2 must be added to both sides.

The same operation must be performed on both sides, or the equation will have been incorrectly changed.

$$x - 2 = 4$$

$$x - 2 + 2 = 4 + 2$$

$$1x = 6$$

Exhibit 2-10 Example Equation

In Exhibit 2-11, the equation is solved much like Exhibit 2-10, except that both sides have 2 subtracted (instead of added) to solve the equation.

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Problem: $x + 2 = 4$

Objective: Get x alone on one side of the equation so that the other side is equal to x .

Solution: To get $x + 2$ alone (to be $1x$), subtract 2 from both sides.

The same operation must be performed on both sides, or the equation will have been incorrectly changed.

$$1x + 2 = 4$$

$$1x + 2 - 2 = 4 - 2$$

$$1x = 2$$

Exhibit 2-11 Example Equation

FACTORS

A **factor** is a number or expression that may be multiplied by another factor to give a mathematical expression. In the mathematical expression 6, factors multiplied by each other to yield 6 may be $[1 * 6]$ or $[2 * 3]$ or $[1.5 * 4]$

<u>Expression</u>	<u>Factors</u>
6	1 * 6 2 * 3 1.5 * 4

In the expression *24 hours* there are several factors. Factors are not always **whole numbers**. They are sometimes words, such as *hours*. These non-number factors are non-integer factors. For simplicity non-integer factors are called “word factors.”

<u>Expression</u>	<u>Factors</u>
24 hours	1 * 24 * hours 2 * 12 * hours 3 * 8 * hours 4 * 6 * hours

A word factor that has no expressed integer always has an implied integer value of 1 (Exhibit 2-12).

x is the same as $1x$
hour is the same as 1 hour
ml is the same as 1 ml
60 minutes/hour is the same as 60 minutes/1 hour
ml/60 drops is the same as 1 ml/60 drops

Exhibit 2-12 Factors

A word factor (such as the word *hour*) is used in arithmetic operations like a number. **Multiplication** and **division** by word factors may be performed on any number or expression. To multiply by a word factor, place it in the numerator. To divide by a word factor, place it in the denominator.

The **numerator** is the top number in a **fraction** and the number to be divided. The **denominator** is the bottom number of a fraction and the number of parts by which the numerator is divided.

Addition and subtraction can only be conducted with expressions that have similar word factors. For example, one cannot subtract 4 apples from 6 oranges. One can only subtract a number of apples from another number of apples and a number of oranges from another number of oranges. Please refer to Exhibit 2-13 for instructions and Exhibit 2-14 for an example equation.

Word factors are not changed by arithmetic operations unless the arithmetic operation uses another word factor. When an arithmetic operation is performed, the number factor may be changed, but not the word factor. A word factor in the denominator will remain there, and a word factor in the numerator will remain there.

To multiply by a word factor, place it in the numerator.
To divide by a word factor, place it in the denominator.
Addition and subtraction can only be conducted with expressions that have the same word factors.

Exhibit 2-13 Working with Word Factors

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Problem: $x = 24 \text{ hours} - 8 \text{ hours}$

Objective: Get x alone on one side of the equation so that the other side is equal to x .

Solution: To solve for x , subtract 8 hours from 24 hours.

The operation is indicated only for one side of the equation and is not a change in the equation.

$$x = 24 \text{ hours} - 8 \text{ hours}$$

$$\begin{array}{r} x = 24 \text{ hours} \\ \quad - 8 \text{ hours} \\ \hline \quad 16 \text{ hours} \\ x = 16 \text{ hours} \end{array}$$

Exhibit 2-14 Example Equation

Word factors may be changed by arithmetic operations that use word factors. When a word factor is divided by another word factor, the answer is a new expression, but not a new number. Division of a word factor by another factor must be written out. If you wished to divide 8 hours by 1 shift, the expression would be written as shown in Exhibit 2-16.

Problem: $x = 24 \text{ hours}/8 \text{ hours}$

Objective: Get x alone on one side of the equation so that the other side is equal to x .

Solution: To solve for x , divide 24 hours by 8 hours as indicated.

The operation is indicated only for one side of the equation and is not a change in the equation.

$$x = \frac{24 \text{ hours}}{8 \text{ hours}}$$

In this example, the factor of hours is divided by hours.

Anything divided by itself is equal to 1.

$$x = \frac{24 \text{ hours}}{8 \text{ hours}}$$

$$x = \frac{24 * 1}{8 * 1}$$

$$x = 3$$

This leaves only
the numerical
factors to solve.

Exhibit 2-15 Example Equation

$$\frac{8 \text{ hours}}{\text{shift}} \text{ (or 8 hours:shift)}$$

This would be reported as 8 hours *per* shift.

Exhibit 2-16 Example Ratio

In a fraction, any number divided by itself is equal to the integer 1. This is also true of word factors.

Any factor (integer or word) divided by itself equals 1.

Exhibit 2-17 Factors

Equations should always be simplified by completing any arithmetic functions indicated. The arithmetic function indicated in Exhibit 2-18 is on the right side and is a multiplication of $\frac{1}{2}$ times 12 eggs. This requires the multiplication of a fraction factor ($\frac{1}{2}$), a whole number factor (12), and a word factor (eggs).

$$x = \frac{1}{2} (12 \text{ eggs})$$

Exhibit 2-18 Example Equation

A whole number is the same as a fraction of itself with a denominator of 1. To multiply a fraction and a whole number, multiply the numerator times the numerator and the denominator times the denominator, as in Exhibit 2-19.

The word factor *eggs* will not work in an arithmetic problem, but it cannot just be ignored for convenience. It must be carried through the equation and written out everywhere the factor (eggs) should be in the arithmetic problem. To work the problem of $\frac{1}{2}$ (12 eggs), multiply as shown in Exhibit 2-19.

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<p>Step 1: Convert the whole number and word factor to fractions.</p>	$12 \text{ eggs} = \frac{12}{1} \text{ eggs} = \frac{12 * \text{eggs}}{1 * 1}$
<p>Step 2: Multiply the fractions (numerators * numerators and denominators * denominators).</p>	$x = \frac{1 * 12 * \text{eggs}}{2 * 1 * 1}$
<p>The answer is derived, but it is not completely simplified.</p>	$x = \frac{1 * 12 * \text{eggs}}{2 * 1 * 1}$
<p>Step 3: Recall that any thing over 1 is a whole expression.</p>	$x = \frac{12 \text{ eggs}}{2}$
	$x = \frac{6 \text{ eggs}}{1} = 6 \text{ eggs}$

Exhibit 2-19 Example Equation**ORDER OF OPERATIONS**

In the previous examples both multiplication and division were required to solve for x . Sometimes solving for the unknown requires multiple operations. The correct answer of a problem depends on the order in which the operations are performed. Exhibit 2-20 illustrates this.

<p>In the operation $x = 5 + 5 * 10$</p> <p>If the addition is performed first, the outcome is:</p> $x = (5 + 5) * 10$ $x = 10 * 10$ $x = 100$ <p>If the multiplication is performed first, the outcome is:</p> $x = 5 + (5 * 10)$ $x = 5 + 50$ $x = 55$ <p><i>So the order of operations clearly makes a difference!</i></p>
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Exhibit 2-20 Order of Operations Example

The standard **order of operations** for mathematical operations follows these rules:

1. Always solve from inside to outside of parentheses.
2. Exponent operations are performed first.
3. Multiplication or division is second. There is no preferred order of operations between these two.
4. Addition or subtraction is performed last. There is no preferred order between these two.

In Exhibit 2-21, it is necessary to divide and to multiply. There is no preferred order between multiplication and division. x represents the unknown factor. Factor the equation to get x alone. The expression that remains on the other side of the equal (=) sign is the solution.

First, multiply both sides by the denominator of x to convert x to a whole expression rather than a fraction	$\frac{3x}{2} = 30$ $\frac{2 * 3x}{2} = 30 * 2$ $3x = 60$
Second, divide both sides by the factor of x to get x alone as $1x$	$3x \div 3 = 60 \div 3$ $x = 20$

Exhibit 2-21 Order of Operations Example

RECIPROCAL NUMBERS

In Exhibit 2-21, both sides of the equation were multiplied and then divided to obtain x with a factor of 1. This may also be done in one operation by multiplying by a fraction.

Every expression other than zero has a matching expression by which it can be multiplied to give a **product** of 1. This matching expression is the **reciprocal**. See Exhibit 2-22 for examples and Exhibit 2-23 for a sample equation.

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Expression	*	Reciprocal	=	Product
$\frac{1}{2}$	*	$\frac{2}{1}$	=	$\frac{2}{2} = 1$
$\frac{1}{4}$	*	$\frac{4}{1}$	=	$\frac{4}{4} = 1$
$\frac{4 \text{ ml}}{1 \text{ min}}$	*	$\frac{1 \text{ min}}{4 \text{ ml}}$	=	$\frac{4 \text{ ml min}}{4 \text{ ml min}} = 1$

Exhibit 2-22 Reciprocal Numbers

$\frac{3x}{2} = 30$	Identify the reciprocal number of x and then go to step 2.
$\frac{3x}{2} * \frac{2}{3} = \frac{30}{1} * \frac{2}{3}$	Multiply both sides by the reciprocal of x (which is $\frac{2}{3}$) to get $1x$ alone.
$\frac{3x * 2}{2 * 3} = \frac{30 * 2}{1 * 3}$	
$\frac{6x}{6} = \frac{60}{3}$	Simplify the fraction.
$x = 20$	

Exhibit 2-23 Reciprocal Number Example

It is important to accurately identify whether word factors are in the numerator or denominator. In 4 ml/minute, *ml* is in the numerator, and *minute* is in the denominator. The denominator of any whole expression is always 1.

Any word factor divided by itself yields a product of 1. The reciprocal of an expression containing a word factor will have a matching word factor in the inverse location.

To factor an expression to 1 in a combined operation:
1. Identify the factor of x .
2. Identify the reciprocal of the factor of x .
3. Multiply both sides of the equation by the reciprocal of the factor of x .

Exhibit 2-24 Using Reciprocal Numbers

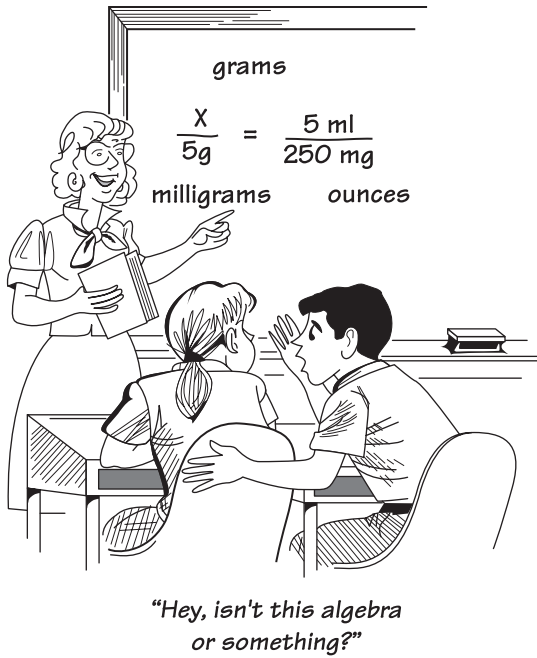


Figure 2-1 Algebra

REVIEW PROBLEMS

Solve the following ratio equations for x .

1. $\frac{4x}{2} = 30$

2. $\frac{4x}{2} = \frac{60}{2}$

3. $\frac{2x}{4} = \frac{15}{2}$

4. $\frac{x}{2} = \frac{15}{2}$

5. $\frac{x}{4} = 5$

6. $\frac{x}{4} = \frac{5}{2}$

7. $\frac{x}{2} = \frac{10}{5}$

8. $\frac{x}{3} = \frac{10}{5}$

9. $\frac{x}{4} = \frac{10}{5}$

10. $\frac{x}{2} = \frac{5}{10}$

11. $\frac{x}{3} = \frac{5}{10}$

12. $\frac{x}{4} = \frac{5}{10}$

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13. $\frac{x}{4} = \frac{4}{1}$

14. $\frac{2x}{3} = \frac{2}{3}$

15. $\frac{3x}{2} = \frac{2}{3}$

16. $\frac{x}{100} = \frac{1 \text{ quarter}}{25}$

17. $\frac{2x}{50\text{¢}} = \frac{1 \text{ quarter}}{25\text{¢}}$

18. $\frac{x}{1 \text{ gal}} = \frac{4 \text{ qt}}{1 \text{ gal}}$

19. $\frac{x}{0.5 \text{ gal}} = \frac{4 \text{ qt}}{1 \text{ gal}}$

20. $\frac{2x}{1 \text{ gal}} = \frac{4 \text{ qt}}{1 \text{ gal}}$

21. $\frac{4x}{1 \text{ gal}} = \frac{4 \text{ qt}}{1 \text{ gal}}$

22. $\frac{x}{12} = \frac{1 \text{ case}}{24}$

23. $\frac{x}{1 \text{ case}} = \frac{12}{24}$

24. $\frac{x}{4 \text{ mg}} = \frac{4 \text{ ml}}{1 \text{ mg}}$

25. $\frac{x}{4 \text{ mg}} = \frac{5 \text{ ml}}{10 \text{ mg}}$

26. $2 \text{ ml } x = 30 \text{ mg}$

27. $2 \text{ mg } x = 30 \text{ ml}$

28. $2x = \frac{15 \text{ mg}}{30 \text{ ml}}$

29. $2x = \frac{15 \text{ ml}}{30 \text{ mg}}$

30. $\frac{x}{2 \text{ ml}} = \frac{15 \text{ mg}}{30 \text{ ml}}$

31. $\frac{x}{2 \text{ mg}} = \frac{15 \text{ ml}}{30 \text{ mg}}$

32. $\frac{x}{15 \text{ ml}} = \frac{2 \text{ mg}}{30 \text{ mg}}$

33. $\frac{x}{10 \text{ ml}} = \frac{5 \text{ mg}}{10 \text{ ml}}$

34. $5x = 25 \text{ mg}$

35. $\frac{x}{10 \text{ L}} = \frac{2 \text{ mg}}{1 \text{ L}}$

36. $\frac{x}{10 \text{ L}} = \frac{1 \text{ mcg}}{1 \text{ L}}$

37. $\frac{x}{10 \text{ L}} = \frac{2 \text{ g}}{1 \text{ L}}$

38. $\frac{x}{10 \text{ L}} = \frac{1 \text{ mg}}{1 \text{ L}}$

39. $\frac{x}{5 \text{ L}} = \frac{1 \text{ g}}{1 \text{ L}}$

40. $\frac{x}{5 \text{ L}} = \frac{5 \text{ g}}{1 \text{ L}}$

$$41. \frac{x}{1 \text{ L}} = \frac{1 \text{ g}}{5 \text{ L}}$$

$$42. \frac{x}{1 \text{ L}} = \frac{5 \text{ g}}{5 \text{ L}}$$

$$43. \frac{x}{2 \text{ L}} = \frac{2 \text{ g}}{4 \text{ L}}$$

$$44. \frac{x}{4 \text{ L}} = \frac{2 \text{ g}}{2 \text{ L}}$$

$$45. \frac{x}{2 \text{ g}} = \frac{2 \text{ L}}{4 \text{ L}}$$

$$46. \frac{x}{2 \text{ g}} = \frac{4 \text{ L}}{2 \text{ L}}$$

$$47. \frac{x}{2 \text{ g}} = \frac{2 \text{ ml}}{4 \text{ ml}}$$

$$48. \frac{x}{2 \text{ g}} = \frac{4 \text{ ml}}{2 \text{ ml}}$$

$$49. \frac{x}{2 \text{ L}} = \frac{2 \text{ g}}{4 \text{ g}}$$

$$50. \frac{x}{2 \text{ L}} = \frac{4 \text{ g}}{2 \text{ g}}$$

