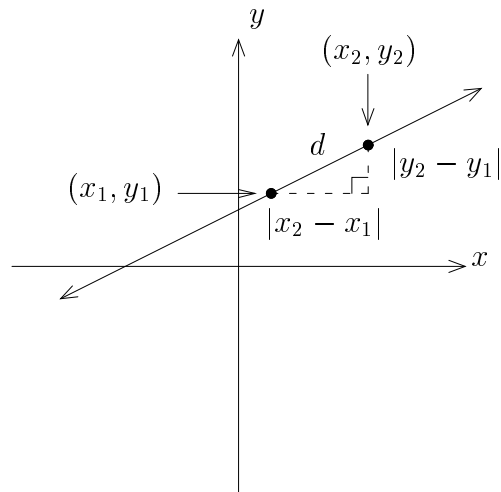


2.8 Distance Formula, Circles, Midpoint Formula

A. Distance Formula

We seek a formula for the distance between two points:



By the Pythagorean Theorem, $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Since distance is positive, we have:

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

B. Example

Find the distance between $(-1, 2)$ and $(3, -4)$

Solution

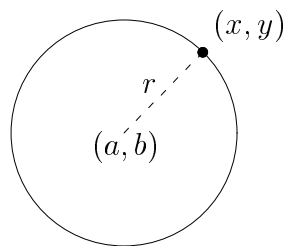
Use the distance formula:

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(3 - (-1))^2 + (-4 - 2)^2} \\&= \sqrt{(4)^2 + (-6)^2} \\&= \sqrt{16 + 36} \\&= \sqrt{52}\end{aligned}$$

Ans $\boxed{2\sqrt{13}}$

C. Circles

A **circle** is the set of points a fixed distance r from a center (a, b) :



By the distance formula, $r = \sqrt{(x - a)^2 + (y - b)^2}$

Eliminating the radical, we get:

Equation of Circle in Standard Form: $(x - a)^2 + (y - b)^2 = r^2$

Note: r is called the **radius** of the circle.

D. Examples

Example 1: Find the equation of a circle with center $(2, -1)$ and radius 4.

Solution

The equation of a circle in standard form: $(x - a)^2 + (y - b)^2 = r^2$

Thus, we have: $(x - 2)^2 + (y + 1)^2 = 4^2$

Ans $(x - 2)^2 + (y + 1)^2 = 16$

Example 2: Given a circle $x^2 + (y - 3)^2 = 5$, find the center and radius.

Solution

Since the equation of a circle in standard form is $(x - a)^2 + (y - b)^2 = r^2$, we have

Ans $\begin{array}{l} \text{center: } (0, 3) \\ \text{radius: } \sqrt{5} \end{array}$

E. Putting the Equation of a Circle in Standard Form

Sometimes the equation of a circle is **not** in standard form.

To put it in standard form, we complete the square in both x and y .

In standard form, it is easy to identify the center and radius of the circle.

Example 1: Put the equation of the circle $x^2 + y^2 - 6x + 2y = 15$ into standard form.

Solution

$$x^2 + y^2 - 6x + 2y = 15$$

$$(x^2 - 6x) + (y^2 + 2y) = 15$$

$$[(x^2 - 6x + 9) - 9] + [y^2 + 2y + 1] - 1 = 15$$

$$(x - 3)^2 - 9 + (y + 1)^2 - 1 = 15$$

Ans $\boxed{(x - 3)^2 + (y + 1)^2 = 25}$

Example 2: Put the equation of the circle $2x^2 + 2y^2 - 10x - 12y = 7$ into standard form.

Solution

$$2x^2 + 2y^2 - 10x - 12y = 7$$

$$(2x^2 - 10x) + (2y^2 - 12y) = 7$$

$$2(x^2 - 5x) + 2(y^2 - 6y) = 7$$

$$\left[2\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{2}\right] + [2(y^2 - 6y + 9) - 18] = 7$$

$$\left[2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2}\right] + [2(y - 3)^2 - 18] = 7$$

$$2\left(x - \frac{5}{2}\right)^2 + 2(y - 3)^2 - \frac{25}{2} - \frac{36}{2} = 7$$

$$2\left(x - \frac{5}{2}\right)^2 + 2(y - 3)^2 - \frac{61}{2} = 7$$

$$2\left(x - \frac{5}{2}\right)^2 + 2(y - 3)^2 = 7 + \frac{61}{2}$$

$$2\left(x - \frac{5}{2}\right)^2 + 2(y - 3)^2 = \frac{14}{2} + \frac{61}{2}$$

$$2\left(x - \frac{5}{2}\right)^2 + 2(y - 3)^2 = \frac{75}{2}$$

Ans $\boxed{\left(x - \frac{5}{2}\right)^2 + (y - 3)^2 = \frac{75}{4}}$

F. Graphing Circles

To graph a circle:

1. Put the equation in standard form.
2. Find the center and radius.
3. Find the x and y intercepts.
4. Plot the x and y intercepts.
Going any direction from the center by a radius amount reaches the circle.
5. Connect the dots.

Example: Find the center, radius, x and y intercepts of the circle, where $x^2 + y^2 - 2x + 8y = -5$. Then graph the circle.

Solution

First, put the circle in standard form:

$$x^2 + y^2 - 2x + 8y = -5$$

$$(x^2 - 2x) + (y^2 + 8y) = -5$$

$$(x^2 - 2x + 1) - 1 + (y^2 + 8y + 16) - 16 = -5$$

$$(x - 1)^2 + (y + 4)^2 - 17 = -5$$

$$(x - 1)^2 + (y + 4)^2 = 12$$

center: $(1, -4)$

radius: $\sqrt{12} = 2\sqrt{3}$

x -intercepts: set $y = 0$:

$$(x - 1)^2 + (0 + 4)^2 = 12$$

$$(x - 1)^2 + 16 = 12$$

$$(x - 1)^2 = -4$$

$$x - 1 = \pm\sqrt{-4}$$

Thus there are no x -intercepts.

***y*-intercepts:** set $x = 0$:

$$(0 - 1)^2 + (y + 4)^2 = 12$$

$$1 + (y + 4)^2 = 12$$

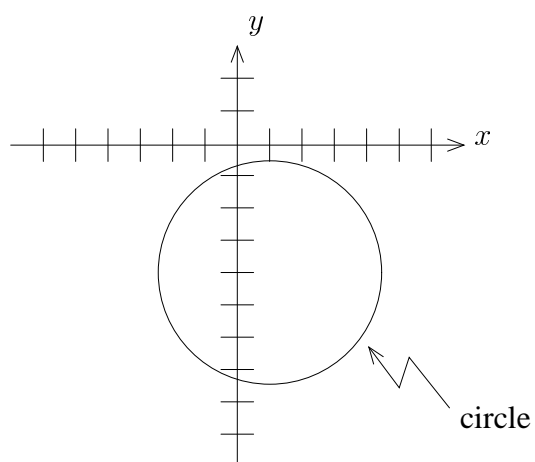
$$(y + 4)^2 = 11$$

$$y + 4 = \pm\sqrt{11}$$

$$y = -4 \pm \sqrt{11}$$

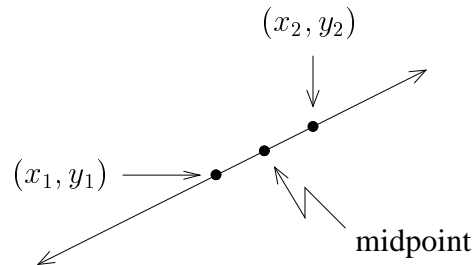
Thus the *y*-intercepts are $-4 \pm \sqrt{11}$.

Graph:



G. Midpoint Formula

The **midpoint** between two points is the point on the line halfway between them.



Midpoint Formula:
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

“average the x -coordinates and average the y -coordinates”

Example: Find the midpoint between $(-1, 2)$ and $(3, -4)$

Solution

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-1 + 3}{2}, \frac{2 + (-4)}{2} \right) = \left(\frac{2}{2}, \frac{-2}{2} \right)$$

Ans $(1, -1)$