GLENCOE MATHEMATICS

Algebra 1

Chapter 8 Resource Masters

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

Study Guide and Intervention Workbook	0-07-827753-1
Study Guide and Intervention Workbook (Spanish)	0-07-827754-X
Skills Practice Workbook	0-07-827747-7
Skills Practice Workbook (Spanish)	0-07-827749-3
Practice Workbook	0-07-827748-5
Practice Workbook (Spanish)	0-07-827750-7

ANSWERS FOR WORKBOOKS The answers for Chapter 8 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Algebra 1
Chapter 8 Resource Masters

2 3 4 5 6 7 8 9 10 024 11 10 09 08 07 06 05 04 03

Contents

Teacher's Guide to Using the Chapter 8 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 8 Resource Masters* includes the core materials needed for Chapter 8. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 1 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 8-1. Encourage them to add these pages to their Algebra Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 1* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 8 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 1. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 470–471. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME _____ DATE _____ PERIOD _____

8 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 8. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
binomial by·NOH·mee·uhl		
constant		
degree of a monomial		
degree of a polynomial		
FOIL method		
monomial mah·NOH·mee·uhl		
negative exponent		
polynomial PAH·luh·NOH·mee·uhl		

(continued on the next page)

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
Power of a Power		
Power of a Product		
Product of Powers		
Power of a Quotient		
Quotient of Powers		
scientific notation		
trinomial try·NOH·mee·uhl		
zero exponent		

NAME _____ DATE _____ PERIOD _____

8-1 Study Guide and Intervention

Multiplying Monomials

Multiply Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify $(3x^6)(5x^2)$.**

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Associative Property

$= (3 \cdot 5)(x^{6+2})$ Product of Powers

$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2 **Simplify $(-4a^3b)(3a^2b^5)$.**

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$

$= -12(a^{3+2})(b^{1+5})$

$= -12a^5b^6$

The product is $-12a^5b^6$.

Exercises

Simplify.

1. $y(y^5)$

2. $n^2 \cdot n^7$

3. $(-7x^2)(x^4)$

4. $x(x^2)(x^4)$

5. $m \cdot m^5$

6. $(-x^3)(-x^4)$

7. $(2a^2)(8a)$

8. $(rs)(rs^3)(s^2)$

9. $(x^2y)(4xy^3)$

10. $\frac{1}{3}(2a^3b)(6b^3)$

11. $(-4x^3)(-5x^7)$

12. $(-3j^2k^4)(2jk^6)$

13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$

14. $(-5xy)(4x^2)(y^4)$

15. $(10x^3yz^2)(-2xy^5z)$

Lesson 8-1

NAME _____ DATE _____ PERIOD _____

8-1 Study Guide and Intervention *(continued)*

Multiplying Monomials

Powers of Monomials An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n, $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n, $(ab)^m = a^m b^m$.

Example **Simplify $(-2ab^2)^3(a^2)^4$.**

$$\begin{aligned}
(-2ab^2)^3(a^2)^4 &= (-2ab^2)^3(a^8) && \text{Power of a Power}\\
&= (-2)^3(a^3)(b^2)^3(a^8) && \text{Power of a Product}\\
&= (-2)^3(a^3)(a^8)(b^2)^3 && \text{Commutative Property}\\
&= (-2)^3(a^{11})(b^2)^3 && \text{Product of Powers}\\
&= -8a^{11}b^6 && \text{Power of a Power}
\end{aligned}$$

The product is $-8a^{11}b^6$.

Exercises

Simplify.

1. $(y^5)^2$

2. $(n^7)^4$

3. $(x^2)^5(x^3)$

4. $-3(ab^4)^3$

5. $(-3ab^4)^3$

6. $(4x^2b)^3$

7. $(4a^2)^2(b^3)$

8. $(4x)^2(b^3)$

9. $(x^2y^4)^5$

10. $(2a^3b^2)(b^3)^2$

11. $(-4xy)^3(-2x^2)^3$

12. $(-3j^2k^3)^2(2j^2k)^3$

13. $(25a^2b)^3\left(\frac{1}{5}abc\right)^2$

14. $(2xy)^2(-3x^2)(4y^4)$

15. $(2x^3y^2z^2)^3(x^2z)^4$

16. $(-2n^6y^5)(-6n^3y^2)(ny)^3$

17. $(-3a^3n^4)(-3a^3n)^4$

18. $-3(2x)^4(4x^5y)^2$

NAME _____ DATE _____ PERIOD _____

8-1 Skills Practice

Multiplying Monomials

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. 11

2. $a - b$

3. $\frac{p^2}{q^2}$

4. y

5. j^3k

6. $2a + 3b$

Simplify.

7. $a^2(a^3)(a^6)$

8. $x(x^2)(x^7)$

9. $(y^2z)(yz^2)$

10. $(\ell^2k^2)(\ell^3k)$

11. $(e^2f^4)(e^2f^2)$

12. $(cd^2)(c^3d^2)$

13. $(2x^2)(3x^5)$

14. $(5a^7)(4a^2)$

15. $(4xy^3)(3x^3y^5)$

16. $(7a^5b^2)(a^2b^3)$

17. $(-5m^3)(3m^8)$

18. $(-2c^4d)(-4cd)$

19. $(10^2)^3$

20. $(p^3)^{12}$

21. $(-6p)^2$

22. $(-3y)^3$

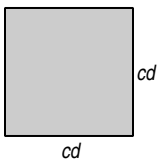
23. $(3pq^2)^2$

24. $(2b^3c^4)^2$

GEOMETRY Express the area of each figure as a monomial.

25.

26.



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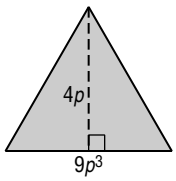


Lesson 8-1

NAME _____ DATE _____ PERIOD _____

8-1 Practice

Multiplying Monomials

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. $\frac{21a^2}{7b}$

2. $\frac{b^3c^2}{2}$

Simplify.

3. $(-5x^2y)(3x^4)$

4. $(2ab^2c^2)(4a^3b^2c^2)$

5. $(3cd^4)(-2c^2)$

6. $(4g^3h)(-2g^5)$

7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$

8. $(-xy)^3(xz)$

9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$

10. $(0.2a^2b^3)^2$

11. $\left(\frac{2}{3}p\right)^2$

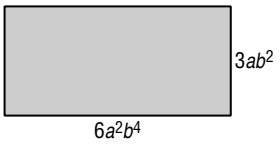
12. $\left(\frac{1}{4}cd^3\right)^2$

13. $(0.4k^3)^3$

14. $[(4^2)^2]^2$

GEOMETRY Express the area of each figure as a monomial.

15.



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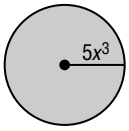

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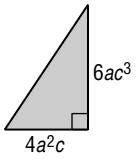
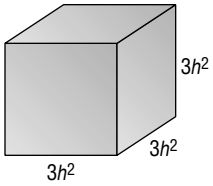


GEOMETRY Express the volume of each solid as a monomial.

18.



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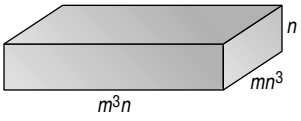

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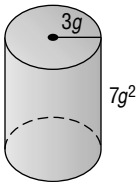


21. COUNTING A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set?

22. HOBBIES Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year?

NAME _____ DATE _____ PERIOD _____

8-1 Reading to Learn Mathematics

Multiplying Monomials

Pre-Activity **Why does doubling speed quadruple braking distance?**

Read the introduction to Lesson 8-1 at the top of page 410 in your textbook.

Find two examples in the table to verify the statement that when speed is doubled, the braking distance is quadrupled. Write your examples in the table.

Speed (miles per hour)	Braking Distance (feet)	Speed Doubled (miles per hour)	Braking Distance Quadrupled (feet)

Lesson 8-1

Reading the Lesson

1. Describe the expression $3xy$ using the terms *monomial*, *constant*, *variable*, and *product*.

2. Complete the chart by choosing the property that can be used to simplify each expression. Then simplify the expression.

Expression	Property	Expression Simplified
$3^5 \cdot 3^2$	Product of Powers Power of a Power Power of a Product	
$(a^3)^4$	Product of Powers Power of a Power Power of a Product	
$(-4xy)^5$	Product of Powers Power of a Power Power of a Product	

Helping You Remember

3. Write an example of each of the three properties of powers discussed in this lesson. Then, using the examples, explain how the property is used to simplify them.

NAME _____ DATE _____ PERIOD _____

8-1 Enrichment

An Wang

An Wang (1920–1990) was an Asian-American who became one of the pioneers of the computer industry in the United States. He grew up in Shanghai, China, but came to the United States to further his studies in science. In 1948, he invented a magnetic pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word processing systems. In 1988, Wang was elected to the National Inventors Hall of Fame.

Digital computers store information as numbers. Because the electronic circuits of a computer can exist in only one of two states, open or closed, the numbers that are stored can consist of only two digits, 0 or 1. Numbers written using only these two digits are called **binary numbers**. To find the decimal value of a binary number, you use the digits to write a *polynomial in 2*. For instance, this is how to find the decimal value of the number 1001101_2. (The subscript 2 indicates that this is a binary number.)

$$\begin{aligned} 1001101_2 &= 1 \times \underline{2^6} + 0 \times \underline{2^5} + 0 \times \underline{2^4} + 1 \times \underline{2^3} + 1 \times \underline{2^2} + 0 \times \underline{2^1} + 1 \times \underline{2^0} \\ &= 1 \times \underline{64} + 0 \times \underline{32} + 0 \times \underline{16} + 1 \times \underline{8} + 1 \times \underline{4} + 0 \times \underline{2} + 1 \times \underline{1} \\ &= 64 + 0 + 0 + 8 + 4 + 0 + 1 \\ &= 77 \end{aligned}$$

Find the decimal value of each binary number.

1. 1111_2 **2.** 10000_2 **3.** 11000011_2 **4.** 10111001_2

Write each decimal number as a binary number.

5. 8 **6.** 11 **7.** 29 **8.** 117

9. The chart at the right shows a set of decimal code numbers that is used widely in storing letters of the alphabet in a computer's memory. Find the code numbers for the letters of your name. Then write the code for your name using binary numbers.

The American Standard Guide for Information Interchange (ASCII)

A	65	**N**	78	**a**	97	**n**	110
B	66	**O**	79	**b**	98	**o**	111
C	67	**P**	80	**c**	99	**p**	112
D	68	**Q**	81	**d**	100	**q**	113
E	69	**R**	82	**e**	101	**r**	114
F	70	**S**	83	**f**	102	**s**	115
G	71	**T**	84	**g**	103	**t**	116
H	72	**U**	85	**h**	104	**u**	117
I	73	**V**	86	**i**	105	**v**	118
J	74	**W**	87	**j**	106	**w**	119
K	75	**X**	88	**k**	107	**x**	120
L	76	**Y**	89	**l**	108	**y**	121
M	77	**Z**	90	**m**	109	**z**	122

NAME _____ DATE _____ PERIOD _____

8-2 Study Guide and Intervention

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume neither a nor b is equal to zero.**

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5 .

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that b is not equal to zero.**

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Lesson 8-2

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{5^5}{5^2}$
2. $\frac{m^6}{m^4}$
3. $\frac{p^5n^4}{p^2n}$
4. $\frac{a^2}{a}$
5. $\frac{x^5y^3}{x^5y^2}$
6. $\frac{-2y^7}{14y^5}$
7. $\frac{xy^6}{y^4x}$
8. $\left(\frac{2a^2b}{a}\right)^3$
9. $\left(\frac{4p^4q^4}{3p^2q^2}\right)^3$
10. $\left(\frac{2v^5w^3}{v^4w^3}\right)^4$
11. $\left(\frac{3r^6s^3}{2r^5s}\right)^4$
12. $\frac{r^7s^7t^2}{s^3r^3t^2}$

NAME _____ DATE _____ PERIOD _____

8-2 Study Guide and Intervention *(continued)*

Dividing Monomials

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that the denominator is not equal to zero.**

$$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$$ Group powers with the same base.

$$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$$ Quotient of Powers and Negative Exponent Properties

$$= \frac{1}{4}a^{-5}b^0c^5$$ Simplify.

$$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$$ Negative Exponent and Zero Exponent Properties

$$= \frac{c^5}{4a^5}$$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{2^2}{2^{-3}}$

2. $\frac{m}{m^{-4}}$

3. $\frac{p^{-8}}{p^3}$

4. $\frac{b^{-4}}{b^{-5}}$

5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$

6. $\frac{(a^2b^3)^2}{(ab)^{-2}}$

7. $\frac{x^4y^0}{x^{-2}}$

8. $\frac{(6a^{-1}b)^2}{(b^2)^4}$

9. $\frac{(3st)^2u^{-4}}{s^{-1}t^2u^7}$

10. $\frac{s^{-3}t^{-5}}{(s^2t^3)^{-1}}$

11. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$

12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$

NAME _____ DATE _____ PERIOD _____

8-2 Skills Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{6^5}{6^4}$

2. $\frac{9^{12}}{9^8}$

3. $\frac{x^4}{x^2}$

4. $\frac{r^3s^2}{r^3s^4}$

5. $\frac{m}{m^3}$

6. $\frac{9d^7}{3d^6}$

7. $\frac{12n^5}{36n}$

8. $\frac{w^4u^3}{w^4u}$

9. $\frac{a^3b^5}{ab^2}$

10. $\frac{m^7n^2}{m^3n^2}$

11. $\frac{-21w^5u^2}{7w^4u^5}$

12. $\frac{32x^3y^2z^5}{-8xyz^2}$

13. $\left(\frac{4p^7}{7s^2}\right)^2$

14. 4^{-4}

15. 8^{-2}

16. $\left(\frac{5}{3}\right)^{-2}$

17. $\left(\frac{9}{11}\right)^{-1}$

18. $\frac{h^3}{h^{-6}}$

19. $k^0(k^4)(k^{-6})$

20. $k^{-1}(\ell^{-6})(m^3)$

21. $\frac{f^{-7}}{f^4}$

22. $\left(\frac{16p^5q^2}{2p^3q^3}\right)^0$

23. $\frac{f^{-5}g^4}{h^{-2}}$

24. $\frac{15x^6y^{-9}}{5xy^{-11}}$

25. $\frac{-15w^0u^{-1}}{5u^3}$

26. $\frac{48x^6y^7z^5}{-6xy^5z^6}$

Lesson 8-2

NAME _____ DATE _____ PERIOD _____

8-2 Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{8^8}{8^4}$

2. $\frac{a^4b^6}{ab^3}$

3. $\frac{xy^2}{xy}$

4. $\frac{m^5np}{m^4p}$

5. $\frac{5c^2d^3}{-4c^2d}$

6. $\frac{8y^7z^6}{4y^6z^5}$

7. $\left(\frac{4f^3g}{3h^6}\right)^3$

8. $\left(\frac{6w^5}{7p^6s^3}\right)^2$

9. $\frac{-4c^2}{24c^5}$

10. $x^3(y^{-5})(x^{-8})$

11. $p(q^{-2})(r^{-3})$

12. 12^{-2}

13. $\left(\frac{3}{7}\right)^{-2}$

14. $\left(\frac{4}{3}\right)^{-4}$

15. $\frac{22r^3s^2}{11r^2s^{-3}}$

16. $\frac{-15w^0u^{-1}}{5u^3}$

17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$

18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$

19. $\frac{6f^{-2}g^3h^5}{54f^{-2}g^{-5}h^3}$

20. $\frac{-12t^{-1}u^5v^{-4}}{2t^{-3}uv^5}$

21. $\frac{r^4}{(3r)^3}$

22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$

23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$

24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$

25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$

26. $\left(\frac{7c^{-3}d^3}{c^5de^{-4}}\right)^{-1}$

27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$

28. BIOLOGY A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells?

29. COUNTING The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"?

NAME _____ DATE _____ PERIOD _____

8-2 Reading to Learn Mathematics

Dividing Monomials

Pre-Activity **How can you compare pH levels?**

Read the introduction to Lesson 8-2 at the top of page 417 in your textbook.

- In the formula $c = \left(\frac{1}{10}\right)^{\text{pH}}$, identify the base and the exponent.

- How do you think c will change as the exponent increases?

Reading the Lesson

1. Explain what the statement $\frac{a^m}{a^n} = a^{m-n}$ means.

2. To find c in the formula $c = \left(\frac{1}{10}\right)^{\text{pH}}$, you can find the power of the numerator, the power of the denominator, and divide. This is an example of what property?

3. Use the Quotient of Powers Property to explain why $3^0 = 1$.

4. Consider the expression 4^{-3}.

a. Explain why the expression 4^{-3} is not simplified.

b. Define the term reciprocal.

c. 4^{-3} is the reciprocal of what power of 4?

d. What is the simplified form of 4^{-3}?

Helping You Remember

5. Describe how you would help a friend who needs to simplify the expression $\frac{4x^2}{2x^5}$.

Lesson 8-2

NAME _____ DATE _____ PERIOD _____

8-2 Enrichment

Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a. $2^{10} =$ _____
$2^9 =$ _____
$2^8 =$ _____
$2^7 =$ _____
$2^6 =$ _____
$2^5 =$ _____
$2^4 =$ _____
$2^3 =$ _____
$2^2 =$ _____
$2^1 =$ _____

b. $5^{10} =$ _____
$5^9 =$ _____
$5^8 =$ _____
$5^7 =$ _____
$5^6 =$ _____
$5^5 =$ _____
$5^4 =$ _____
$5^3 =$ _____
$5^2 =$ _____
$5^1 =$ _____

c. $4^{10} =$ _____
$4^9 =$ _____
$4^8 =$ _____
$4^7 =$ _____
$4^6 =$ _____
$4^5 =$ _____
$4^4 =$ _____
$4^3 =$ _____
$4^2 =$ _____
$4^1 =$ _____

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.

2. Describe the pattern of the powers from the top of the column to the bottom.

3. What would you expect the following powers to be?
2^0 5^0 4^0

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using 22, 25, and 24 as bases.

Study the pattern below. Then answer the questions.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 =$ __?__ 0^{-1} does not exist. 0^{-2} does not exist. 0^{-3} does not exist.

5. Why do 0^{-1}, 0^{-2}, and 0^{-3} not exist?

6. Based upon the pattern, can you determine whether 0^0 exists?

7. The symbol 0^0 is called an **indeterminate**, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that 0^0 cannot equal 1?

NAME _____ DATE _____ PERIOD _____

8-3 Study Guide and Intervention

Scientific Notation

Scientific Notation Keeping track of place value in very large or very small numbers written in standard form may be difficult. It is more efficient to write such numbers in scientific notation. A number is expressed in scientific notation when it is written as a product of two factors, one factor that is greater than or equal to 1 and less than 10 and one factor that is a power of ten.

Scientific Notation	A number is in scientific notation when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

Example 1 **Express 3.52×10^4 in standard notation.**

$3.52 \times 10^4 = 3.52 \times 10{,}000$
$= 35{,}200$

The decimal point moved 4 places to the right.

Example 2 **Express 6.21×10^{-5} in standard notation.**

$6.21 \times 10^{-5} = 6.21 \times \frac{1}{10^5}$
$= 6.21 \times 0.00001$
$= 0.0000621$

The decimal point moved 5 places to the left.

Example 3 **Express 37,600,000 in scientific notation.**

$37{,}600{,}000 = 3.76 \times 10^7$

The decimal point moved 7 places so that it is between the 3 and the 7. Since $37{,}600{,}000 > 1$, the exponent is positive.

Example 4 **Express 0.0000549 in scientific notation.**

$0.0000549 = 5.49 \times 10^{-5}$

The decimal point moved 5 places so that it is between the 5 and the 4. Since $0.0000549 < 1$, the exponent is negative.

Exercises

Express each number in standard notation.

1. 3.65×10^5 **2.** 7.02×10^{-4} **3.** 8.003×10^8

4. 7.451×10^6 **5.** 5.91×10^0 **6.** 7.99×10^{-1}

7. 8.9354×10^{10} **8.** 8.1×10^{-9} **9.** 4×10^{15}

Express each number in scientific notation.

10. 0.0000456 **11.** 0.00001 **12.** 590,000,000

13. 0.00000000012 **14.** 0.000080436 **15.** 0.03621

16. 433×10^4 **17.** 0.0042×10^{-3} **18.** 50,000,000,000

Lesson 8-3

NAME _____ DATE _____ PERIOD _____

8-3 Study Guide and Intervention *(continued)*

Scientific Notation

Products and Quotients with Scientific Notation You can use properties of powers to compute with numbers written in scientific notation.

Example 1 **Evaluate $(6.7 \times 10^3)(2 \times 10^{-5})$. Express the result in scientific and standard notation.**

$$\begin{aligned}(6.7 \times 10^3)(2 \times 10^{-5}) &= (6.7 \times 2)(10^3 \times 10^{-5}) && \text{Associative Property}\\ &= 13.4 \times 10^{-2} && \text{Product of Powers}\\ &= (1.34 \times 10^1) \times 10^{-2} && 13.4 = 1.34 \times 10^1\\ &= 1.34 \times (10^1 \times 10^{-2}) && \text{Associative Property}\\ &= 1.34 \times 10^{-1} \text{ or } 0.134 && \text{Product of Powers}\end{aligned}$$

The solution is 1.34×10^{-1} or 0.134.

Example 2 **Evaluate $\frac{1.5088 \times 10^8}{4.1 \times 10^5}$. Express the result in scientific and standard notation.**

$$\begin{aligned}\frac{1.5088 \times 10^8}{4.1 \times 10^5} &= \left(\frac{1.5088}{4.1}\right)\left(\frac{10^8}{10^5}\right) && \text{Associative Property}\\ &= 0.368 \times 10^3 && \text{Quotient of Powers}\\ &= (3.68 \times 10^{-1}) \times 10^3 && 0.368 = 3.68 \times 10^{-1}\\ &= 3.68 \times (10^{-1} \times 10^3) && \text{Associative Property}\\ &= 3.68 \times 10^2 \text{ or } 368 && \text{Product of Powers}\end{aligned}$$

The solution is 3.68×10^2 or 368.

Exercises

Evaluate. Express each result in scientific and standard notation.

1. $\frac{1.4 \times 10^4}{2 \times 10^2}$

2. $\frac{3 \times 10^{-12}}{2 \times 10^{-15}}$

3. $(3.2 \times 10^{-2})(2.0 \times 10^2)$

4. $\frac{1.2672 \times 10^{-8}}{2.4 \times 10^{-12}}$

5. $(7.7 \times 10^5)(2.1 \times 10^2)$

6. $\frac{9.72 \times 10^8}{7.2 \times 10^{10}}$

7. $(3.3 \times 10^5)(1.5 \times 10^{-4})$

8. $\frac{3.3 \times 10^{-12}}{1.1 \times 10^{-14}}$

9. $\frac{4 \times 10^{-4}}{2.5 \times 10^2}$

10. FUEL CONSUMPTION North America burned 4.5×10^{16} BTU of petroleum in 1998. At this rate, how many BTU's will be burned in 9 years? **Source:** *The New York Times 2001 Almanac*

11. OIL PRODUCTION If the United States produced 6.25×10^9 barrels of crude oil in 1998, and Canada produced 1.98×10^9 barrels, what is the quotient of their production rates? Write a statement using this quotient. **Source:** *The New York Times 2001 Almanac*

NAME _____ DATE _____ PERIOD _____

8-3 Skills Practice

Scientific Notation

Express each number in standard notation.

1. 4×10^3

2. 2×10^8

3. 3.2×10^5

4. 3×10^{-6}

5. 9×10^{-2}

6. 4.7×10^{-7}

ASTRONOMY Express the number in each statement in standard notation.

7. The diameter of Jupiter is 1.42984×10^5 kilometers.

8. The surface density of the main ring around Jupiter is 5×10^{-6} grams per centimeter squared.

9. The minimum distance from Mars to Earth is 5.45×10^7 kilometers.

Express each number in scientific notation.

10. 41,000,000

11. 65,100

12. 283,000,000

13. 264,701

14. 0.019

15. 0.000007

16. 0.000010035

17. 264.9

18. 150×10^2

Evaluate. Express each result in scientific and standard notation.

19. $(3.1 \times 10^7)(2 \times 10^{-5})$

20. $(5 \times 10^{-2})(1.4 \times 10^{-4})$

21. $(3 \times 10^3)(4.2 \times 10^{-1})$

22. $(3 \times 10^{-2})(5.2 \times 10^9)$

23. $(2.4 \times 10^2)(4 \times 10^{-10})$

24. $(1.5 \times 10^{-4})(7 \times 10^{-5})$

25. $\dfrac{5.1 \times 10^6}{1.5 \times 10^2}$

26. $\dfrac{7.2 \times 10^{-5}}{4 \times 10^{-3}}$

Lesson 8-3

NAME _____ DATE _____ PERIOD _____

8-3 Practice

Scientific Notation

Express each number in standard notation.

1. 7.3×10^{7} **2.** 2.9×10^{3} **3.** 9.821×10^{12}

4. 3.54×10^{-1} **5.** 7.3642×10^{4} **6.** 4.268×10^{-6}

PHYSICS Express the number in each statement in standard notation.

7. An electron has a negative charge of 1.6×10^{-19} Coulomb.

8. In the middle layer of the sun's atmosphere, called the chromosphere, the temperature averages 2.78×10^{4} degrees Celsius.

Express each number in scientific notation.

9. 915,600,000,000 **10.** 6387 **11.** 845,320 **12.** 0.00000000814

13. 0.00009621 **14.** 0.003157 **15.** 30,620 **16.** 0.0000000000112

17. 56×10^{7} **18.** 4740×10^{5} **19.** 0.076×10^{-3} **20.** 0.0057×10^{3}

Evaluate. Express each result in scientific and standard notation.

21. $(5 \times 10^{-2})(2.3 \times 10^{12})$ **22.** $(2.5 \times 10^{-3})(6 \times 10^{15})$

23. $(3.9 \times 10^{3})(4.2 \times 10^{-11})$ **24.** $(4.6 \times 10^{-4})(3.1 \times 10^{-1})$

25. $\dfrac{3.12 \times 10^{3}}{1.56 \times 10^{-3}}$ **26.** $\dfrac{6.72 \times 10^{3}}{4.2 \times 10^{8}}$ **27.** $\dfrac{1.17 \times 10^{2}}{5 \times 10^{-1}}$

28. $\dfrac{1.82 \times 10^{5}}{9.1 \times 10^{7}}$ **29.** $\dfrac{1.68 \times 10^{4}}{8.4 \times 10^{-4}}$ **30.** $\dfrac{2.015 \times 10^{-3}}{3.1 \times 10^{2}}$

31. BIOLOGY A cubic millimeter of human blood contains about 5×10^{6} red blood cells. An adult human body may contain about 5×10^{6} cubic millimeters of blood. About how many red blood cells does such a human body contain?

32. POPULATION The population of Arizona is about 4.778×10^{6} people. The land area is about 1.14×10^{5} square miles. What is the population density per square mile?

NAME _____ DATE _____ PERIOD _____

8-3 Reading to Learn Mathematics

Scientific Notation

Pre-Activity **Why is scientific notation important in astronomy?**

Read the introduction to Lesson 8-3 at the top of page 425 in your textbook.

In the table, each mass is written as the product of a number and a power of 10. Look at the first factor in each product. How are these factors alike?

Reading the Lesson

1. Is the number 0.0543×10^4 in scientific notation? Explain.

2. Complete each sentence to change from scientific notation to standard notation.

a. To express 3.64×10^6 in standard notation, move the decimal point _____ places to the _____.

b. To express 7.825×10^{-3} in standard notation, move the decimal point _____ places to the _____.

3. Complete each sentence to change from standard notation to scientific notation.

a. To express 0.0007865 in scientific notation, move the decimal point _____ places to the right and write _____.

b. To express 54,000,000,000 in scientific notation, move the decimal point _____ places to the left and write _____.

4. Write *positive* or *negative* to complete each sentence.

a. _____ powers of 10 are used to express very large numbers in scientific notation.

b. _____ powers of 10 are used to express very small numbers in scientific notation.

Helping You Remember

5. Describe the method you would use to estimate how many times greater the mass of Saturn is than the mass of Pluto.

Lesson 8-3

NAME _____ DATE _____ PERIOD _____

8-3 Enrichment

Converting Metric Units

Scientific notation is convenient to use for unit conversions in the metric system.

Example 1 **How many kilometers are there in 4,300,000 meters?**

Divide the measure by the number of meters (1000) in one kilometer. Express both numbers in scientific notation.

$$\frac{4.3 \times 10^6}{1 \times 10^3} = 4.3 \times 10^3$$

The answer is 4.3 3 103 km.

Example 2 **Convert 3700 grams into milligrams.**

Multiply by the number of milligrams (1000) in 1 gram.

$(3.7 \times 10^3)(1 \times 10^3) = 3.7 \times 10^6$

There are 3.7×10^6 mg in 3700 g.

Complete the following. Express each answer in scientific notation.

1. 250,000 m = _____ km

2. 375 km = _____ m

3. 247 m = _____ cm

4. 5000 m = _____ mm

5. 0.0004 km = _____ m

6. 0.01 mm = _____ m

7. 6000 m = _____ mm

8. 340 cm = _____ km

9. 52,000 mg = _____ g

10. 420 kL = _____ L

Solve.

11. The planet Mars has a diameter of 6.76×10^3 km. What is the diameter of Mars in meters? Express the answer in both scientific and decimal notation.

12. The distance from earth to the sun is 149,590,000 km. Light travels 3.0×10^8 meters per second. How long does it take light from the sun to reach the earth in minutes? Round to the nearest hundredth.

13. A light-year is the distance that light travels in one year. (See Exercise 12.) How far is a light year in kilometers? Express your answer in scientific notation. Round to the nearest hundredth.

NAME _____ DATE _____ PERIOD _____

8-4 Study Guide and Intervention

Polynomials

Degree of a Polynomial A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The **degree** of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

Example **State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*. Then give the degree of the polynomial.**

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	Yes. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	Yes. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	No. $3n^{-4} = \frac{3}{n^4}$, which is not a monomial	none of these	—
$9x^3 + 4x + x + 4 + 2x$	Yes. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*.

1. 36

2. $\frac{3}{q^2} + 5$

3. $7x - x + 5$

4. $8g^2h - 7gh + 2$

5. $\frac{1}{4y^2} + 5y - 8$

6. $6x + x^2$

Find the degree of each polynomial.

7. $4x^2y^3z$

8. $-2abc$

9. $15m$

10. $s + 5t$

11. 22

12. $18x^2 + 4yz - 10y$

13. $x^4 - 6x^2 - 2x^3 - 10$

14. $2x^3y^2 - 4xy^3$

15. $-2r^8s^4 + 7r^2s - 4r^7s^6$

16. $9x^2 + yz^8$

17. $8b + bc^5$

18. $4x^4y - 8zx^2 + 2x^5$

19. $4x^2 - 1$

20. $9abc + bc - d^5$

21. $h^3m + 6h^4m^2 - 7$

Lesson 8-4

NAME _____ DATE _____ PERIOD _____

8-4 Study Guide and Intervention *(continued)*

Polynomials

Write Polynomials in Order The terms of a polynomial are usually arranged so that the powers of one variable are in **ascending** (increasing) order or **descending** (decreasing) order.

Example 1 **Arrange the terms of each polynomial so that the powers of x are in ascending order.**

a. $x^4 - x^2 + 5x^3$

$-x^2 + 5x^3 + x^4$

b. $8x^3y - y^2 + 6x^2y + xy^2$

$-y^2 + xy^2 + 6x^2y + 8x^3y$

Example 2 **Arrange the terms of each polynomial so that the powers of x are in descending order.**

a. $x^4 + 4x^5 - x^2$

$4x^5 + x^4 - x^2$

b. $-6xy + y^3 - x^2y^2 + x^4y^2$

$x^4y^2 - x^2y^2 - 6xy + y^3$

Exercises

Arrange the terms of each polynomial so that the powers of x are in ascending order.

1. $5x + x^2 + 6$

2. $6x + 9 - 4x^2$

3. $4xy + 2y + 6x^2$

4. $6y^2x - 6x^2y + 2$

5. $x^4 + x^3 + x^2$

6. $2x^3 - x + 3x^7$

7. $-5cx + 10c^2x^3 + 15cx^2$

8. $-4nx - 5n^3x^3 + 5$

9. $4xy + 2y + 5x^2$

Arrange the terms of each polynomial so that the powers of x are in descending order.

10. $2x + x^2 - 5$

11. $20x - 10x^2 + 5x^3$

12. $x^2 + 4yx - 10x^5$

13. $9bx + 3bx^2 - 6x^3$

14. $x^3 + x^5 - x^2$

15. $ax^2 + 8a^2x^5 - 4$

16. $3x^3y - 4xy^2 - x^4y^2 + y^5$

17. $x^4 + 4x^3 - 7x^5 + 1$

18. $-3x^6 - x^5 + 2x^8$

19. $-15cx^2 + 8c^2x^5 + cx$

20. $24x^2y - 12x^3y^2 + 6x^4$

21. $-15x^3 + 10x^4y^2 + 7xy^2$

NAME _____ DATE _____ PERIOD _____

8-4 Skills Practice

Polynomials

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

1. $5mn + n^2$

2. $4by + 2b - by$

3. -32

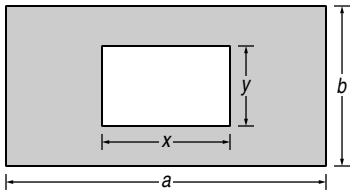
4. $\frac{3x}{7}$

5. $5x^2 - 3x^{-4}$

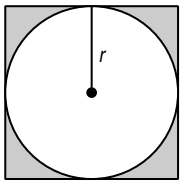
6. $2c^2 + 8c + 9 - 3$

GEOMETRY Write a polynomial to represent the area of each shaded region.

7.



8.



Find the degree of each polynomial.

9. 12

10. $3r^4$

11. $b + 6$

12. $4a^3 - 2a$

13. $5abc - 2b^2 + 1$

14. $8x^5y^4 - 2x^8$

Arrange the terms of each polynomial so that the powers of x are in ascending order.

15. $3x + 1 + 2x^2$

16. $5x - 6 + 3x^2$

17. $9x^2 + 2 + x^3 + x$

18. $-3 + 3x^3 - x^2 + 4x$

19. $7r^5x + 21r^4 - r^2x^2 - 15x^3$

20. $3a^2x^4 + 14a^2 - 10x^3 + ax^2$

Arrange the terms of each polynomial so that the powers of x are in descending order.

21. $x^2 + 3x^3 + 27 - x$

22. $25 - x^3 + x$

23. $x - 3x^2 + 4 + 5x^3$

24. $x^2 + 64 - x + 7x^3$

25. $2cx + 32 - c^3x^2 + 6x^3$

26. $13 - x^3y^3 + x^2y^2 + x$

Lesson 8-4

NAME _____ DATE _____ PERIOD _____

8-4 Practice

Polynomials

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

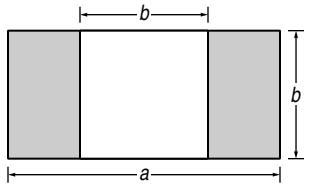
1. $7a^2b + 3b^2 - a^2b$

2. $\frac{1}{5}y^3 + y^2 - 9$

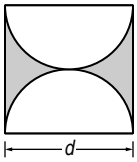
3. $6g^2h^3k$

GEOMETRY Write a polynomial to represent the area of each shaded region.

4.



5.



Find the degree of each polynomial.

6. $x + 3x^4 - 21x^2 + x^3$

7. $3g^2h^3 + g^3h$

8. $-2x^2y + 3xy^3 + x^2$

9. $5n^3m - 2m^3 + n^2m^4 + n^2$

10. $a^3b^2c + 2a^5c + b^3c^2$

11. $10s^2t^2 + 4st^2 - 5s^3t^2$

Arrange the terms of each polynomial so that the powers of x are in ascending order.

12. $8x^2 - 15 + 5x^5$

13. $10bx - 7b^2 + x^4 + 4b^2x^3$

14. $-3x^3y + 8y^2 + xy^4$

15. $7ax - 12 + 3ax^3 + a^2x^2$

Arrange the terms of each polynomial so that the powers of x are in descending order.

16. $13x^2 - 5 + 6x^3 - x$

17. $4x + 2x^5 - 6x^3 + 2$

18. $g^2x - 3gx^3 + 7g^3 + 4x^2$

19. $-11x^2y^3 + 6y - 2xy + 2x^4$

20. $7a^2x^2 + 17 - a^3x^3 + 2ax$

21. $12rx^3 + 9r^6 + r^2x + 8x^6$

22. MONEY Write a polynomial to represent the value of t ten-dollar bills, f fifty-dollar bills, and h one-hundred-dollar bills.

23. GRAVITY The height above the ground of a ball thrown up with a velocity of 96 feet per second from a height of 6 feet is $6 + 96t - 16t^2$ feet, where t is the time in seconds. According to this model, how high is the ball after 7 seconds? Explain.

NAME _____ DATE _____ PERIOD _____

8-4 Reading to Learn Mathematics

Polynomials

Pre-Activity **How are polynomials useful in modeling data?**

Read the introduction to Lesson 8-4 at the top of page 432 in your textbook.

- How many terms does $t^4 - 9t^3 + 26t - 18t + 76$ have?

- What could you call a polynomial with just one term?

Reading the Lesson

1. What is the meaning of the prefixes *mono-*, *bi-*, and *tri-*?

2. Write examples of words that begin with the prefixes *mono-*, *bi-*, and *tri-*.

3. Complete the table.

	monomial	binomial	trinomial	polynomial with more than three terms
Example	$3r^2t$	$2x^2 + 3x$	$5x^2 + 3x + 2$	$7s^2 + s^4 + 2s^3 - s + 5$
Number of Terms				

4. What is the degree of the monomial $3xy^2z$?

5. What is the degree of the polynomial $4x^4 + 2x^3y^3 + y^2 + 14$? Explain how you found your answer.

Helping You Remember

6. Use a dictionary to find the meaning of the terms *ascending* and *descending*. Write their meanings and then describe a situation in your everyday life that relates to them.

Lesson 8-4

NAME _____ DATE _____ PERIOD _____

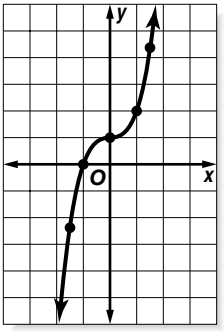
8-4 Enrichment

Polynomial Functions

Suppose a linear equation such as $23x + y = 4$ is solved for y. Then an equivalent equation, $y = 3x + 4$, is found. Expressed in this way, y is a function of x, or $f(x) = 3x + 4$. Notice that the right side of the equation is a binomial of degree 1.

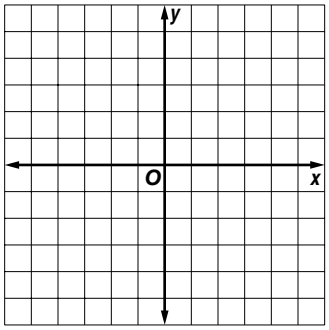
Higher-degree polynomials in x may also form functions. An example is $f(x) = x^3 + 1$, which is a polynomial function of degree 3. You can graph this function using a table of ordered pairs, as shown at the right.

x	y
$-1\frac{1}{2}$	$-2\frac{3}{8}$
-1	0
0	1
1	2
$1\frac{1}{2}$	$4\frac{3}{8}$

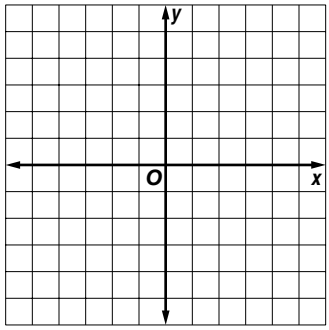


For each of the following polynomial functions, make a table of values for x and $y = f(x)$. Then draw the graph on the grid.

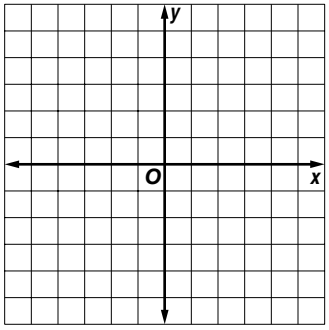
1. $f(x) = 1 - x^2$



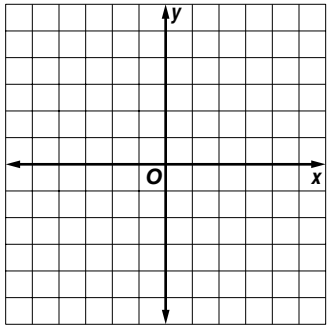
2. $f(x) = x^2 - 5$



3. $f(x) = x^2 + 4x - 1$



4. $f(x) = x^3$



NAME _____ DATE _____ PERIOD _____

8-5 Study Guide and Intervention

Adding and Subtracting Polynomials

Add Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

Example 1 **Find $(2x^2 + x - 8) + (3x - 4x^2 + 2)$.**

Horizontal Method

Group like terms.

$(2x^2 + x - 8) + (3x - 4x^2 + 2)$

$= [(2x^2 + (-4x^2)] + (x + 3x) + [(-8) + 2)]$

$= -2x^2 + 4x - 6$.

The sum is $-2x^2 + 4x - 6$.

Example 2 **Find $(3x^2 + 5xy) + (xy + 2x^2)$.**

Vertical Method

Align like terms in columns and add.

$$\begin{array}{rl} & 3x^2 + 5xy \\ (+) & 2x^2 + \ xy \\ \hline & 5x^2 + 6xy \end{array}$$

Put the terms in descending order.

The sum is $5x^2 + 6xy$.

Exercises

Find each sum.

1. $(4a - 5) + (3a + 6)$

2. $(6x + 9) + (4x^2 - 7)$

3. $(6xy + 2y + 6x) + (4xy - x)$

4. $(x^2 + y^2) + (-x^2 + y^2)$

5. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$

6. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$

7. $(5p + 2q) + (2p^2 - 8q + 1)$

8. $(4x^2 - x + 4) + (5x + 2x^2 + 2)$

9. $(6x^2 + 3x) + (x^2 - 4x - 3)$

10. $(x^2 + 2xy + y^2) + (x^2 - xy - 2y^2)$

11. $(2a - 4b - c) + (-2a - b - 4c)$

12. $(6xy^2 + 4xy) + (2xy - 10xy^2 + y^2)$

13. $(2p - 5q) + (3p + 6q) + (p - q)$

14. $(2x^2 - 6) + (5x^2 + 2) + (-x^2 - 7)$

15. $(3z^2 + 5z) + (z^2 + 2z) + (z - 4)$

16. $(8x^2 + 4x + 3y^2 + y) + (6x^2 - x + 4y)$

Lesson 8-5

NAME _____ DATE _____ PERIOD _____

8-5 Study Guide and Intervention *(continued)*

Adding and Subtracting Polynomials

Subtract Polynomials You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Example **Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.**

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$
$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$
$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$
$= 2x^2 + (-9)$
$= 2x^2 - 9$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 3x^2 + 2x - 6 \\ (-) & x^2 + 2x + 3 \\ \hline & 3x^2 + 2x - 6 \\ (+) & -x^2 - 2x - 3 \\ \hline & 2x^2 \quad - 9 \end{array}$$

The difference is $2x^2 - 9$.

Exercises

Find each difference.

1. $(3a - 5) - (5a + 1)$

2. $(9x + 2) - (-3x^2 - 5)$

3. $(9xy + y - 2x) - (6xy - 2x)$

4. $(x^2 + y^2) - (-x^2 + y^2)$

5. $(6p^2 + 4p + 5) - (2p^2 - 5p + 1)$

6. $(6x^2 + 5xy - 2y^2) - (-xy - 2x^2 - 4y^2)$

7. $(8p - 5q) - (-6p^2 + 6q - 3)$

8. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$

9. $(3x^2 - 2x) - (3x^2 + 5x - 1)$

10. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$

11. $(2h - 6j - 2k) - (-7h - 5j - 4k)$

12. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$

13. $(2a - 8b) - (-3a + 5b)$

14. $(2x^2 - 8) - (-2x^2 - 6)$

15. $(6z^2 + 4z + 2) - (4z^2 + z)$

16. $(6x^2 - 5x + 1) - (-7x^2 - 2x + 4)$

NAME _____ DATE _____ PERIOD _____

8-5 Skills Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(2x + 3y) + (4x + 9y)$

2. $(6s + 5t) + (4t + 8s)$

3. $(5a + 9b) - (2a + 4b)$

4. $(11m - 7n) - (2m + 6n)$

5. $(m^2 - m) + (2m + m^2)$

6. $(x^2 - 3x) - (2x^2 + 5x)$

7. $(d^2 - d + 5) - (2d + 5)$

8. $(2e^2 - 5e) + (7e - 3e^2)$

9. $(5f + g - 2) + (-2f + 3)$

10. $(6k^2 + 2k + 9) + (4k^2 - 5k)$

11. $(x^3 - x + 1) - (3x - 1)$

12. $(b^2 + ab - 2) - (2b^2 + 2ab)$

13. $(7z^2 + 4 - z) - (-5 + 3z^2)$

14. $(5 + 4n + 2m) + (-6m - 8)$

15. $(4t^2 + 2) + (-4 + 2t)$

16. $(3g^3 + 7g) - (4g + 8g^3)$

17. $(2a^2 + 8a + 4) - (a^2 - 3)$

18. $(3x^2 - 7x + 5) - (-x^2 + 4x)$

19. $(7z^2 + z + 1) - (-4z + 3z^2 - 3)$

20. $(2c^2 + 7c + 4) + (c^2 + 1 - 9c)$

21. $(n^2 + 3n + 2) - (2n^2 - 6n - 2)$

22. $(a^2 + ab - 3b^2) + (b^2 + 4a^2 - ab)$

23. $(\ell^2 - 5\ell - 6) + (2\ell^2 + 5 + \ell)$

24. $(2m^2 + 5m + 1) - (4m^2 - 3m - 3)$

25. $(x^2 - 6x + 2) - (-5x^2 + 7x - 4)$

26. $(5b^2 - 9b - 5) + (b^2 - 6 + 2b)$

27. $(2x^2 - 6x - 2) + (x^2 + 4x) + (3x^2 + x + 5)$

Lesson 8-5

NAME _____ DATE _____ PERIOD _____

8-5 Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(4y + 5) + (-7y - 1)$

2. $(-x^2 + 3x) - (5x + 2x^2)$

3. $(4k^2 + 8k + 2) - (2k + 3)$

4. $(2m^2 + 6m) + (m^2 - 5m + 7)$

5. $(2w^2 - 3w + 1) + (4w - 7)$

6. $(g^3 + 2g^2) - (6g - 4g^2 + 2g^3)$

7. $(5a^2 + 6a + 2) - (7a^2 - 7a + 5)$

8. $(-4p^2 - p + 9) + (p^2 + 3p - 1)$

9. $(x^3 - 3x + 1) - (x^3 + 7 - 12x)$

10. $(6c^2 - c + 1) - (-4 + 2c^2 + 8c)$

11. $(-b^3 + 8bc^2 + 5) - (7bc^2 - 2 + b^3)$

12. $(5n^2 - 3n + 2) + (-n + 2n^2 - 4)$

13. $(4y^2 + 2y - 8) - (7y^2 + 4 - y)$

14. $(w^2 - 4w - 1) + (-5 + 5w^2 - 3w)$

15. $(4u^2 - 2u - 3) + (3u^2 - u + 4)$

16. $(5b^2 - 8 + 2b) - (b + 9b^2 + 5)$

17. $(4d^2 + 2d + 2) + (5d^2 - 2 - d)$

18. $(8x^2 + x - 6) - (-x^2 + 2x - 3)$

19. $(3h^2 + 7h - 1) - (4h + 8h^2 + 1)$

20. $(4m^2 - 3m + 10) + (m^2 + m - 2)$

21. $(x^2 + y^2 - 6) - (5x^2 - y^2 - 5)$

22. $(7t^2 + 2 - t) + (t^2 - 7 - 2t)$

23. $(k^3 - 2k^2 + 4k + 6) - (-4k + k^2 - 3)$

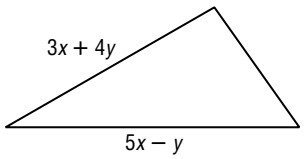
24. $(9j^2 + j + jk) + (-3j^2 - jk - 4j)$

25. $(2x + 6y - 3z) + (4x + 6z - 8y) + (x - 3y + z)$

26. $(6f^2 - 7f - 3) - (5f^2 - 1 + 2f) - (2f^2 - 3 + f)$

27. BUSINESS The polynomial $s^3 - 70s^2 + 1500s - 10{,}800$ models the profit a company makes on selling an item at a price s. A second item sold at the same price brings in a profit of $s^3 - 30s^2 + 450s - 5000$. Write a polynomial that expresses the total profit from the sale of both items.

28. GEOMETRY The measures of two sides of a triangle are given. If P is the perimeter, and $P = 10x + 5y$, find the measure of the third side.



NAME _____ DATE _____ PERIOD _____

8-5 Reading to Learn Mathematics

Adding and Subtracting Polynomials

Pre-Activity **How can adding polynomials help you model sales?**

Read the introduction to Lesson 8-5 at the top of page 439 in your textbook.

What operation would you use to find how much more the traditional toy sales R were than the video games sales V?

Reading the Lesson

1. Use the example $(-3x^3 + 4x^2 + 5x + 1) + (-5x^3 - 2x^2 + 2x - 7)$.

a. Show what is meant by grouping like terms horizontally.

b. Show what is meant by aligning like terms vertically.

c. Choose one method, then add the polynomials.

2. How is subtracting a polynomial like subtracting a rational number?

3. An algebra student got the following exercise wrong on his homework. What was his error?

$(3x^5 - 3x^4 + 2x^3 - 4x^2 + 5) - (2x^5 - x^3 + 2x^2 - 4)$

$= [3x^5 + (-2x^5)] + (-3x^4) + [2x^3 + (-x^3)] + [-4x^2 + (-2x^2)] + (5 + 4)$

$= x^5 - 3x^4 + x^3 - 6x^2 + 9$

Helping You Remember

4. How is adding and subtracting polynomials vertically like adding and subtracting decimals vertically?

Lesson 8-5

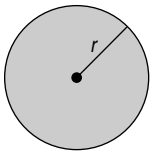
NAME _____ DATE _____ PERIOD _____

8-5 Enrichment

Circular Areas and Volumes

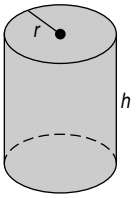
Area of Circle

$A = \pi r^2$



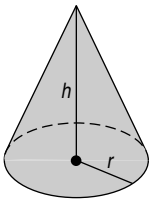
Volume of Cylinder

$V = \pi r^2 h$



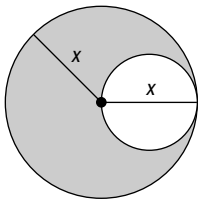
Volume of Cone

$V = \frac{1}{3}\pi r^2 h$

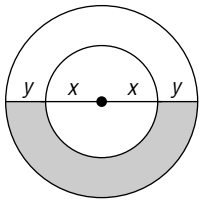


Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)

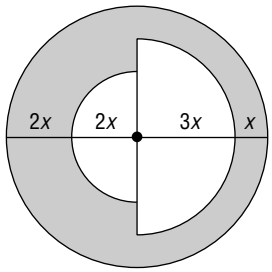
1.



2.

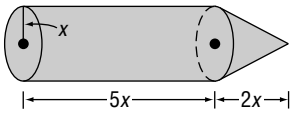


3.

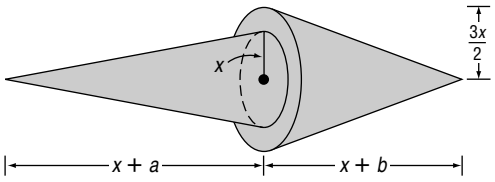


Write an algebraic expression for the total volume of each figure.

4.

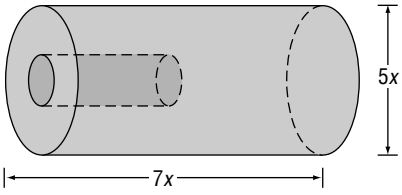


5.

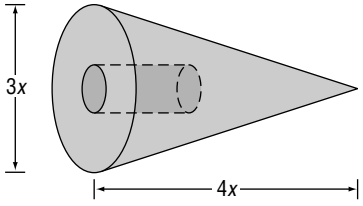


Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.

6.



7.



NAME _____ DATE _____ PERIOD _____

8-6 Study Guide and Intervention

Multiplying a Polynomial by a Monomial

Product of Monomial and Polynomial The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

Example 1 **Find $-3x^2(4x^2 + 6x - 8)$.**

Horizontal Method

$$\begin{aligned}&-3x^2(4x^2 + 6x - 8)\\ &= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)\\ &= -12x^4 + (-18x^3) - (-24x^2)\\ &= -12x^4 - 18x^3 + 24x^2\end{aligned}$$

Vertical Method

$$\begin{array}{rr} & 4x^2 + 6x - 8\\ (\times) & \qquad -3x^2\\ \hline & -12x^4 - 18x^3 + 24x^2\end{array}$$

The product is $-12x^4 - 18x^3 + 24x^2$.

Example 2 **Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.**

$$\begin{aligned}&-2(4x^2 + 5x) - x(x^2 + 6x)\\ &= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)\\ &= -8x^2 + (-10x) + (-x^3) + (-6x^2)\\ &= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)\\ &= -x^3 - 14x^2 - 10x\end{aligned}$$

Exercises

Find each product.

1. $x(5x + x^2)$

2. $x(4x^2 + 3x + 2)$

3. $-2xy(2y + 4x^2)$

4. $-2g(g^2 - 2g + 2)$

5. $3x(x^4 + x^3 + x^2)$

6. $-4x(2x^3 - 2x + 3)$

7. $-4cx(10 + 3x)$

8. $3y(-4x - 6x^3 - 2y)$

9. $2x^2y^2(3xy + 2y + 5x)$

Simplify.

10. $x(3x - 4) - 5x$

11. $-x(2x^2 - 4x) - 6x^2$

12. $6a(2a - b) + 2a(-4a + 5b)$

13. $4r(2r^2 - 3r + 5) + 6r(4r^2 + 2r + 8)$

14. $4n(3n^2 + n - 4) - n(3 - n)$

15. $2b(b^2 + 4b + 8) - 3b(3b^2 + 9b - 18)$

16. $-2z(4z^2 - 3z + 1) - z(3z^2 + 2z - 1)$

17. $2(4x^2 - 2x) - 3(-6x^2 + 4) + 2x(x - 1)$

NAME _____ DATE _____ PERIOD _____

8-6 Study Guide and Intervention *(continued)*

Multiplying a Polynomial by a Monomial

Solve Equations with Polynomial Expressions Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example **Solve $4(n - 2) + 5n = 6(3 - n) + 19$.**

$4(n - 2) + 5n = 6(3 - n) + 19$	Original equation
$4n - 8 + 5n = 18 - 6n + 19$	Distributive Property
$9n - 8 = 37 - 6n$	Combine like terms.
$15n - 8 = 37$	Add $6n$ to both sides.
$15n = 45$	Add 8 to both sides.
$n = 3$	Divide each side by 15.

The solution is 3.

Exercises

Solve each equation.

1. $2(a - 3) = 3(-2a + 6)$

2. $3(x + 5) - 6 = 18$

3. $3x(x - 5) - 3x^2 = -30$

4. $6(x^2 + 2x) = 2(3x^2 + 12)$

5. $4(2p + 1) - 12p = 2(8p + 12)$

6. $2(6x + 4) + 2 = 4(x - 4)$

7. $-2(4y - 3) - 8y + 6 = 4(y - 2)$

8. $c(c + 2) - c(c - 6) = 10c - 12$

9. $3(x^2 - 2x) = 3x^2 + 5x - 11$

10. $2(4x + 3) + 2 = -4(x + 1)$

11. $3(2h - 6) - (2h + 1) = 9$

12. $3(y + 5) - (4y - 8) = -2y + 10$

13. $3(2a - 6) - (-3a - 1) = 4a - 2$

14. $5(2x^2 - 1) - (10x^2 - 6) = -(x + 2)$

15. $3(x + 2) + 2(x + 1) = -5(x - 3)$

16. $4(3p^2 + 2p) - 12p^2 = 2(8p + 6)$

NAME _____ DATE _____ PERIOD _____

8-6 Skills Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $a(4a + 3)$

2. $-c(11c + 4)$

3. $x(2x - 5)$

4. $2y(y - 4)$

5. $-3n(n^2 + 2n)$

6. $4h(3h - 5)$

7. $3x(5x^2 - x + 4)$

8. $7c(5 - 2c^2 + c^3)$

9. $-4b(1 - 9b - 2b^2)$

10. $6y(-5 - y + 4y^2)$

11. $2m^2(2m^2 + 3m - 5)$

12. $-3n^2(-2n^2 + 3n + 4)$

Simplify.

13. $w(3w + 2) + 5w$

14. $f(5f - 3) - 2f$

15. $-p(2p - 8) - 5p$

16. $y^2(-4y + 5) - 6y^2$

17. $2x(3x^2 + 4) - 3x^3$

18. $4a(5a^2 - 4) + 9a$

19. $4b(-5b - 3) - 2(b^2 - 7b - 4)$

20. $3m(3m + 6) - 3(m^2 + 4m + 1)$

Solve each equation.

21. $3(a + 2) + 5 = 2a + 4$

22. $2(4x + 2) - 8 = 4(x + 3)$

23. $5(y + 1) + 2 = 4(y + 2) - 6$

24. $4(b + 6) = 2(b + 5) + 2$

25. $6(m - 2) + 14 = 3(m + 2) - 10$

26. $3(c + 5) - 2 = 2(c + 6) + 2$

NAME _____ DATE _____ PERIOD _____

8-6 Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $2h(-7h^2 - 4h)$

2. $6pq(3p^2 + 4q)$

3. $-2u^2n(4u - 2n)$

4. $5jk(3jk + 2k)$

5. $-3rs(-2s^2 + 3r)$

6. $4mg^2(2mg + 4g)$

7. $-\frac{1}{4}m(8m^2 + m - 7)$

8. $-\frac{2}{3}n^2(-9n^2 + 3n + 6)$

Simplify.

9. $-2\ell(3\ell - 4) + 7\ell$

10. $5w(-7w + 3) + 2w(-2w^2 + 19w + 2)$

11. $6t(2t - 3) - 5(2t^2 + 9t - 3)$

12. $-2(3m^3 + 5m + 6) + 3m(2m^2 + 3m + 1)$

13. $-3g(7g - 2) + 3(g^2 + 2g + 1) - 3g(-5g + 3)$

14. $4z^2(z - 7) - 5z(z^2 - 2z - 2) + 3z(4z - 2)$

Solve each equation.

15. $5(2s - 1) + 3 = 3(3s + 2)$

16. $3(3u + 2) + 5 = 2(2u - 2)$

17. $4(8n + 3) - 5 = 2(6n + 8) + 1$

18. $8(3b + 1) = 4(b + 3) - 9$

19. $h(h - 3) - 2h = h(h - 2) - 12$

20. $w(w + 6) + 4w = -7 + w(w + 9)$

21. $t(t + 4) - 1 = t(t + 2) + 2$

22. $u(u - 5) + 8u = u(u + 2) - 4$

23. NUMBER THEORY Let x be an integer. What is the product of twice the integer added to three times the next consecutive integer?

INVESTMENTS For Exercises 24–26, use the following information.

Kent invested \$5,000 in a retirement plan. He allocated x dollars of the money to a bond account that earns 4% interest per year and the rest to a traditional account that earns 5% interest per year.

24. Write an expression that represents the amount of money invested in the traditional account.

25. Write a polynomial model in simplest form for the total amount of money T Kent has invested after one year. (*Hint:* Each account has $A + IA$ dollars, where A is the original amount in the account and I is its interest rate.)

26. If Kent put \$500 in the bond account, how much money does he have in his retirement plan after one year?

NAME _____ DATE _____ PERIOD _____

8-6 Reading to Learn Mathematics

Multiplying a Polynomial by a Monomial

Pre-Activity **How is finding the product of a monomial and a polynomial related to finding the area of a rectangle?**

Read the introduction to Lesson 8-6 at the top of page 444 in your textbook. You may recall that the formula for the area of a rectangle is $A = \ell w$. In this rectangle, $\ell =$ _____ and $w =$ _____. How would you substitute these values in the area formula?

Reading the Lesson

1. Refer to Lesson 8-6.

a. How is the Distributive Property used to multiply a polynomial by a monomial?

b. Use the Distributive Property to complete the following.

$2y^2(3y^2 + 2y - 7) = 2y^2($_____$) + 2y^2($_____$) - 2y^2($_____$)$

$= $ _____ $+$ _____ $-$ _____

$-3x^3(x^3 - 2x^2 + 3) =$ _____ $-$ _____ $+$ _____

$= $ _____ $+$ _____ $-$ _____

2. What is the difference between simplifying an expression and solving an equation?

Helping You Remember

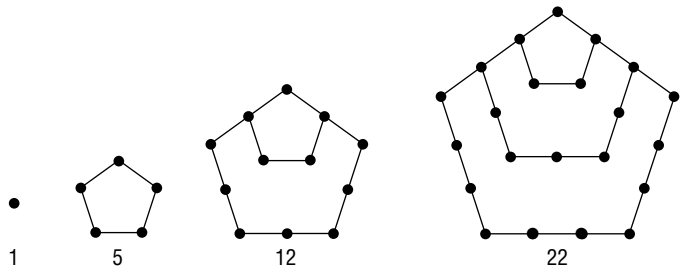
3. Use the equation $2x(x - 5) + 3x(x + 3) = 5x(x + 7) - 9$ to show how you would explain the process of solving equations with polynomial expressions to another algebra student.

NAME _____ DATE _____ PERIOD _____

8-6 Enrichment

Figurate Numbers

The numbers below are called **pentagonal numbers**. They are the numbers of dots or disks that can be arranged as pentagons.



1. Find the product $\frac{1}{2}n(3n - 1)$.

2. Evaluate the product in Exercise 1 for values of n from 1 through 4.

3. What do you notice?

4. Find the next six pentagonal numbers.

5. Find the product $\frac{1}{2}n(n + 1)$.

6. Evaluate the product in Exercise 5 for values of n from 1 through 5. On another sheet of paper, make drawings to show why these numbers are called the triangular numbers.

7. Find the product $n(2n - 1)$.

8. Evaluate the product in Exercise 7 for values of n from 1 through 5. Draw these hexagonal numbers.

9. Find the first 5 square numbers. Also, write the general expression for any square number.

The numbers you have explored above are called the plane figurate numbers because they can be arranged to make geometric figures. You can also create solid figurate numbers.

10. If you pile 10 oranges into a pyramid with a triangle as a base, you get one of the tetrahedral numbers. How many layers are there in the pyramid? How many oranges are there in the bottom layers?

11. Evaluate the expression $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$ for values of n from 1 through 5 to find the first five tetrahedral numbers.

NAME _____ DATE _____ PERIOD _____

8-7 Study Guide and Intervention

Multiplying Polynomials

Multiply Binomials To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

Example 1 **Find $(x + 3)(x - 4)$.**

Horizontal Method

$(x + 3)(x - 4)$
$= x(x - 4) + 3(x - 4)$
$= (x)(x) + x(-4) + 3(x) + 3(-4)$
$= x^2 - 4x + 3x - 12$
$= x^2 - x - 12$

Vertical Method

$$\begin{array}{rr} & x + 3 \\ (\times) & x - 4 \\ \hline & -4x - 12 \\ x^2 + 3x & \\ \hline x^2 - x - 12 & \end{array}$$

The product is $x^2 - x - 12$.

Example 2 **Find $(x - 2)(x + 5)$ using the FOIL method.**

$(x - 2)(x + 5)$

First Outer Inner Last

$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$
$= x^2 + 5x + (-2x) - 10$
$= x^2 + 3x - 10$

The product is $x^2 + 3x - 10$.

Lesson 8-7

Exercises

Find each product.

1. $(x + 2)(x + 3)$

2. $(x - 4)(x + 1)$

3. $(x - 6)(x - 2)$

4. $(p - 4)(p + 2)$

5. $(y + 5)(y + 2)$

6. $(2x - 1)(x + 5)$

7. $(3n - 4)(3n - 4)$

8. $(8m - 2)(8m + 2)$

9. $(k + 4)(5k - 1)$

10. $(3x + 1)(4x + 3)$

11. $(x - 8)(-3x + 1)$

12. $(5t + 4)(2t - 6)$

13. $(5m - 3n)(4m - 2n)$

14. $(a - 3b)(2a - 5b)$

15. $(8x - 5)(8x + 5)$

16. $(2n - 4)(2n + 5)$

17. $(4m - 3)(5m - 5)$

18. $(7g - 4)(7g + 4)$

NAME _____ DATE _____ PERIOD _____

8-7 Study Guide and Intervention *(continued)*

Multiplying Polynomials

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

Example **Find $(3x + 2)(2x^2 - 4x + 5)$.**

$(3x + 2)(2x^2 - 4x + 5)$

$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$ Distributive Property

$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$ Distributive Property

$= 6x^3 - 8x^2 + 7x + 10$ Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Exercises

Find each product.

1. $(x + 2)(x^2 - 2x + 1)$

2. $(x + 3)(2x^2 + x - 3)$

3. $(2x - 1)(x^2 - x + 2)$

4. $(p - 3)(p^2 - 4p + 2)$

5. $(3k + 2)(k^2 + k - 4)$

6. $(2t + 1)(10t^2 - 2t - 4)$

7. $(3n - 4)(n^2 + 5n - 4)$

8. $(8x - 2)(3x^2 + 2x - 1)$

9. $(2a + 4)(2a^2 - 8a + 3)$

10. $(3x - 4)(2x^2 + 3x + 3)$

11. $(n^2 + 2n - 1)(n^2 + n + 2)$

12. $(t^2 + 4t - 1)(2t^2 - t - 3)$

13. $(y^2 - 5y + 3)(2y^2 + 7y - 4)$

14. $(3b^2 - 2b + 1)(2b^2 - 3b - 4)$

NAME _____ DATE _____ PERIOD _____

8-7 Skills Practice

Multiplying Polynomials

Find each product.

1. $(m + 4)(m + 1)$

2. $(x + 2)(x + 2)$

3. $(b + 3)(b + 4)$

4. $(t + 4)(t - 3)$

5. $(r + 1)(r - 2)$

6. $(z - 5)(z + 1)$

7. $(3c + 1)(c - 2)$

8. $(2x - 6)(x + 3)$

9. $(d - 1)(5d - 4)$

10. $(2\ell + 5)(\ell - 4)$

11. $(3n - 7)(n + 3)$

12. $(q + 5)(5q - 1)$

13. $(3b + 3)(3b - 2)$

14. $(2m + 2)(3m - 3)$

15. $(4c + 1)(2c + 1)$

16. $(5a - 2)(2a - 3)$

17. $(4h - 2)(4h - 1)$

18. $(x - y)(2x - y)$

19. $(e + 4)(e^2 + 3e - 6)$

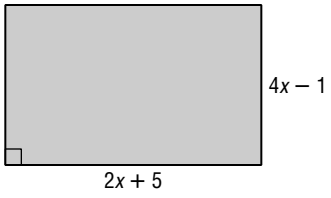
20. $(t + 1)(t^2 + 2t + 4)$

21. $(k + 4)(k^2 + 3k - 6)$

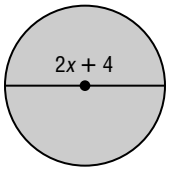
22. $(m + 3)(m^2 + 3m + 5)$

GEOMETRY Write an expression to represent the area of each figure.

23.



24.



Lesson 8-7

NAME _____ DATE _____ PERIOD _____

8-7 Practice

Multiplying Polynomials

Find each product.

1. $(q + 6)(q + 5)$

2. $(x + 7)(x + 4)$

3. $(s + 5)(s - 6)$

4. $(n - 4)(n - 6)$

5. $(a - 5)(a - 8)$

6. $(w - 6)(w - 9)$

7. $(4c + 6)(c - 4)$

8. $(2x - 9)(2x + 4)$

9. $(4d - 5)(2d - 3)$

10. $(4b + 3)(3b - 4)$

11. $(4m + 2)(4m - 3)$

12. $(5c - 5)(7c + 9)$

13. $(6a - 3)(7a - 4)$

14. $(6h - 3)(4h - 2)$

15. $(2x - 2)(5x - 4)$

16. $(3a - b)(2a - b)$

17. $(4g + 3h)(2g + 3h)$

18. $(4x + y)(4x + y)$

19. $(m + 5)(m^2 + 4m - 8)$

20. $(t + 3)(t^2 + 4t + 7)$

21. $(2h + 3)(2h^2 + 3h + 4)$

22. $(3d + 3)(2d^2 + 5d - 2)$

23. $(3q + 2)(9q^2 - 12q + 4)$

24. $(3r + 2)(9r^2 + 6r + 4)$

25. $(3c^2 + 2c - 1)(2c^2 + c + 9)$

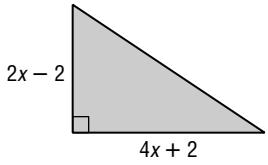
26. $(2\ell^2 + \ell + 3)(4\ell^2 + 2\ell - 2)$

27. $(2x^2 - 2x - 3)(2x^2 - 4x + 3)$

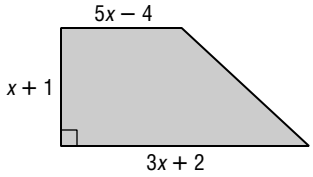
28. $(3y^2 + 2y + 2)(3y^2 - 4y - 5)$

GEOMETRY Write an expression to represent the area of each figure.

29.



30.



31. NUMBER THEORY Let x be an even integer. What is the product of the next two consecutive even integers?

32. GEOMETRY The volume of a rectangular pyramid is one third the product of the area of its base and its height. Find an expression for the volume of a rectangular pyramid whose base has an area of $3x^2 + 12x + 9$ square feet and whose height is $x + 3$ feet.

NAME _____ DATE _____ PERIOD _____

8-7 Reading to Learn Mathematics

Multiplying Polynomials

Pre-Activity **How is multiplying binomials similar to multiplying two-digit numbers?**

Read the introduction to Lesson 8-7 at the top of page 452 in your textbook.

In your own words, explain how the distributive property is used twice to multiply two-digit numbers.

Reading the Lesson

1. How is multiplying binomials similar to multiplying two-digit numbers?

2. Complete the table using the FOIL method.

		Product of First Terms	+	Product of Outer Terms	+	Product of Inner Terms	+	Product of Last Terms
$(x + 5)(x - 3)$	=		+		+		+	
	=		+		+		+	
	=		+		−			
$(3y + 6)(y - 2)$	=		+		+		+	
	=		+		+		+	
	=		−					

Helping You Remember

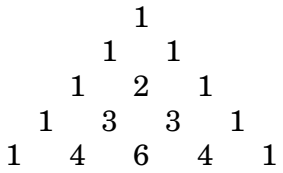
3. Think of a method for remembering all the product combinations used in the FOIL method for multiplying two binomials. Describe your method using words or a diagram.

NAME _____ DATE _____ PERIOD _____

8-7 Enrichment

Pascal's Triangle

This arrangement of numbers is called Pascal's Triangle. It was first published in 1665, but was known hundreds of years earlier.



1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row?

2. Describe how to create the 6th row of Pascal's Triangle.

3. Write the numbers for rows 6 through 10 of the triangle.

Row 6:

Row 7:

Row 8:

Row 9:

Row 10:

Multiply to find the expanded form of each product.

4. $(a + b)^2$

5. $(a + b)^3$

6. $(a + b)^4$

Now compare the coefficients of the three products in Exercises 4–6 with Pascal's Triangle.

7. Describe the relationship between the expanded form of $(a + b)^n$ and Pascal's Triangle.

8. Use Pascal's Triangle to write the expanded form of $(a + b)^6$.

NAME _____ DATE _____ PERIOD _____

8-8 Study Guide and Intervention

Special Products

Squares of Sums and Differences Some pairs of binomials have products that follow specific patterns. One such pattern is called the *square of a sum*. Another is called the *square of a difference*.

Square of a sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example 1 **Find $(3a + 4)(3a + 4)$.**

Use the square of a sum pattern, with $a = 3a$ and $b = 4$.

$$(3a + 4)(3a + 4) = (3a)^2 + 2(3a)(4) + (4)^2$$
$$= 9a^2 + 24a + 16$$

The product is $9a^2 + 24a + 16$.

Example 2 **Find $(2z - 9)(2z - 9)$.**

Use the square of a difference pattern with $a = 2z$ and $b = 9$.

$$(2z - 9)(2z - 9) = (2z)^2 - 2(2z)(9) + (9)(9)$$
$$= 4z^2 - 36z + 81$$

The product is $4z^2 - 36z + 81$.

Exercises

Find each product.

1. $(x - 6)^2$ **2.** $(3p + 4)^2$ **3.** $(4x - 5)^2$

4. $(2x - 1)^2$ **5.** $(2h + 3)^2$ **6.** $(m + 5)^2$

7. $(c + 3)^2$ **8.** $(3 - p)^2$ **9.** $(x - 5y)^2$

10. $(8y + 4)^2$ **11.** $(8 + x)^2$ **12.** $(3a - 2b)^2$

13. $(2x - 8)^2$ **14.** $(x^2 + 1)^2$ **15.** $(m^2 - 2)^2$

16. $(x^3 - 1)^2$ **17.** $(2h^2 - k^2)^2$ **18.** $\left(\frac{1}{4}x + 3\right)^2$

19. $(x - 4y^2)^2$ **20.** $(2p + 4q)^2$ **21.** $\left(\frac{2}{3}x - 2\right)^2$

Lesson 8-8

NAME _____ DATE _____ PERIOD _____

8-8 Study Guide and Intervention *(continued)*

Special Products

Product of a Sum and a Difference There is also a pattern for the product of a sum and a difference of the same two terms, $(a + b)(a - b)$. The product is called the **difference of squares**.

Product of a Sum and a Difference	$(a + b)(a - b) = a^2 - b^2$

Example **Find $(5x + 3y)(5x - 3y)$.**

$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and a Difference

$(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$ $a = 5x$ and $b = 3y$

$= 25x^2 - 9y^2$ Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

1. $(x - 4)(x + 4)$

2. $(p + 2)(p - 2)$

3. $(4x - 5)(4x + 5)$

4. $(2x - 1)(2x + 1)$

5. $(h + 7)(h - 7)$

6. $(m - 5)(m + 5)$

7. $(2c - 3)(2c + 3)$

8. $(3 - 5q)(3 + 5q)$

9. $(x - y)(x + y)$

10. $(y - 4x)(y + 4x)$

11. $(8 + 4x)(8 - 4x)$

12. $(3a - 2b)(3a + 2b)$

13. $(3y - 8)(3y + 8)$

14. $(x^2 - 1)(x^2 + 1)$

15. $(m^2 - 5)(m^2 + 5)$

16. $(x^3 - 2)(x^3 + 2)$

17. $(h^2 - k^2)(h^2 + k^2)$

18. $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$

19. $(3x - 2y^2)(3x + 2y^2)$

20. $(2p - 5s)(2p + 5s)$

21. $\left(\frac{4}{3}x - 2y\right)\left(\frac{4}{3}x + 2y\right)$

NAME _____ DATE _____ PERIOD _____

8-8 Skills Practice

Special Products

Find each product.

1. $(n + 3)^2$

2. $(x + 4)(x + 4)$

3. $(y - 7)^2$

4. $(t - 3)(t - 3)$

5. $(b + 1)(b - 1)$

6. $(a - 5)(a + 5)$

7. $(p - 4)^2$

8. $(z + 3)(z - 3)$

9. $(\ell + 2)(\ell + 2)$

10. $(r - 1)(r - 1)$

11. $(3g + 2)(3g - 2)$

12. $(2m - 3)(2m + 3)$

13. $(6 + u)^2$

14. $(r + s)^2$

15. $(3q + 1)(3q - 1)$

16. $(c - e)^2$

17. $(2k - 2)^2$

18. $(w + 3h)^2$

19. $(3p - 4)(3p + 4)$

20. $(t + 2u)^2$

21. $(x - 4y)^2$

22. $(3b + 7)(3b - 7)$

23. $(3y - 3g)(3y + 3g)$

24. $(s^2 + r^2)^2$

25. $(2k + m^2)^2$

26. $(3u^2 - n)^2$

27. GEOMETRY The length of a rectangle is the sum of two whole numbers. The width of the rectangle is the difference of the same two whole numbers. Using these facts, write a verbal expression for the area of the rectangle.

Lesson 8-8

NAME _____ DATE _____ PERIOD _____

8-8 Practice

Special Products

Find each product.

1. $(n + 9)^2$

2. $(q + 8)^2$

3. $(\ell - 10)^2$

4. $(r - 11)^2$

5. $(p + 7)^2$

6. $(b + 6)(b - 6)$

7. $(z + 13)(z - 13)$

8. $(4e + 2)^2$

9. $(5w - 4)^2$

10. $(6h - 1)^2$

11. $(3s + 4)^2$

12. $(7v - 2)^2$

13. $(7k + 3)(7k - 3)$

14. $(4d - 7)(4d + 7)$

15. $(3g + 9h)(3g - 9h)$

16. $(4q + 5t)(4q - 5t)$

17. $(a + 6u)^2$

18. $(5r + s)^2$

19. $(6c - m)^2$

20. $(k - 6y)^2$

21. $(u - 7p)^2$

22. $(4b - 7v)^2$

23. $(6n + 4p)^2$

24. $(5q + 6s)^2$

25. $(6a - 7b)(6a + 7b)$

26. $(8h + 3d)(8h - 3d)$

27. $(9x + 2y^2)^2$

28. $(3p^3 + 2m)^2$

29. $(5a^2 - 2b)^2$

30. $(4m^3 - 2t)^2$

31. $(6e^3 - c)^2$

32. $(2b^2 - g)(2b^2 + g)$

33. $(2v^2 + 3e^2)(2v^2 + 3e^2)$

34. GEOMETRY Janelle wants to enlarge a square graph that she has made so that a side of the new graph will be 1 inch more than twice the original side s. What trinomial represents the area of the enlarged graph?

GENETICS For Exercises 35 and 36, use the following information.

In a guinea pig, pure black hair coloring B is dominant over pure white coloring b. Suppose two hybrid Bb guinea pigs, with black hair coloring, are bred.

35. Find an expression for the genetic make-up of the guinea pig offspring.

36. What is the probability that two hybrid guinea pigs with black hair coloring will produce a guinea pig with white hair coloring?

NAME _____ DATE _____ PERIOD _____

8-8 Reading to Learn Mathematics

Special Products

Pre-Activity **When is the product of two binomials also a binomial?**

Read the introduction to Lesson 8-8 at the top of page 458 in your textook.

What is a meant by the term *trinomial product*?

Reading the Lesson

1. Refer to the Key Concepts boxes on pages 458, 459, and 460.

a. When multiplying two binomials, there are three special products. What are the three special products that may result when multiplying two binomials?

b. Explain what is meant by the name of each special product.

c. Use the examples in the Key Concepts boxes to complete the table.

	Symbols	Product	Example	Product
Square of a Sum				
Square of a Difference				
Product of a Sum and a Difference				

2. What is another phrase that describes the product of the sum and difference of two terms?

Helping You Remember

3. Explain how FOIL can help you remember how many terms are in the special products studied in this lesson.

Lesson 8-8

NAME _____ DATE _____ PERIOD _____

8-8 Enrichment

Sums and Differences of Cubes

Recall the formulas for finding some special products:

Perfect-square trinomials: $(a + b)^2 = a^2 + 2ab + b^2$ or
$(a - b)^2 = a^2 - 2ab + b^2$

Difference of two squares: $(a + b)(a - b) = a^2 - b^2$

A pattern also exists for finding the cube of a sum $(a + b)^3$.

1. Find the product of $(a + b)(a + b)(a + b)$.

2. Use the pattern from Exercise 1 to evaluate $(x + 2)^3$.

3. Based on your answer to Exercise 1, predict the pattern for the cube of a difference $(a - b)^3$.

4. Find the product of $(a - b)(a - b)(a - b)$ and compare it to your answer for Exercise 3.

5. Use the pattern from Exercise 4 to evaluate $(x + 4)^3$.

Find each product.

6. $(x + 6)^3$

7. $(x - 10)^3$

8. $(3x - y)^3$

9. $(2x - y)^3$

10. $(4x + 3y)^3$

11. $(5x + 2)^3$

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Test, Form 1

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Simplify $y^5 \cdot y^3$.

A. y^2 **B.** y^8 **C.** y^{15} **D.** $2y^8$ **1.** _____

2. Simplify $(6x^3)(x^2)$.

A. $6x^5$ **B.** $6x^6$ **C.** $7x^5$ **D.** $7x^6$ **2.** _____

3. Simplify $(b^4)^3$.

A. b^7 **B.** $3b^4$ **C.** b^{12} **D.** $3b^7$ **3.** _____

4. Simplify $\frac{a^7}{a^4}$. Assume the denominator is not equal to zero.

A. a^{11} **B.** a^{28} **C.** a^3 **D.** 1 **4.** _____

5. Simplify $(5x^2)(5^2x^3)$.

A. $25x^6$ **B.** $25x^5$ **C.** $125x^6$ **D.** $125x^5$ **5.** _____

6. Simplify $\left(\frac{2c}{d}\right)^3$. Assume the denominator is not equal to zero.

A. $\frac{6c^6}{d^3}$ **B.** $\frac{8c}{d}$ **C.** $\frac{24c^3}{d^3}$ **D.** $\frac{8c^3}{d^3}$ **6.** _____

7. Simplify $\frac{m^5n^2}{m^2n^3}$. Assume the denominator is not equal to zero.

A. m^7n^5 **B.** $\frac{m^3}{n}$ **C.** m^3n **D.** $\frac{n}{m^3}$ **7.** _____

8. Simplify $(x^2)(x^{-3})(x^{-2})$.

A. x^{-12} **B.** x^7 **C.** $\frac{1}{x^3}$ **D.** $\frac{1}{x^{-3}}$ **8.** _____

9. Express 3851 in scientific notation.

A. 3.851×10^3 **B.** 38.51×10^2 **C.** 385.1×10 **D.** 3.851×10^{-3} **9.** _____

10. Express 5.9×10^3 in standard notation.

A. 5900 **B.** 0.0059 **C.** 59,000 **D.** 0.00059 **10.** _____

11. Evaluate $(3 \times 10^4)(3 \times 10^5)$.

A. 6×10^9 **B.** 9×10^9 **C.** 6×10^{20} **D.** 9×10^{20} **11.** _____

12. Find the degree of the polynomial $b^5 + 2b^3 + 7$.

A. 3 **B.** 8 **C.** 5 **D.** 7 **12.** _____

Assessment

13. Which of the following polynomials shows the terms of $x^2 + 5x^3 - 4 - 2x$ arranged so that the powers of x are in descending order?

A. $5x^3 - 2x + x^2 - 4$ **B.** $-4 - 2x + x^2 + 5x^3$

C. $5x^3 - 4 - 2x + x^2$ **D.** $5x^3 + x^2 - 2x - 4$ **13.** _____

14. MONEY Write a polynomial to represent the value of d dimes and n nickels.

A. $10d + 5n$ **B.** $0.1d + 0.5n$ **C.** $0.1d + 0.05n$ **D.** $d + n$ **14.** _____

15. Find $(n^2 + 3n) + (2n^2 - n)$.

A. $3n^2 + 2n$ **B.** $3n^2 - 2n$ **C.** $n^2 + 4n$ **D.** $n^2 - 2n$ **15.** _____

16. Find $(2a - 5) - (3a + 1)$.

A. $5a + 6$ **B.** $a - 4$ **C.** $-a - 6$ **D.** $-a - 4$ **16.** _____

17. Find $3m^2(2m^2 - m)$.

A. $5m^4 - 3m^3$ **B.** $6m^4 - 3m^2$ **C.** $5m^4 - 3m$ **D.** $6m^4 - 3m^3$ **17.** _____

18. Simplify $3(x^2 + 2x) - x(x - 1)$.

A. $4x^2 + x$ **B.** $2x^2 + 7x$ **C.** $2x^2 + 3x$ **D.** $2x^2 + 5x$ **18.** _____

19. Solve $3(2n - 6) = -4(n - 3)$.

A. 3 **B.** $\frac{3}{5}$ **C.** 6 **D.** $1\frac{4}{5}$ **19.** _____

20. Find $(x + 3)(x + 5)$.

A. $x^2 + 8x + 15$ **B.** $x^2 + 15$ **C.** $2x + 8$ **D.** $2x + 15$ **20.** _____

21. Find $(2n - 3)(n + 4)$.

A. $3n + 1$ **B.** $2n^2 - 12$

C. $2n^2 + 5n - 12$ **D.** $2n^2 + 11n + 1$ **21.** _____

22. Find $(x + 3)(2x^2 - 4x + 8)$.

A. $2x^3 + 10x^2 + 20x + 24$ **B.** $4x^2 - 4x + 24$

C. $12x^2 + 20x + 24$ **D.** $2x^3 + 2x^2 - 4x + 24$ **22.** _____

23. Find $(y + 5)^2$.

A. $y^2 + 25$ **B.** $2y + 10$ **C.** $y^2 + 10y + 10$ **D.** $y^2 + 10y + 25$ **23.** _____

24. Find $(3y - 1)^2$.

A. $6y^2 - 6y + 1$ **B.** $9y^2 - 6y + 1$ **C.** $9y^2 - 3y + 1$ **D.** $9y^2 - 6y - 1$ **24.** _____

25. Find $(2x - 5)(2x + 5)$.

A. $4x$ **B.** $4x^2 - 25$ **C.** $4x^2 - 20x - 25$ **D.** $4x^2 + 25$ **25.** _____

Bonus Simplify $(3^{n+1})(3^{2n})^4$. **B:** _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Test, Form 2A

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Simplify $(9d^4)(-6d^5)$.

A. $3d^9$ **B.** $3d$ **C.** $-54d^9$ **D.** $-54d^{20}$ **1.** _____

2. Simplify $(x^3)^8$.

A. x^{24} **B.** x^{11} **C.** $8x^{24}$ **D.** $8x^{11}$ **2.** _____

3. Simplify $(-2hk)^4(4h^3k^5)^2$.

A. $2h^{24}k^{40}$ **B.** $-64h^9k^{11}$ **C.** $-256h^{10}k^{14}$ **D.** $256h^{10}k^{14}$ **3.** _____

4. Simplify $\frac{a^9}{a^3}$. Assume the denominator is not equal to zero.

A. a^3 **B.** a^{12} **C.** a^6 **D.** a^{27} **4.** _____

5. Simplify $\frac{36b^4c^2}{9b^{-1}c^5}$. Assume the denominator is not equal to zero.

A. $\frac{27b^4}{c^3}$ **B.** $\frac{4b^4}{c^3}$ **C.** $\frac{27b^3}{c^3}$ **D.** $\frac{4b^5}{c^3}$ **5.** _____

6. Simplify $\frac{(3y^4n^6)^2}{(y^2n^{-3})^4}$. Assume the denominator is not equal to zero.

A. $\frac{9}{y^{16}}$ **B.** $\frac{9}{n^{24}}$ **C.** $9y^{16}$ **D.** $9n^{24}$ **6.** _____

7. Express 4173 in scientific notation.

A. 4.173×10^3 **B.** 41.73×10^2 **C.** 417.3×10 **D.** 4.173×10^{-3} **7.** _____

8. Evaluate $(4 \times 10^{-2})(2 \times 10^8)$.

A. 8×10^{-16} **B.** 8×10^6 **C.** 6×10^6 **D.** 6×10^{-16} **8.** _____

9. Find the degree of the polynomial $3xy - 8x^2y^5 + x^7y$.

A. 2 **B.** 7 **C.** 8 **D.** 10 **9.** _____

10. Arrange the terms of $4x^3y^2 - 6xy^3 + 2x^5 + 3y$ so that the powers of x are in descending order.

A. $3y - 6xy^3 + 4x^3y^2 + 2x^5$ **B.** $4x^3y^2 + 3y + 2x^5 - 6xy^3$

C. $2x^5 + 4x^3y^2 - 6xy^3 + 3y$ **D.** $-6xy^3 + 4x^3y^2 + 3y + 2x^5$ **10.** _____

11. Find $(9t^2 + 4t - 6) - (t^2 - 2t + 4)$.

A. $8t^2 + 6t - 10$ **B.** $8t^2 + 2t - 2$ **C.** $9t^2 + 6t - 2$ **D.** $9t^2 + 6t - 10$ **11.** _____

Assessment

8

Chapter 8 Test, Form 2A *(continued)*

12. CARS The number of domestic car sales from 1981–1999 in millions is modeled by the expression $y = -0.01n^2 + 0.11n + 6.51$ where n is the number of years since 1981. The number of import car sales from 1981–1999 in millions is modeled by the expression $y = 0.02n + 2.57$. Find an expression that models the total number of car sales n years since 1981.

A. $-0.01n^2 - 0.09n + 3.94$ **B.** $0.01n^2 - 0.13n - 9.08$
C. $-0.01n^2 + 0.31n + 8.08$ **D.** $-0.01n^2 + 0.13n + 9.08$ **12.** _____

13. Simplify $2a^2(5a - 6) - 5a(a^2 - 3a + 4) - 7(a - 5)$.

A. $5a^3 + 3a^2 - 27a + 35$ **B.** $5a^3 - 27a^2 + 13a - 35$
C. $5a^3 - 10a - 7$ **D.** none of these **13.** _____

14. Find $(c - 5)(c - 7)$.

A. $c^2 + 12c + 35$ **B.** $c^2 - 12c - 35$
C. $c^2 - 12c + 35$ **D.** $c^2 + 35$ **14.** _____

15. Find $(3y - 4)(2y^2 + y - 1)$.

A. $6y^3 - 5y^2 - 7y - 4$ **B.** $6y^3 - 5y^2 - 7y + 4$
C. $6y^3 - 7y^2 - 7y + 4$ **D.** $6y^3 - 5y^2 + 7y + 4$ **15.** _____

16. Find $(3a - 2b)(3a + 2b)$.

A. $9a^2 - 4b^2$ **B.** $9a^2 + 4b^2$
C. $9a^2 - 12ab + 4b^2$ **D.** $9a^2 + 12ab + 4b^2$ **16.** _____

17. Find $(4a^2 + b)^2$.

A. $16a^4 + b^2$ **B.** $8a^4 + b^2$
C. $16a^4 + 8a^2b + b^2$ **D.** $4a^4 + 8a^2b + b^2$ **17.** _____

18. Solve $6(n - 11) = 12 + 4(2n - 3)$.

A. -11 **B.** 11 **C.** -33 **D.** 33 **18.** _____

19. Solve $5x^2 - 3x = (7x^2 + 5x) - (2x^2 + 16)$.

A. 2 **B.** -2 **C.** 8 **D.** -8 **19.** _____

20. GEOMETRY The length of a rectangle is 4 units less than twice the width. If the length is decreased by 3 units and the width is increased by 1 unit, the area is decreased by 16 square units. If w is the original width, which equation must be true?

A. $(2w - 3)(w - 3) = w(2w - 4) - 16$
B. $2(2w - 3) + 2(w - 3) = 2w + 2(2w - 4) - 16$
C. $(2w - 7)(w + 1) = w(2w - 4) - 16$
D. $2(2w - 7) + 2(w + 1) = 2w + (2w - 4) - 16$ **20.** _____

Bonus Simplify $\frac{7^{x-3}}{7^{3x-1}}$. **B:** _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Test, Form 2B

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

1. Simplify $(5d^3)(-2d^2)$.

A. $10d^5$ **B.** $-10d^5$ **C.** $10d^6$ **D.** $-10d^6$ **1.** _____

2. Simplify $(m^4)^2$.

A. $6m$ **B.** m^8 **C.** m^6 **D.** $2m^4$ **2.** _____

3. Simplify $(-2xy^2)^4(2x^3y^4)^2$.

A. $4x^{24}y^{32}$ **B.** $-8x^9y^6$ **C.** $64x^{10}y^{16}$ **D.** $-4x^{10}y^{16}$ **3.** _____

4. Simplify $\frac{a^{12}}{a^4}$. Assume the denominator is not equal to zero.

A. a^3 **B.** a^{16} **C.** a^{48} **D.** a^8 **4.** _____

5. Simplify $\frac{6n^{-3}y}{2n^{-1}y^{-3}}$. Assume the denominator is not equal to zero.

A. $\frac{4y^3}{n^2}$ **B.** $\frac{3y^4}{n^2}$ **C.** $\frac{3}{n^4y^2}$ **D.** $\frac{3n^2}{y^4}$ **5.** _____

6. Simplify $\frac{(a^{-2}b^4)^{-6}}{(a^4b^{-8})^3}$. Assume the denominator is not equal to zero.

A. ab^3 **B.** 1 **C.** $\frac{a^{24}}{b^{48}}$ **D.** $\frac{b^{48}}{a^{24}}$ **6.** _____

7. Express 5.43×10^{-4} in standard notation.

A. 54,300 **B.** 0.00543 **C.** 5430 **D.** 0.000543 **7.** _____

8. Evaluate $\frac{2.88 \times 10^3}{2.4 \times 10^{-7}}$.

A. 0.48×10^{-4} **B.** 1.2×10^{-4} **C.** 1.2×10^{10} **D.** 0.48×10^{10} **8.** _____

9. Find the degree of the polynomial $2x^3y - 4xy^2 + 9x^3y^2$.

A. 4 **B.** 3 **C.** 12 **D.** 5 **9.** _____

10. Arrange the terms of $x^2y^3 + 4xy^2 - 3x^3y + 6$ so that the powers of x are in ascending order.

A. $6 + 4xy^2 + x^2y^3 - 3x^3y$ **B.** $x^2y^3 - 3x^3y + 4xy^2 + 6$

C. $6 + 4xy^2 - 3x^3y + x^2y^3$ **D.** $6 - 3x^3y + 4xy^2 + x^2y^3$ **10.** _____

11. Find $(3c^2 - 8c + 5) + (c^2 - 8c - 6)$.

A. $3c^2 - 1$ **B.** $4c^2 + 11$ **C.** $4c^2 - 16c - 1$ **D.** $2c^2 - 16c - 1$ **11.** _____

Assessment

8

Chapter 8 Test, Form 2B *(continued)*

12. CLUBS The number of girls in a local 4-H club is modeled by the expression $4n^2 - n + 52$ where n is the number of years after 1990. The number of girls in 4H who are age 8 and under is modeled by the expression $n^2 + 13$. Find an expression that models the number of girls older than 8 in the club.

A. $3n^2 + n - 39$ **B.** $3n^2 - n + 39$

C. $-3n^2 - n - 39$ **D.** $-3n^2 + n + 39$ **12.** _____

13. Simplify $3b^2(4b + 7) - 2b(b^2 - 5b - 3) - 6(b - 2)$.

A. $14b^3 + 11b^2 - 12b - 12$ **B.** $41b^2 + 12$

C. $14b^3 + 31b^2 + 12b + 12$ **D.** $10b^3 + 31b^2 + 12$ **13.** _____

14. Find $(x + 2)(x + 4)$.

A. $x^2 + 8$ **B.** $x^2 + 2x + 6$

C. $x^2 + 2x + 8$ **D.** $x^2 + 6x + 8$ **14.** _____

15. Find $(3x + 2)(4x^2 - 2x - 7)$.

A. $12x^3 + 2x^2 - 25x - 14$ **B.** $12x^3 + 14x^2 + 25x + 14$

C. $7x^3 + 9x^2 - 25x - 14$ **D.** $7x^3 + 7x^2 - 4x - 5$ **15.** _____

16. Find $(3y + 4z)(3y - 4z)$.

A. $9y^2 - 16z^2$ **B.** $9y^2 - 24yz - 16z^2$

C. $9y^2 + 16z^2$ **D.** $9y^2 - 24yz + 16z^2$ **16.** _____

17. Find $(-2r^2 + s)^2$.

A. $4r^4 + s^2$ **B.** $4r^4 - 4r^2s + s^2$

C. $-4r^4 + s^2$ **D.** $-4r^4 - 4r^2s + s^2$ **17.** _____

18. Solve $-4(5 - 2n) = 8(-6 - 5n)$.

A. $-\frac{1}{9}$ **B.** $-\frac{28}{3}$ **C.** $-\frac{7}{8}$ **D.** $-\frac{7}{12}$ **18.** _____

19. Solve $x(x + 3) - 2 = 2 + x(x + 1)$.

A. 2 **B.** -2 **C.** 1 **D.** 0 **19.** _____

20. ART A picture is 4 inches longer than it is wide. It is surrounded by a mat that is 2 inches wide. The total area of the mat is 112 square inches. If w is the width of the picture, which equation is true?

A. $(w + 4)(w + 8) + w(w + 4) = 112$

B. $(w + 2)(w + 6) - w(w + 4) = 112$

C. $(w + 4)(w + 8) - w(w + 4) = 112$

D. $(w + 2)(w + 6) + w(w + 4) = 112$ **20.** _____

Bonus Simplify $3^{2n-1} \cdot 3^{5n}$. **B:** _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Test, Form 2C

SCORE _____

Simplify.

1. $3y^5 \cdot y^3$ **1.** _____

2. $(9m^3n^5)(-2mn^2)$ **2.** _____

3. $(w^5y^4)^3$ **3.** _____

4. $4a^3n^6 + 4(a^3n)^6 + 4(an^2)^3$ **4.** _____

For Questions 5–7, simplify. Assume that no denominator is equal to zero.

5. $\frac{p^6q^2}{p^3q}$ **5.** _____

6. $\frac{16r^3s^{-5}}{4r^{-1}s^2}$ **6.** _____

7. $\frac{(-8x^2y^2)^2}{(4x^3y)^3}$ **7.** _____

8. Express 0.000498 in scientific notation. **8.** _____

9. Express 1.27×10^5 in standard notation. **9.** _____

For Questions 10 and 11, express each result in scientific and standard notation.

10. Evaluate $(2.5 \times 10^{-2})(4 \times 10^6)$. **10.** _____

11. The radius of Earth is approximately 2.51×10^8 inches. The radius of the Sun is approximately 2.74×10^{10} inches. How many times greater is the radius of the Sun than the radius of Earth? **11.** _____

12. Find the degree of the polynomial $2x^3y^3 + 4xy - 10x^3y$. **12.** _____

13. Arrange the terms of the polynomial $4 + 3x^3y^3 - x^5y + xy$ so that the powers of x are in descending order. **13.** _____

Assessment

Chapter 8 Test, Form 2C *(continued)*

Find each sum or difference.

14. $(5n^2 - 2ny + 3y^2) - (9n^2 - 8ny - 10y^2)$ **14.** _____

15. $(11m^2 - 2mn + 8n^2) + (8m^2 + 4mn - 2n^2)$ **15.** _____

16. $(x^2 + 5y) - (2x^2 + 6y)$ **16.** _____

Find each product.

17. $5hk^2(2h^2k - hk^3 + 4h^2k^2)$ **17.** _____

18. $(4x^2 + 2y^2)(2x^2 - y^2)$ **18.** _____

19. $(3s + 5)(2s^2 - 8s + 6)$ **19.** _____

20. $(5c - 4)^2$ **20.** _____

21. $(7a - 3b)(7a + 3b)$ **21.** _____

22. $(4n + 1)^2$ **22.** _____

For Questions 23 and 24, solve each equation.

23. $-6(3n - 2) = 4(-3 - 2n)$ **23.** _____

24. $8n + 11 = 4 + 5(2n - 1)$ **24.** _____

25. GARDENING The length of a rectangular garden is 8 feet longer than the width. The garden is surrounded by a 4-foot sidewalk. The sidewalk has an area of 320 square feet. Find the dimensions of the garden. **25.** _____

Bonus If you multiply $(x + 1)^{20}$, how many terms will there be? (*Hint*: Look for a pattern in the smaller powers of $(x + 1)$.) **B:** _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Test, Form 2D

SCORE _____

Simplify.

1. $5x^4 \cdot x^3$ **1.** _____

2. $(3a^2b^5)(-2ab^3)$ **2.** _____

3. $(w^3z^7)^3$ **3.** _____

4. $4a^4b^8 + 2(ab^2)^4 + 4(a^2b^4)^2$ **4.** _____

For Questions 5–7, simplify. Assume that no denominator is equal to zero.

5. $\frac{-12m^3}{4m^5}$ **5.** _____

6. $\frac{4b^{-3}d^2}{8b^2d^{-5}}$ **6.** _____

7. $\frac{(3r^3s^5)^3}{(-3r^2s^7)^2}$ **7.** _____

8. Express 12,556 in scientific notation. **8.** _____

9. Express 7.43×10^{-3} in standard notation. **9.** _____

For Questions 10 and 11, express each result in scientific and standard notation.

10. The minimum distance from Earth to the Moon is approximately 2.26×10^5 miles. There are approximately 6.34×10^4 inches in one mile. What is the minimum distance from Earth to the Moon in inches? **10.** _____

11. $\frac{5.4 \times 10^4}{2.7 \times 10^{-3}}$ **11.** _____

12. Find the degree of the polynomial $2x^2y - 4x^5 + 6xy^3$. **12.** _____

13. Arrange the terms of the polynomial $3x^2y - xy^2 - 3y^3 + x^3$ so that the powers of x are in descending order. **13.** _____

Assessment

Find each sum or difference.

14. $(7m^2 + 3m - 4) - (3m^2 + 9m - 5)$ **14.** _____

15. $(4y^2 + 3y - 7) + (4y^2 - 7y - 2)$ **15.** _____

16. $(3a^3 - 4b) - (2a^3 - b)$ **16.** _____

Find each product.

17. $3x^2y(2x^2y - 5xy^2 + 8y^3x^2)$ **17.** _____

18. $(3r^2 + 5s^2)(3r^2 - 5s^2)$ **18.** _____

19. $(2n + 3)(3n^2 - 4n + 1)$ **19.** _____

20. $(5y + 6)^2$ **20.** _____

21. $(2k + 5r)(2k - 5r)$ **21.** _____

22. $(2c - 1)^2$ **22.** _____

For Questions 23 and 24, solve each equation.

23. $5x + 8 = 3 + 2(3x - 4)$ **23.** _____

24. $-5(2n - 3) = 7(3 - n)$ **24.** _____

25. GARDENING The length of a rectangular garden is 5 feet longer than its width. The garden is surrounded by a 2-foot-wide sidewalk. The sidewalk has an area of 76 square feet. Find the dimensions of the garden. **25.** _____

Bonus If $(x + 1)^{10}$ is multiplied out, how many terms will there be? (*Hint*: Look for a pattern in the smaller powers of $(x + 1)$.) **B:** _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Test, Form 3

SCORE _____

Simplify.

1. $(ab^8)(3a^6b^2)$ **1.** _____

2. $\left(\frac{2}{3}h^3\right)^4$ **2.** _____

3. $\left(\frac{2}{3}r\right)^3(27r)(5s)^2\left(\frac{1}{2}s^4\right)^3$ **3.** _____

4. $(4^{3x+7})(4^{2x-9})$ **4.** _____

For Questions 5–7, simplify. Assume that no denominator is equal to zero.

5. $\frac{-54c^2d^5}{9c^6d^2}$ **5.** _____

6. $\frac{(-2mx^{-3})^{-4}}{8m^{-5}x^0}$ **6.** _____

7. $\left(\frac{-3a^2b^{-3}}{6a^3b^{-4}}\right)^2\left(\frac{-5b}{4a^{-3}}\right)^{-3}$ **7.** _____

8. Express 196,783 in scientific notation. **8.** _____

For Questions 9 and 10, express each result in scientific and standard notation.

9. $(7.2 \times 10^{-3})(8.1 \times 10^{-2})$ **9.** _____

10. $\frac{4.59 \times 10^{-3}}{5.1 \times 10^2}$ **10.** _____

11. Arrange the terms of the polynomial $2xy - 6 + 4x^5y^2 + 7x^6y^3$ so that the powers of x are in descending order. **11.** _____

12. Find the degree of the polynomial $m^2np^2 + mn^3p^2 - 4m^4n$. **12.** _____

Assessment

8

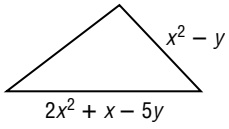
Chapter 8 Test, Form 3 *(continued)*

For Questions 13 and 14, find each sum or difference.

13. $(8w^2 + 4w - 2) + (2w^2 - w + 6)$ **13.** _____

14. $(7u^2v - 3uv + 4uv^2) - (4uv - 3u^2v - 2uv^2)$ **14.** _____

15. GEOMETRY The measures of two sides of a triangle are given on the triangle at the right. If the perimeter of the triangle is $6x^2 + 8y$, find the measure of the third side. **15.** _____



16. Simplify $5n^2(n - 6) - 2n(3n^2 + n - 6) + 7(n^2 - 3)$. **16.** _____

For Questions 17–20, find each product.

17. $(2y - 7)(4y + 4)$ **17.** _____

18. $\left(\frac{2}{3}m - 1\right)\left(\frac{1}{2}m - 2\right)$ **18.** _____

19. $(4x + y)(2x^2 - xy + 5y^2)$ **19.** _____

20. $(5r^2 + 3s^2)(5r^2 - 3s^2)$ **20.** _____

21. A square has sides of length $(3x - 1)$ feet. Write an expression that represents the area of the square. **21.** _____

22. Solve $y(y - 6) = (5y^2 - 36) - (4y^2 - 3y)$. **22.** _____

23. If $f(x) = x^2 + 5x$ and $g(x) = 2x - x^2$, find $f(a + 1) - g(a + 1)$. **23.** _____

24. If $a^2 + b^2 = 11$ and $ab = 3$, find the value of $(a - b)^2$. **24.** _____

25. GEOMETRY The length of a rectangle is 4 centimeters more than its width. If the length is increased by 8 centimeters and the width is decreased by 4 centimeters, the area will remain unchanged. Find the original dimensions of the rectangle. **25.** _____

Bonus Graph the solution set of $(x + 3)(x + 5) - (x + 1)^2 < 2(x + 1)$. **B:**

NAME _____ DATE _____ PERIOD _____

8 Chapter 8 Open-Ended Assessment

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. For each equation determine why the statement is incorrect. Explain how to change the right side of the equation to make a true statement.

a. $(4u - 5v) - (u + v) = 3u - 4v$

b. $x^2y(3x^3 + 4) = 3x^6y + 4x^2y$

c. $(3a + 5b)^2 = 9a^2 + 25b^2$

2. a. State whether the number 23.4×10^8 is in scientific notation. Explain how you determined your answer.

b. Explain why scientific notation is helpful when dividing 22,100,000 by 0.00000013. Find the quotient $22{,}100{,}000 \div 0.00000013$.

3. Give a counterexample for each statement.

a. The degree of a polynomial is always 2 or greater.

b. The degree of a trinomial is always greater than the degree of a monomial.

4. a. Simplify $\left(\frac{4a^3}{2a^{-2}}\right)^4$ by using the Quotient of Powers property first, then use the Power of a Power property.

b. Simplify $\left(\frac{4a^3}{2a^{-2}}\right)^4$ by using the Power of a Quotient property first, then use the Quotient of Powers property.

c. Write a statement that generalizes the results of part **a** and part **b**.

5. Draw an area model demonstrating the product $(3x + y)(x + 2y)$. Find the product algebraically. Explain how the algebraic product verifies the area model.

Assessment

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Vocabulary Test/Review

SCORE _____

binomial	monomial	Power of a Quotient
constant	negative exponent	Quotient of Powers
degree of a monomial	polynomial	scientific notation
degree of a polynomial	Power of a Power	trinomial
difference of squares	Power of a Product	zero exponent
FOIL method	Product of Powers	

Write the letter of the term that best matches each statement or phrase.

_____ **1.** constant

_____ **2.** Power of a Power

_____ **3.** Product of Powers

_____ **4.** FOIL method

_____ **5.** Quotient of Powers

_____ **6.** Power of a Quotient

_____ **7.** monomial

_____ **8.** zero exponent

_____ **9.** trinomial

_____ **10.** binomial

a. property that tells us to find the power of the numerator and the power of the denominator

b. property that tells us that any nonzero number raised to the zero power is 1

c. property that tells us to multiply exponents

d. the sum of three monomials

e. monomials that are real numbers

f. used to multiply two binomials

g. property that tell us to add exponents when multiplying two powers that have the same base

h. the sum of two monomials

i. property that tells us to subtract exponents when dividing two powers that have the same base

j. a number, a variable, or a product of a number and one or more variables

In your own words—
Define each term.

11. degree of a monomial

12. degree of a polynomial

13. scientific notation

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Quiz

(Lessons 8–1 and 8–2)

SCORE _____

Simplify.

1. $(r^3)(2r^5)$ 1. _____
2. $(x^5)^4$ 2. _____
3. $(-4m^2n^3)(3mn^4)$ 3. _____
4. $(-5x^4y^2)^3$ 4. _____
5. $(2cd)^2(-4c^3)^2$ 5. _____
6. $(5y^2w^4)^2 + 2(yw^2)^4$ 6. _____

Simplify. Assume that no denominator is equal to zero.

7. $\frac{6^{15}}{6^9}$ 7. _____
8. $\frac{y^8}{y^3}$ 8. _____
9. $\frac{r^6n^{-7}}{r^4n^2}$ 9. _____

10. **STANDARDIZED TEST PRACTICE** Write the ratio of the area of a circle with radius r to the circumference of the same circle.

A. $\frac{2}{r}$ **B.** 2 **C.** $\frac{r}{2}$ **D.** $\frac{1}{2r}$ 10. _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Quiz

(Lessons 8–3 and 8–4)

SCORE _____

For Questions 1 and 2, express each number in scientific notation.

1. 57,600 1. _____
2. 0.0000061 2. _____
3. Express 6.4871×10^{-3} in standard notation. 3. _____

Evaluate. Express each result in scientific and standard notation.

4. $\frac{7.2 \times 10^{-5}}{4.5 \times 10^2}$ 4. _____
5. $(3.5 \times 10^6)(8.2 \times 10^3)$ 5. _____

Find the degree of each polynomial.

6. $5a - 2b^2 + 1$ 6. _____
7. $24xy - xy^3 + x^2$ 7. _____

For Questions 8 and 9, arrange the terms of each polynomial so that the powers of x are in descending order.

8. $4x^2 - 3x^3 + 2x + 12$ 8. _____
9. $5x^3y + 3xy^4 - x^2y^3 + y^4$ 9. _____
10. Arrange the terms of the polynomial $-3 + x^2 + 4x$ so that the powers of x are in ascending order. 10. _____

Assessment

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Quiz

SCORE _____

(Lessons 8–5 and 8–6)

For Questions 1 and 2, find each sum or difference.

1. $(-2x^2 + x + 6) + (5x^2 - 4x - 2)$ **1.** _____

2. $(5a + 9b) - (2a + 4b)$ **2.** _____

3. Find $3xy(2x^2 + 5xy - 7y^2)$. **3.** _____

4. Simplify $5c^2(c + 10) - 4c(2c^2 - 6c + 1)$. **4.** _____

5. Solve $3(2p + 1) = 7(5 - p) - 6$. **5.** _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Quiz

SCORE _____

(Lesson 8–7)

Find each product.

1. $(4m - 1)(m + 2)$ **1.** _____

2. $(2c - 3)(c^2 - 4c + 5)$ **2.** _____

3. $(4h + 3)^2$ **3.** _____

4. $(a - 9)^2$ **4.** _____

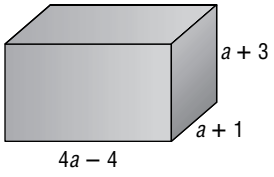
5. $(2x + 6y)(2x - 6y)$ **5.** _____

6. $(m^2 + 2n)^2$ **6.** _____

7. $(9x - 7)(9x + 7)$ **7.** _____

8. $(a - 3b)(a + 3b)$ **8.** _____

For Exercises 9 and 10, use the figure.



9. Write an expression to represent the area of the top side of the prism. **9.** _____

10. Write an expression to represent the volume of the prism. **10.** _____

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Mid-Chapter Test

SCORE _____

(Lessons 8–1 through 8–4)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Simplify $(n^5)(n^2)(m^3)(m^4)$.

A. $n^{10}m^{12}$ **B.** n^7m^7 **C.** nm^7 **D.** nm^{14} **1.** _____

2. Simplify $(3w^2v)^2(-2w^5v^2)^3$.

A. $-72w^{19}v^8$ **B.** $72w^{12}v^7$ **C.** $-36w^{32}v^{10}$ **D.** $36w^{19}v^6$ **2.** _____

For Questions 3 and 4, simplify. Assume the denominator is not equal to zero.

3. $\frac{m^6n^3}{m^2n^6}$

A. $\frac{m^4}{n^3}$ **B.** $-\frac{m^4}{n^3}$ **C.** $-\frac{m^8}{n^3}$ **D.** $\frac{m^8}{n^3}$ **3.** _____

4. $\frac{(z^2w^{-1})^3}{(z^3w^2)^2}$

A. $\frac{1}{w^7}$ **B.** $\frac{z^{12}}{w^7}$ **C.** w **D.** $\frac{1}{w}$ **4.** _____

5. Express 31,000,000 in scientific notation.

A. 31×10^6 **B.** 3.1×10^{-7}

C. 3.1×10^7 **D.** 31×10^{-6} **5.** _____

6. Find the degree of the polynomial $4x^2y^3 + 2xy^2 - 5x^3y$.

A. 4 **B.** 3 **C.** 6 **D.** 5 **6.** _____

7. Express 4.02×10^{-3} in standard notation.

A. 0.00402 **B.** 402,000 **C.** 4020 **D.** 402 **7.** _____

Part II

Evaluate. Express each result in scientific and standard notation.

8. $\frac{1.17 \times 10^2}{5 \times 10^{-1}}$ **9.** $(8.3 \times 10^2)(9.1 \times 10^{-7})$

8. _____

9. _____

For Questions 10 and 11, simplify. Assume that no denominator is equal to zero.

10. $\frac{2x^3y^5}{5(x^4y^2)^3}$ **11.** $\frac{-10m^{-1}y^0r}{-14m^{-7}y^{-3}r^{-4}}$

10. _____

11. _____

12. Find the degree of $5c^2 - 3c^4 + 2c^3 - 1$. **12.** _____

Arrange the terms of each polynomial so that the powers of x are in descending order. **13.** _____

13. $6 + 2x^2 - 3x + 4x^3$

14. $2a^2x^3 + 8x^4 - 3a^4x^2 + 8x$ **14.** _____

Assessment

NAME _____ DATE _____ PERIOD _____

8

Chapter 8 Cumulative Review

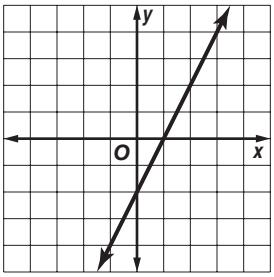
(Chapters 1–8)

1. Find a counterexample for the conditional statement.
If a number is divisible by 5, then the one's digit is a 5.
(Lesson 1-7)

1. _____

2. Solve $a(b - c) = \frac{a}{b}$ for c. (Lesson 3-8)

2. _____



3. Write an equation in function notation for the relation.
(Lesson 4-8)

3. _____

4. TUITION The average cost of tuition and fees at a four-year college was \$3,356 for the year 1999–2000. For the year 1997–1998 the average cost was \$3,111. Find the rate of change between the year 1997–1998 and the year 1999–2000. (Lesson 5-1)

4. _____

5. Write $y + 1 = 4(x - 2)$ in slope intercept form. (Lesson 5-5)

5. _____

6. Solve $14u + 9 - 10u > 21$. (Lesson 6-3)

6. _____

7. Solve $3y - 4 \leq 7$. (Lesson 6-5)

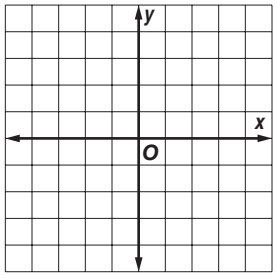
7. _____

8. RECREATION Barrington needs to buy snorkels and fins for his family for their annual beach vacation. Snorkels cost \$8 a set and fins cost \$12 a pair. Write an inequality that represents this situation if Barrington has \$62 to spend. (Lesson 6-6)

8. _____

9. Graph the system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it. (Lesson 7-1)
$x + y = 4$
$y = x$

9. _____



10. Determine the best method to solve the system of equations. Then solve the system. (Lesson 7-4)
$5x - 2y = 7$
$2x + 5y = -3$

10. _____

11. Simplify $\frac{51x^{-1}y^3}{17x^2y}$. Assume the denominator is not equal to zero. (Lesson 8-2)

11. _____

12. Evaluate $(6 \times 10^5)(2.3 \times 10^3)$. Express the result in scientific and standard notation. (Lesson 8-3)

12. _____

13. Find $\left(\frac{2}{3}a^2 - \frac{1}{4}a - \frac{1}{5}\right) - \left(\frac{1}{3}a^2 + \frac{3}{4}a + \frac{2}{5}\right)$. (Lesson 8-5)

13. _____

14. Find $(3x + 2)^2$. (Lesson 8-8)

14. _____

NAME _____ DATE _____ PERIOD _____

8 Standardized Test Practice

(Chapters 1–8)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Write $y - 11 = \frac{1}{2}(x - 12)$ in standard form. (Lesson 5-5)

A. $y = \frac{1}{2}x + 5$ **B.** $y = \frac{1}{2}x - 17$ **C.** $x - 2y = -10$ **D.** $x - 2y = 17$ **1.** Ⓐ Ⓑ Ⓒ Ⓓ

2. Ashanti wants to collect more than 50 food items for the local homeless shelter. If he has already collected 13, how many more items must he collect? (Lesson 6-1)

E. 37 **F.** at least 38 **G.** at least 37 **H.** more than 36 **2.** Ⓔ Ⓕ Ⓖ Ⓗ

3. Solve $-5n + 22 \geq -73$. (Lesson 6-3)

A. $\{n \mid n \leq 19\}$ **B.** $\{n \mid n \geq 19\}$ **C.** $\left\{n \mid n \leq -\frac{51}{5}\right\}$ **D.** $\{n \mid n \geq 90\}$ **3.** Ⓐ Ⓑ Ⓒ Ⓓ

4. Solve $|5k + 2| > 1$. (Lesson 6-5)

E. $\left\{k \mid k < -\frac{3}{5} \text{ or } k > -\frac{1}{5}\right\}$ **F.** $\left\{k \mid k - \frac{3}{5} < k < \frac{1}{5}\right\}$

G. $\varnothing$ **H.** $\{k \mid k \text{ is a real number.}\}$ **4.** Ⓔ Ⓕ Ⓖ Ⓗ

5. Use substitution to solve the system of equations. (Lesson 7-2)

$y = -2x$

$5x + 3y = 4$

A. $(-4, 8)$ **B.** $(8, -4)$ **C.** $(-8, 4)$ **D.** $(4, -8)$ **5.** Ⓐ Ⓑ Ⓒ Ⓓ

6. Use elimination to solve the system of equations. (Lesson 7-3)

$x + 3y = -6$

$2x + 3y = -9$

E. $(-2, -4)$ **F.** $(-6, 1)$ **G.** $(-3, -1)$ **H.** $(3, -5)$ **6.** Ⓔ Ⓕ Ⓖ Ⓗ

7. Simplify $\frac{(3^2)(3^3)}{(3^{-2})(3^{-3})}$. (Lesson 8-2)

A. 3^{10} **B.** 3^{12} **C.** -1 **D.** $\frac{1}{3}$ **7.** Ⓐ Ⓑ Ⓒ Ⓓ

8. Express 0.46×10^3 in scientific notation. (Lesson 8-3)

E. 0.46×10^3 **F.** 4.6×10^2 **G.** 4.6×10^3 **H.** 46×10^1 **8.** Ⓔ Ⓕ Ⓖ Ⓗ

9. Find $(3n^2 - 6n) + (5n^3 + 2n) - (2n^3 - n^2)$. (Lesson 8-5)

A. $3n^3 + 2n^2 - 8n$ **B.** $6n^3 - n^2 - 4n$

C. $3n^3 + 4n^2 - 4n$ **D.** $6n^3 + 4n^2 - 8n$ **9.** Ⓐ Ⓑ Ⓒ Ⓓ

10. Find $(7b - 11)(b + 3)$. (Lesson 8-7)

E. $7b^2 + 10b - 33$ **F.** $7b^2 - 11b - 33$

G. $7b^2 - 11b + 8$ **H.** $7b^2 + 10b + 8$ **10.** Ⓔ Ⓕ Ⓖ Ⓗ

Assessment

NAME _____ DATE _____ PERIOD _____

8 Standardized Test Practice *(continued)*

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

11. Evaluate $4x(y^2 - z^2)$ if $x = 3$, $y = 5$ and $z = 4$. (Lesson 1-2)

11.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12. After receiving his allowance, Spencer spent half of his money on a card for his mother. He bought 2 toy cars for \$0.98 to give to his brothers, and a soft drink for \$0.35. How much did Spencer receive for his allowance if he has \$0.42 left over? (Lesson 3-4)

12.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13. At the end of the 2001 football season, the Dallas Cowboys and the Denver Broncos had won a total of 7 Super Bowls. The Cowboys had won 2.5 times as many Super Bowls as the Broncos. How many Super Bowls had the Dallas Cowboys won? (Lesson 7-2)

13.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14. Express 5.7×10^{-2} in standard notation. (Lesson 8-3)

14.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Part 3: Quantitative Comparison

Instructions: Compare the quantities in columns A and B. Shade in
(A) if the quantity in column A is greater;
(B) if the quantity in column B is greater;
(C) if the quantities are equal; or
(D) if the relationship cannot be determined from the information given.

	Column A	Column B	
15.	$\sqrt{\frac{5}{7}}$	$\sqrt{\frac{9}{11}}$	**15.** (A) (B) (C) (D)

(Lesson 2-7)

16. $y = 3x - 10$
$y = -2x + 15.$

	x	y	**16.** (A) (B) (C) (D)

(Lesson 7-2)

17.	the degree of $7x^5 + 4x^3 - x^6$	the degree of $3x^2y^2 - 6x^2y^3 + 8x^3y^2$	**17.** (A) (B) (C) (D)

(Lesson 8-4)

NAME _____ DATE _____ PERIOD _____

8 Standardized Test Practice

Student Record Sheet *(Use with pages 470–471 of the Student Edition.)*

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	6 Ⓐ Ⓑ Ⓒ Ⓓ		

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 11 and 13, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11 _____ (grid in)

12 _____

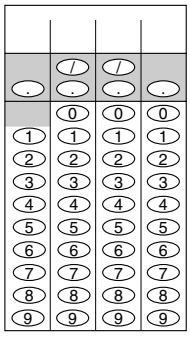
13 _____ (grid in)

14 _____

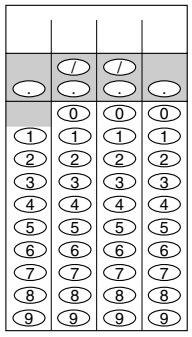
15 _____

16 _____

11



13



Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

17 Ⓐ Ⓑ Ⓒ Ⓓ	21 Ⓐ Ⓑ Ⓒ Ⓓ
18 Ⓐ Ⓑ Ⓒ Ⓓ	22 Ⓐ Ⓑ Ⓒ Ⓓ
19 Ⓐ Ⓑ Ⓒ Ⓓ	23 Ⓐ Ⓑ Ⓒ Ⓓ
20 Ⓐ Ⓑ Ⓒ Ⓓ	

Part 4 Open-Ended

Record your answers for Question 24 on the back of this paper.

Answers

NAME _____ DATE _____ PERIOD ____

8-1 Study Guide and Intervention

Multiplying Monomials

Multiply Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify $(3x^6)(5x^2)$.**

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Associative Property
$= (3 \cdot 5)(x^{6+2})$ Product of Powers
$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2 **Simplify $(-4a^3b)(3a^2b^5)$.**

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$
$= -12(a^{3+2})(b^{1+5})$
$= -12a^5b^6$

The product is $-12a^5b^6$.

Exercises

Simplify.

1. $y(y^5)$ — y^6
2. $n^2 \cdot n^7$ — n^9
3. $(-7x^2)(x^4)$ — $-7x^6$
4. $x(x^2)(x^4)$ — x^7
5. $m \cdot m^5$ — m^6
6. $(-x^3)(-x^4)$ — x^7
7. $(2a^2)(8a)$ — $16a^3$
8. $(rs)(rs^3)(s^2)$ — r^2s^6
9. $(x^2y)(4xy^3)$ — $4x^3y^4$
10. $\frac{1}{3}(2a^3b)(6b^3)$ — $4a^3b^4$
11. $(-4x^3)(-5x^7)$ — $20x^{10}$
12. $(-3j^2k^4)(2jk^6)$ — $-6j^3k^{10}$
13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$ — $a^3b^2c^7$
14. $(-5xy)(4x^2)(y^4)$ — $-20x^3y^5$
15. $(10x^3yz^2)(-2xy^5z)$ — $-20x^4y^6z^3$

NAME _____ DATE _____ PERIOD ____

8-1 Study Guide and Intervention (continued)

Multiplying Monomials

Powers of Monomials An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n, $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n, $(ab)^m = a^m b^m$.

Example **Simplify $(-2ab^2)^3(a^2)^4$.**

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power
$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product
$= (-2)^3(a^3)(a^8)(b^2)^3$ Commutative Property
$= (-2)^3(a^{11})(b^2)^3$ Product of Powers
$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Exercises

Simplify.

1. $(y^5)^2$ — y^{10}
2. $(n^7)^4$ — n^{28}
3. $(x^2)^5(x^3)$ — x^{13}
4. $-3(ab^4)^3$ — $-3a^3b^{12}$
5. $(-3ab^4)^3$ — $-27a^3b^{12}$
6. $(4x^2b)^3$ — $64x^6b^3$
7. $(4a^2)^2(b^3)$ — $16a^4b^3$
8. $(4x)^2(b^3)$ — $16x^2b^3$
9. $(x^2y^4)^5$ — $x^{10}y^{20}$
10. $(2a^3b^2)(b^3)^2$ — $2a^3b^8$
11. $(-4xy)^3(-2x^2)^3$ — $512x^9y^3$
12. $(-3j^2k^3)^2(2j^2k)^3$ — $72j^{10}k^9$
13. $(25a^2b)^3\left(\frac{1}{5}abc\right)^2$ — $625a^8b^5c^2$
14. $(2xy)^2(-3x^2)(4y^4)$ — $-48x^4y^6$
15. $(2x^3y^2z^2)^3(x^2z)^4$ — $8x^{17}y^6z^{10}$
16. $(-2n^6y^5)(-6n^3y^2)(ny)^3$ — $12n^{12}y^{10}$
17. $(-3a^3n^4)(-3a^3n)^4$ — $-243a^{15}n^8$
18. $-3(2x)^4(4x^5y)^2$ — $-768x^{14}y^2$

NAME _____ DATE _____ PERIOD ____

8-1 Skills Practice

Multiplying Monomials

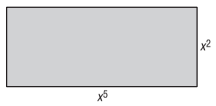
Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. 11 Yes; 11 is a real number and an example of a constant.
2. $a - b$ No; This is the difference, not the product, of two variables.
3. $\frac{p^2}{q^2}$ No; This is the quotient, not the product, of two variables.
4. y Yes; Single variables are monomials.
5. j^3k Yes; This is the product of two variables.
6. $2a + 3b$ No; This is the sum of two monomials.

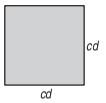
Simplify.

7. $a^2(a^3)(a^6)$ a^{11}
8. $x(x^2)(x^7)$ x^{10}
9. $(y^2z)(yz^2)$ y^3z^3
10. $(\ell^2k^2)(\ell^3k)$ ℓ^5k^3
11. $(e^2f^4)(e^2f^2)$ e^4f^6
12. $(cd^2)(c^3d^2)$ c^4d^4
13. $(2x^2)(3x^5)$ $6x^7$
14. $(5a^7)(4a^2)$ $20a^9$
15. $(4xy^3)(3x^3y^5)$ $12x^4y^8$
16. $(7a^5b^2)(a^2b^3)$ $7a^7b^5$
17. $(-5m^3)(3m^8)$ $-15m^{11}$
18. $(-2c^4d)(-4cd)$ $8c^5d^2$
19. $(10^2)^3$ 10^6 or 1,000,000
20. $(p^3)^{12}$ p^{36}
21. $(-6p)^2$ $36p^2$
22. $(-3y)^3$ $-27y^3$
23. $(3pq^2)^2$ $9p^2q^4$
24. $(2b^3c^4)^2$ $4b^6c^8$

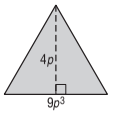
GEOMETRY Express the area of each figure as a monomial.

25. 

x^7

26. 

c^2d^2

27. 

$18p^4$

NAME _____ DATE _____ PERIOD ____

8-1 Practice (Average)

Multiplying Monomials

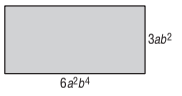
Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. $\frac{21a^2}{7b}$ No; this involves the quotient, not the product, of variables.
2. $\frac{b^3c^2}{2}$ Yes; this is the product of a number, $\frac{1}{2}$, and two variables.

Simplify.

3. $(-5x^2y)(3x^4)$ $-15x^6y$
4. $(2ab^2c^2)(4a^3b^2c^2)$ $8a^4b^4c^4$
5. $(3cd^4)(-2c^2)$ $-6c^3d^4$
6. $(4g^3h)(-2g^5)$ $-8g^8h$
7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$ $5x^2y^7$
8. $(-xy)^3(xz)$ $-x^4y^3z$
9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$ $-54m^5n^4$
10. $(0.2a^2b^3)^2$ $0.04a^4b^6$
11. $\left(\frac{2}{3}p\right)^2$ $\frac{4}{9}p^2$
12. $\left(\frac{1}{4}cd^3\right)^2$ $\frac{1}{16}c^2d^6$
13. $(0.4k^3)^3$ $0.064k^9$
14. $[(4^2)^2]^2$ 4^8 or 65,536

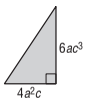
GEOMETRY Express the area of each figure as a monomial.

15. 

$18a^3b^6$

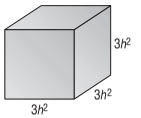
16. 

$(25x^6)\pi$

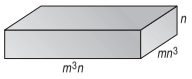
17. 

$12a^3c^4$

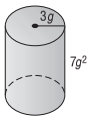
GEOMETRY Express the volume of each solid as a monomial.

18. 

$27h^6$

19. 

m^4n^5

20. 

$(63g^4)\pi$

21. **COUNTING** A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set? 2^5 or 32

22. **HOBBIES** Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year? 2^6 or 64

Answers

NAME _____ DATE _____ PERIOD ____

8-1

Reading to Learn Mathematics

Multiplying Monomials

Pre-Activity **Why does doubling speed quadruple braking distance?**

Read the introduction to Lesson 8-1 at the top of page 410 in your textbook.

Find two examples in the table to verify the statement that when speed is doubled, the braking distance is quadrupled. Write your examples in the table.

Speed (miles per hour)	Braking Distance (feet)	Speed Doubled (miles per hour)	Braking Distance Quadrupled (feet)
20	20	40	80
30	45	60	180

Reading the Lesson

1. Describe the expression $3xy$ using the terms *monomial*, *constant*, *variable*, and *product*.

The monomial $3xy$ is the product of the constant 3 and the variables x and y.

2. Complete the chart by choosing the property that can be used to simplify each expression. Then simplify the expression.

Expression	Property	Expression Simplified
$3^5 \cdot 3^2$	Product of Powers Power of a Power Power of a Product	3^7 or 2187
$(a^3)^4$	Product of Powers Power of a Power Power of a Product	a^{12}
$(-4xy)^5$	Product of Powers Power of a Power Power of a Product	$-1024x^5y^5$

Helping You Remember

3. Write an example of each of the three properties of powers discussed in this lesson. Then, using the examples, explain how the property is used to simplify them.

Sample answer: For $z^2 \cdot z^5$, since the bases are the same, use the Product of Powers Property and add the exponents to get z^7. For $(a^4)^3$, use the Power of a Power Property. Multiply the exponents to get a^{12}. For $(3rs)^3$, use the Power of a Product Property. Raise the constant and each variable to the power to get $27r^3s^3$.

NAME _____ DATE _____ PERIOD ____

8-1

Enrichment

An Wang

An Wang (1920–1990) was an Asian-American who became one of the pioneers of the computer industry in the United States. He grew up in Shanghai, China, but came to the United States to further his studies in science. In 1948, he invented a magnetic pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word processing systems. In 1988, Wang was elected to the National Inventors Hall of Fame.

Digital computers store information as numbers. Because the electronic circuits of a computer can exist in only one of two states, open or closed, the numbers that are stored can consist of only two digits, 0 or 1. Numbers written using only these two digits are called **binary numbers**. To find the decimal value of a binary number, you use the digits to write a *polynomial in 2*. For instance, this is how to find the decimal value of the number 1001101_2. (The subscript 2 indicates that this is a binary number.)

$$\begin{aligned} 1001101_2 &= 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 64 + 0 \times 32 + 0 \times 16 + 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 64 + 0 + 0 + 8 + 4 + 0 + 1 \\ &= 77 \end{aligned}$$

Find the decimal value of each binary number.

1. 1111_2 15 **2.** 10000_2 16 **3.** 11000011_2 195 **4.** 10111001_2 185

Write each decimal number as a binary number.

5. 8 1000 **6.** 11 1011 **7.** 29 11101 **8.** 117 1110101

9. The chart at the right shows a set of decimal code numbers that is used widely in storing letters of the alphabet in a computer's memory. Find the code numbers for the letters of your name. Then write the code for your name using binary numbers. Answers will vary.

The American Standard Guide for Information Interchange (ASCII)							
A	65	N	78	a	97	n	110
B	66	O	79	b	98	o	111
C	67	P	80	c	99	p	112
D	68	Q	81	d	100	q	113
E	69	R	82	e	101	r	114
F	70	S	83	f	102	s	115
G	71	T	84	g	103	t	116
H	72	U	85	h	104	u	117
I	73	V	86	i	105	v	118
J	74	W	87	j	106	w	119
K	75	X	88	k	107	x	120
L	76	Y	89	l	108	y	121
M	77	Z	90	m	109	z	122

NAME _____ DATE _____ PERIOD ____

8-2 Study Guide and Intervention

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume neither a nor b is equal to zero.**

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5.

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that b is not equal to zero.**

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{5^5}{5^2}$ 5^3 or 125
2. $\frac{m^6}{m^4}$ m^2
3. $\frac{p^5n^4}{p^2n}$ p^3n^3
4. $\frac{a^2}{a}$ a
5. $\frac{x^5y^3}{x^5y^2}$ y
6. $\frac{-2y^7}{14y^5}$ $-\frac{1}{7}y^2$
7. $\frac{xy^6}{y^4x}$ y^2
8. $\left(\frac{2a^2b}{a}\right)^3$ $8a^3b^3$
9. $\left(\frac{4p^4q^4}{3p^2q^2}\right)^3$ $\frac{64}{27}p^6q^6$
10. $\left(\frac{2v^5w^3}{v^4w^3}\right)^4$ $16v^4$
11. $\left(\frac{3r^6s^3}{2r^5s}\right)^4$ $\frac{81}{16}r^4s^8$
12. $\frac{r^7s^7t^2}{s^3r^3t^2}$ r^4s^4

NAME _____ DATE _____ PERIOD ____

8-2 Study Guide and Intervention *(continued)*

Dividing Monomials

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that the denominator is not equal to zero.**

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{2^2}{2^{-3}}$ 2^5 or 32
2. $\frac{m}{m^{-4}}$ m^5
3. $\frac{p^{-8}}{p^3}$ $\frac{1}{p^{11}}$
4. $\frac{b^{-4}}{b^{-5}}$ b
5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$ $\frac{w}{4y^2}$
6. $\frac{(a^2b^3)^2}{(ab)^{-2}}$ a^6b^8
7. $\frac{x^4y^0}{x^{-2}}$ x^6
8. $\frac{(6a^{-1}b)^2}{(b^2)^4}$ $\frac{36}{a^2b^6}$
9. $\frac{(3st)^2u^{-4}}{s^{-1}t^2u^7}$ $\frac{9s^3}{u^{11}}$
10. $\frac{s^{-3}t^{-5}}{(s^2t^3)^{-1}}$ $\frac{1}{st^2}$
11. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$ 1
12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$ $-\frac{m^3}{32n^{10}}$

Answers

NAME _____ DATE _____ PERIOD ____

8-2 Skills Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{6^5}{6^4}$ 6^1 or 6
2. $\frac{9^{12}}{9^8}$ 9^4 or 6561
3. $\frac{x^4}{x^2}$ x^2
4. $\frac{r^3s^2}{r^3s^4}$ $\frac{1}{s^2}$
5. $\frac{m}{m^3}$ $\frac{1}{m^2}$
6. $\frac{9d^7}{3d^6}$ $3d$
7. $\frac{12n^5}{36n}$ $\frac{n^4}{3}$
8. $\frac{w^4u^3}{w^4u}$ u^2
9. $\frac{a^3b^5}{ab^2}$ a^2b^3
10. $\frac{m^7n^2}{m^3n^2}$ m^4
11. $\frac{-21w^5u^2}{7w^4u^5}$ $-\frac{3w}{u^3}$
12. $\frac{32x^3y^2z^5}{-8xyz^2}$ $-4x^2yz^3$
13. $\left(\frac{4p^7}{7s^2}\right)^2$ $\frac{16p^{14}}{49s^4}$
14. 4^{-4} $\frac{1}{256}$
15. 8^{-2} $\frac{1}{64}$
16. $\left(\frac{5}{3}\right)^{-2}$ $\frac{9}{25}$
17. $\left(\frac{9}{11}\right)^{-1}$ $\frac{11}{9}$
18. $\frac{h^3}{h^{-6}}$ h^9
19. $k^0(k^4)(k^{-6})$ $\frac{1}{k^2}$
20. $k^{-1}(\ell^{-6})(m^3)$ $\frac{m^3}{k\ell^6}$
21. $\frac{f^{-7}}{f^4}$ $\frac{1}{f^{11}}$
22. $\left(\frac{16p^5q^2}{2p^3q^3}\right)^0$ 1
23. $\frac{f^{-5}g^4}{h^{-2}}$ $\frac{g^4h^2}{f^5}$
24. $\frac{15x^6y^{-9}}{5xy^{-11}}$ $3x^5y^2$
25. $\frac{-15w^0u^{-1}}{5u^3}$ $-\frac{3}{u^4}$
26. $\frac{48x^6y^7z^5}{-6xy^5z^6}$ $-\frac{8x^5y^2}{z}$

NAME _____ DATE _____ PERIOD ____

8-2 Practice (Average)

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{8^8}{8^4}$ 8^4 or 4096
2. $\frac{a^4b^6}{ab^3}$ a^3b^3
3. $\frac{xy^2}{xy}$ y
4. $\frac{m^5np}{m^4p}$ mn
5. $\frac{5c^2d^3}{-4c^2d}$ $-\frac{5d^2}{4}$
6. $\frac{8y^7z^6}{4y^6z^5}$ $2yz$
7. $\left(\frac{4f^3g}{3h^6}\right)^3$ $\frac{64f^9g^3}{27h^{18}}$
8. $\left(\frac{6w^5}{7p^6s^3}\right)^2$ $\frac{36w^{10}}{49p^{12}s^6}$
9. $\frac{-4c^2}{24c^5}$ $-\frac{1}{6c^3}$
10. $x^3(y^{-5})(x^{-8})$ $\frac{1}{x^5y^5}$
11. $p(q^{-2})(r^{-3})$ $\frac{p}{q^2r^3}$
12. 12^{-2} $\frac{1}{144}$
13. $\left(\frac{3}{7}\right)^{-2}$ $\frac{49}{9}$
14. $\left(\frac{4}{3}\right)^{-4}$ $\frac{81}{256}$
15. $\frac{22r^3s^2}{11r^2s^{-3}}$ $2rs^5$
16. $\frac{-15w^0u^{-1}}{5u^3}$ $-\frac{3}{u^4}$
17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$ $2c^4f^7$
18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$ 1
19. $\frac{6f^{-2}g^3h^5}{54f^{-2}g^{-5}h^3}$ $\frac{g^8h^2}{9}$
20. $\frac{-12t^{-1}u^5v^{-4}}{2t^{-3}uv^5}$ $-\frac{6t^2u^4}{v^9}$
21. $\frac{r^4}{(3r)^3}$ $\frac{r}{27}$
22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$ $\frac{m^2}{n^2}$
23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$ $\frac{j}{k^{15}}$
24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$ $\frac{a^4}{40b^7}$
25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$ $\frac{q^{10}}{r^{25}}$
26. $\left(\frac{7c^{-3}d^3}{c^5de^{-4}}\right)^{-1}$ $\frac{c^8}{7d^2e^4}$
27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$ $\frac{9x^2}{4y^2z^6}$

28. **BIOLOGY** A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells? $\frac{1}{484}$

29. **COUNTING** The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"? 676

NAME _____ DATE _____ PERIOD ____

8-2 Reading to Learn Mathematics

Dividing Monomials

Pre-Activity **How can you compare pH levels?**

Read the introduction to Lesson 8-2 at the top of page 417 in your textbook.

- In the formula $c = \left(\frac{1}{10}\right)^{\text{pH}}$, identify the base and the exponent.

  base = $\frac{1}{10}$, exponent = pH

- How do you think c will change as the exponent increases?

  c will decrease.

Reading the Lesson

1. Explain what the statement $\frac{a^m}{a^n} = a^{m-n}$ means.

   To divide two powers that have the same base, subtract the exponents.

2. To find c in the formula $c = \left(\frac{1}{10}\right)^{\text{pH}}$, you can find the power of the numerator, the power of the denominator, and divide. This is an example of what property?

   Power of a Quotient Property

3. Use the Quotient of Powers Property to explain why $3^0 = 1$. Sample answer: $\frac{3^4}{3^4} = 1$. The Quotient of Powers Property says that when you divide two powers that have the same base, you subtract the exponents. So $\frac{3^4}{3^4} = 3^0$.

4. Consider the expression 4^{-3}.

   a. Explain why the expression 4^{-3} is not simplified. An expression involving exponents is not considered simplified if the expression contains negative exponents.

   b. Define the term reciprocal. The reciprocal of a number is 1 divided by the number.

   c. 4^{-3} is the reciprocal of what power of 4? 4^3

   d. What is the simplified form of 4^{-3}? $\frac{1}{4^3}$ or $\frac{1}{64}$

Helping You Remember

5. Describe how you would help a friend who needs to simplify the expression $\frac{4x^2}{2x^5}$.

   Divide the constants and group powers with the same base to get $\left(\frac{4}{2}\right)\left(\frac{x^2}{x^5}\right)$. Use the Quotient of Powers Property to get $(2)(x^{2-5})$ or $(2)(x^{-3})$. To simplify $(2)(x^{-3})$, use the Negative Exponent Property to get $(2)\left(\frac{1}{x^3}\right)$, or $\frac{2}{x^3}$.

NAME _____ DATE _____ PERIOD ____

8-2 Enrichment

Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a.		b.		c.	
2^{10} =	1024	5^{10} =	9,765,625	4^{10} =	1,048,576
2^9 =	512	5^9 =	1,953,125	4^9 =	262,144
2^8 =	256	5^8 =	390,625	4^8 =	65,536
2^7 =	128	5^7 =	78,125	4^7 =	16,384
2^6 =	64	5^6 =	15,625	4^6 =	4096
2^5 =	32	5^5 =	3125	4^5 =	1024
2^4 =	16	5^4 =	625	4^4 =	256
2^3 =	8	5^3 =	125	4^3 =	64
2^2 =	4	5^2 =	25	4^2 =	16
2^1 =	2	5^1 =	5	4^1 =	4

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.

   The exponents decrease by one from each row to the one below.

2. Describe the pattern of the powers from the top of the column to the bottom. To get each power, divide the power on the row above by the base (2, 5, or 4).

3. What would you expect the following powers to be?

   2^0 1 5^0 1 4^0 1

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using 22, 25, and 24 as bases. Any nonzero number to the zero power equals one.

Study the pattern below. Then answer the questions.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 =$? 0^{-1} does not exist. 0^{-2} does not exist. 0^{-3} does not exist.

5. Why do 0^{-1}, 0^{-2}, and 0^{-3} not exist?

   Negative exponents are not defined unless the base is nonzero.

6. Based upon the pattern, can you determine whether 0^0 exists?

   No, since the pattern $0^n = 0$ breaks down for $n < 1$.

7. The symbol 0^0 is called an **indeterminate**, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that 0^0 cannot equal 1?

   Answers will vary. One answer is that if $0^0 = 1$, then $1 = \frac{1}{1} = \frac{1^0}{0^0} = \left(\frac{1}{0}\right)^0$, which is a false result, since division by zero is not allowed. Thus, 0^0 cannot equal 1.

Answers

NAME _____ DATE _____ PERIOD ____

8-3 Study Guide and Intervention

Scientific Notation

Scientific Notation Keeping track of place value in very large or very small numbers written in standard form may be difficult. It is more efficient to write such numbers in scientific notation. A number is expressed in scientific notation when it is written as a product of two factors, one factor that is greater than or equal to 1 and less than 10 and one factor that is a power of ten.

Scientific Notation	A number is in scientific notation when it is in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Example 1 **Express 3.52×10^4 in standard notation.**

$3.52 \times 10^4 = 3.52 \times 10{,}000$
$= 35{,}200$

The decimal point moved 4 places to the right.

Example 2 **Express 6.21×10^{-5} in standard notation.**

$6.21 \times 10^{-5} = 6.21 \times \frac{1}{10^5}$
$= 6.21 \times 0.00001$
$= 0.0000621$

The decimal point moved 5 places to the left.

Example 3 **Express 37,600,000 in scientific notation.**

$37{,}600{,}000 = 3.76 \times 10^7$

The decimal point moved 7 places so that it is between the 3 and the 7. Since $37{,}600{,}000 > 1$, the exponent is positive.

Example 4 **Express 0.0000549 in scientific notation.**

$0.0000549 = 5.49 \times 10^{-5}$

The decimal point moved 5 places so that it is between the 5 and the 4. Since $0.0000549 < 1$, the exponent is negative.

Exercises

Express each number in standard notation.

1. 3.65×10^5 — 365,000
2. 7.02×10^{-4} — 0.000702
3. 8.003×10^8 — 800,300,000
4. 7.451×10^6 — 7,451,000
5. 5.91×10^0 — 5.91
6. 7.99×10^{-1} — 0.799
7. 8.9354×10^{10} — 89,354,000,000
8. 8.1×10^{-9} — 0.0000000081
9. 4×10^{15} — 4,000,000,000,000,000

Express each number in scientific notation.

10. 0.0000456 — 4.56×10^{-5}
11. 0.00001 — 1×10^{-5}
12. 590,000,000 — 5.9×10^8
13. 0.00000000012 — 1.2×10^{-10}
14. 0.000080436 — 8.0436×10^{-5}
15. 0.03621 — 3.621×10^{-2}
16. 433×10^4 — 4.33×10^6
17. 0.0042×10^{-3} — 4.2×10^{-6}
18. 50,000,000,000 — 5×10^{10}

NAME _____ DATE _____ PERIOD ____

8-3 Study Guide and Intervention (continued)

Scientific Notation

Products and Quotients with Scientific Notation You can use properties of powers to compute with numbers written in scientific notation.

Example 1 **Evaluate $(6.7 \times 10^3)(2 \times 10^{-5})$. Express the result in scientific and standard notation.**

$(6.7 \times 10^3)(2 \times 10^{-5}) = (6.7 \times 2)(10^3 \times 10^{-5})$ Associative Property
$= 13.4 \times 10^{-2}$ Product of Powers
$= (1.34 \times 10^1) \times 10^{-2}$ $13.4 = 1.34 \times 10^1$
$= 1.34 \times (10^1 \times 10^{-2})$ Associative Property
$= 1.34 \times 10^{-1}$ or 0.134 Product of Powers

The solution is 1.34×10^{-1} or 0.134.

Example 2 **Evaluate $\frac{1.5088 \times 10^8}{4.1 \times 10^5}$. Express the result in scientific and standard notation.**

$\frac{1.5088 \times 10^8}{4.1 \times 10^5} = \left(\frac{1.5088}{4.1}\right)\left(\frac{10^8}{10^5}\right)$ Associative Property
$= 0.368 \times 10^3$ Quotient of Powers
$= (3.68 \times 10^{-1}) \times 10^3$ $0.368 = 3.68 \times 10^{-1}$
$= 3.68 \times (10^{-1} \times 10^3)$ Associative Property
$= 3.68 \times 10^2$ or 368 Product of Powers

The solution is 3.68×10^2 or 368.

Exercises

Evaluate. Express each result in scientific and standard notation.

1. $\frac{1.4 \times 10^4}{2 \times 10^2}$ — 7×10; 70
2. $\frac{3 \times 10^{-12}}{2 \times 10^{-15}}$ — 1.5×10^3; 1500
3. $(3.2 \times 10^{-2})(2.0 \times 10^2)$ — 6.4×10^0; 6.4
4. $\frac{1.2672 \times 10^{-8}}{2.4 \times 10^{-12}}$ — 5.28×10^3; 5280
5. $(7.7 \times 10^5)(2.1 \times 10^2)$ — 1.617×10^8; 161,700,000
6. $\frac{9.72 \times 10^8}{7.2 \times 10^{10}}$ — 1.35×10^{-2}; 0.0135
7. $(3.3 \times 10^5)(1.5 \times 10^{-4})$ — 4.95×10^1; 49.5
8. $\frac{3.3 \times 10^{-12}}{1.1 \times 10^{-14}}$ — 3×10^2; 300
9. $\frac{4 \times 10^{-4}}{2.5 \times 10^2}$ — 1.6×10^{-6}; 0.0000016

10. **FUEL CONSUMPTION** North America burned 4.5×10^{16} BTU of petroleum in 1998. At this rate, how many BTU's will be burned in 9 years? **Source:** *The New York Times 2001 Almanac*
4.05×10^{17}

11. **OIL PRODUCTION** If the United States produced 6.25×10^9 barrels of crude oil in 1998, and Canada produced 1.98×10^9 barrels, what is the quotient of their production rates? Write a statement using this quotient. **Source:** *The New York Times 2001 Almanac*
About 3.16; Sample answer: The United States produces more than 3 times the crude oil of Canada.

NAME _____ DATE _____ PERIOD ____

8-3 Skills Practice

Scientific Notation

Express each number in standard notation.

1. 4×10^3 — 4000
2. 2×10^8 — 200,000,000
3. 3.2×10^5 — 320,000
4. 3×10^{-6} — 0.000003
5. 9×10^{-2} — 0.09
6. 4.7×10^{-7} — 0.00000047

ASTRONOMY Express the number in each statement in standard notation.

7. The diameter of Jupiter is 1.42984×10^5 kilometers. 142,984
8. The surface density of the main ring around Jupiter is 5×10^{-6} grams per centimeter squared. 0.000005
9. The minimum distance from Mars to Earth is 5.45×10^7 kilometers. 54,500,000

Express each number in scientific notation.

10. 41,000,000 — 4.1×10^7
11. 65,100 — 6.51×10^4
12. 283,000,000 — 2.83×10^8
13. 264,701 — 2.64701×10^5
14. 0.019 — 1.9×10^{-2}
15. 0.000007 — 7×10^{-6}
16. 0.000010035 — 1.0035×10^{-5}
17. 264.9 — 2.649×10^2
18. 150×10^2 — 1.5×10^4

Evaluate. Express each result in scientific and standard notation.

19. $(3.1 \times 10^7)(2 \times 10^{-5})$ — 6.2×10^2; 620
20. $(5 \times 10^{-2})(1.4 \times 10^{-4})$ — 7.0×10^{-6}; 0.000007
21. $(3 \times 10^3)(4.2 \times 10^{-1})$ — 1.26×10^3; 1260
22. $(3 \times 10^{-2})(5.2 \times 10^9)$ — 1.56×10^8; 156,000,000
23. $(2.4 \times 10^2)(4 \times 10^{-10})$ — 9.6×10^{-8}; 0.000000096
24. $(1.5 \times 10^{-4})(7 \times 10^{-5})$ — 1.05×10^{-8}; 0.0000000105
25. $\frac{5.1 \times 10^6}{1.5 \times 10^2}$ — 3.4×10^4; 34,000
26. $\frac{7.2 \times 10^{-5}}{4 \times 10^{-3}}$ — 1.8×10^{-2}; 0.018

NAME _____ DATE _____ PERIOD ____

8-3 Practice (Average)

Scientific Notation

Express each number in standard notation.

1. 7.3×10^7 — 73,000,000
2. 2.9×10^3 — 2900
3. 9.821×10^{12} — 9,821,000,000,000
4. 3.54×10^{-1} — 0.354
5. 7.3642×10^4 — 73,642
6. 4.268×10^{-6} — 0.000004268

PHYSICS Express the number in each statement in standard notation.

7. An electron has a negative charge of 1.6×10^{-19} Coulomb. 0.00000000000000000016
8. In the middle layer of the sun's atmosphere, called the chromosphere, the temperature averages 2.78×10^4 degrees Celsius. 27,800

Express each number in scientific notation.

9. 915,600,000,000 — 9.156×10^{11}
10. 6387 — 6.387×10^3
11. 845,320 — 8.4532×10^5
12. 0.00000000814 — 8.14×10^{-9}
13. 0.00009621 — 9.621×10^{-5}
14. 0.003157 — 3.157×10^{-3}
15. 30,620 — 3.062×10^4
16. 0.0000000000112 — 1.12×10^{-11}
17. 56×10^7 — 5.6×10^8
18. 4740×10^5 — 4.74×10^8
19. 0.076×10^{-3} — 7.6×10^{-5}
20. 0.0057×10^3 — 5.7 or 5.7×10^0

Evaluate. Express each result in scientific and standard notation.

21. $(5 \times 10^{-2})(2.3 \times 10^{12})$ — 1.15×10^{11}; 115,000,000,000
22. $(2.5 \times 10^{-3})(6 \times 10^{15})$ — 1.5×10^{13}; 15,000,000,000,000
23. $(3.9 \times 10^3)(4.2 \times 10^{-11})$ — 1.638×10^{-7}; 0.0000001638
24. $(4.6 \times 10^{-4})(3.1 \times 10^{-1})$ — 1.426×10^{-4}; 0.0001426
25. $\frac{3.12 \times 10^3}{1.56 \times 10^{-3}}$ — 2.0×10^6; 2,000,000
26. $\frac{6.72 \times 10^3}{4.2 \times 10^8}$ — 1.6×10^{-5}; 0.000016
27. $\frac{1.17 \times 10^2}{5 \times 10^{-1}}$ — 2.34×10^2; 234
28. $\frac{1.82 \times 10^5}{9.1 \times 10^7}$ — 2.0×10^{-3}; 0.002
29. $\frac{1.68 \times 10^4}{8.4 \times 10^{-4}}$ — 2.0×10^7; 20,000,000
30. $\frac{2.015 \times 10^{-3}}{3.1 \times 10^2}$ — 6.5×10^{-6}; 0.0000065

31. **BIOLOGY** A cubic millimeter of human blood contains about 5×10^6 red blood cells. An adult human body may contain about 5×10^6 cubic millimeters of blood. About how many red blood cells does such a human body contain? about 2.5×10^{13} or 25 trillion
32. **POPULATION** The population of Arizona is about 4.778×10^6 people. The land area is about 1.14×10^5 square miles. What is the population density per square mile? about 42 people per square mile

Answers

NAME _____ DATE _____ PERIOD ____

8-3

Reading to Learn Mathematics

Scientific Notation

Pre-Activity **Why is scientific notation important in astronomy?**

Read the introduction to Lesson 8-3 at the top of page 425 in your textbook.

In the table, each mass is written as the product of a number and a power of 10. Look at the first factor in each product. How are these factors alike?

They are all greater than 1 and less than 10.

Reading the Lesson

1. Is the number 0.0543×10^4 in scientific notation? Explain.

No; the first factor is less than 1.

2. Complete each sentence to change from scientific notation to standard notation.

a. To express 3.64×10^6 in standard notation, move the decimal point 6 places to the right.

b. To express 7.825×10^{-3} in standard notation, move the decimal point 3 places to the left.

3. Complete each sentence to change from standard notation to scientific notation.

a. To express 0.0007865 in scientific notation, move the decimal point 4 places to the right and write 7.865×10^{-4}.

b. To express 54,000,000,000 in scientific notation, move the decimal point 10 places to the left and write 5.4×10^{10}.

4. Write *positive* or *negative* to complete each sentence.

a. Positive powers of 10 are used to express very large numbers in scientific notation.

b. Negative powers of 10 are used to express very small numbers in scientific notation.

Helping You Remember

5. Describe the method you would use to estimate how many times greater the mass of Saturn is than the mass of Pluto.

Divide 5.69×10^{26} by 1.27×10^{22}. Since $5.69 \div 1.27 \approx 4.48$ and $10^{26} \div 10^{22}$ is 10^4, the mass of Mars is about 4.48×10^4 times the mass of Pluto.

NAME _____ DATE _____ PERIOD ____

8-3

Enrichment

Converting Metric Units

Scientific notation is convenient to use for unit conversions in the metric system.

Example 1 **How many kilometers are there in 4,300,000 meters?**

Divide the measure by the number of meters (1000) in one kilometer. Express both numbers in scientific notation.

$\frac{4.3 \times 10^6}{1 \times 10^3} = 4.3 \times 10^3$

The answer is 4.3 3 103 km.

Example 2 **Convert 3700 grams into milligrams.**

Multiply by the number of milligrams (1000) in 1 gram.

$(3.7 \times 10^3)(1 \times 10^3) = 3.7 \times 10^6$

There are 3.7×10^6 mg in 3700 g.

Complete the following. Express each answer in scientific notation.

1. 250,000 m = 2.5×10^2 km

2. 375 km = 3.75×10^5 m

3. 247 m = 2.47×10^4 cm

4. 5000 m = 5×10^6 mm

5. 0.0004 km = 4×10^{-1} m

6. 0.01 mm = 1×10^{-5} m

7. 6000 m = 6×10^6 mm

8. 340 cm = 3.4×10^{-3} km

9. 52,000 mg = 5.2×10^1 g

10. 420 kL = 4.2×10^5 L

Solve.

11. The planet Mars has a diameter of 6.76×10^3 km. What is the diameter of Mars in meters? Express the answer in both scientific and decimal notation. 6,760,000 m; 6.76×10^6 m

12. The distance from earth to the sun is 149,590,000 km. Light travels 3.0×10^8 meters per second. How long does it take light from the sun to reach the earth in minutes? Round to the nearest hundredth. 8.31 min

13. A light-year is the distance that light travels in one year. (See Exercise 12.) How far is a light year in kilometers? Express your answer in scientific notation. Round to the nearest hundredth.
9.46×10^{12} km

NAME _____ DATE _____ PERIOD ____

8-4 Study Guide and Intervention

Polynomials

Degree of a Polynomial A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The **degree** of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

Example **State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*. Then give the degree of the polynomial.**

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	Yes. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	Yes. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	No. $3n^{-4} = \frac{3}{n^4}$, which is not a monomial	none of these	—
$9x^3 + 4x + x + 4 + 2x$	Yes. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*.

1. 36 yes; monomial
2. $\frac{3}{q^2} + 5$ no
3. $7x - x + 5$ yes; binomial
4. $8g^2h - 7gh + 2$ yes; trinomial
5. $\frac{1}{4y^2} + 5y - 8$ no
6. $6x + x^2$ yes; binomial

Find the degree of each polynomial.

7. $4x^2y^3z$ 6
8. $-2abc$ 3
9. $15m$ 1
10. $s + 5t$ 1
11. 22 0
12. $18x^2 + 4yz - 10y$ 2
13. $x^4 - 6x^2 - 2x^3 - 10$ 4
14. $2x^3y^2 - 4xy^3$ 5
15. $-2r^8s^4 + 7r^2s - 4r^7s^6$ 13
16. $9x^2 + yz^8$ 9
17. $8b + bc^5$ 6
18. $4x^4y - 8zx^2 + 2x^5$ 5
19. $4x^2 - 1$ 2
20. $9abc + bc - d^5$ 5
21. $h^3m + 6h^4m^2 - 7$ 6

NAME _____ DATE _____ PERIOD ____

8-4 Study Guide and Intervention *(continued)*

Polynomials

Write Polynomials in Order The terms of a polynomial are usually arranged so that the powers of one variable are in **ascending** (increasing) order or **descending** (decreasing) order.

Example 1 **Arrange the terms of each polynomial so that the powers of x are in ascending order.**

a. $x^4 - x^2 + 5x^3$
$-x^2 + 5x^3 + x^4$

b. $8x^3y - y^2 + 6x^2y + xy^2$
$-y^2 + xy^2 + 6x^2y + 8x^3y$

Example 2 **Arrange the terms of each polynomial so that the powers of x are in descending order.**

a. $x^4 + 4x^5 - x^2$
$4x^5 + x^4 - x^2$

b. $-6xy + y^3 - x^2y^2 + x^4y^2$
$x^4y^2 - x^2y^2 - 6xy + y^3$

Exercises

Arrange the terms of each polynomial so that the powers of x are in ascending order.

1. $5x + x^2 + 6$
$6 + 5x + x^2$
2. $6x + 9 - 4x^2$
$9 + 6x - 4x^2$
3. $4xy + 2y + 6x^2$
$2y + 4xy + 6x^2$
4. $6y^2x - 6x^2y + 2$
$2 + 6y^2x - 6x^2y$
5. $x^4 + x^3 + x^2$
$x^2 + x^3 + x^4$
6. $2x^3 - x + 3x^7$
$-x + 2x^3 + 3x^7$
7. $-5cx + 10c^2x^3 + 15cx^2$
$-5cx + 15cx^2 + 10c^2x^3$
8. $-4nx - 5n^3x^3 + 5$
$5 - 4nx - 5n^3x^3$
9. $4xy + 2y + 5x^2$
$2y + 4xy + 5x^2$

Arrange the terms of each polynomial so that the powers of x are in descending order.

10. $2x + x^2 - 5$
$x^2 + 2x - 5$
11. $20x - 10x^2 + 5x^3$
$5x^3 - 10x^2 + 20x$
12. $x^2 + 4yx - 10x^5$
$-10x^5 + x^2 + 4yx$
13. $9bx + 3bx^2 - 6x^3$
$-6x^3 + 3bx^2 + 9bx$
14. $x^3 + x^5 - x^2$
$x^5 + x^3 - x^2$
15. $ax^2 + 8a^2x^5 - 4$
$8a^2x^5 + ax^2 - 4$
16. $3x^3y - 4xy^2 - x^4y^2 + y^5$
$-x^4y^2 + 3x^3y - 4xy^2 + y^5$
17. $x^4 + 4x^3 - 7x^5 + 1$
$-7x^5 + x^4 + 4x^3 + 1$
18. $-3x^6 - x^5 + 2x^8$
$2x^8 - 3x^6 - x^5$
19. $-15cx^2 + 8c^2x^5 + cx$
$8c^2x^5 - 15cx^2 + cx$
20. $24x^2y - 12x^3y^2 + 6x^4$
$6x^4 - 12x^3y^2 + 24x^2y$
21. $-15x^3 + 10x^4y^2 + 7xy^2$
$10x^4y^2 - 15x^3 + 7xy^2$

Answers

NAME _____ DATE _____ PERIOD ____

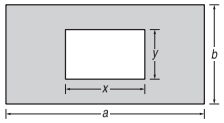
8-4 Skills Practice

Polynomials

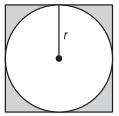
State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

1. $5mn + n^2$
   yes; binomial
2. $4by + 2b - by$
   yes; binomial
3. -32
   yes; monomial
4. $\frac{3x}{7}$
   yes; monomial
5. $5x^2 - 3x^{-4}$
   no
6. $2c^2 + 8c + 9 - 3$
   yes; trinomial

GEOMETRY Write a polynomial to represent the area of each shaded region.

7. 

$ab - xy$

8. 

$4r^2 - \pi r^2$

Find the degree of each polynomial.

9. 12 0
10. $3r^4$ 4
11. $b + 6$ 1
12. $4a^3 - 2a$ 3
13. $5abc - 2b^2 + 1$ 3
14. $8x^5y^4 - 2x^8$ 9

Arrange the terms of each polynomial so that the powers of x are in ascending order.

15. $3x + 1 + 2x^2$ $1 + 3x + 2x^2$
16. $5x - 6 + 3x^2$ $-6 + 5x + 3x^2$
17. $9x^2 + 2 + x^3 + x$ $2 + x + 9x^2 + x^3$
18. $-3 + 3x^3 - x^2 + 4x$ $-3 + 4x - x^2 + 3x^3$
19. $7r^5x + 21r^4 - r^2x^2 - 15x^3$
    $21r^4 + 7r^5x - r^2x^2 - 15x^3$
20. $3a^2x^4 + 14a^2 - 10x^3 + ax^2$
    $14a^2 + ax^2 - 10x^3 + 3a^2x^4$

Arrange the terms of each polynomial so that the powers of x are in descending order.

21. $x^2 + 3x^3 + 27 - x$
    $3x^3 + x^2 - x + 27$
22. $25 - x^3 + x$
    $-x^3 + x + 25$
23. $x - 3x^2 + 4 + 5x^3$
    $5x^3 - 3x^2 + x + 4$
24. $x^2 + 64 - x + 7x^3$
    $7x^3 + x^2 - x + 64$
25. $2cx + 32 - c^3x^2 + 6x^3$
    $6x^3 - c^3x^2 + 2cx + 32$
26. $13 - x^3y^3 + x^2y^2 + x$
    $-x^3y^3 + x^2y^2 + x + 13$

NAME _____ DATE _____ PERIOD ____

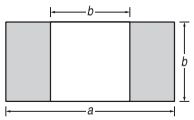
8-4 Practice (Average)

Polynomials

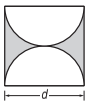
State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

1. $7a^2b + 3b^2 - a^2b$
   yes; binomial
2. $\frac{1}{5}y^3 + y^2 - 9$
   yes; trinomial
3. $6g^2h^3k$
   yes; monomial

GEOMETRY Write a polynomial to represent the area of each shaded region.

4. 

$ab - b^2$

5. 

$d^2 - \frac{1}{4}\pi d^2$

Find the degree of each polynomial.

6. $x + 3x^4 - 21x^2 + x^3$ 4
7. $3g^2h^3 + g^3h$ 5
8. $-2x^2y + 3xy^3 + x^2$ 4
9. $5n^3m - 2m^3 + n^2m^4 + n^2$ 6
10. $a^3b^2c + 2a^5c + b^3c^2$ 6
11. $10s^2t^2 + 4st^2 - 5s^3t^2$ 5

Arrange the terms of each polynomial so that the powers of x are in ascending order.

12. $8x^2 - 15 + 5x^5$
    $-15 + 8x^2 + 5x^5$
13. $10bx - 7b^2 + x^4 + 4b^2x^3$
    $-7b^2 + 10bx + 4b^2x^3 + x^4$
14. $-3x^3y + 8y^2 + xy^4$
    $8y^2 + xy^4 - 3x^3y$
15. $7ax - 12 + 3ax^3 + a^2x^2$
    $-12 + 7ax + a^2x^2 + 3ax^3$

Arrange the terms of each polynomial so that the powers of x are in descending order.

16. $13x^2 - 5 + 6x^3 - x$
    $6x^3 + 13x^2 - x - 5$
17. $4x + 2x^5 - 6x^3 + 2$
    $2x^5 - 6x^3 + 4x + 2$
18. $g^2x - 3gx^3 + 7g^3 + 4x^2$
    $-3gx^3 + 4x^2 + g^2x + 7g^3$
19. $-11x^2y^3 + 6y - 2xy + 2x^4$
    $2x^4 - 11x^2y^3 - 2xy + 6y$
20. $7a^2x^2 + 17 - a^3x^3 + 2ax$
    $-a^3x^3 + 7a^2x^2 + 2ax + 17$
21. $12rx^3 + 9r^6 + r^2x + 8x^6$
    $8x^6 + 12rx^3 + r^2x + 9r^6$

22. **MONEY** Write a polynomial to represent the value of t ten-dollar bills, f fifty-dollar bills, and h one-hundred-dollar bills. $10t + 50f + 100h$

23. **GRAVITY** The height above the ground of a ball thrown up with a velocity of 96 feet per second from a height of 6 feet is $6 + 96t - 16t^2$ feet, where t is the time in seconds. According to this model, how high is the ball after 7 seconds? Explain.
    -106 ft; The height is negative because the model does not account for the ball hitting the ground when the height is 0 feet.

NAME _____ DATE _____ PERIOD ____

8-4 Reading to Learn Mathematics

Polynomials

Pre-Activity **How are polynomials useful in modeling data?**

Read the introduction to Lesson 8-4 at the top of page 432 in your textbook.

- How many terms does $t^4 - 9t^3 + 26t - 18t + 76$ have?

  five

- What could you call a polynomial with just one term?

  a monomial

Reading the Lesson

1. What is the meaning of the prefixes *mono-*, *bi-*, and *tri-*?

   Mono- means one, *bi-* means two, and *tri-* means three.

2. Write examples of words that begin with the prefixes *mono-*, *bi-*, and *tri-*.

   Sample answer: monocycle (one wheel), bicycle (two wheels), tricycle (three wheels)

3. Complete the table.

	monomial	binomial	trinomial	polynomial with more than three terms
Example	$3r^2t$	$2x^2 + 3x$	$5x^2 + 3x + 2$	$7s^2 + s^4 + 2s^3 - s + 5$
Number of Terms	1	2	3	5

4. What is the degree of the monomial $3xy^2z$? 4

5. What is the degree of the polynomial $4x^4 + 2x^3y^3 + y^2 + 14$? Explain how you found your answer.

   6; Since $0 + 4 = 4$, $4x^4$ has degree 4; since $0 + 3 + 3 = 6$, $2x^3y^3$ has degree 6; y^2 has degree 2; and 14 has degree 0. The highest degree of these terms is 6.

Helping You Remember

6. Use a dictionary to find the meaning of the terms *ascending* and *descending*. Write their meanings and then describe a situation in your everyday life that relates to them.

   ascending: going, growing, or moving upward; descending: moving from a higher to a lower place; Sample answer: climbing stairs, hiking

NAME _____ DATE _____ PERIOD ____

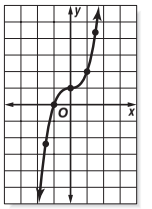
8-4 Enrichment

Polynomial Functions

Suppose a linear equation such as $23x + y = 4$ is solved for y. Then an equivalent equation, $y = 3x + 4$, is found. Expressed in this way, y is a function of x, or $f(x) = 3x + 4$. Notice that the right side of the equation is a binomial of degree 1.

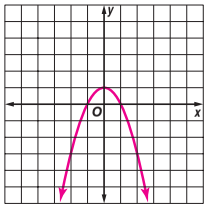
Higher-degree polynomials in x may also form functions. An example is $f(x) = x^3 + 1$, which is a polynomial function of degree 3. You can graph this function using a table of ordered pairs, as shown at the right.

x	y
$-1\frac{1}{2}$	$-2\frac{3}{8}$
-1	0
0	1
1	2
$1\frac{1}{2}$	$4\frac{3}{8}$

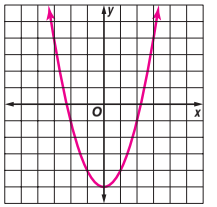


For each of the following polynomial functions, make a table of values for x and $y = f(x)$. Then draw the graph on the grid.

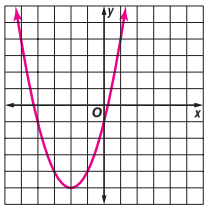
1. $f(x) = 1 - x^2$



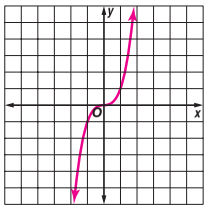
2. $f(x) = x^2 - 5$



3. $f(x) = x^2 + 4x - 1$



4. $f(x) = x^3$



Answers

NAME _____ DATE _____ PERIOD ____

8-5 Study Guide and Intervention

Adding and Subtracting Polynomials

Add Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

Example 1 **Find $(2x^2 + x - 8) + (3x - 4x^2 + 2)$.**

Horizontal Method

Group like terms.

$(2x^2 + x - 8) + (3x - 4x^2 + 2)$
$= [(2x^2 + (-4x^2)] + (x + 3x) + [(-8) + 2)]$
$= -2x^2 + 4x - 6.$

The sum is $-2x^2 + 4x - 6$.

Example 2 **Find $(3x^2 + 5xy) + (xy + 2x^2)$.**

Vertical Method

Align like terms in columns and add.

$$\begin{array}{r} 3x^2 + 5xy \\ (+)\ 2x^2 + xy \\ \hline 5x^2 + 6xy \end{array}$$

Put the terms in descending order.

The sum is $5x^2 + 6xy$.

Exercises

Find each sum.

1. $(4a - 5) + (3a + 6)$
   $7a + 1$
2. $(6x + 9) + (4x^2 - 7)$
   $4x^2 + 6x + 2$
3. $(6xy + 2y + 6x) + (4xy - x)$
   $10xy + 5x + 2y$
4. $(x^2 + y^2) + (-x^2 + y^2)$
   $2y^2$
5. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$
   $4p^2 - 9p + 10$
6. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$
   $-4x^2 + 4xy + 6y^2$
7. $(5p + 2q) + (2p^2 - 8q + 1)$
   $2p^2 + 5p - 6q + 1$
8. $(4x^2 - x + 4) + (5x + 2x^2 + 2)$
   $6x^2 + 4x + 6$
9. $(6x^2 + 3x) + (x^2 - 4x - 3)$
   $7x^2 - x - 3$
10. $(x^2 + 2xy + y^2) + (x^2 - xy - 2y^2)$
    $2x^2 + xy - y^2$
11. $(2a - 4b - c) + (-2a - b - 4c)$
    $-5b - 5c$
12. $(6xy^2 + 4xy) + (2xy - 10xy^2 + y^2)$
    $-4xy^2 + 6xy + y^2$
13. $(2p - 5q) + (3p + 6q) + (p - q)$
    $6p$
14. $(2x^2 - 6) + (5x^2 + 2) + (-x^2 - 7)$
    $6x^2 - 11$
15. $(3z^2 + 5z) + (z^2 + 2z) + (z - 4)$
    $4z^2 + 8z - 4$
16. $(8x^2 + 4x + 3y^2 + y) + (6x^2 - x + 4y)$
    $14x^2 + 3x + 3y^2 + 5y$

NAME _____ DATE _____ PERIOD ____

8-5 Study Guide and Intervention *(continued)*

Adding and Subtracting Polynomials

Subtract Polynomials You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Example **Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.**

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$
$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$
$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$
$= 2x^2 + (-9)$
$= 2x^2 - 9$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{r} 3x^2 + 2x - 6 \\ (-)\ x^2 + 2x + 3 \\ \hline 3x^2 + 2x - 6 \\ (+) -x^2 - 2x - 3 \\ \hline 2x^2 \quad - 9 \end{array}$$

The difference is $2x^2 - 9$.

Exercises

Find each difference.

1. $(3a - 5) - (5a + 1)$
   $-2a - 6$
2. $(9x + 2) - (-3x^2 - 5)$
   $3x^2 + 9x + 7$
3. $(9xy + y - 2x) - (6xy - 2x)$
   $3xy + y$
4. $(x^2 + y^2) - (-x^2 + y^2)$
   $2x^2$
5. $(6p^2 + 4p + 5) - (2p^2 - 5p + 1)$
   $4p^2 + 9p + 4$
6. $(6x^2 + 5xy - 2y^2) - (-xy - 2x^2 - 4y^2)$
   $8x^2 + 6xy + 2y^2$
7. $(8p - 5q) - (-6p^2 + 6q - 3)$
   $6p^2 + 8p - 11q + 3$
8. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$
   $9x^2 - 2x - 8$
9. $(3x^2 - 2x) - (3x^2 + 5x - 1)$
   $-7x + 1$
10. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$
    $5x^2 + 4xy + 7y^2$
11. $(2h - 6j - 2k) - (-7h - 5j - 4k)$
    $9h - j + 2k$
12. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$
    $17xy^2 + 7xy$
13. $(2a - 8b) - (-3a + 5b)$
    $5a - 13b$
14. $(2x^2 - 8) - (-2x^2 - 6)$
    $4x^2 - 2$
15. $(6z^2 + 4z + 2) - (4z^2 + z)$
    $2z^2 + 3z + 2$
16. $(6x^2 - 5x + 1) - (-7x^2 - 2x + 4)$
    $13x^2 - 3x - 3$

NAME _____ DATE _____ PERIOD ____

8-5 Skills Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(2x + 3y) + (4x + 9y)$ $6x + 12y$
2. $(6s + 5t) + (4t + 8s)$ $14s + 9t$
3. $(5a + 9b) - (2a + 4b)$ $3a + 5b$
4. $(11m - 7n) - (2m + 6n)$ $9m - 13n$
5. $(m^2 - m) + (2m + m^2)$ $2m^2 + m$
6. $(x^2 - 3x) - (2x^2 + 5x)$ $-x^2 - 8x$
7. $(d^2 - d + 5) - (2d + 5)$ $d^2 - 3d$
8. $(2e^2 - 5e) + (7e - 3e^2)$ $-e^2 + 2e$
9. $(5f + g - 2) + (-2f + 3)$ $3f + g + 1$
10. $(6k^2 + 2k + 9) + (4k^2 - 5k)$ $10k^2 - 3k + 9$
11. $(x^3 - x + 1) - (3x - 1)$ $x^3 - 4x + 2$
12. $(b^2 + ab - 2) - (2b^2 + 2ab)$ $-b^2 - ab - 2$
13. $(7z^2 + 4 - z) - (-5 + 3z^2)$ $4z^2 - z + 9$
14. $(5 + 4n + 2m) + (-6m - 8)$ $-3 + 4n - 4m$
15. $(4t^2 + 2) + (-4 + 2t)$ $4t^2 + 2t - 2$
16. $(3g^3 + 7g) - (4g + 8g^3)$ $-5g^3 + 3g$
17. $(2a^2 + 8a + 4) - (a^2 - 3)$ $a^2 + 8a + 7$
18. $(3x^2 - 7x + 5) - (-x^2 + 4x)$ $4x^2 - 11x + 5$
19. $(7z^2 + z + 1) - (-4z + 3z^2 - 3)$ $4z^2 + 5z + 4$
20. $(2c^2 + 7c + 4) + (c^2 + 1 - 9c)$ $3c^2 - 2c + 5$
21. $(n^2 + 3n + 2) - (2n^2 - 6n - 2)$ $-n^2 + 9n + 4$
22. $(a^2 + ab - 3b^2) + (b^2 + 4a^2 - ab)$ $5a^2 - 2b^2$
23. $(\ell^2 - 5\ell - 6) + (2\ell^2 + 5 + \ell)$ $3\ell^2 - 4\ell - 1$
24. $(2m^2 + 5m + 1) - (4m^2 - 3m - 3)$ $-2m^2 + 8m + 4$
25. $(x^2 - 6x + 2) - (-5x^2 + 7x - 4)$ $6x^2 - 13x + 6$
26. $(5b^2 - 9b - 5) + (b^2 - 6 + 2b)$ $6b^2 - 7b - 11$
27. $(2x^2 - 6x - 2) + (x^2 + 4x) + (3x^2 + x + 5)$ $6x^2 - x + 3$

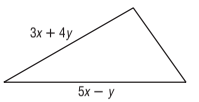
NAME _____ DATE _____ PERIOD ____

8-5 Practice (Average)

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(4y + 5) + (-7y - 1)$ $-3y + 4$
2. $(-x^2 + 3x) - (5x + 2x^2)$ $-3x^2 - 2x$
3. $(4k^2 + 8k + 2) - (2k + 3)$ $4k^2 + 6k - 1$
4. $(2m^2 + 6m) + (m^2 - 5m + 7)$ $3m^2 + m + 7$
5. $(2w^2 - 3w + 1) + (4w - 7)$ $2w^2 + w - 6$
6. $(g^3 + 2g^2) - (6g - 4g^2 + 2g^3)$ $-g^3 + 6g^2 - 6g$
7. $(5a^2 + 6a + 2) - (7a^2 - 7a + 5)$ $-2a^2 + 13a - 3$
8. $(-4p^2 - p + 9) + (p^2 + 3p - 1)$ $-3p^2 + 2p + 8$
9. $(x^3 - 3x + 1) - (x^3 + 7 - 12x)$ $9x - 6$
10. $(6c^2 - c + 1) - (-4 + 2c^2 + 8c)$ $4c^2 - 9c + 5$
11. $(-b^3 + 8bc^2 + 5) - (7bc^2 - 2 + b^3)$ $-2b^3 + bc^2 + 7$
12. $(5n^2 - 3n + 2) + (-n + 2n^2 - 4)$ $7n^2 - 4n - 2$
13. $(4y^2 + 2y - 8) - (7y^2 + 4 - y)$ $-3y^2 + 3y - 12$
14. $(w^2 - 4w - 1) + (-5 + 5w^2 - 3w)$ $6w^2 - 7w - 6$
15. $(4u^2 - 2u - 3) + (3u^2 - u + 4)$ $7u^2 - 3u + 1$
16. $(5b^2 - 8 + 2b) - (b + 9b^2 + 5)$ $-4b^2 + b - 13$
17. $(4d^2 + 2d + 2) + (5d^2 - 2 - d)$ $9d^2 + d$
18. $(8x^2 + x - 6) - (-x^2 + 2x - 3)$ $9x^2 - x - 3$
19. $(3h^2 + 7h - 1) - (4h + 8h^2 + 1)$ $-5h^2 + 3h - 2$
20. $(4m^2 - 3m + 10) + (m^2 + m - 2)$ $5m^2 - 2m + 8$
21. $(x^2 + y^2 - 6) - (5x^2 - y^2 - 5)$ $-4x^2 + 2y^2 - 1$
22. $(7t^2 + 2 - t) + (t^2 - 7 - 2t)$ $8t^2 - 3t - 5$
23. $(k^3 - 2k^2 + 4k + 6) - (-4k + k^2 - 3)$ $k^3 - 3k^2 + 8k + 9$
24. $(9j^2 + j + jk) + (-3j^2 - jk - 4j)$ $6j^2 - 3j$
25. $(2x + 6y - 3z) + (4x + 6z - 8y) + (x - 3y + z)$ $7x - 5y + 4z$
26. $(6f^2 - 7f - 3) - (5f^2 - 1 + 2f) - (2f^2 - 3 + f)$ $-f^2 - 10f + 1$
27. **BUSINESS** The polynomial $s^3 - 70s^2 + 1500s - 10{,}800$ models the profit a company makes on selling an item at a price s. A second item sold at the same price brings in a profit of $s^3 - 30s^2 + 450s - 5000$. Write a polynomial that expresses the total profit from the sale of both items. $2s^3 - 100s^2 + 1950s - 15{,}800$
28. **GEOMETRY** The measures of two sides of a triangle are given. If P is the perimeter, and $P = 10x + 5y$, find the measure of the third side. $2x + 2y$



Answers

NAME _____ DATE _____ PERIOD ____

8-5

Reading to Learn Mathematics

Adding and Subtracting Polynomials

Pre-Activity **How can adding polynomials help you model sales?**

Read the introduction to Lesson 8-5 at the top of page 439 in your textbook.

What operation would you use to find how much more the traditional toy sales R were than the video games sales V?

subtraction

Reading the Lesson

1. Use the example $(-3x^3 + 4x^2 + 5x + 1) + (-5x^3 - 2x^2 + 2x - 7)$.

a. Show what is meant by grouping like terms horizontally.

$[-3x^3 + (-5x^3)] + [(4x^2 + (-2x^2)] + (5x + 2x) + [1 + (-7)]$

b. Show what is meant by aligning like terms vertically.

$$\begin{array}{r} -3x^3 + 4x^2 + 5x + 1 \\ (+)\ -5x^3 - 2x^2 + 2x - 7 \\ \hline \end{array}$$

c. Choose one method, then add the polynomials.

$-8x^3 + 2x^2 + 7x - 6$

2. How is subtracting a polynomial like subtracting a rational number?

You subtract by adding the additive inverse.

3. An algebra student got the following exercise wrong on his homework. What was his error?

$(3x^5 - 3x^4 + 2x^3 - 4x^2 + 5) - (2x^5 - x^3 + 2x^2 - 4)$
$= [3x^5 + (-2x^5)] + (-3x^4) + [2x^3 + (-x^3)] + [-4x^2 + (-2x^2)] + (5 + 4)$
$= x^5 - 3x^4 + x^3 - 6x^2 + 9$

He did not add the additive inverse of $-x^3$.

Helping You Remember

4. How is adding and subtracting polynomials vertically like adding and subtracting decimals vertically?

Aligning like terms when adding or subtracting polynomials is like using place value to align digits when you add or subtract decimals.

NAME _____ DATE _____ PERIOD ____

8-5

Enrichment

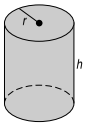
Circular Areas and Volumes

Area of Circle

$A = \pi r^2$

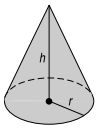
Volume of Cylinder

$V = \pi r^2 h$



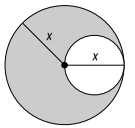
Volume of Cone

$V = \frac{1}{3}\pi r^2 h$



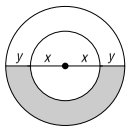
Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)

1.



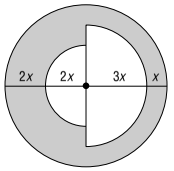
$\pi x^2 - \pi\left(\frac{x}{2}\right)^2 = \pi x^2$

2.



$\frac{\pi}{2}(y^2 + 2xy)$

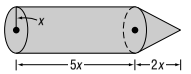
3.



$\frac{19}{2}\pi x^2$

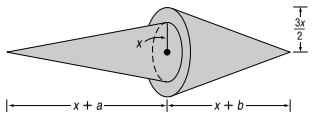
Write an algebraic expression for the total volume of each figure.

4.



$5\frac{2}{3}\pi x^3$

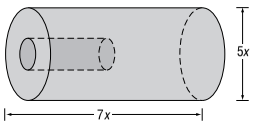
5.



$\frac{\pi}{12}[13x^3 + (4a + 9b)x^2]$

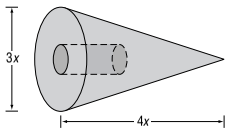
Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.

6.



$\frac{175\pi}{4}x^3 - 20\pi$ in^3

7.



$3\pi x^3 - 20\pi$ in^3

NAME _____ DATE _____ PERIOD ____

8-6 Study Guide and Intervention

Multiplying a Polynomial by a Monomial

Product of Monomial and Polynomial The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

Example 1 **Find $-3x^2(4x^2 + 6x - 8)$.**

Horizontal Method

$-3x^2(4x^2 + 6x - 8)$
$= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)$
$= -12x^4 + (-18x^3) - (-24x^2)$
$= -12x^4 - 18x^3 + 24x^2$

Vertical Method

$4x^2 + 6x - 8$
$(\times) \quad -3x^2$
$-12x^4 - 18x^3 + 24x^2$

The product is $-12x^4 - 18x^3 + 24x^2$.

Example 2 **Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.**

$-2(4x^2 + 5x) - x(x^2 + 6x)$
$= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)$
$= -8x^2 + (-10x) + (-x^3) + (-6x^2)$
$= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)$
$= -x^3 - 14x^2 - 10x$

Exercises

Find each product.

1. $x(5x + x^2)$ — $5x^2 + x^3$
2. $x(4x^2 + 3x + 2)$ — $4x^3 + 3x^2 + 2x$
3. $-2xy(2y + 4x^2)$ — $-4xy^2 - 8x^3y$
4. $-2g(g^2 - 2g + 2)$ — $-2g^3 + 4g^2 - 4g$
5. $3x(x^4 + x^3 + x^2)$ — $3x^5 + 3x^4 + 3x^3$
6. $-4x(2x^3 - 2x + 3)$ — $-8x^4 + 8x^2 - 12x$
7. $-4cx(10 + 3x)$ — $-40cx - 12cx^2$
8. $3y(-4x - 6x^3 - 2y)$ — $-12xy - 18x^3y - 6y^2$
9. $2x^2y^2(3xy + 2y + 5x)$ — $6x^3y^3 + 4x^2y^3 + 10x^3y^2$

Simplify.

10. $x(3x - 4) - 5x$ — $3x^2 - 9x$
11. $-x(2x^2 - 4x) - 6x^2$ — $-2x^3 - 2x^2$
12. $6a(2a - b) + 2a(-4a + 5b)$ — $4a^2 + 4ab$
13. $4r(2r^2 - 3r + 5) + 6r(4r^2 + 2r + 8)$ — $32r^3 + 68r$
14. $4n(3n^2 + n - 4) - n(3 - n)$ — $12n^3 + 5n^2 - 19n$
15. $2b(b^2 + 4b + 8) - 3b(3b^2 + 9b - 18)$ — $-7b^3 - 19b^2 + 70b$
16. $-2z(4z^2 - 3z + 1) - z(3z^2 + 2z - 1)$ — $-11z^3 + 4z^2 - z$
17. $2(4x^2 - 2x) - 3(-6x^2 + 4) + 2x(x - 1)$ — $28x^2 - 6x - 12$

NAME _____ DATE _____ PERIOD ____

8-6 Study Guide and Intervention *(continued)*

Multiplying a Polynomial by a Monomial

Solve Equations with Polynomial Expressions Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example **Solve $4(n - 2) + 5n = 6(3 - n) + 19$.**

$4(n - 2) + 5n = 6(3 - n) + 19$	Original equation
$4n - 8 + 5n = 18 - 6n + 19$	Distributive Property
$9n - 8 = 37 - 6n$	Combine like terms.
$15n - 8 = 37$	Add $6n$ to both sides.
$15n = 45$	Add 8 to both sides.
$n = 3$	Divide each side by 15.

The solution is 3.

Exercises

Solve each equation.

1. $2(a - 3) = 3(-2a + 6)$ — 3
2. $3(x + 5) - 6 = 18$ — 3
3. $3x(x - 5) - 3x^2 = -30$ — 2
4. $6(x^2 + 2x) = 2(3x^2 + 12)$ — 2
5. $4(2p + 1) - 12p = 2(8p + 12)$ — -1
6. $2(6x + 4) + 2 = 4(x - 4)$ — $-3\frac{1}{2}$
7. $-2(4y - 3) - 8y + 6 = 4(y - 2)$ — 1
8. $c(c + 2) - c(c - 6) = 10c - 12$ — 6
9. $3(x^2 - 2x) = 3x^2 + 5x - 11$ — 1
10. $2(4x + 3) + 2 = -4(x + 1)$ — -1
11. $3(2h - 6) - (2h + 1) = 9$ — 7
12. $3(y + 5) - (4y - 8) = -2y + 10$ — -13
13. $3(2a - 6) - (-3a - 1) = 4a - 2$ — $2\frac{3}{5}$
14. $5(2x^2 - 1) - (10x^2 - 6) = -(x + 2)$ — -3
15. $3(x + 2) + 2(x + 1) = -5(x - 3)$ — $\frac{7}{10}$
16. $4(3p^2 + 2p) - 12p^2 = 2(8p + 6)$ — $-\frac{3}{2}$

Answers

NAME _____ DATE _____ PERIOD ____

8-6 Skills Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $a(4a + 3)$ — $4a^2 + 3a$
2. $-c(11c + 4)$ — $-11c^2 - 4c$
3. $x(2x - 5)$ — $2x^2 - 5x$
4. $2y(y - 4)$ — $2y^2 - 8y$
5. $-3n(n^2 + 2n)$ — $-3n^3 - 6n^2$
6. $4h(3h - 5)$ — $12h^2 - 20h$
7. $3x(5x^2 - x + 4)$ — $15x^3 - 3x^2 + 12x$
8. $7c(5 - 2c^2 + c^3)$ — $35c - 14c^3 + 7c^4$
9. $-4b(1 - 9b - 2b^2)$ — $-4b + 36b^2 + 8b^3$
10. $6y(-5 - y + 4y^2)$ — $-30y - 6y^2 + 24y^3$
11. $2m^2(2m^2 + 3m - 5)$ — $4m^4 + 6m^3 - 10m^2$
12. $-3n^2(-2n^2 + 3n + 4)$ — $6n^4 - 9n^3 - 12n^2$

Simplify.

13. $w(3w + 2) + 5w$ — $3w^2 + 7w$
14. $f(5f - 3) - 2f$ — $5f^2 - 5f$
15. $-p(2p - 8) - 5p$ — $-2p^2 + 3p$
16. $y^2(-4y + 5) - 6y^2$ — $-4y^3 - y^2$
17. $2x(3x^2 + 4) - 3x^3$ — $3x^3 + 8x$
18. $4a(5a^2 - 4) + 9a$ — $20a^3 - 7a$
19. $4b(-5b - 3) - 2(b^2 - 7b - 4)$ — $-22b^2 + 2b + 8$
20. $3m(3m + 6) - 3(m^2 + 4m + 1)$ — $6m^2 + 6m - 3$

Solve each equation.

21. $3(a + 2) + 5 = 2a + 4$ -7
22. $2(4x + 2) - 8 = 4(x + 3)$ 4
23. $5(y + 1) + 2 = 4(y + 2) - 6$ -5
24. $4(b + 6) = 2(b + 5) + 2$ -6
25. $6(m - 2) + 14 = 3(m + 2) - 10$ -2
26. $3(c + 5) - 2 = 2(c + 6) + 2$ 1

NAME _____ DATE _____ PERIOD ____

8-6 Practice (Average)

Multiplying a Polynomial by a Monomial

Find each product.

1. $2h(-7h^2 - 4h)$ — $-14h^3 - 8h^2$
2. $6pq(3p^2 + 4q)$ — $18p^3q + 24pq^2$
3. $-2u^2n(4u - 2n)$ — $-8u^3n + 4u^2n^2$
4. $5jk(3jk + 2k)$ — $15j^2k^2 + 10jk^2$
5. $-3rs(-2s^2 + 3r)$ — $6rs^3 - 9r^2s$
6. $4mg^2(2mg + 4g)$ — $8m^2g^3 + 16mg^3$
7. $-\frac{1}{4}m(8m^2 + m - 7)$ — $-2m^3 - \frac{1}{4}m^2 + \frac{7}{4}m$
8. $-\frac{2}{3}n^2(-9n^2 + 3n + 6)$ — $6n^4 - 2n^3 - 4n^2$

Simplify.

9. $-2\ell(3\ell - 4) + 7\ell$ — $-6\ell^2 + 15\ell$
10. $5w(-7w + 3) + 2w(-2w^2 + 19w + 2)$ — $-4w^3 + 3w^2 + 19w$
11. $6t(2t - 3) - 5(2t^2 + 9t - 3)$ — $2t^2 - 63t + 15$
12. $-2(3m^3 + 5m + 6) + 3m(2m^2 + 3m + 1)$ — $9m^2 - 7m - 12$
13. $-3g(7g - 2) + 3(g^2 + 2g + 1) - 3g(-5g + 3)$ $-3g^2 + 3g + 3$
14. $4z^2(z - 7) - 5z(z^2 - 2z - 2) + 3z(4z - 2)$ $-z^3 - 6z^2 + 4z$

Solve each equation.

15. $5(2s - 1) + 3 = 3(3s + 2)$ 8
16. $3(3u + 2) + 5 = 2(2u - 2)$ -3
17. $4(8n + 3) - 5 = 2(6n + 8) + 1$ $\frac{1}{2}$
18. $8(3b + 1) = 4(b + 3) - 9$ $-\frac{1}{4}$
19. $h(h - 3) - 2h = h(h - 2) - 12$ 4
20. $w(w + 6) + 4w = -7 + w(w + 9)$ -7
21. $t(t + 4) - 1 = t(t + 2) + 2$ $\frac{3}{2}$
22. $u(u - 5) + 8u = u(u + 2) - 4$ -4

23. **NUMBER THEORY** Let x be an integer. What is the product of twice the integer added to three times the next consecutive integer? $5x + 3$

INVESTMENTS For Exercises 24–26, use the following information.

Kent invested \$5,000 in a retirement plan. He allocated x dollars of the money to a bond account that earns 4% interest per year and the rest to a traditional account that earns 5% interest per year.

24. Write an expression that represents the amount of money invested in the traditional account. $5{,}000 - x$
25. Write a polynomial model in simplest form for the total amount of money T Kent has invested after one year. (*Hint:* Each account has $A + IA$ dollars, where A is the original amount in the account and I is its interest rate.) $T = 5{,}250 - 0.01x$
26. If Kent put \$500 in the bond account, how much money does he have in his retirement plan after one year? \$5,245

NAME _____ DATE _____ PERIOD ____

8-6 Reading to Learn Mathematics

Multiplying a Polynomial by a Monomial

Pre-Activity **How is finding the product of a monomial and a polynomial related to finding the area of a rectangle?**

Read the introduction to Lesson 8-6 at the top of page 444 in your textbook.

You may recall that the formula for the area of a rectangle is $A = \ell w$. In this rectangle, $\ell =$ ___x + 3___ and $w =$ ___2x___. How would you substitute these values in the area formula?

$A = (x + 3)(2x)$

Reading the Lesson

1. Refer to Lesson 8-6.

a. How is the Distributive Property used to multiply a polynomial by a monomial?

The monomial is multiplied by each term in the polynomial.

b. Use the Distributive Property to complete the following.

$2y^2(3y^2 + 2y - 7) = 2y^2($ ___3y²___ $) + 2y^2($ ___2y___ $) - 2y^2($ ___7___ $)$

$=$ ___6y⁴___ $+$ ___4y³___ $-$ ___14y²___

$-3x^3(x^3 - 2x^2 + 3) =$ ___−3x³(x³)___ $-$ ___(−3x³)(2x²)___ $+$ ___(−3x³)(3)___

$=$ ___−3x⁶___ $+$ ___6x⁵___ $-$ ___9x³___

2. What is the difference between simplifying an expression and solving an equation?

Simplifying an expression is combining like terms. Solving an equation is finding the value of the variable that makes the equation true.

Helping You Remember

3. Use the equation $2x(x - 5) + 3x(x + 3) = 5x(x + 7) - 9$ to show how you would explain the process of solving equations with polynomial expressions to another algebra student.

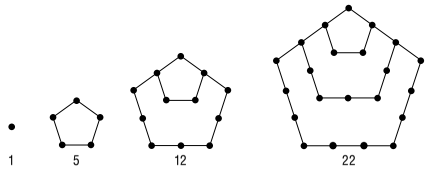
Use the Distributive Property.	$2x^2 - 10x + 3x^2 + 9x = 5x^2 + 35x - 9$
Combine like terms.	$5x^2 - x = 5x^2 + 35x - 9$
Subtract $5x^2$ from both sides.	$-x = 35x - 9$
Subtract $35x$ from both sides.	$-36x = -9$
Divide each side by −36.	$x = 0.25$

NAME _____ DATE _____ PERIOD ____

8-6 Enrichment

Figurate Numbers

The numbers below are called **pentagonal numbers**. They are the numbers of dots or disks that can be arranged as pentagons.



1. Find the product $\frac{1}{2}n(3n - 1)$. $\frac{3n^2}{2} - \frac{n}{2}$

2. Evaluate the product in Exercise 1 for values of n from 1 through 4. 1, 5, 12, 22

3. What do you notice? They are the first four pentagonal numbers.

4. Find the next six pentagonal numbers. 35, 51, 70, 92, 117, 145

5. Find the product $\frac{1}{2}n(n + 1)$. $\frac{n^2}{2} + \frac{n}{2}$

6. Evaluate the product in Exercise 5 for values of n from 1 through 5. On another sheet of paper, make drawings to show why these numbers are called the triangular numbers. 1, 3, 6, 10, 15

7. Find the product $n(2n - 1)$. $2n^2 - n$

8. Evaluate the product in Exercise 7 for values of n from 1 through 5. Draw these hexagonal numbers. 1, 6, 15, 28, 45

9. Find the first 5 square numbers. Also, write the general expression for any square number. 1, 4, 9, 16, 25; n^2

The numbers you have explored above are called the plane figurate numbers because they can be arranged to make geometric figures. You can also create solid figurate numbers.

10. If you pile 10 oranges into a pyramid with a triangle as a base, you get one of the tetrahedral numbers. How many layers are there in the pyramid? How many oranges are there in the bottom layers? 3 layers; 6

11. Evaluate the expression $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$ for values of n from 1 through 5 to find the first five tetrahedral numbers. 1, 4, 10, 20, 35

Answers

NAME _____ DATE _____ PERIOD ____

8-7 Study Guide and Intervention

Multiplying Polynomials

Multiply Binomials To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

Example 1 **Find $(x + 3)(x - 4)$.**

Horizontal Method

$(x + 3)(x - 4)$
$= x(x - 4) + 3(x - 4)$
$= (x)(x) + x(-4) + 3(x) + 3(-4)$
$= x^2 - 4x + 3x - 12$
$= x^2 - x - 12$

Vertical Method

$$\begin{array}{r} x + 3 \\ (\times)\quad x - 4 \\ \hline -4x - 12 \\ x^2 + 3x \\ \hline x^2 - x - 12 \end{array}$$

The product is $x^2 - x - 12$.

Example 2 **Find $(x - 2)(x + 5)$ using the FOIL method.**

$(x - 2)(x + 5)$

First Outer Inner Last

$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$
$= x^2 + 5x + (-2x) - 10$
$= x^2 + 3x - 10$

The product is $x^2 + 3x - 10$.

Lesson 8-7

Exercises

Find each product.

1. $(x + 2)(x + 3)$ — $x^2 + 5x + 6$
2. $(x - 4)(x + 1)$ — $x^2 - 3x - 4$
3. $(x - 6)(x - 2)$ — $x^2 - 8x + 12$
4. $(p - 4)(p + 2)$ — $p^2 - 2p - 8$
5. $(y + 5)(y + 2)$ — $y^2 + 7y + 10$
6. $(2x - 1)(x + 5)$ — $2x^2 + 9x - 5$
7. $(3n - 4)(3n - 4)$ — $9n^2 - 24n + 16$
8. $(8m - 2)(8m + 2)$ — $64m^2 - 4$
9. $(k + 4)(5k - 1)$ — $5k^2 + 19k - 4$
10. $(3x + 1)(4x + 3)$ — $12x^2 + 13x + 3$
11. $(x - 8)(-3x + 1)$ — $-3x^2 + 25x - 8$
12. $(5t + 4)(2t - 6)$ — $10t^2 - 22t - 24$
13. $(5m - 3n)(4m - 2n)$ — $20m^2 - 22mn + 6n^2$
14. $(a - 3b)(2a - 5b)$ — $2a^2 - 11ab + 15b^2$
15. $(8x - 5)(8x + 5)$ — $64x^2 - 25$
16. $(2n - 4)(2n + 5)$ — $4n^2 + 2n - 20$
17. $(4m - 3)(5m - 5)$ — $20m^2 - 35m + 15$
18. $(7g - 4)(7g + 4)$ — $49g^2 - 16$

NAME _____ DATE _____ PERIOD ____

8-7 Study Guide and Intervention *(continued)*

Multiplying Polynomials

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

Example **Find $(3x + 2)(2x^2 - 4x + 5)$.**

$(3x + 2)(2x^2 - 4x + 5)$
$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$ Distributive Property
$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$ Distributive Property
$= 6x^3 - 8x^2 + 7x + 10$ Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Exercises

Find each product.

1. $(x + 2)(x^2 - 2x + 1)$ — $x^3 - 3x + 2$
2. $(x + 3)(2x^2 + x - 3)$ — $2x^3 + 7x^2 - 9$
3. $(2x - 1)(x^2 - x + 2)$ — $2x^3 - 3x^2 + 5x - 2$
4. $(p - 3)(p^2 - 4p + 2)$ — $p^3 - 7p^2 + 14p - 6$
5. $(3k + 2)(k^2 + k - 4)$ — $3k^3 + 5k^2 - 10k - 8$
6. $(2t + 1)(10t^2 - 2t - 4)$ — $20t^3 + 6t^2 - 10t - 4$
7. $(3n - 4)(n^2 + 5n - 4)$ — $3n^3 + 11n^2 - 32n + 16$
8. $(8x - 2)(3x^2 + 2x - 1)$ — $24x^3 + 10x^2 - 12x + 2$
9. $(2a + 4)(2a^2 - 8a + 3)$ — $4a^3 - 8a^2 - 26a + 12$
10. $(3x - 4)(2x^2 + 3x + 3)$ — $6x^3 + x^2 - 3x - 12$
11. $(n^2 + 2n - 1)(n^2 + n + 2)$ — $n^4 + 3n^3 + 3n^2 + 3n - 2$
12. $(t^2 + 4t - 1)(2t^2 - t - 3)$ — $2t^4 + 7t^3 - 9t^2 - 11t + 3$
13. $(y^2 - 5y + 3)(2y^2 + 7y - 4)$ — $2y^4 - 3y^3 - 33y^2 + 41y - 12$
14. $(3b^2 - 2b + 1)(2b^2 - 3b - 4)$ — $6b^4 - 13b^3 - 4b^2 + 5b - 4$

NAME _____ DATE _____ PERIOD ____

8-7 Skills Practice

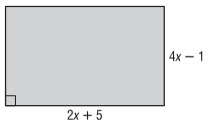
Multiplying Polynomials

Find each product.

1. $(m + 4)(m + 1)$
   $m^2 + 5m + 4$
2. $(x + 2)(x + 2)$
   $x^2 + 4x + 4$
3. $(b + 3)(b + 4)$
   $b^2 + 7b + 12$
4. $(t + 4)(t - 3)$
   $t^2 + t - 12$
5. $(r + 1)(r - 2)$
   $r^2 - r - 2$
6. $(z - 5)(z + 1)$
   $z^2 - 4z - 5$
7. $(3c + 1)(c - 2)$
   $3c^2 - 5c - 2$
8. $(2x - 6)(x + 3)$
   $2x^2 - 18$
9. $(d - 1)(5d - 4)$
   $5d^2 - 9d + 4$
10. $(2\ell + 5)(\ell - 4)$
    $2\ell^2 - 3\ell - 20$
11. $(3n - 7)(n + 3)$
    $3n^2 + 2n - 21$
12. $(q + 5)(5q - 1)$
    $5q^2 + 24q - 5$
13. $(3b + 3)(3b - 2)$
    $9b^2 + 3b - 6$
14. $(2m + 2)(3m - 3)$
    $6m^2 - 6$
15. $(4c + 1)(2c + 1)$
    $8c^2 + 6c + 1$
16. $(5a - 2)(2a - 3)$
    $10a^2 - 19a + 6$
17. $(4h - 2)(4h - 1)$
    $16h^2 - 12h + 2$
18. $(x - y)(2x - y)$
    $2x^2 - 3xy + y^2$
19. $(e + 4)(e^2 + 3e - 6)$
    $e^3 + 7e^2 + 6e - 24$
20. $(t + 1)(t^2 + 2t + 4)$
    $t^3 + 3t^2 + 6t + 4$
21. $(k + 4)(k^2 + 3k - 6)$
    $k^3 + 7k^2 + 6k - 24$
22. $(m + 3)(m^2 + 3m + 5)$
    $m^3 + 6m^2 + 14m + 15$

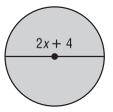
GEOMETRY Write an expression to represent the area of each figure.

23.



$8x^2 + 18x - 5$ units^2

24.



$(x^2 + 4x + 4)\pi$ units^2

NAME _____ DATE _____ PERIOD ____

8-7 Practice (Average)

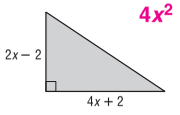
Multiplying Polynomials

Find each product.

1. $(q + 6)(q + 5)$
   $q^2 + 11q + 30$
2. $(x + 7)(x + 4)$
   $x^2 + 11x + 28$
3. $(s + 5)(s - 6)$
   $s^2 - s - 30$
4. $(n - 4)(n - 6)$
   $n^2 - 10n + 24$
5. $(a - 5)(a - 8)$
   $a^2 - 13a + 40$
6. $(w - 6)(w - 9)$
   $w^2 - 15w + 54$
7. $(4c + 6)(c - 4)$
   $4c^2 - 10c - 24$
8. $(2x - 9)(2x + 4)$
   $4x^2 - 10x - 36$
9. $(4d - 5)(2d - 3)$
   $8d^2 - 22d + 15$
10. $(4b + 3)(3b - 4)$
    $12b^2 - 7b - 12$
11. $(4m + 2)(4m - 3)$
    $16m^2 - 4m - 6$
12. $(5c - 5)(7c + 9)$
    $35c^2 + 10c - 45$
13. $(6a - 3)(7a - 4)$
    $42a^2 - 45a + 12$
14. $(6h - 3)(4h - 2)$
    $24h^2 - 24h + 6$
15. $(2x - 2)(5x - 4)$
    $10x^2 - 18x + 8$
16. $(3a - b)(2a - b)$
    $6a^2 - 5ab + b^2$
17. $(4g + 3h)(2g + 3h)$
    $8g^2 + 18gh + 9h^2$
18. $(4x + y)(4x + y)$
    $16x^2 + 8xy + y^2$
19. $(m + 5)(m^2 + 4m - 8)$
    $m^3 + 9m^2 + 12m - 40$
20. $(t + 3)(t^2 + 4t + 7)$
    $t^3 + 7t^2 + 19t + 21$
21. $(2h + 3)(2h^2 + 3h + 4)$
    $4h^3 + 12h^2 + 17h + 12$
22. $(3d + 3)(2d^2 + 5d - 2)$
    $6d^3 + 21d^2 + 9d - 6$
23. $(3q + 2)(9q^2 - 12q + 4)$
    $27q^3 - 18q^2 - 12q + 8$
24. $(3r + 2)(9r^2 + 6r + 4)$
    $27r^3 + 36r^2 + 24r + 8$
25. $(3c^2 + 2c - 1)(2c^2 + c + 9)$
    $6c^4 + 7c^3 + 27c^2 + 17c - 9$
26. $(2\ell^2 + \ell + 3)(4\ell^2 + 2\ell - 2)$
    $8\ell^4 + 8\ell^3 + 10\ell^2 + 4\ell - 6$
27. $(2x^2 - 2x - 3)(2x^2 - 4x + 3)$
    $4x^4 - 12x^3 + 8x^2 + 6x - 9$
28. $(3y^2 + 2y + 2)(3y^2 - 4y - 5)$
    $9y^4 - 6y^3 - 17y^2 - 18y - 10$

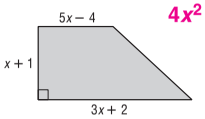
GEOMETRY Write an expression to represent the area of each figure.

29.



$4x^2 - 2x - 2$ units^2

30.



$4x^2 + 3x - 1$ units^2

31. **NUMBER THEORY** Let x be an even integer. What is the product of the next two consecutive even integers? $x^2 + 6x + 8$

32. **GEOMETRY** The volume of a rectangular pyramid is one third the product of the area of its base and its height. Find an expression for the volume of a rectangular pyramid whose base has an area of $3x^2 + 12x + 9$ square feet and whose height is $x + 3$ feet.
    $x^3 + 7x^2 + 15x + 9$ feet^3

Answers

NAME _____ DATE _____ PERIOD ____

8-7

Reading to Learn Mathematics

Multiplying Polynomials

Pre-Activity **How is multiplying binomials similar to multiplying two-digit numbers?**

Read the introduction to Lesson 8-7 at the top of page 452 in your textbook.

In your own words, explain how the distributive property is used twice to multiply two-digit numbers.

The ones of the first factor are multiplied by the tens and the ones of the other factor. Then the tens of the first factor are multiplied by the tens and ones of the other factor.

Reading the Lesson

1. How is multiplying binomials similar to multiplying two-digit numbers?

Binomials have two terms and each term of one binomial is multiplied by each term of the other binomial.

2. Complete the table using the FOIL method.

		Product of First Terms	+	Product of Outer Terms	+	Product of Inner Terms	+	Product of Last Terms
$(x + 5)(x - 3)$	=	$(x)(x)$	+	$(x)(-3)$	+	$(5)(x)$	+	$(5)(-3)$
	=	x^2	+	$-3x$	+	$5x$	+	-15
	=	x^2	+	$2x$	−	15		
$(3y + 6)(y - 2)$	=	$(3y)(y)$	+	$(3y)(-2)$	+	$(6)(y)$	+	$(6)(-2)$
	=	$3y^2$	+	$-6y$	+	$6y$	+	-12
	=	$3y^2$	−	12				

Helping You Remember

3. Think of a method for remembering all the product combinations used in the FOIL method for multiplying two binomials. Describe your method using words or a diagram.

Sample answer: Imagine that the two binomials are written on the floor. For FOIL, think of all the possible ways you could have your left foot on a term of the first binomial and your right foot on a term of the second binomial. Your feet could be on the first terms, the outer terms, the inner terms, or the last terms.

NAME _____ DATE _____ PERIOD ____

8-7

Enrichment

Pascal's Triangle

This arrangement of numbers is called Pascal's Triangle. It was first published in 1665, but was known hundreds of years earlier.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
```

1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row?

3 and 3

2. Describe how to create the 6th row of Pascal's Triangle.

The first and last numbers are 1. Evaluate 1 + 4, 4 + 6, 6 + 4, and 4 + 1 to find the other numbers.

3. Write the numbers for rows 6 through 10 of the triangle.

Row 6: **1 5 10 10 5 1**

Row 7: **1 6 15 20 15 6 1**

Row 8: **1 7 21 35 35 21 7 1**

Row 9: **1 8 28 56 70 56 28 8 1**

Row 10: **1 9 36 84 126 126 84 36 9 1**

Multiply to find the expanded form of each product.

4. $(a + b)^2$ $a^2 + 2ab + b^2$

5. $(a + b)^3$ $a^3 + 3a^2b + 3ab^2 + b^3$

6. $(a + b)^4$ $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Now compare the coefficients of the three products in Exercises 4–6 with Pascal's Triangle.

7. Describe the relationship between the expanded form of $(a + b)^n$ and Pascal's Triangle.

The coefficients of the expanded form are found in row $n + 1$ of Pascal's Triangle.

8. Use Pascal's Triangle to write the expanded form of $(a + b)^6$.

$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

NAME _____ DATE _____ PERIOD ____

8-8 Study Guide and Intervention

Special Products

Squares of Sums and Differences Some pairs of binomials have products that follow specific patterns. One such pattern is called the *square of a sum*. Another is called the *square of a difference*.

Square of a sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example 1 **Find $(3a + 4)(3a + 4)$.**

Use the square of a sum pattern, with $a = 3a$ and $b = 4$.

$(3a + 4)(3a + 4) = (3a)^2 + 2(3a)(4) + (4)^2$
$= 9a^2 + 24a + 16$

The product is $9a^2 + 24a + 16$.

Example 2 **Find $(2z - 9)(2z - 9)$.**

Use the square of a difference pattern with $a = 2z$ and $b = 9$.

$(2z - 9)(2z - 9) = (2z)^2 - 2(2z)(9) + (9)(9)$
$= 4z^2 - 36z + 81$

The product is $4z^2 - 36z + 81$.

Exercises

Find each product.

1. $(x - 6)^2$ — $x^2 - 12x + 36$
2. $(3p + 4)^2$ — $9p^2 + 24p + 16$
3. $(4x - 5)^2$ — $16x^2 - 40x + 25$
4. $(2x - 1)^2$ — $4x^2 - 4x + 1$
5. $(2h + 3)^2$ — $4h^2 + 12h + 9$
6. $(m + 5)^2$ — $m^2 + 10m + 25$
7. $(c + 3)^2$ — $c^2 + 6c + 9$
8. $(3 - p)^2$ — $9 - 6p + p^2$
9. $(x - 5y)^2$ — $x^2 - 10xy + 25y^2$
10. $(8y + 4)^2$ — $64y^2 + 64y + 16$
11. $(8 + x)^2$ — $64 + 16x + x^2$
12. $(3a - 2b)^2$ — $9a^2 - 12ab + 4b^2$
13. $(2x - 8)^2$ — $4x^2 - 32x + 64$
14. $(x^2 + 1)^2$ — $x^4 + 2x^2 + 1$
15. $(m^2 - 2)^2$ — $m^4 - 4m^2 + 4$
16. $(x^3 - 1)^2$ — $x^6 - 2x^3 + 1$
17. $(2h^2 - k^2)^2$ — $4h^4 - 4h^2k^2 + k^4$
18. $\left(\frac{1}{4}x + 3\right)^2$ — $\frac{1}{16}x^2 + \frac{3}{2}x + 9$
19. $(x - 4y^2)^2$ — $x^2 - 8xy^2 + 16y^4$
20. $(2p + 4q)^2$ — $4p^2 + 16pq + 16q^2$
21. $\left(\frac{2}{3}x - 2\right)^2$ — $\frac{4}{9}x^2 - \frac{8}{3}x + 4$

NAME _____ DATE _____ PERIOD ____

8-8 Study Guide and Intervention *(continued)*

Special Products

Product of a Sum and a Difference There is also a pattern for the product of a sum and a difference of the same two terms, $(a + b)(a - b)$. The product is called the **difference of squares**.

Product of a Sum and a Difference	$(a + b)(a - b) = a^2 - b^2$

Example **Find $(5x + 3y)(5x - 3y)$.**

$(a + b)(a - b) = a^2 - b^2$ Product of a Sum and a Difference
$(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$ $a = 5x$ and $b = 3y$
$= 25x^2 - 9y^2$ Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

1. $(x - 4)(x + 4)$ — $x^2 - 16$
2. $(p + 2)(p - 2)$ — $p^2 - 4$
3. $(4x - 5)(4x + 5)$ — $16x^2 - 25$
4. $(2x - 1)(2x + 1)$ — $4x^2 - 1$
5. $(h + 7)(h - 7)$ — $h^2 - 49$
6. $(m - 5)(m + 5)$ — $m^2 - 25$
7. $(2c - 3)(2c + 3)$ — $4c^2 - 9$
8. $(3 - 5q)(3 + 5q)$ — $9 - 25q^2$
9. $(x - y)(x + y)$ — $x^2 - y^2$
10. $(y - 4x)(y + 4x)$ — $y^2 - 16x^2$
11. $(8 + 4x)(8 - 4x)$ — $64 - 16x^2$
12. $(3a - 2b)(3a + 2b)$ — $9a^2 - 4b^2$
13. $(3y - 8)(3y + 8)$ — $9y^2 - 64$
14. $(x^2 - 1)(x^2 + 1)$ — $x^4 - 1$
15. $(m^2 - 5)(m^2 + 5)$ — $m^4 - 25$
16. $(x^3 - 2)(x^3 + 2)$ — $x^6 - 4$
17. $(h^2 - k^2)(h^2 + k^2)$ — $h^4 - k^4$
18. $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$ — $\frac{1}{16}x^2 - 4$
19. $(3x - 2y^2)(3x + 2y^2)$ — $9x^2 - 4y^4$
20. $(2p - 5s)(2p + 5s)$ — $4p^2 - 25s^2$
21. $\left(\frac{4}{3}x - 2y\right)\left(\frac{4}{3}x + 2y\right)$ — $\frac{16}{9}x^2 - 4y^2$

Answers

NAME _____ DATE _____ PERIOD ____

8-8 Skills Practice

Special Products

Find each product.

1. $(n + 3)^2$
   $n^2 + 6n + 9$
2. $(x + 4)(x + 4)$
   $x^2 + 8x + 16$
3. $(y - 7)^2$
   $y^2 - 14y + 49$
4. $(t - 3)(t - 3)$
   $t^2 - 6t + 9$
5. $(b + 1)(b - 1)$
   $b^2 - 1$
6. $(a - 5)(a + 5)$
   $a^2 - 25$
7. $(p - 4)^2$
   $p^2 - 8p + 16$
8. $(z + 3)(z - 3)$
   $z^2 - 9$
9. $(\ell + 2)(\ell + 2)$
   $\ell^2 + 4\ell + 4$
10. $(r - 1)(r - 1)$
    $r^2 - 2r + 1$
11. $(3g + 2)(3g - 2)$
    $9g^2 - 4$
12. $(2m - 3)(2m + 3)$
    $4m^2 - 9$
13. $(6 + u)^2$
    $36 + 12u + u^2$
14. $(r + s)^2$
    $r^2 + 2rs + s^2$
15. $(3q + 1)(3q - 1)$
    $9q^2 - 1$
16. $(c - e)^2$
    $c^2 - 2ce + e^2$
17. $(2k - 2)^2$
    $4k^2 - 8k + 4$
18. $(w + 3h)^2$
    $w^2 + 6wh + 9h^2$
19. $(3p - 4)(3p + 4)$
    $9p^2 - 16$
20. $(t + 2u)^2$
    $t^2 + 4tu + 4u^2$
21. $(x - 4y)^2$
    $x^2 - 8xy + 16y^2$
22. $(3b + 7)(3b - 7)$
    $9b^2 - 49$
23. $(3y - 3g)(3y + 3g)$
    $9y^2 - 9g^2$
24. $(s^2 + r^2)^2$
    $s^4 + 2s^2r^2 + r^4$
25. $(2k + m^2)^2$
    $4k^2 + 4km^2 + m^4$
26. $(3u^2 - n)^2$
    $9u^4 - 6u^2n + n^2$

27. **GEOMETRY** The length of a rectangle is the sum of two whole numbers. The width of the rectangle is the difference of the same two whole numbers. Using these facts, write a verbal expression for the area of the rectangle. **The area is the square of the larger number minus the square of the smaller number.**

NAME _____ DATE _____ PERIOD ____

8-8 Practice (Average)

Special Products

Find each product.

1. $(n + 9)^2$
   $n^2 + 18n + 81$
2. $(q + 8)^2$
   $q^2 + 16q + 64$
3. $(\ell - 10)^2$
   $\ell^2 - 20\ell + 100$
4. $(r - 11)^2$
   $r^2 - 22r + 121$
5. $(p + 7)^2$
   $p^2 + 14p + 49$
6. $(b + 6)(b - 6)$
   $b^2 - 36$
7. $(z + 13)(z - 13)$
   $z^2 - 169$
8. $(4e + 2)^2$
   $16e^2 + 16e + 4$
9. $(5w - 4)^2$
   $25w^2 - 40w + 16$
10. $(6h - 1)^2$
    $36h^2 - 12h + 1$
11. $(3s + 4)^2$
    $9s^2 + 24s + 16$
12. $(7v - 2)^2$
    $49v^2 - 28v + 4$
13. $(7k + 3)(7k - 3)$
    $49k^2 - 9$
14. $(4d - 7)(4d + 7)$
    $16d^2 - 49$
15. $(3g + 9h)(3g - 9h)$
    $9g^2 - 81h^2$
16. $(4q + 5t)(4q - 5t)$
    $16q^2 - 25t^2$
17. $(a + 6u)^2$
    $a^2 + 12au + 36u^2$
18. $(5r + s)^2$
    $25r^2 + 10rs + s^2$
19. $(6c - m)^2$
    $36c^2 - 12cm + m^2$
20. $(k - 6y)^2$
    $k^2 - 12ky + 36y^2$
21. $(u - 7p)^2$
    $u^2 - 14up + 49p^2$
22. $(4b - 7v)^2$
    $16b^2 - 56bv + 49v^2$
23. $(6n + 4p)^2$
    $36n^2 + 48np + 16p^2$
24. $(5q + 6s)^2$
    $25q^2 + 60qs + 36s^2$
25. $(6a - 7b)(6a + 7b)$
    $36a^2 - 49b^2$
26. $(8h + 3d)(8h - 3d)$
    $64h^2 - 9d^2$
27. $(9x + 2y^2)^2$
    $81x^2 + 36xy^2 + 4y^4$
28. $(3p^3 + 2m)^2$
    $9p^6 + 12p^3m + 4m^2$
29. $(5a^2 - 2b)^2$
    $25a^4 - 20a^2b + 4b^2$
30. $(4m^3 - 2t)^2$
    $16m^6 - 16m^3t + 4t^2$
31. $(6e^3 - c)^2$
    $36e^6 - 12e^3c + c^2$
32. $(2b^2 - g)(2b^2 + g)$
    $4b^4 - g^2$
33. $(2v^2 + 3e^2)(2v^2 + 3e^2)$
    $4v^4 + 12v^2e^2 + 9e^4$

34. **GEOMETRY** Janelle wants to enlarge a square graph that she has made so that a side of the new graph will be 1 inch more than twice the original side s. What trinomial represents the area of the enlarged graph? $4s^2 + 4s + 1$

GENETICS For Exercises 35 and 36, use the following information.

In a guinea pig, pure black hair coloring B is dominant over pure white coloring b. Suppose two hybrid Bb guinea pigs, with black hair coloring, are bred.

35. Find an expression for the genetic make-up of the guinea pig offspring.
    $0.25BB + 0.50Bb + 0.25bb$
36. What is the probability that two hybrid guinea pigs with black hair coloring will produce a guinea pig with white hair coloring? 25%

NAME _____ DATE _____ PERIOD ____

8-8

Reading to Learn Mathematics

Special Products

Pre-Activity **When is the product of two binomials also a binomial?**

Read the introduction to Lesson 8-8 at the top of page 458 in your textook.

What is a meant by the term *trinomial product*?

a three-term polynomial answer when multiplying polynomials

Reading the Lesson

1. Refer to the Key Concepts boxes on pages 458, 459, and 460.

a. When multiplying two binomials, there are three special products. What are the three special products that may result when multiplying two binomials?

square of a sum, square of a difference, product of a sum and a difference

b. Explain what is meant by the name of each special product.

square of a sum: squaring the sum of two monomials; square of a difference: squaring the difference of two monomials; product of a sum and a difference: the product of the sum and the difference of the same two terms

c. Use the examples in the Key Concepts boxes to complete the table.

	Symbols	Product	Example	Product
Square of a Sum	$(a + b)^2$	$a^2 + 2ab + b^2$	$(x + 7)^2$	$x^2 + 14x + 49$
Square of a Difference	$(a - b)^2$	$a^2 - 2ab + b^2$	$(x - 4)^2$	$x^2 - 8x + 16$
Product of a Sum and a Difference	$(a + b)(a - b)$	$a^2 - b^2$	$(x + 9)(x - 9)$	$x^2 - 81$

2. What is another phrase that describes the product of the sum and difference of two terms? **difference of squares**

Helping You Remember

3. Explain how FOIL can help you remember how many terms are in the special products studied in this lesson. **For the square of a sum and the square of a difference, the inner and outer products are equal, so there are are only three terms. For the product of the sum and difference of two terms, two of the products for FOIL are opposites. That means that the final product has only two terms.**

NAME _____ DATE _____ PERIOD ____

8-8

Enrichment

Sums and Differences of Cubes

Recall the formulas for finding some special products:

Perfect-square trinomials: $(a + b)^2 = a^2 + 2ab + b^2$ or $(a - b)^2 = a^2 - 2ab + b^2$

Difference of two squares: $(a + b)(a - b) = a^2 - b^2$

A pattern also exists for finding the cube of a sum $(a + b)^3$.

1. Find the product of $(a + b)(a + b)(a + b)$.

$a^3 + 3a^2b + 3ab^2 + b^3$

2. Use the pattern from Exercise 1 to evaluate $(x + 2)^3$.

$x^3 + 6x^2 + 12x + 8$

3. Based on your answer to Exercise 1, predict the pattern for the cube of a difference $(a - b)^3$.

$a^3 - 3a^2b + 3ab^2 - b^3$

4. Find the product of $(a - b)(a - b)(a - b)$ and compare it to your answer for Exercise 3.

$a^3 - 3a^2b + 3ab^2 - b^3$

5. Use the pattern from Exercise 4 to evaluate $(x + 4)^3$.

$x^3 + 12x^2 + 48x + 64$

Find each product.

6. $(x + 6)^3$

$x^3 + 18x^2 + 108x + 216$

7. $(x - 10)^3$

$x^3 - 30x^2 + 300x - 1000$

8. $(3x - y)^3$

$27x^3 - 27x^2y + 9xy^2 - y^3$

9. $(2x - y)^3$

$8x^3 - 12x^2y + 6xy^2 - y^3$

10. $(4x + 3y)^3$

$64x^3 + 144x^2y + 108xy^2 + 27y^3$

11. $(5x + 2)^3$

$125x^3 + 150x^2 + 60x + 8$

Chapter 8 Assessment Answer Key

Form 1

Page 503

1. B
2. A
3. C
4. C
5. D
6. D
7. B
8. C
9. A
10. A
11. B
12. C

Page 504

13. D
14. C
15. A
16. C
17. D
18. B
19. A
20. A
21. C
22. D
23. D
24. B
25. B

B: $3^{9n + 1}$

Form 2A

Page 505

1. C
2. A
3. D
4. C
5. D
6. D
7. A
8. B
9. C
10. C
11. A

(continued on the next page)

Chapter 8 Assessment Answer Key

Form 2A *(continued)*
Page 506

12. D

13. A

14. C

15. B

16. A

17. C

18. C

19. A

20. C

B: 7^{-2x-2}

Form 2B
Page 507

1. B

2. B

3. C

4. D

5. B

6. B

7. D

8. C

9. D

10. A

11. C

Page 508

12. B

13. D

14. D

15. A

16. A

17. B

18. D

19. A

20. C

B: 3^{7n-1}

Answers

Chapter 8 Assessment Answer Key

Form 2C

Page 509

1. $3y^8$
2. $-18m^4n^7$
3. $w^{15}y^{12}$
4. $8a^3n^6 + 4a^{18}n^6$
5. p^3q
6. $\frac{4r^4}{s^7}$
7. $\frac{y}{x^5}$
8. 4.98×10^{-4}
9. 127,000
10. 1.0×10^5; 100,000
11. about 1.09×10^2 or 109 times greater
12. 6
13. $-x^5y + 3x^3y^3 + xy + 4$

Page 510

14. $-4n^2 + 6ny + 13y^2$
15. $19m^2 + 2mn + 6n^2$
16. $-x^2 - y$
17. $10h^3k^3 - 5h^2k^5 + 20h^3k^4$
18. $8x^4 - 2y^4$
19. $6s^3 - 14s^2 - 22s + 30$
20. $25c^2 - 40c + 16$
21. $49a^2 - 9b^2$
22. $16n^2 + 8n + 1$
23. $2\frac{2}{5}$
24. 6
25. length is 20 ft; width is 12 ft

B: 21 terms

Chapter 8 Assessment Answer Key

Form 2D

Page 511

1. $5x^7$
2. $-6a^3b^8$
3. w^9z^{21}
4. $10a^4b^8$
5. $-\frac{3}{m^2}$
6. $\frac{d^7}{2b^5}$
7. $3r^5s$
8. 1.2556×10^4
9. 0.00743
10. about 1.43×10^{10} or 14,300,000,000 in.
11. 2×10^7; 20,000,000
12. 5
13. $x^3 + 3x^2y - xy^2 - 3y^3$

Page 512

14. $4m^2 - 6m + 1$
15. $8y^2 - 4y - 9$
16. $a^3 - 3b$
17. $6x^4y^2 - 15x^3y^3 + 24x^4y^4$
18. $9r^4 - 25s^4$
19. $6n^3 + n^2 - 10n + 3$
20. $25y^2 + 60y + 36$
21. $4k^2 - 25r^2$
22. $4c^2 - 4c + 1$
23. 13
24. –2
25. length is 10 ft; width is 5 ft

B: 11 terms

Answers

Chapter 8 Assessment Answer Key

Form 3

Page 513

1. $3a^7b^{10}$

2. $\frac{16}{81}h^{12}$

3. $25r^4s^{14}$

4. 4^{5x-2}

5. $-\frac{6d^3}{c^4}$

6. $\frac{mx^{12}}{128}$

7. $-\frac{16}{125a^{11}b}$

8. 1.96783×10^5

9. 5.832×10^{-4}; 0.0005832

10. 9.0×10^{-6}; 0.000009

11. $7x^6y^3 + 4x^5y^2 + 2xy - 6$

12. 6

Page 514

13. $10w^2 + 3w + 4$

14. $10u^2v - 7uv + 6uv^2$

15. $3x^2 + x - 5y$

16. $-n^3 - 25n^2 + 12n - 21$

17. $8y^2 - 20y - 28$

18. $\frac{1}{3}m^2 - \frac{11}{6}m + 2$

19. $8x^3 - 2x^2y + 19xy^2 + 5y^3$

20. $25r^4 - 9s^4$

21. $(9x^2 - 6x + 1)$ ft^2

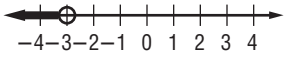
22. 4

23. $2a^2 + 7a + 5$

24. 5

25. length is 16 cm; width is 12 cm

B:



Answers

Chapter 8 Assessment Answer Key

Page 515, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *multiplying and dividing monomials, using scientific notation, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *multiplying and dividing monomials, using scientific notation, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *multiplying and dividing monomials, using scientific notation, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Graphs may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *multiplying and dividing monomials, using scientific notation, the degree of a polynomial,* and *adding, subtracting, and multiplying polynomials.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 8 Assessment Answer Key

Page 515, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A31, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1a. Students should recognize that the v terms were not subtracted correctly. Since $-5v - (+v) = -5v - v = -6v$, the right side of the equation should be $3u - 6v$.

1b. Students should recognize that the Product of a Power property was not used correctly to multiply $x^2y(3x^3)$. The right side of the equation should be $3x^5y + 4x^2y$.

1c. Students should recognize that the pattern for the Square of a Sum was not used correctly. The middle term $2(3a)(5b)$ was omitted. Since $(3a + 5b)^2 = (3a)^2 + 2(3a)(5b) + (5b)^2$, the right side of the equation should be $9a^2 + 30ab + 25b^2$.

2a. The number 23.4×10^8 is not in scientific notation. A number in scientific notation is of the form $a \times 10^b$ where $1 \leq a < 10$. The number needs to be adjusted to fit the form of scientific notation.
$23.4 \times 10^8 = 2.34 \times 10 \times 10^8$
$= 2.34 \times 10^9$

2b. Using scientific notation for division of very large and small numbers results in the division of two numbers that are between one and ten. The correct decimal place is found by using the Quotient of Powers property on two powers of ten. Thus, $22{,}100{,}000 \div 0.00000013 = 2.21 \times 10^7 \div 1.3 \times 10^{-7}$
$= \dfrac{2.21 \times 10^7}{1.3 \times 10^{-7}} = \dfrac{2.21}{1.3} = \dfrac{10^7}{10^{-7}} = 1.7 \times 10^{14}$

3a. Sample Answer: The binomial $x + 1$ has degree 1.

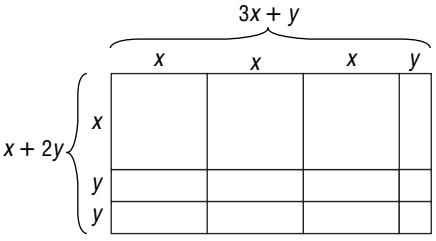
3b. Sample Answer: The trinomial $x^2 + x + 1$ has degree 2, yet the monomial x^3 has degree 3.

4a. $\left(\dfrac{4a^3}{2a^{-2}}\right)^4 = (2a^5)^4 = 16a^{20}$

4b. $\left(\dfrac{4a^3}{2a^{-2}}\right)^4 = \dfrac{(4a^3)^4}{(2a^{-2})^4} = \dfrac{256a^{12}}{16a^{-8}} = 16a^{20}$

4c. Sample answer: When simplifying monomials, the order of applying the Quotient of Powers property and Power of a Quotient property does not matter.

5a.



$3x^2 + 7xy + 2y^2$
The length of the rectangular region is $3x + y$. The width of the region is $x + 2y$.The rectangular area is made up of 3 x^2-areas, 7 xy-areas, and 2 y^2-areas. When added together the area of the rectangular region is exactly equal to the product of the two binomials.

Chapter 8 Assessment Answer Key

Vocabulary Test/Review
Page 516

1. e
2. c
3. g
4. f
5. i
6. a
7. j
8. b
9. d
10. h
11. the sum of the exponents of all the variables
12. the greatest degree of any term
13. a number written as a product of a factor greater than or equal to 1 and less than 10 and a power of 10

Quiz (Lessons 8–1 and 8–2)
Page 517

1. $2r^8$
2. x^{20}
3. $-12m^3n^7$
4. $-125x^{12}y^6$
5. $64c^8d^2$
6. $27y^4w^8$
7. 6^6 or 46,656
8. y^5
9. $\frac{r^2}{n^9}$
10. C

Quiz (Lessons 8–3 and 8–4)
Page 517

1. 5.76×10^4
2. 6.1×10^{-6}
3. 0.0064871
4. 1.6×10^{-7}; 0.00000016
5. 2.87×10^{10}; 28,700,000,000
6. 2
7. 4
8. $-3x^3 + 4x^2 + 2x + 12$
9. $5x^3y - x^2y^3 + 3xy^4 + y^4$
10. $-3 + 4x + x^2$

Quiz (Lessons 8–5 and 8–6)
Page 518

1. $3x^2 - 3x + 4$
2. $3a + 5b$
3. $6x^3y + 15x^2y^2 - 21xy^3$
4. $-3c^3 + 74c^2 - 4c$
5. 2

Quiz (Lesson 8–7)
Page 518

1. $4m^2 + 7m - 2$
2. $2c^3 - 11c^2 + 22c - 15$
3. $16h^2 + 24h + 9$
4. $a^2 - 18a + 81$
5. $4x^2 - 36y^2$
6. $m^4 + 4m^2n + 4n^2$
7. $81x^2 - 49$
8. $a^2 - 9b^2$
9. $(4a^2 - 4)$ units^2
10. $(4a^3 + 12a^2 - 4a - 12)$ units^3

Answers

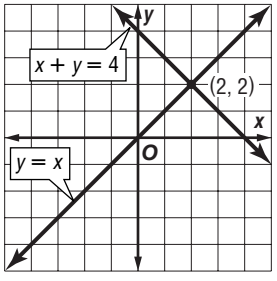
Chapter 8 Assessment Answer Key

Mid-Chapter Test
Page 519

1. B
2. A
3. A
4. A
5. C
6. D
7. A
8. 2.34×10^2; 234
9. 7.553×10^{-4}; 0.0007553
10. $\frac{2}{5x^9y}$
11. $\frac{5m^6y^3r^5}{7}$
12. 4
13. $4x^3 + 2x^2 - 3x + 6$
14. $8x^4 + 2a^2x^3 - 3a^4x^2 + 8x$

Cumulative Review
Page 520

1. 10
2. $c = b - \frac{1}{b}$
3. $f(x) = 2x - 2$
4. \$122.50 per yr.
5. $y = 4x - 9$
6. $\{u \mid u > 3\}$
7. $\left\{y \mid y \leq 3\frac{2}{3}\right\}$
8. $8x + 12y \leq 62$
9. one solution; (2, 2)



10. elimination (x); (1, –1)
11. $\frac{3y^2}{x^3}$
12. 1.38×10^9; 1,380,000,000
13. $\frac{1}{3}a^2 - a - \frac{3}{5}$
14. $9x^2 + 12x + 4$

Chapter 8 Assessment Answer Key

Standardized Test Practice

Page 521

1. (A) (B) **(C)** (D)

2. (E) **(F)** (G) (H)

3. **(A)** (B) (C) (D)

4. (E) (F) (G) **(H)**

5. **(A)** (B) (C) (D)

6. (E) (F) **(G)** (H)

7. **(A)** (B) (C) (D)

8. (E) **(F)** (G) (H)

9. (A) (B) **(C)** (D)

10. **(E)** (F) (G) (H)

Page 522

11.

1	0	8	
	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	**8**	8
9	9	9	9

12.

3	.	5	0
	/	/	
.	.	.	.
	0	0	**0**
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	**5**	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13.

5			
	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14.

.	0	5	7
	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	**5**	5
6	6	6	6
7	7	7	**7**
8	8	8	8
9	9	9	9

15. (A) **(B)** (C) (D)

16. (A) (B) **(C)** (D)

17. **(A)** (B) (C) (D)

Answers