# Fractions, decimals and percentages 

This chapter is about working with fractions, decimals and percentages. If you cook a batch of 40 pancakes, you need these skills to help share them equally with 5 friends.

## Вefare you start this chapter

1 Write down the common factors of 18 and 27.
2 Write down the highest common factor of 48 and 36 .
3 Simplify
a $\frac{4}{8}$
b $\frac{8}{20}$
C $\frac{28}{32}$

4 Find the missing number in each of these conversions.
a 7.5 litres $=\square \mathrm{ml}$
b $32 \mathrm{~mm}=\square \mathrm{cm}$
c $0.6 \mathrm{~km}=\square \mathrm{m}=\square \mathrm{cm} \quad$ d $\frac{1}{2}$ hour $=\square$ minutes

5 Work out $15 \%$ of $£ 50$.

6 Anna needs $£ 250$ to buy a new bicycle.
She saves $£ 18$ each week.
For how many weeks must she save so that she has enough money?

7 Use your calculator to work out $\frac{125}{800}$ as a decimal.
8 a Convert $24 \%$ to a decimal.
b Convert $124 \%$ to a decimal.
9 Calculate $40 \times \frac{60}{100}$.

## 2.1 Frectian of en emaunt

## Why learn thas?

Understanding fractions helps you to understand musical note lengths.

## Qbjectives

D Find a fraction of an amount with a calculator in complex situations

## Skills ㄷheck

1 Use the $\triangle$ and $\because$ keys on your calculator to calculate these.
a $\frac{2}{5} \times 720 \mathrm{ml}$
b $\frac{7}{9} \times 8109$
c $\frac{17}{35} \times 655 \mathrm{~kg}$

2 Use the $\triangle$ and $\because$ keys on your calculator to calculate each of the missing numbers.
Give your answers as mixed numbers.
a $\frac{1}{25} \times 1$ hour $=\square$ minutes
b $\frac{5}{12} \times 1$ litre $=\square \mathrm{ml}$

## Finding a fraction of an amaunt using <br> a calculator

There are many makes and models of calculators available.
Most scientific calculators have a fractions key.
The fractions key may look like this $\square$ ab or this $\square$.

Make sure you know which is the fractions key on your calculator, and how to use it!

## Example 1

Use a calculator to work out
a $\frac{1}{5}$ of 4 cookies b $\frac{7}{15}$ of 314 cars
a $\frac{1}{5} \times 4=\frac{4}{5}$ cookies
b $\frac{7}{15} \times 314=146 \frac{8}{15}$ cars

> Using the ab key:

1 ab $5 \times 4=4.5$
Using the ab key:
7 ab $15 \times 314=445 \_8 \_15$
Not all calculators will give the answer as a mixed number - you'll have to press another button to change it to this form.

## Exercise 2A

1 Ryan runs a dog rehoming centre.
On average he feeds each $\operatorname{dog} \frac{3}{4}$ of a tin of food per day.
This week there are 23 dogs at the centre.
How many tins of dog food does Ryan need for this week?
2 Heather buys 80 duvets for $£ 595$.
She sells $\frac{2}{5}$ of them for $£ 12$ each and $\frac{7}{16}$ of them for $£ 10$ each.
a How much profit does Heather make?
b How many duvets does she have left over?

3 In a school election Jack got $\frac{2}{9}$ of the votes and Rebecca got $\frac{4}{9}$ of the votes.
Rebecca had 42 more votes than Jack.
How many students didn't vote for Jack or Rebecca?
4 There are 48 members in a scout group.
Of these members, $\frac{5}{8}$ are boys. Of the boys, $\frac{1}{3}$ are 12 years old or over.
Half of all the scouts are 12 years old or over.
How many of the scouts are girls under the age of 12 ?

## 2.2 <br> Qne quantity es a frection of anather

Why learn this?
This topic often comes up in an exam.

## Objectives

## (D) Write one quantity as a fraction of another

## Skills check

1 Find the missing number in each of these conversions.
a $1 \mathrm{~kg}=\square \mathrm{g}$
b $1 \mathrm{~km}=\square \mathrm{m}$
c $\quad 1 \mathrm{~m}=\square \mathrm{cm}$

2 Simplify each fraction to its lowest terms.
a $\frac{2}{6}$
b $\frac{8}{12}$
C $\frac{15}{20}$

## Writing ane quantity as a fractian of anather

To write one quantity as a fraction of another

- first write both quantities in the same units
- then write the first quantity over the second quantity
- finally, simplify the fraction.


## Example 2

a Write 40 p as a fraction of $£ 6$.
b In Diane's class there are 13 boys and 14 girls.
What fraction of the class are boys?


Write $£ 6$ as 600 p and then write 40 over 600. Remember to simplify the fraction to its lowest terms.
$13+14=27$ students altogether. 13 out of the 27 are boys.

## Exercise 2B

1 In Polly's dog training class, three of the dogs pass their elementary certificate.
The other four do not pass.
What fraction of the dogs pass?

2 During April it was sunny for 14 days.
What fraction of the days in April were not sunny?
3 Donna said, 'I've got 5 red sweets and 9 blue sweets so the fraction of my sweets that are red is $\frac{5}{9}$.'
Is Donna correct? Explain your answer.
4 In each case, write the first quantity as a fraction of the second.
a $£ 2, £ 7$
b 1 hour, 24 hours
c 6 weeks, 8 weeks

5 In each case, write the first quantity as a fraction of the second.
a 20 p, $£ 1$
b 5 minutes, 1 hour
c 5 days, 2 weeks
d $30 \mathrm{~cm}, 1 \mathrm{~m}$
e $750 \mathrm{~g}, 1 \mathrm{~kg}$
f $12 \mathrm{~m}, 1 \mathrm{~km}$

6 Lauren gets $£ 5$ pocket money per week.
Each week she buys a magazine for $£ 1.75$ and she saves $£ 1.50$.
The rest is left over for her to spend on other things.
What fraction of her pocket money does Lauren
a spend on a magazine
b save
c have left over?

## 2.3 Calculating with freations

## Why learn this?

abjectives
By learning how to enter fractions into
a calculator you can save a lot of time.

## Skills check

1 Use the fractions key on a calculator to simplify these fractions.
a $\frac{24}{36}$
b $\frac{55}{75}$
C $\frac{52}{65}$

2 Use the fractions key on a calculator to convert these mixed numbers to improper fractions.
a $3 \frac{8}{9}$
b $4 \frac{12}{17}$
C $16 \frac{18}{25}$

3 Use the fractions key on a calculator to convert these improper fractions to mixed numbers.
a $\frac{130}{7}$
b $\frac{196}{15}$
C $\frac{223}{32}$

## Calculating with fractions using a calculator

To carry out any calculations involving fractions, enter the fractions on your calculator using the fractions key. Make sure you practise using your calculator so you become skilled.

## Example 3

Calculate
a $3 \frac{1}{2}-1 \frac{2}{5}$
b $4 \frac{3}{4} \div \frac{2}{5}$
c $15 \times 2 \frac{3}{8}$


Using the $\quad$ ab
3 ab 1 ab $2-1$ ab 2 ab $5=\square$ コ」 I I

Using the ab key：
4 ab 3 ab $4 \div 2$ ab $5=\square$ 1」7」日

Using the ab key：
$15 \times 2$ aba 3 ab $8=$ コ5」5」ロ

## Exercise 2C

1 Work out these．Give all your answers as mixed numbers．
a $3 \frac{1}{4}+6 \frac{2}{9}$
b $5 \frac{1}{4}-2 \frac{3}{8}$
c $8 \frac{2}{15}+9 \frac{11}{12}$

2 A football stadium has 72000 seats．Each seat is $\frac{4}{5}$ m wide．
If all 72000 seats were placed next to each other in a line，how long would the line be？ Give your answer in
a metres
b kilometres
c miles．
$1 \mathrm{~km} \approx \frac{5}{8}$ mile

3 Fred added four identical mixed numbers．
He got an answer of $8 \frac{3}{4}$ ．
What were the mixed numbers that Fred added？
4 When Joe was born he weighed 3.5 kg ．
Three months later he weighed 4.75 kg ．
What is his increase in weight as a fraction of his birth weight？
5 A water－butt contains $123 \frac{3}{4}$ litres of water．
The tap is opened so that $4 \frac{1}{2}$ litres of water pour out every minute．
At this rate，how long will the water－butt take to empty？
Give your answer in minutes and seconds．

6 Which calculation gives the largest answer？
a $24 \times \frac{7}{8}$
b $15 \times 1 \frac{2}{5}$
c $8 \frac{1}{4} \times 2 \frac{6}{11}$

7 A basketball bounces to $\frac{4}{5}$ of the height from which it was dropped．
How high is the second bounce if it is dropped initially from $2 \frac{1}{2} \mathrm{~m}$ ？


## Metric－Imperial conversions

1 litre is approximately $1 \frac{3}{4}$ pints
1 pint of water weighs approximately $1 \frac{1}{4}$ pounds（b）
1 kg is approximately $2 \frac{1}{5} \mathrm{lb}$

8 Anders buys a fish tank．
The tank holds 120 litres of water when full．
Anders fills the tank with water．
a What is the mass of the water in the tank？ Give your answer in kilograms．
b What do you notice about your answer？

$9 £ 10$ is $\frac{1}{3}$ of $\frac{1}{2}$ of a sum of money．
What is the sum of money？
$10 £ 5$ is $\frac{3}{4}$ of $\frac{2}{3}$ of a sum of money．
What is the sum of money？
$11 £ 5$ is half of four－sevenths of a sum of money．
What is the sum of money？

What is the sum of money？

## ロпе qபentity as a percentage of enather

## Why learn this？

## ロ冋jectives

So you can work
out your test results as a percentage．
（D）Write one quantity as a percentage of another
Write one quantity as a percentage of another in more complex situations

## Skills check

1 Write down how many
a cm are in 1 m
b $\mathrm{m} l$ are in 1 litre
d $g$ are in 1 kg
e hours are in a day
c cl are in 1 litre
f months are in a year．

2 Copy and complete these divisions．
a $\frac{3}{5}=3 \div 5=\square$
b $\frac{7}{28}=7 \div \square=\square \mathbf{c}$
c $\frac{25}{40}=\square$ $\square$ $\square=\square$

## Writing ane quantity as a percentage af anather

To write one quantity as a percentage of another
－write the first quantity as a fraction of the second．
－multiply the fraction by 100 to convert it to a percentage．

## Example 4

a Express $£ 5$ as a percentage of $£ 25$.
b Express 8 mm as a percentage of 2 cm .
c Daren bought a car for $£ 8000$ and sold it for $£ 6500$.
What percentage of the price that he paid has he lost?


Write 5 as a fraction of 25 then multiply by 100 .

The units must be the same so convert 2 cm into 20 mm .
Then write 8 as a fraction of 20 and multiply by 100 .

First work out how much he has lost: $8000-6500=1500$
Then write 1500 as a fraction of 8000 and multiply by 100 .

## Exercise 2D

1 In each case, express the first quantity as a percentage of the second.
a $£ 5, £ 50$
b $£ 5, £ 80$
c $£ 25, £ 75$
d 4 hours, 1 day
e 36 minutes, 1 hour
f $125 \mathrm{~g}, 1 \mathrm{~kg}$
g 13 weeks, 1 year
h $14.5 \mathrm{~cm}, 1 \mathrm{~m}$
i $275 \mathrm{ml}, 1$ litre

Make sure the units of both quantities are the same.

2 Fathe goes to the shops with $£ 12$. He buys a book for $£ 4.50$.
What percentage of his money has he got left?
3 Hari scores 26 out of 30 in an English test.
What percentage of the test does Hari get wrong?
4 A badminton club has 63 members.
The table shows the membership numbers.

| Men | Women | Girls | Boys |
| :---: | :---: | :---: | :---: |
| 17 | 21 | 12 | 13 |

What percentage of the members are
a men
b female?

6 Last year Llanreath Divers took $£ 1800$ in membership fees.
This year they took $£ 2100$ in membership fees.
What is the percentage increase in the amount taken in membership fees?
7 Jon Brower Minnoch was the heaviest man recorded in history.
In 16 months he lost 419 kg in weight.
His final weight was 216 kg .
What percentage of his starting weight did he lose?
8 In 2007 Little Haven won the South Pembrokeshire short mat bowls league.
Out of the 20 games they played, they won 11 and drew 2.
They scored 813 shots for and had 515 shots against.
They won the league with a total of 102 points.
What percentage of the games that they played did they lose?
9 Mosel bought a car for $£ 2500$, spent $£ 400$ on improvements and sold it for $£ 3750$. What was his percentage profit?

10 A shop owner buys in 2000 chocolate bars at 37 p each and sells 1777 of them for 45p each.
The rest were not sold and were discarded.
Work out the shop owner's percentage profit.


Percenteqe increase and decrease

## Keywords

## [D] $\operatorname{lactives~}$

## (D) Calculate a percentage increase or decrease

D Perform calculations using VAT you could save in a sale.

## Skills check

1 Convert 73\% to a decimal.
2 Convert 119\% to a decimal.
3 Work out a $5 \%$ of $£ 200$ b $13 \%$ of $£ 420$ c $17 \%$ of $£ 333$

## Percentage increase and decrease <br> Method A <br> Method B

1 Work out the value of the increase (or decrease).
2 Add it to (or subtract it from) the original amount.
This method is most commonly used when working without a calculator.

1 Add the percentage increase to 100\% (or subtract the percentage decrease from 100\%).
2 Convert this percentage to a decimal.
3 Multiply it by the original amount.
This method is especially useful when using a calculator.

## Example 5

John used to earn $£ 320$ a week. He has had an $8 \%$ pay rise.
What does he earn now?

| MethodA |  | Method B |  |
| ---: | :--- | ---: | :--- |
| 1\% of $£ 320$ | $=£ 320 \div 100$ |  | Rise $=8 \%$ |
|  | $=£ 3.20$ |  | New salary |$=8 \%+100 \%$

Divide by 100 to convert a percentage to a decimal.

## Example 6

The price of a bike is reduced by $20 \%$ in a sale. The original price was $£ 150$.
What is the sale price?


## Exercise 2E

1 a Increase 20 by $5 \%$
c Decrease 3400 ml by $17 \%$
e Increase $£ 135$ by $7 \frac{1}{2} \%$
b Increase 56 by 12\%
d Decrease 480 by $32 \%$
f Decrease 890 ml by $8.4 \%$

2 Firefighters are given a $4 \%$ pay rise. If John earned $£ 420$ per week, how much will he earn after the pay rise?

3 There has been a $6 \%$ decrease in the number of reported thefts in Walton this year. There were 350 reported thefts last year. How many were there this year?

4 A new car costs $£ 8450$. After two years the value of the car will have decreased by $43 \%$. How much will the car be worth?

5 Sales of a magazine, costing $£ 1.95$, decreased by $11 \%$ this week. They sold 4300 copies last week. How much money have they lost on their sales this week?

6 Jane starts a new job earning $£ 12000$ per year, increasing by $3 \%$ after 3 months. Milly starts on $£ 11500$ per year, increasing by $5 \%$ after 3 months.
Who has the greater salary after 3 months?
7 Ali weighed 84 kg . He lost $4 \%$ of his body weight when he started running but then put $2 \%$ of his new weight back on.
a How much was Ali's lowest weight?
b How much does Ali weigh now?
8 In 2007 the number of pairs of breeding sparrows was estimated as 200000.
This decreased by $38 \%$ in 2008. In 2009 there was a slight recovery with an increase of $4 \%$. How many breeding pairs were estimated at the end of 2009 ?

## VAT

VAT stands for value added tax. It is a tax that is added to the price of most items in shops and to many other services.
VAT is calculated as a percentage. Generally it is $17.5 \%$ in the UK.

VAT at 17.5\% can be worked out by finding $10 \%+5 \%+2.5 \%$.

## Example 7

A digital camera is advertised for sale at $£ 240$ (excluding VAT).
How much will you have to pay?
'Excluding' means that VAT must be added on to the advertised price.


## Exercise 2F

For questions 1 to 5 , assume the rate of VAT is $17.5 \%$.
1 A DVD player costs $£ 130$ (excluding VAT).
A phone costs $£ 40$ (excluding VAT).
Work out a the VAT and $\mathbf{b}$ the total cost of each item.
2 What is the total cost of a TV sold for $£ 126$ + VAT?

3 One car is sold at $£ 4000$ (including VAT) and another is sold at $£ 3800$ (excluding VAT). Which car is the more expensive?

4 A meal for four costs $£ 73.60$ plus VAT. If the four people decide to share the bill equally, how much will each pay?

5 The cost of a scooter is $£ 376+$ VAT. Jas decides to pay by credit. He pays an initial deposit of $15 \%$ and then 12 monthly payments of $£ 35$. How much more does Jas pay by buying on credit?

# 己. $\quad$ Index nபmbers 

## Keywords

index number, base, retail prices index

## Why learn this?

## Dbjectives

The retail prices index is used to work out the interest rate on student loans.

## Skills check

1 Work out
a 100-79
b $165-100$
c $235-100$
d $100-36$

2 Calculate
a $45 \times \frac{90}{100}$
b $80 \times \frac{112}{100}$
c $34 \times \frac{186}{100}$
d $120 \times \frac{214}{100}$

## HELP Section 2.1

## Retall prices index

An index number compares one quantity, usually a price, with another.
The figure that the quantities are compared with is called the base.
The index number is a percentage of the base, but the percentage sign is left out.
The base usually starts at 100 .
The UK retail prices index started at base 100 in 1987.
In May 2009 the UK retail prices index was 211.3.
This means that average retail prices increased by $111.3 \%$ between 1987 and 2009.

In 2000 the price of a litre of petrol was 69p.
Using the year 2000 as the base year, the price indices of petrol for 1999, 2001, 2002 and 2003 are given in the table.

| Year | 1999 | 2000 | 2001 | 2002 | 2003 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Index | 92 | 100 | 103 | 107 | 110 |
| Price |  | $69 p$ |  |  |  |

Work out the price of petrol in 1999 and in 2001 to 2003.
Give your answers to one decimal place.
$1999: 69 p \times \frac{92}{100}=63.5 p$
$2001: 69 p \times \frac{103}{100}=71.1 p$
$2002: 69 p \times \frac{107}{100}=73.8 p$
$2003: 69 p \times \frac{110}{100}=75.9 p$

In 1999 the index was less than the base, so the price of a litre of petrol must be less. In the years 2001 to 2003 the index was more than the base, so the price must be more.

Notice that for each year's calculation, you always use 69p as this is the price of petrol in the base year. So in the calculation for 2002, you don't use the 71.1p from 2001.

## Exercise 2G

1 In 2006 the price of a litre of petrol was 95p.
The table shows the price index of petrol for the next three years, using 2006 as the base year.

| Year | 2006 | 2007 | 2008 | 2009 |
| :--- | :---: | :---: | :---: | :---: |
| Index | 100 | 95 | 100 | 110 |
| Price | $95 p$ |  |  |  |

An index of 95 means that the value has gone down by $5 \%$. An index of 110 means that the value has gone up by $10 \%$.
Work out the price of petrol from 2007 to 2009. Give your answers to one decimal place.

2 The index for the price of laptop computers, compared with 2005 as base, is 74 .
a Has the price of laptop computers gone up or down?
b By what percentage has the price of laptop computers changed?
3 This year the price of a 4 GB memory stick is $40 \%$ lower than last year. What is the index for the price of a 4 GB memory stick this year, using last year as base?

4 In 1990 an average box of tissues cost 30 p .
Taking 1990 as the base year of 100, work out how much an average box of tissues costs today, when the price index is 240 .

5 The retail prices index was introduced in January 1987.
It was given a base number of 100 .
In May 2009 the index number was 211.3.
In January 1987 the 'standard weekly shopping basket' cost $£ 38.50$.
How much did the 'standard weekly shopping basket' cost in May 2009?
6 In 1990 the price of 1 kg of bananas was $£ 1.14$.
Using 1990 as the base year, the price index of 1 kg of bananas in 2008 was 75. Peter says, 'The price of bananas in 2008 is $\frac{1}{4}$ of the price they were in 1990.' Is Peter correct? Explain your answer.

7 The graph shows the exchange rates for the euro $(€)$ and the pound $(£)$ in 2008.
Exchange rate for euro to pound in 2008

a What was the exchange rate in January?
b Using January 2008 as the base of 100, work out the index for December 2008.
8 A toy factory produced 455000 toys in November 2008.
This represented an index of 78, using November 2002 as the base year with an index of 100 .
How many toys did this factory produce in November 2002?

## Review exercise

1 Rosa says ' 2 mm as a fraction of 50 cm can be written like this: $\frac{2}{50}=\frac{1}{25}$.' Is Rosa correct?
Explain your answer.

2 Nigel is given $£ 40$ for his birthday.
He spends $£ 16.40$ on clothes.
What percentage of his money has he got left?
3 This year the price of a DVD is $8 \%$ lower than last year.
What is the index for the price of a DVD this year, using last year as base? [2 marks]
4 In 19781 kg of mushrooms cost $£ 1.53$.
Taking 1978 as the base year of 100, work out how much 1 kg of mushrooms cost in 2008, when the price index was 172.

5 All train fares are to increase by $7 \frac{1}{2} \%$ next year. At the moment, Jane pays $£ 28$ for her ticket. How much will she have to pay next year?

6 Gaynor plants 60 daffodil bulbs.
48 of the bulbs grow.
What percentage of the bulbs do not grow?
[2 marks]
7 Heather sleeps for 8 hours every day. What percentage of a week is Heather awake?

9 Two posters advertise the same TV. Which method is cheaper? Find how much can be saved using the cheaper method.


10 Last year a summer fête raised $£ 1420$ for charity.
This year it raised $£ 1650$.
What is the percentage increase in the amount raised for charity?
Give your answer to the nearest whole number.
[3 marks]
11 Last year Amy bought a laptop computer.
This year the price of the same laptop computer fell by $£ 80$ to $£ 370$.
By what percentage of last year's price has the price fallen?
Give your answer to one decimal place.

## Chapter summary

In this chapter you have learned how to

- find a fraction of an amount with a calculator in complex situations D
- write one quantity as a fraction of another
- calculate a percentage increase or decrease
- perform calculations using VAT D
- write one quantity as a percentage of another D C
- understand and use the retail prices index D C
- use the fractions key on a calculator with mixed numbers D C

