

CHAPTER TEN: DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

Unlike other standardized admissions tests, you do not have to remember a copious number of rules and endless amounts of material for the SAT. The College Board uses a finite number of skills to test your ability to reason critically. Therefore, the SAT math sections can be conquered given intense study and repeated practice. The more exposure you have to real SAT questions, the fewer surprises await you on test day.

As we discussed earlier, much of the test content and many of the required skills are quite basic—such as remainders or fractions. You may find that some of the content review in the following chapters seems elementary and inappropriate to high school students. We urge you to read through it, however, as tips, tricks, and shortcuts are mentioned in the content area discussion. We often advise you to solve problems much differently than the ways you were previously taught, so skipping content can cause you to miss important information.

Each content section (such as Counting Problems or Sequences) contains two parts: the “Required Knowledge and Skill Set” and “Application on the SAT.” The “Required Knowledge and Skill Set” is a review of basic concepts and applications involving the content area. The “Application on the SAT” section examines how this basic content may be presented on the test.

At the beginning of each content area, in the notes column near the edge of the page, is a frequency guide where the content is assigned a number. This number indicates the likelihood that this content will be tested on your SAT. Based on PowerScore’s extensive analysis of real tests, you can use the following key to predict the general frequency of each question type:

Rating	Frequency
5	Extremely High: 3 or more questions typically appear on every SAT
4	High: at least 2 questions typically appear on every SAT
3	Moderate: at least one question typically appears on every SAT
2	Low: one question typically appears on every two or three SATs
1	Extremely Low: one question appears infrequently and without a pattern

A Frequency Guide is provided for every type of problem so you can prioritize the content you need to study most.

This book contains many examples and explanations of multiple-choice and student-produced response questions. It is important to understand how these questions are numbered throughout the book so that you can learn to judge a question's difficulty. All of the multiple-choice questions are numbered 1 through 20, just as they are on the longest multiple-choice section of the SAT. The first 5 to 7 questions are typically Easy and the last 2 to 4 are usually Hard. The questions in between have a Medium difficulty level. Keep in mind that question number 8 is probably easier than number 15, even though they are both considered Medium level questions.

Examples of student-produced response questions are numbered 9 through 18, just as they are on the test. Questions 9 and 10 should be considered Easy, while those numbered 11 through 16 are Medium. A question that is listed as 17 or 18 is Hard.

Following each individual content review is a short problem set. An answer key is provided at the end of the chapter, where each question is assigned a degree of difficulty. If you struggle with any Easy or Medium difficulty level questions, reread the content section and then use the map to find additional practice questions in the "Blue Book Database," located on the book website (www.powerscore.com/satmathbible). It is only through repeated practice that you can become confident with a specific question type.

Remember that there are eight solution strategies you can employ on SAT math questions:

1. ANALYZE the Answer Choices
2. BACKPLUG the Answer Choices
3. SUPPLY Numbers
4. TRANSLATE from English to Math
5. RECORD What You Know
6. SPLIT the Question into Parts
7. DIAGRAM the Question
8. SIZE UP the Figures

Be sure to check your answers in the answer key following the chapter. It is important to understand why you missed a particular question, in order to avoid making this same mistake on the SAT.

Score increases are dependent upon repeated practice! Be sure to complete the questions in each Problem Set.

The PowerScore SAT Solutions Strategies are fully explained in Chapter Four.

The final math content area of the SAT includes data analysis, statistics, and probability. Because many high school students have not taken a statistics course, they find these questions especially intimidating. However, these test takers make ideal PowerScore students, as there are no bad habits to unlearn or excess information to disregard. You simply need to learn the concepts in this chapter to be prepared to face any statistics or probability question on the SAT.

This chapter will explore the following concepts and explain how they are tested on the SAT:

1. Data Analysis
2. Average, Median, Mode
3. Counting Problems
 - A. Combinations
 - B. Permutations
4. Probability
5. Sequences
6. Overlapping Groups
7. Logical Reasoning

Data Analysis, Statistics, and Probability questions typically account for 10% to 20% of the SAT Math questions. Understanding this content is essential to your success on the SAT, so be sure to tackle the questions in the Blue Book Database for extra practice upon completing the problem sets in this section of the book. Good luck!

Do not be intimidated by this section; we will give you all the tools you need to succeed!

Data Analysis

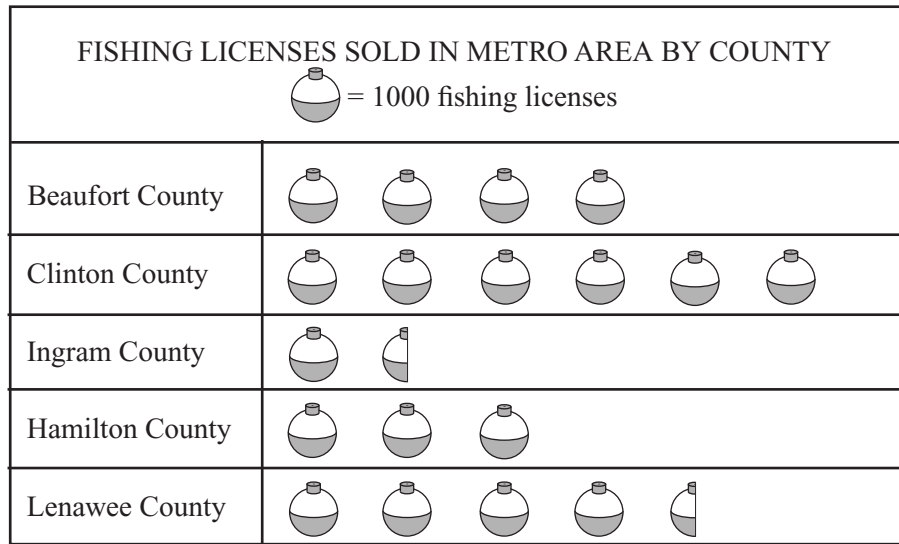
Frequency Guide: 4

Data analysis questions use diagrams, figures, tables, or graphs in conjunction with arithmetic and algebra.

Required Knowledge and Skill Set

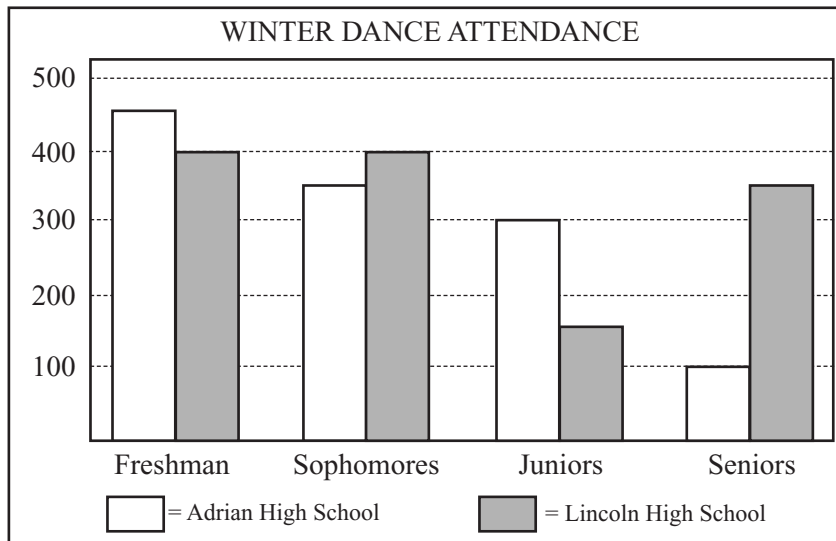
- You must be able to read and interpret several graphs, including a pictograph, bar graph, pie graph, line graph, and scatterplot.

A pictograph uses pictures to represent data:



How many fewer fishing licenses were sold in Hamilton County than Clinton County? Answers to these questions appear in the margin on page 400.

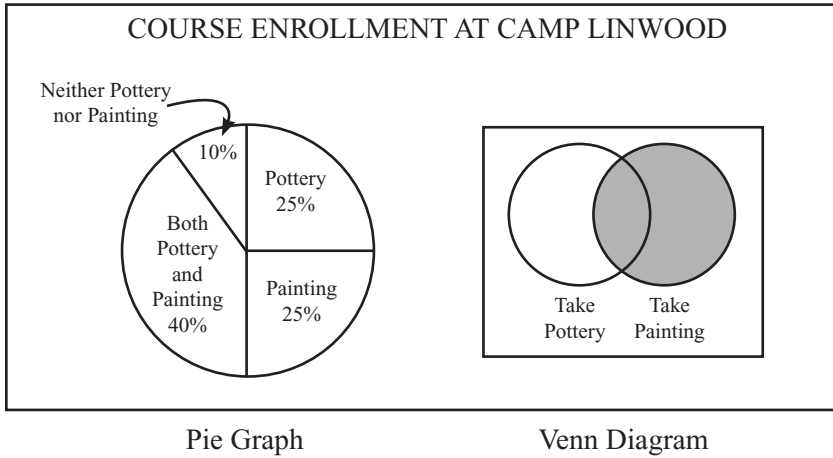
A bar graph uses horizontal or vertical bars to represent data:



Which school had the most students attend the Winter Dance?

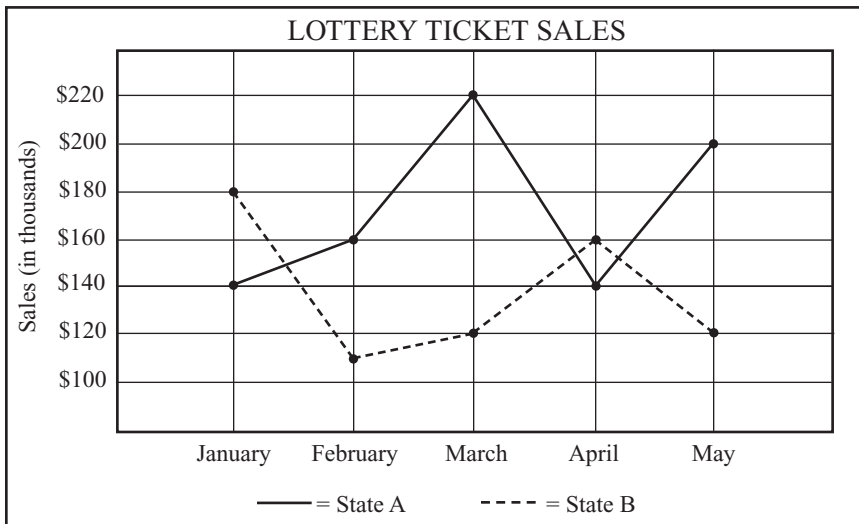
DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

A pie graph, often used to represent percentages, uses a circle to display data. A Venn Diagram uses two circles to represent data:



If there are 500 students at Camp Linwood, how many are in Pottery class only?

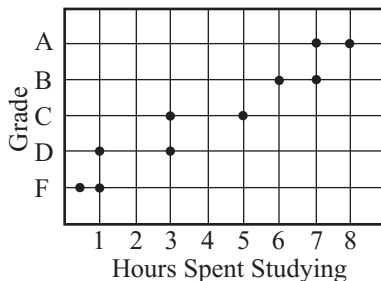
A line graph plots data on a graph and connects the points to form a line:



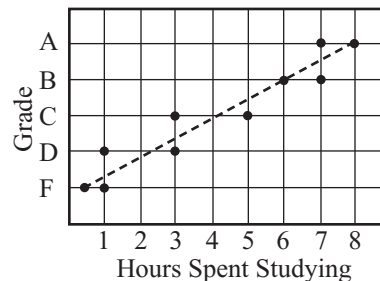
What was the difference in lottery sales between State A and State B in March?

A scatterplot also plots data on the graph, but the data is not connected by a line. If the points are clustered closely together, you may be able to draw a line to show a trend in data:

HOURS STUDIED PER GRADE FOR 10 STUDENTS



HOURS STUDIED PER GRADE FOR 10 STUDENTS



Scatterplots tend to be the most difficult Data Analysis questions because fewer students are exposed to them.

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

2. Read graphs very carefully! Take note of titles, headings, scale, units, and other information presented.
3. For line graphs, bar graphs, and scatterplots, be prepared to **SIZE UP** the figure and estimate an answer. Visually accurate figures can often be solved just by analyzing the graph or the diagram.
4. Tables organize information in an easy-to-read format:

EMPLOYEE BENEFITS		
Years Worked	Vacation Days	Commission Percentage
1 to 5	10	4%
5 to 9	14	9%
10 to 20	17	16%
20 or more	21	25%

What is the difference between the month with the most tourists and the month with the fewest tourists?

Month	Number of Tourists
May	40,000
June	80,000
July	150,000
August	110,000
September	30,000
October	15,000

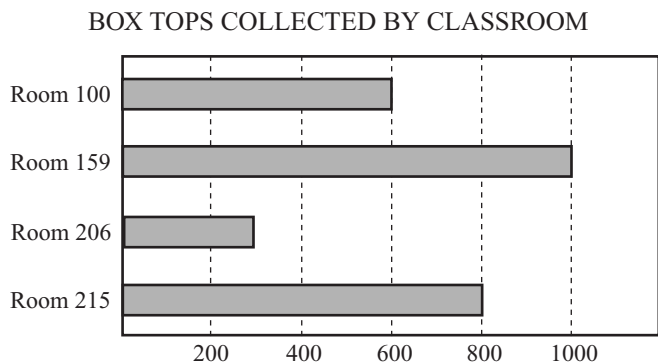
CLIFFORD HIGH SCHOOL'S JV BASEBALL RESULTS					
Date	Home Team	Score	Away Team	Score	Record
Mar. 28	Clifford	5	Lorraine	2	1-0
Apr. 2	Westbrook	8	Clifford	9	2-0
Apr. 6	Clifford	2	Highlands	4	2-1
Apr. 8	Clifford	5	Chesterfield	0	3-1
Apr. 16	Clifford	3	Lawrence	2	4-1
Apr. 21	Northfield	7	Clifford	2	4-2
Apr. 27	Jamestown	12	Clifford	0	4-3
May 1	Clifford	6	Canton	5	5-3
May 6	Naleen	0	Clifford	1	6-3
May 9	Clifford	7	Washington	4	7-3

ANSWERS

Fishing: 3000
 Dance: Lincoln
 Camp: 125
 Lottery: \$100,000
 Tourists: 135,000

Application on the SAT

Most graph questions on the SAT simply ask you to interpret data in the figure. Be prepared to apply arithmetic or algebra when interpreting the graph. Consider an example:



12. The graph above illustrates the number of box tops collected by four classrooms during a school contest. The sum of the box tops collected by the two rooms with the fewest box tops is approximately what percent of the sum of the box tops collected by the two rooms with the most box tops?
- (A) 30%
 - (B) 50%
 - (C) 65%
 - (D) 70%
 - (E) 80%

To begin, find the sum of the box tops collected by the two rooms with the least number of box tops:

$$\begin{array}{r} \text{Room 100:} \quad 600 \\ \text{Room 206:} \quad \underline{300} \quad (\text{approximately}) \\ \hline 900 \end{array}$$

Now find the sum of the box tops collected by the two rooms with the largest number of box tops:

$$\begin{array}{r} \text{Room 159:} \quad 1000 \\ \text{Room 215:} \quad \underline{800} \\ \hline 1800 \end{array}$$

Now TRANSLATE:

The sum of the fewest is what percent of the sum of the most

$$900 = \frac{x}{100} \times 1800 \rightarrow 90,000 = 1800x \rightarrow 50 = x$$

The correct answer is (B), 50%.

Data Analysis questions test not only your ability to read tables and graphs, but also your ability to apply arithmetic, algebra, probability, or statistics to the information in the figure.

Some questions will provide data and ask you to choose the graph that illustrates that data. These are usually line graph questions:

LENGTH OF A FISH

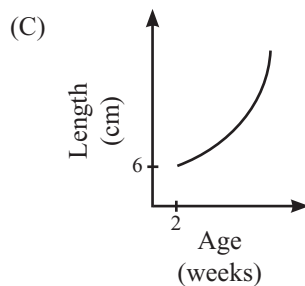
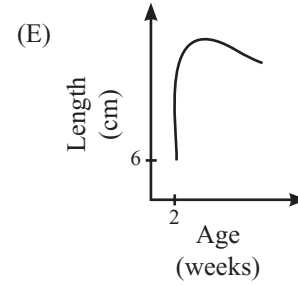
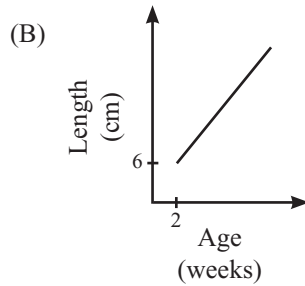
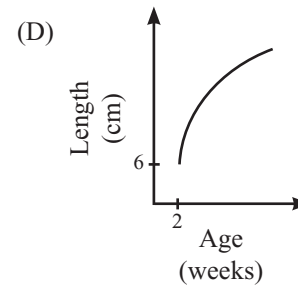
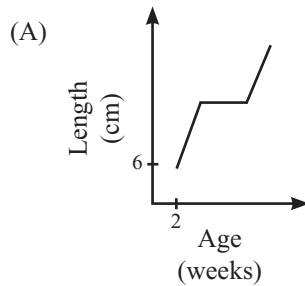
Age (in weeks)	2	3	4	5	6
Length (in centimeters)	6	15	20	22	23

12. The measurements of a certain fish at different ages are given in the table above. Which of the following graphs could represent the information in the table?

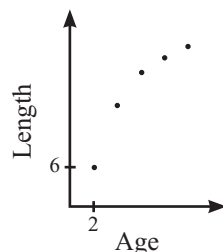
Confidence Quotation

"We gain strength and courage and confidence by each experience in which we really stop to look fear in the face. We must do that which we think we cannot."

—Eleanor Roosevelt,
First Lady of the United States and civil rights activist



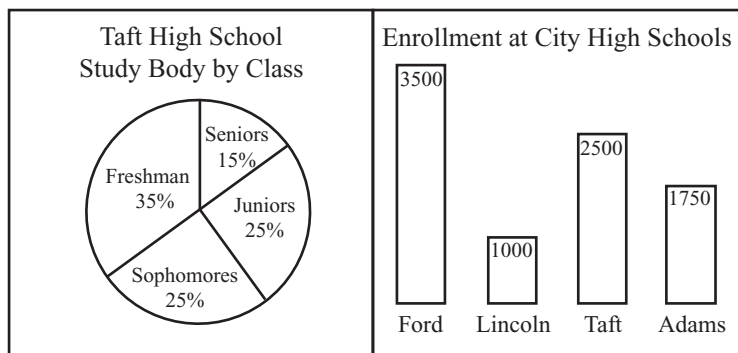
To solve this question, draw a rough sketch of the graph and plot the points:



If you drew a line through the points, which of the answer choices would your sketch resemble?

The correct answer is choice (D).

You should be prepared to compare two graphs displaying the same or related information:



7. According to the graphs above, how many seniors are enrolled at Taft High School?
- (A) 375
 - (B) 525
 - (C) 750
 - (D) 1500
 - (E) 2125

You must use information from both graphs to solve this question. First, find the total student enrollment at Taft High School. According to the bar graph, total enrollment is 2500 students.

Now use the information in the pie graph. Fifteen percent of the students at Taft High School are seniors:

$$15\% \text{ of } 2500 = ?$$

$$0.15 \times 2500 = 375$$

The correct answer is (A).

Data Analysis questions are sometimes accompanied by two or three questions. The text above the graph will alert you to this situation by saying something like "Questions 7 and 8 refer to the information in the following graphs."

8. According to the information in the graphs, how many more freshman are enrolled than sophomores at Taft High School?
- (A) 125
 - (B) 250
 - (C) 375
 - (D) 500
 - (E) 575

$$\begin{array}{l} \text{Freshman: } 35\% \text{ of } 2500 = 875 \\ \text{Sophomores: } 25\% \text{ of } 2500 = \underline{-625} \\ \hline 250 \end{array}$$

The correct answer is (B).

Expect at least one question that compares two or more graphs or tables. In fact, these types of questions often have two questions associated with the same figure.

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

Tables are the most common figure in Data Analysis questions.

Tables are prominently featured on the SAT. Some questions with tables will ask you to use arithmetic to solve a problem using information in a completed table:

SALES OF WILDLIFE TOURS

Price of Tour	Number of Purchased Tours
\$5.00	120,000
\$10.00	95,000
\$20.00	65,000

10. A wildlife company offered tours for three different prices during a single year. Based on the information above, how much more money did the company make when the price was \$20.00 than when the price was \$5.00 ?
- (A) \$35,000
 - (B) \$70,000
 - (C) \$350,000
 - (D) \$700,000
 - (E) \$1,050,000

Find the total sales of the \$5.00 tickets and the \$20.00 tickets:

$$\begin{aligned} \$5.00 \times 120,000 &= \$600,000 \\ \$20.00 \times 65,000 &= \$1,300,000 \end{aligned}$$

Now simply subtract the smaller amount from the larger amount:

$$\$1,300,000 - \$600,000 = \$700,000$$

The correct answer is (D).

Later in this chapter we will cover averages, medians, modes, counting problems, and probability. All of these statistics topics have appeared on the SAT in questions with tables, so expect to see some completed tables testing your ability with these subject areas.

Questions with completed tables usually require arithmetic.

Other table questions will require you to use algebra to find missing information:

LISA'S HORSE EXPENSES

	Boarding	Lessons	Total
January		\$50	
February		\$40	
March		\$80	
Total			\$260

9. The table above, which is missing some information, shows Lisa's expenses for keeping a horse. If her boarding costs were the same each month, what were her total expenses for February?
- (A) \$30
 - (B) \$50
 - (C) \$70
 - (D) \$80
 - (E) \$110

Use a variable to represent the boarding expenses. Since the cost is the same every month, use the same variable:

LISA'S HORSE EXPENSES

	Boarding	Lessons	Total
January	b	\$50	
February	b	\$40	
March	b	\$80	
Total			\$260

Now, write an algebraic equation that solves for b :

$$(b + 50) + (b + 40) + (b + 80) = 170$$

$$3b + 170 = 260$$

$$3b = 90$$

$$b = 30$$

Many students would select answer choice (A) and move on. But they would be wrong; the question asks for the total expenses in February:

LISA'S HORSE EXPENSES

	Boarding	Lessons	Total
January	\$30	\$50	
February	\$30	\$40	\$70
March	\$30	\$80	
Total			\$260

The correct answer is (C).

Questions with incomplete tables usually require algebra.

Learning to reread each question after finding a solution is an invaluable practice on the SAT.

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

Data Analysis Problem Set

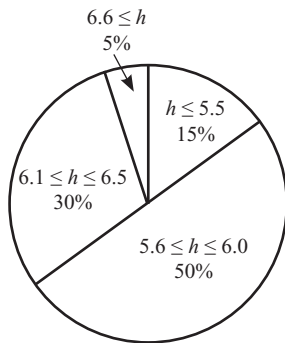
Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 450.

APPROXIMATE CONVERSIONS

Number of Gallons	2	4	x
Number of Liters	7.6	15.2	30.4

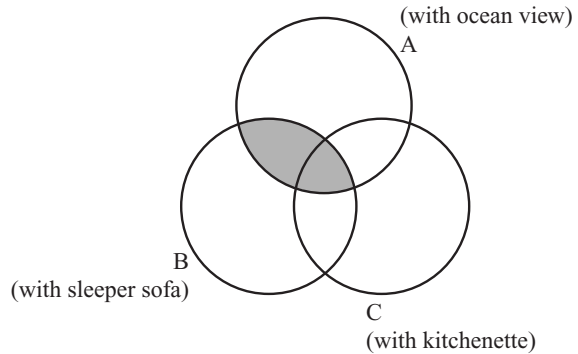
1. The table above shows approximate conversions from gallons to liters. What is the value of x ?
- (A) 6
 (B) 8
 (C) 10
 (D) 12
 (E) 16

Height of Seniors



2. The seniors at Woodhaven High School are being measured for their caps and gowns for graduation. The figure above shows their height (h), in feet. For example, 30% of the seniors are 6.1 feet to 6.5 feet tall. If there are 760 seniors at Woodhaven High School, how many are 6.0 feet tall or less?
- (A) 114
 (B) 228
 (C) 380
 (D) 494
 (E) 722

CONDOMINIUMS

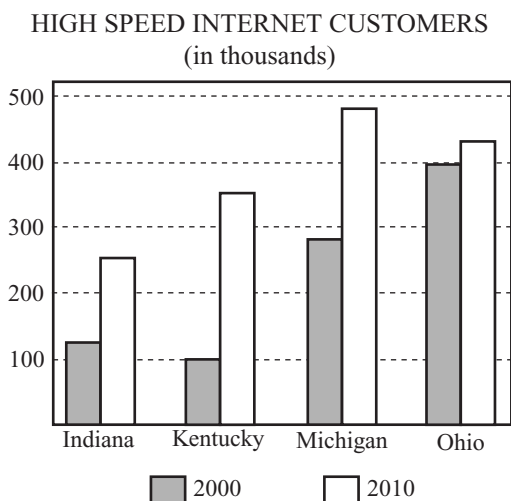


3. In the figure above, three circles represent condominiums on the beach. Circle A represents condominiums with an ocean view, Circle B represents condominiums with a sleeper sofa, and Circle C represents condominiums with a kitchenette. What does the shaded region represent?
- (A) Condominiums with an ocean view, sleeper sofa, and kitchenette
 (B) Condominiums with an ocean view and sleeper sofa, but without kitchenettes
 (C) Condominiums with an ocean view and sleeper sofa (some possibly with kitchenettes)
 (D) Condominiums with an ocean view and kitchenette (some possibly with a sleeper sofa)
 (E) Condominiums with a sleeper sofa and kitchenette (some possibly with an ocean view)

Data Analysis Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 450.

Questions 4 and 5 refer to the following figure:



4. The table above shows a cable company's high speed internet customers in four states in 2000 and 2010. The number of customers in Kentucky in 2000 was approximately what percent of the number of customers in Ohio in 2000?
 - (A) 15%
 - (B) 20%
 - (C) 25%
 - (D) 30%
 - (E) 35%

5. From 2000 to 2010, the total number of high speed internet customers in the four states was increased by approximately what percent?
 - (A) 25%
 - (B) 33%
 - (C) 50%
 - (D) 58%
 - (E) 67%

Color	Number of People Choosing Color
Blue	176
Green	x
Red	97
Yellow	y

6. In a survey, 500 people were asked to choose their favorite color among blue, green, red, and yellow. Each person chose exactly one color. The results of the survey are given in the table above. If x and y are positive integers, what is the greatest possible value of x ?
 - (A) 77
 - (B) 114
 - (C) 226
 - (D) 227
 - (E) 500

Average, Median, and Mode

Frequency Guide: 4

The average, median, and mode are statistical data that occur frequently on the SAT. They are easy to remember if you study the following information.

Required Knowledge and Skill Set

1. The average is always referred to as the “average (arithmetic mean)” on the SAT, so you do not have to memorize that the mean is the average.
2. The formula for averages is needed for every average problem on the SAT:

$$\frac{\text{sum of the numbers}}{\text{number of numbers}} = \text{average}$$

Use a shorthand version of this formula to save time on the test:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average}$$

This formula can also be used to find the sum or the number of numbers in a set:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \rightarrow \text{sum} = (\text{average})(\# \text{ of \#s})$$

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \rightarrow \# \text{ of \#s} = \frac{\text{sum}}{\text{average}}$$

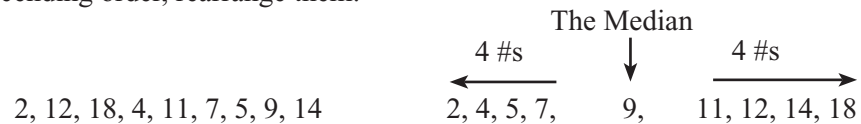
MEMORY MARKER:

This formula is required for every average question, and it's even used for some problems that do not appear to involve averages!

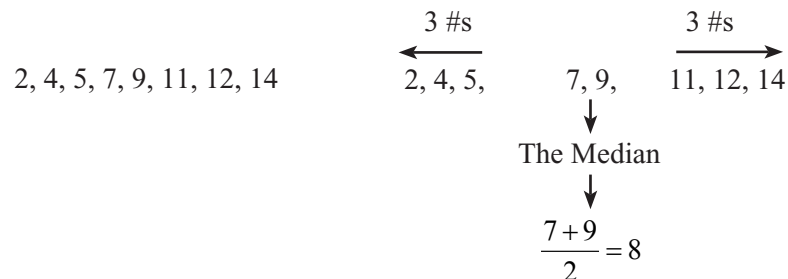
MEMORY MARKER:

Some students remember the median by thinking about the word “medium.” “Medium” is in the middle of “small” and “large,” just as the median is in the middle of small and large numbers.

3. The median is the middle number in a list of numbers placed in ascending or descending order. On the SAT, if the numbers are not listed in ascending or descending order, rearrange them:



4. Nearly all SAT questions concerning the median will use an odd number of items in the list. If you are presented with an even number of items, the median is the average of the two numbers in the middle:



5. In a patterned set of consecutive integers, the average and the median are the same:

5, 6, 7, 8, 9, 10, 11 average = 8, median = 8
 Pattern: increase by 1

36, 38, 40, 42, 44, 46 average = 41, median = 41
 Pattern: increase by 2

1055, 1060, 1065 average = 1060, median = 1060
 Pattern: increase by 5


-12, -6, 0, 6, 12, 18 average = 3, median = 3
 Pattern: increase by 6

The average and the median are not the same if any numbers repeat or if a pattern changes:



2, 3, 4, 5, 11 average = 5, median = 4

36, 38, 38, 40, 42 average = 38.8, median = 38

6. The mode is the most common number in a series:

The Mode
 3 occurrences

 7, 8, 8, 8, 11, 13, 16, 19, 19, 22

On the SAT, the mode is tested less often than the average and the median. If the mode is tested, there is usually only one in the series. However, there can be two modes. If another 19 appeared in the list above, the mode would be 8 *and* 19:

The Mode The Mode
 3 occurrences 3 occurrences
 
 7, 8, 8, 8, 11, 13, 16, 19, 19, 19, 22

7. Average, median, and mode questions are often combined with Data Analysis questions, as the series of numbers can be neatly organized in a table.

≧ARITHMETRICK≦
 The average and the median are always the same in a patterned list of consecutive integers!

MEMORY MARKER:
 You can remember that the mode is the most common number in a series by remembering "The mode is the most!" Both "mode" and "most" start with "mo-" and both have four letters.

Confidence Quotation

“They can conquer who believe they can.”

—Virgil, Roman poet

Variables are meant to intimidate you. Do not let them. Use your formula and treat the variables like any real value.

Application on the SAT

Average questions come in many varieties, all of which assess your understanding of how averages work. What happens to the average when a smaller number is added to a series? What is the largest possible value of one of the numbers if the list is made up of integers? Understanding these questions is important on the SAT.

An occasional average question is simple and straightforward, asking you to find the average given a list of integers. Since this is the same type of question asked in classroom math, you should have no problem using the average formula. But the SAT usually makes averages more intimidating by using variables:

8. The average (arithmetic mean) of 1, 2, 5, 9, and x is 4. The average (arithmetic mean) of 2, 3 and y is 6. What is the value of $x + y$?
- (A) 3
 - (B) 5
 - (C) 10
 - (D) 13
 - (E) 16

This question consists of two averages. Solve each separately, using the average formula:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average}$$

$$\frac{1+2+5+9+x}{5} = 4 \rightarrow 17+x=20 \rightarrow x=3$$

$$\frac{2+3+y}{3} = 6 \rightarrow 5+y=18 \rightarrow y=13$$

Now find $x + y$:

$$x + y = 3 + 13 = 16$$

The correct answer is (E).

Do not let variables intimidate you. By using the formula and inserting the information from the problem into the appropriate places in the formula, this question becomes a simple algebra problem.

You must understand what happens to an average question when numbers are added to or deleted from a set. Consider the following:

9. The average (arithmetic mean) of a set of seven numbers is 8. When an eighth number is added to the set, the average of the eight numbers is still 8. What number was added to the set?
- (A) 6
 (B) 7
 (C) 8
 (D) 9
 (E) 10

Remember, the average formula is needed for all average questions. Plug in the information you have about the set of seven numbers:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \quad \rightarrow \quad \frac{\text{sum}}{7} = 8 \quad \rightarrow \quad \text{sum} = 56$$

Now find the sum of the eight number set:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \quad \rightarrow \quad \frac{\text{sum}}{8} = 8 \quad \rightarrow \quad \text{sum} = 64$$

When an extra number was added to the set, the sum changed by 8 ($56 + 8 = 64$). Therefore, the number added to the set must be 8. The correct answer is (C).

An overwhelming majority of average problems will depend on you working with or finding the sum. Consider a more difficult grid-in question:

18. In a set of 5 positive integers, 56, 138, x , y , and z , all five integers are different and the average (arithmetic mean) is 300. If the integers x , y , and z are greater than 138, what is the greatest possible value for any of the integers?

Use the average formula to find the sum of x , y , and z :

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \quad \rightarrow$$

$$\frac{56 + 138 + x + y + z}{5} = 300 \quad \rightarrow \quad 194 + x + y + z = 1500 \quad \rightarrow \quad x + y + z = 1306$$

Say that z is the integer with the greatest possible value. That means x and y must be as small as possible. Since they are greater than 138, the smallest they can be is 139 and 140:

$$139 + 140 + z = 1306 \quad \rightarrow \quad 279 + z = 1306 \quad \rightarrow \quad z = 1027$$

The greatest possible value for any of the integers is 1027. You must understand that in order for one number to be large as it can be, the others must be as small as possible without violating any of the rules set forth in the question.

The sum is the key to many average questions on the SAT.

Remember, use the average formula any time you see the words "average (arithmetic mean)" on the SAT.

In classroom math, these questions are referred to as “weighted averages.”

Another difficult problem concerns combining averages. This type of question occurs less frequently than the previous example.

17. Two classes were given a math test. The first class had 25 students and the average test score was 86%. The second class had 15 students and their average score was 94%. If the teacher combined the test scores of both classes, what is the average of both classes together?
- (A) 88%
 (B) 89%
 (C) 90%
 (D) 91%
 (E) 92%

Can you guess what we must find for each class? That’s right—the sum!

$$\frac{\text{sum}}{\# \text{ of } \#s} = \text{average} \rightarrow \frac{\text{sum}_{\text{Class1}}}{25} = 0.86 \rightarrow \text{sum}_{\text{Class1}} = 21.5$$

$$\frac{\text{sum}}{\# \text{ of } \#s} = \text{average} \rightarrow \frac{\text{sum}_{\text{Class2}}}{15} = 0.94 \rightarrow \text{sum}_{\text{Class2}} = 14.1$$

Once you have the sum for each class, you can combine the classes.

$$\text{Total sum: } 21.5 + 14.1 = 35.6$$

$$\text{Total students: } 25 + 15 = 40$$

Now find the new average:

$$\frac{\text{sum}}{\# \text{ of } \#s} = \text{average} \rightarrow \frac{35.6}{40} \rightarrow 0.89 \rightarrow 89\%$$

The correct answer is (B).

Do you know the most common wrong answer? It is (C), 90%. Too many students simply add 86% and 94% and divide by 2. However, this is a combined average question, so each individual sum must be found in order to find the sum of the combined average.

⚠ CAUTION: SAT TRAP!

The most common wrong answer for a weighted average question is the simple average of the two averages without taking into account the number of elements in each group.

One time-saving average question concerns the average and the median of a list of patterned numbers. As revealed in the Required Knowledge and Skill Set, a list of patterned integers has the same average and median. Let’s examine how this knowledge can gain you valuable time on the SAT:

13. If x is the average (arithmetic mean) of 5 consecutive odd integers, what is the median of this set of integers?
- (A) 0
 - (B) 1
 - (C) $x - 2$
 - (D) x
 - (E) $x + 2$

In a list of consecutive or patterned numbers, the median is the same as the average, so the answer is (D). Most students would spend at least 30 seconds running numbers through the average formula on this question. You can answer this one without any calculations.

The final type of average question to review does not appear as an average question at all. These questions deal with the sum of consecutive integers and the word “average” does not occur in the text :

13. The sum of 7 consecutive even integers is 224. What integer has the least value in the list?
- (A) 16
 - (B) 18
 - (C) 26
 - (D) 29
 - (E) 32

When a question has the word “sum” and “consecutive” in it, pull out the average formula, as the question is likely a disguised average problem:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \quad \rightarrow \quad \frac{224}{7} \quad \rightarrow \quad 32$$

The average of the 7 integers is 32. Since the average and the median are the same in consecutive sets, you can find the other numbers by counting down and up from 32:

$$\underline{\quad}, \underline{\quad}, \underline{\quad}, 32, \underline{\quad}, \underline{\quad}, \underline{\quad}$$

$$26, 28, 30, 32, 34, 36, 38$$

The number with the least value is 26, so answer (C) is correct.

Do you know the most common wrong answer? Some students forget that the list is consecutive EVEN numbers. If you just study a consecutive pattern, the integer with the least value is 29:

$$29, 30, 31, 32, 33, 34, 35 \quad (\text{Incorrect})$$

≥ARITHMETRICK≤

If the words “sum” and “consecutive” appear in a question, the question is likely an average problem and will require the average formula.

⚠ CAUTION: SAT TRAP!

Be sure to read any question involving the word “consecutive” to verify whether you were using consecutive, consecutive even, or consecutive odd numbers.

Median questions are often used with tables.

Median questions can be divided into two basic types. The first simply asks you to find the median. To solve these questions, put the set of numbers in order from least to greatest, and then locate the middle number. Note that you may need to use arithmetic or algebra to generate the set of numbers, as in the following question:

DOGS ADOPTED AT PET STORES

	Start	End	Adopted
The Dog Park	16	8	
Super Pets	20	16	
Wags	18	6	
Bark Avenue	12	8	
Paws and Claws	15	4	
Pet Emporium	9	8	
Fido's	18	11	

17. The local humane society recently hosted a dog adoption event at 7 local pet stores. Each pet store started and ended the day with the number of dogs shown in the table of above. The number in the "Adopted" column is defined by the number of dogs at the start of the day minus the number of dogs at the end of the day. What is the median of the missing values in the "Adopted" column?
- (A) 4
 (B) 7
 (C) 8
 (D) 11
 (E) 12

Before you can find the median, you must generate the numbers in the "Adopted" column:

	Start	End		Adopted
The Dog Park	16	8	$16 - 8 =$	8
Super Pets	20	16	$20 - 16 =$	4
Wags	18	6	$18 - 6 =$	12
Bark Avenue	12	8	$12 - 8 =$	4
Paws and Claws	15	4	$15 - 4 =$	11
Pet Emporium	9	8	$9 - 8 =$	1
Fido's	18	11	$18 - 11 =$	7

In a large set of numbers, it may help to cross out one number on the left side of the median for every number on the right side of the median.

Now write the set of numbers from that column in order from least to greatest:

1, 4, 4, 7, 8, 11, 12

\leftarrow 1, 4, 4, **(7)**, 8, 11, 12 \rightarrow

Which number is in the middle? There are three numbers to the left of 7 and three numbers to the right of 7, so 7 is the median. The correct answer is (B).

Average, Median, and Mode Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 452.

1. Which answer choice contains a set of numbers in which the median is greater than the average (arithmetic mean)?
- (A) {3, 4, 5, 6, 7}
 - (B) {3, 4, 5, 6, 8}
 - (C) {3, 5, 5, 5, 7}
 - (D) {-2, 4, 5, 6, 7}
 - (E) {-2, 4, 5, 6, 12}

3. Eight consecutive odd integers are arranged in ascending order, from smallest to largest. The sum of the last four integers is 232. What is the sum of the first four integers?

	7	7	
	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

2. The sum of five consecutive even integers, $a, b, c, d,$ and e , respectively, is 50. Which of the following is equal to the median of the set?

- (A) $\frac{a+b+c+d+e}{30}$
- (B) $\frac{30}{c}$
- (C) $b+2$
- (D) $e-a$
- (E) $\frac{b+d}{5}$

4. Which of the following answer choices is equal to the sum of three consecutive odd integers?

- (A) 153
- (B) 154
- (C) 155
- (D) 156
- (E) 157

Average, Median, and Mode Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 452.

5. In 7 days, Mario cooked 98 pounds of spaghetti. Each day after the first, he cooked 2 more pounds than he cooked than the day before. What is the difference between the average (arithmetic mean) number of pounds of spaghetti he cooked per day and the median number of pounds he cooked during the 7 days?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

7. Five numbers, x , $2x$, $2x + 6$, $3x - 1$ and $4x - 8$, are in a set. If the average (arithmetic mean) of the five numbers is 9, what is the value of the mode in this set?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

6. The average (arithmetic mean) of five different positive integers is 30. What is the greatest possible value of one of these integers?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Counting Problems

Frequency Guide: 4

Unless you have already taken a statistics course, you probably have not encountered counting problems. These questions are mainly made up of combinations and permutations, which have complex explanations and special formulas. However, on the SAT, they are quite basic, and can be solved without formulas.

Required Knowledge and Skill Set

- Counting problems require you to do exactly what their name implies—count! The most basic counting problems ask you to count the number of possibilities presented in a word problem. These problems often deal with sums and products, which we will examine more closely in the next section.
- Permutations and combinations are arrangements of groups of numbers. In a permutation, the order of the items is important; in a combination, the order of the items is not important. There are more possible arrangements in a permutation than a combination.
- Combinations combine two or more elements. To understand combinations, let's consider an example. At a restaurant, there are three flavors of ice cream and four choices for toppings. If each ice cream sundae consists of one ice cream flavor and one topping, how many different combinations of sundaes are possible?

Because these are counting problems, you can always just count:

Three Flavors: 1, 2, and 3
Four Toppings: A, B, C, and D

1-A	2-A	3-A
1-B	2-B	3-B
1-C	2-C	3-C
1-D	2-D	3-D

There are 12 combinations. The order of the items is not important; chocolate with sprinkles is the same as sprinkles with chocolate.

But counting is not the most efficient solution method. To easily find the number of possibilities in a combination, simply multiply the number of elements:

$$3 \text{ flavors} \times 4 \text{ toppings} = 12 \text{ combinations}$$

This works no matter how many elements are present. Say we added 5 syrups to the menu, and each sundae consisted of one flavor of ice cream, one topping, and one syrup. How many combinations are possible now?

$$3 \text{ flavors} \times 4 \text{ toppings} \times 5 \text{ syrups} = 60 \text{ combinations}$$

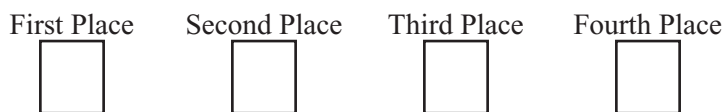
Permutations deal with many possibilities; combinations deal with fewer possibilities.

MEMORY MARKER:

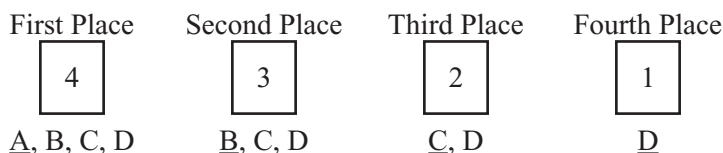
To find the possible number of combinations, multiply the elements.

4. Now look at an example of a permutation. In gym class, four students are running a race. How many different finishing orders are possible at the end of the race?

We often refer to permutations as “card questions,” because we use blank “cards” to set up the problems. Draw and label four blank cards, each one representing a specific finishing order:



Say the four runners are named A, B, C, and D. How many possibilities are there for first place? Four (A, B, C, or D). Assign one of them first place. For the ease of discussion, we will go in alphabetical order. Runner A receives first place. How many runners are now eligible for second place? Three (B, C, or D). If B finishes in second place, how many runners are available for third place? Two (C or D). If C finishes third, only D is left to come in fourth place:



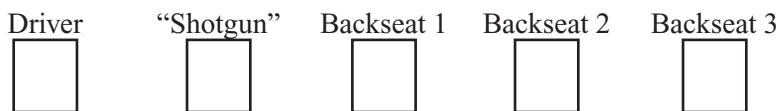
To find the number of finishing orders, multiply the cards together:

First Place	Second Place	Third Place	Fourth Place				
<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="4"/>	×	<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="3"/>	×	<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="2"/>	×	<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="1"/>	= 24

There are 24 possible finishing orders.

5. Permutations often come with restrictions that dictate rules about the order of the elements. For example, five people are in a car. If only 3 people can drive, how many different seating arrangements are possible?

Set up five cards, one for each position in the car. Always put the restriction at the front of the list:



Call the five passengers A, B, C, D, and E. Only A, B, and C can drive:

Driver	“Shotgun”	Backseat 1	Backseat 2	Backseat 3					
<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="3"/>	×	<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="4"/>	×	<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="3"/>	×	<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="2"/>	×	<input style="width: 40px; height: 40px; text-align: center; border: 1px solid black;" type="text" value="1"/>	= 72
<u>A</u> , B, C		<u>B</u> , C, D, E		<u>C</u> , D, E		<u>D</u> , E		<u>E</u>	

There are 72 possible seating arrangements.

Permutations are ordered arrangements; ABC, BCA, and CBA, are considered different items, so there are more permutations than combinations.

Combinations are unordered selections; ABC, BCA, and CBA are all the same item, so there are fewer combinations than permutations.

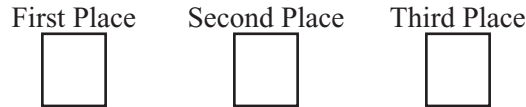
MEMORY MARKER:
To find the possible number of arrangements when order is involved, set up cards and multiply the items in the cards.

When B and C are not selected to drive, they go back into the “general population” and are now eligible to ride shotgun.

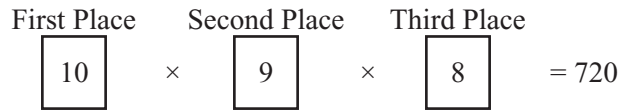
Combinations and permutations with restrictions are usually the hardest questions in a math section.

6. Permutations might also restrict the number of positions. Returning to the race question, the gym class now consists of 10 students, all of whom are running the race. The first place finisher will receive a blue ribbon, the second place runner will be given a red ribbon, and the third place contestant will receive a green ribbon. How many different possibilities are there for the top three spots?

For this question, there are only three cards, even though there are 10 runners:



But there are still 10 people who can finish in first place, leaving 9 people a chance at second place, and 8 with a shot at third:



There are 720 possible arrangements for ribbon winners.

7. Combinations can also carry restrictions. For example, there are three detectives and four uniformed officers in a new police program. Every incident must be responded to by a team of one detective and two uniformed officers. How many combinations of teams are possible?

The formula for solving this problem is complicated. Because the SAT uses small groups for these questions, we recommend that you count the teams. There are two ways to do this. You can count every possibility:

- Three Detectives: 1, 2, and 3
 Four Uniformed Officers: A, B, C, and D
- | | | |
|-------|-------|-------|
| 1-A-B | 2-A-B | 3-A-B |
| 1-A-C | 2-A-C | 3-A-C |
| 1-A-D | 2-A-D | 3-A-D |
| 1-B-C | 2-B-C | 3-B-C |
| 1-B-D | 2-B-D | 3-B-D |
| 1-C-D | 2-C-D | 3-C-D |

If combinations allow elements to repeat, forget formulas and simply count the number of combinations.

Or, for a faster solution, count the number of possibilities for one detective, and then multiply that number by 3. Detective 1 has 6 possible arrangements. That means that Detectives 2 and 3 also have 6 arrangements each:

$$6 \text{ arrangements} \times 3 \text{ detectives} = 18 \text{ possible arrangements}$$

PowerScore recommends that you use the second strategy for restricted combinations, as it is more efficient.

Application on the SAT

Counting problems come in all difficulty levels on the SAT. The most basic question simply asks you to count items, and often these items are sums or products of a limited quantity of numbers. Consider an example:

1, 3, 5, 8, 10

13. How many different sums can be made by adding any two different numbers from the list above?
- (A) 6
 - (B) 8
 - (C) 10
 - (D) 12
 - (E) 25

Pay close attention to the word “different” in the question. It gives two very important pieces of information.

First, two different numbers are added together. The order of these two numbers is not important, because the sum of $1 + 3$ is the same as the sum of $3 + 1$. This is a combination problem, and there will be a total of 10 combinations:

$$5 \text{ numbers} \times 2 \text{ added together} = 10 \text{ combinations}$$

But do not be tricked into taking choice (C). The problem requests the number of *different* sums; while there will be 10 results, some of those results may be the same value. Without finding all of the sums, we cannot predict if there are any that are the same.

Find the sums of all 10 combinations:

$$\begin{array}{llll} 1 + 3 = 4 & 3 + 5 = 8 & 5 + 8 = 13 & 8 + 10 = 18 \\ 1 + 5 = 6 & 3 + 8 = 11 & 5 + 10 = 15 & \\ 1 + 8 = 9 & 3 + 10 = 13 & & \\ 1 + 10 = 11 & & & \end{array}$$

List the sums, in ascending order:

4, 6, 8, 9, 11, 11, 13, 13, 15, 18

Notice that two of the sums repeat. Count the number of sums, excluding any repeats:

4, 6, 8, 9, 11, 13, 15, 18

There are 8 *different* sums. The correct answer is (B).

⚠ CAUTION: SAT TRAP!

We cannot overemphasize the importance of reading carefully! The SAT math sections test both math and reading comprehension!

Some difficult counting problems present such a large field of countable items, that it helps to study a small sample and apply your findings to the entire group.

18. In a list of 57 consecutive integers, the median is 70. What is the largest integer in the list?
- (A) 96
 - (B) 97
 - (C) 98
 - (D) 99
 - (E) 200

If you had three hours to take each math section, you could write out every consecutive integer and find the largest number in the list:

..., 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78,

But because you only have 25 minutes per section, you do not have time to list all 57 numbers. Instead, study a smaller set of numbers:

1, 2, 3, 4, 5

This is a set of 5 consecutive integers with a median of 3.

5 integers – 1 median = 4 integers

4 integers ÷ 2 sides of the median (right and left) = 2 numbers per side

$$\begin{array}{ccc} 2 \text{ \#s} & & 2 \text{ \#s} \\ \leftarrow & & \rightarrow \\ 1, 2, \underline{3}, 4, 5 \end{array}$$

the median + the numbers per side = the largest integer in the list
 $3 + 2 = 5$

Apply the knowledge you gained from the smaller list to the larger list:

57 integers – 1 median = 56 integers

56 integers ÷ 2 sides of the median (right and left) = 28 numbers per side

$$\begin{array}{ccc} 28 \text{ \#s} & & 28 \text{ \#s} \\ \leftarrow & & \rightarrow \\ \dots, 67, 68, 69, \underline{70}, 71, 72, 73, \dots \end{array}$$

the median + the numbers per side = the largest integer in the list
 $70 + 28 = 98$

The correct answer is (C).

▷ARITHMETRICK◁

When asked to find a number in a large list, study a small sample and apply your findings to the entire list.

Expect to see simple combinations and permutations on the SAT. However, because most students have never seen problems like these, the questions have exaggerated difficulty levels:

17. Mary has three necklaces, four bracelets, and three rings. If she wears one necklace, one bracelet, and one ring, how many different combinations can Mary make?
- (A) 4
 (B) 10
 (C) 24
 (D) 36
 (E) 48

This is a combination problem. You know this, because the order of the items does not matter. Wearing a gold necklace, blue bracelet, red ring is the same as wearing a blue bracelet, red ring, and gold necklace.

You can count the number of combinations, but this is an inefficient solution. It is faster to just multiply the number of elements together:

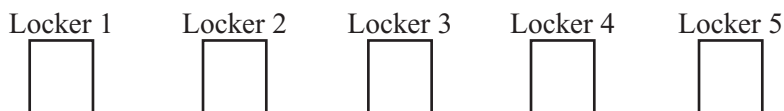
$$3 \text{ necklaces} \times 4 \text{ bracelets} \times 3 \text{ rings} = 36 \text{ combinations}$$

The correct answer is (D).

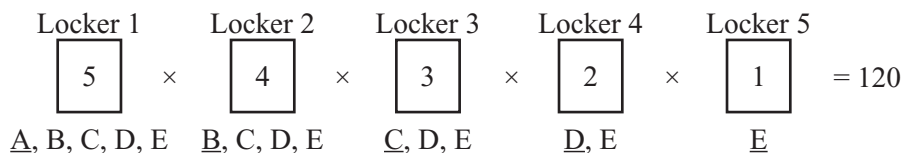
Simple permutations are also present:

19. Five lockers are to be assigned to five students. How many different arrangements of lockers are possible?
- (A) 5
 (B) 25
 (C) 50
 (D) 100
 (E) 120

To solve this problem, draw five blank cards—or in this case, five blank lockers:



Call the students A, B, C, D, and E. How many possibilities are there for each locker?



There are 120 possible arrangements, so answer (E) is correct.

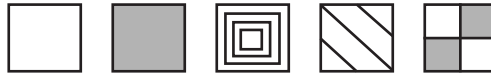
Combination problems often use the words “select” or “combinations.”

Confidence Quotation

“I have learned to use the word ‘impossible’ with the greatest caution.”

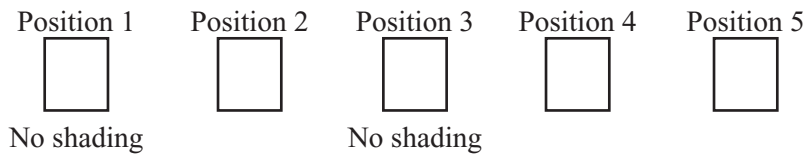
—Wernher von Braun, rocket scientist

Expect to see some combination and permutations problems involving restrictions. These are usually the most difficult questions in a section:

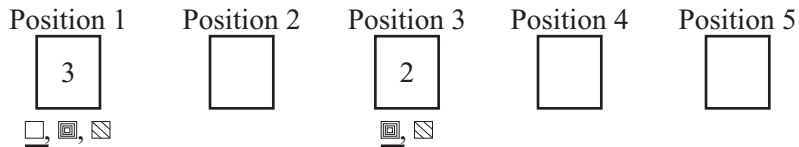


20. The five blocks shown above are to be placed in a line on a shelf. Two of the blocks, currently in the second and fifth position, have shading. If the blocks with shading can never be in first position or the center position, how many different arrangements are possible?
- (A) 36
 (B) 54
 (C) 72
 (D) 96
 (E) 120

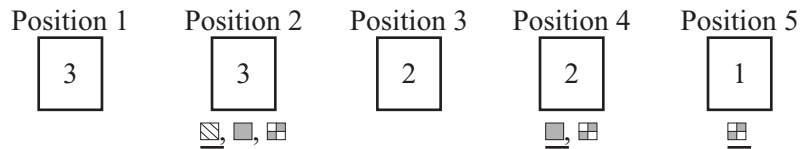
This is a permutation with restrictions. Draw five blank cards:



Always start with the restrictions. There are three possible candidates without shading for Position 1, leaving two candidates without shading for Position 3:



Then return to the other positions and determine the possibilities with all remaining elements:



Multiply each of the possibilities:

$$\begin{array}{ccccccccc}
 \text{Position 1} & & \text{Position 2} & & \text{Position 3} & & \text{Position 4} & & \text{Position 5} & & \\
 \boxed{3} & \times & \boxed{3} & \times & \boxed{2} & \times & \boxed{2} & \times & \boxed{1} & = & 36
 \end{array}$$

There are 36 possible arrangements that keep shaded blocks out of the first and third position. The correct answer is (A).

DIAGRAMMING
 the question is an important skill for many concepts in the Data Analysis, Statistics, and Probability content area.

The specific positions of the restrictions do not matter, but you should always start by analyzing the restricted positions in permutation questions.

Counting Problems Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 455.

3, 4, 5, 8, 9

1. Two different numbers are selected from the list above and their product is determined. How many different pairs of numbers with a product greater than 30 can be selected?

- (A) 5
- (B) 6
- (C) 7
- (D) 8
- (E) 9

2. A restaurant is offering a new buffet with six types of sandwiches, four sides, and five desserts. If customers are allowed to select one sandwich, one side, and one dessert, how many meal combinations are possible?

	1	1	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

3. A hot dog vendor offers three choices of condiments: mustard, ketchup, and horseradish. If a customer can select one, two, or all three condiments, how many different combinations of condiments are possible?

- (A) 5
- (B) 6
- (C) 7
- (D) 8
- (E) 9

4. Five dogs are in a dog show. They are to be lined up in a single row, and the dog with the most ribbons is to be placed in the first position. The two dogs with the fewest ribbons are to be placed in the last two positions. If none of the dogs have the same amount of ribbons, how many different arrangements of dogs are possible?

	1	1	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Counting Problems Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 455.

5. Robbie has to schedule five different meetings during the five day work week. If exactly one meeting is held each day, how many different arrangements of meetings are possible for the five day work week?

	/	/		
.
	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

6. The sum of the first 50 consecutive positive even integers is x and the sum of the first 50 consecutive positive integers is y . What is x in terms of y ?
- (A) $2y^2$
- (B) y^2
- (C) $2y$
- (D) $\frac{2}{y}$
- (E) $\frac{y}{2}$



7. How many different ways can 6 people arrange themselves in the 6 seats at a bridal party table shown above if the bride and groom must be sitting in the two center seats?

	/	/			
.
	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Probability

The College Board tests basic probability concepts on the SAT.

Frequency Guide: 3

Required Knowledge and Skill Set

1. Probability indicates the likelihood that a specific event will occur. The probability of something happening can be expressed using any number from 0 to 1. A probability of 0 means the event will *never* happen. A probability of 1 indicates that an event will *always* happen. A probability of $\frac{1}{3}$ signifies that the event has a 1 in 3 chance of occurring.
2. The probability of an occurrence can be expressed by a simple formula:

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

If there are 6 green socks, 4 blue socks, and 2 red socks in a drawer, what is the probability that you randomly select a blue sock?

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{4 \text{ blue socks}}{12 \text{ total socks}} \rightarrow \frac{1}{3}$$

There is a one in three chance that you randomly choose a blue sock.

3. The probability of something *not* occurring is 1 minus the probability that it will occur:

$$\text{Probability of an event not occurring} = 1 - \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

If there are 6 green socks, 4 blue socks, and 2 red socks in a drawer, what is the probability that you do *not* select a blue sock? You have two ways to solve this question. You can use the formula for the probability of an event not occurring:

$$\begin{aligned} \text{Probability of an event not occurring} &= 1 - \frac{\text{favorable}}{\text{possible}} \\ 1 - \frac{4 \text{ blue socks}}{12 \text{ total socks}} &\rightarrow 1 - \frac{1}{3} \rightarrow \frac{2}{3} \end{aligned}$$

Or you can find the probability of selecting a green or red sock:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{6 \text{ green} + 2 \text{ red}}{12 \text{ total socks}} \rightarrow \frac{8}{12} \rightarrow \frac{2}{3}$$

MEMORY MARKER:

You should memorize the formula for finding probability.

MEMORY MARKER:

Once you memorize the formula for the probability of an event occurring, the formula for the probability of an event not occurring is easy!

MEMORY MARKER:

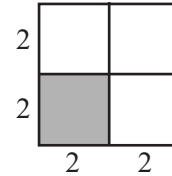
The formula for Geometric Probability is an altered version of the formula for probability.

4. Probability is often applied to geometric figures on the SAT. The area of a shaded region and the total area of a figure provide the information for the probability formula. To make these questions a little easier, slightly alter the probability formula as follows:

$$\text{Geometric Probability} = \frac{\text{shaded area}}{\text{total possible area}}$$

Consider an example:

If a point was to be selected at random from the square to the right, what is the probability that the point would be in the shaded area?



To solve, find the area of the shaded square and the total area of the large square:

$$\begin{aligned} \text{Shaded area} &= 2 \times 2 = 4 \\ \text{Total possible area} &= 4 \times 4 = 16 \end{aligned}$$

Then apply this information to the altered formula:

$$\text{Geometric Probability} = \frac{\text{shaded area}}{\text{total possible area}} \rightarrow \frac{4}{16} \rightarrow \frac{1}{4}$$

There is a 1 in 4 chance the randomly selected point will come from the shaded region.

5. Advanced probability questions entail two or more occurrences. These questions are rare, but have on occasion appeared as the most difficult questions in a section.

The probability of two or more non-related or independent events occurring is the product of the individual probabilities of those events. For example, what is the probability of flipping a penny and getting a “heads” and rolling a standard 6-sided die and getting a 2?

Find the probability of each independent event:

$$\text{Probability of flipping heads} = \frac{1 \text{ side with heads}}{2 \text{ possible sides}} = \frac{1}{2}$$

$$\text{Probability of rolling a 2} = \frac{1 \text{ side with a '2'}}{6 \text{ possible sides}} = \frac{1}{6}$$

And then multiply the individual probabilities to find the probability of both events occurring:

$$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \quad \text{There is a 1 in 12 chance that both events occur.}$$

MEMORY MARKER:

To find the probability of two or more non-related events, multiply the probability of each individual event.

This question would not occur on the SAT without an explanation about the number of sides of a penny and a standard die. We use it here, though, for ease of discussion.

6. When the probability of two events is being calculated, pay careful attention to whether the first event changes the probability of the second event.

For example, if a drawer contains 4 blue socks and 6 green socks, and you randomly select two socks, what is the probability that both socks are blue?

Start with the first sock. There are 10 socks in the drawer and 4 are blue:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{4}{10} \rightarrow \frac{2}{5}$$

The second sock is a bit trickier. How many blue socks are left in the drawer? Only 3, because you have already pulled one out. So how many total socks are left in the drawer? Only 9. Find the probability of the second sock being blue:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{3}{9} \rightarrow \frac{1}{3}$$

Now find the probability of both events occurring by finding the product of the two independent events:

$$\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$$

There is a 2 in 15 chance that both socks pulled from the drawer will be blue.

Application on the SAT

Probability questions are usually combined with Arithmetic, Algebra, Geometry, or Data Analysis questions on the SAT. One simple arithmetic question provides the probability, but asks for the number of favorable or possible outcomes. Let's look at a grid-in example:

11. There are 496 employees in a company, one of whom is to be selected at random to win a car. If the probability that a supervisor will be selected is $\frac{3}{16}$, how many supervisors work at the company?

Set up your equation just as you would if you were looking for probability, but supply the probability in order to find the number of favorable outcomes:

$$\begin{aligned} \text{Probability} &= \frac{\text{favorable}}{\text{possible}} \\ \frac{3}{16} &= \frac{\text{supervisors}}{496} \rightarrow 1488 = 16s \rightarrow 93 = s \end{aligned}$$

There are 93 supervisors at the company.

When two events occur in the same question, be sure to take into account how the first event affects the second event.

Always use the probability formula when given a question with the word "probability."

Probability is often combined with ratio questions. These questions have an exaggerated difficulty level because students forget how to set up the ratio. Consider another grid-in example:

13. A box contains only pens and pencils. There are three times as many pens as pencils in the box. If one writing utensil is to be selected at random from the box, what is the probability that the utensil is a pen?

The question provides a ratio for the numbers of pens to the number of pencils. The key to ratio questions is to find the denominator:

$$\begin{array}{l} \text{Pens : Pencils} \\ 3 : 1 \qquad \text{Pens + Pencils = Denominator} \rightarrow 3 + 1 = 4 \\ \\ \frac{3}{4}, \frac{1}{4} \qquad \frac{3}{4} \text{ of the utensils are pens, } \frac{1}{4} \text{ of the utensils are pencils} \end{array}$$

The probability of drawing a pen is the same as the fraction of pens in the box:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{3 \text{ pens}}{4 \text{ utensils}} \rightarrow \frac{3}{4}$$

The test makers may make this problem more difficult by adding another ratio:

18. A box contains only pens and pencils. There are three times as many pens as pencils. The pens are either green or blue, and 5 times as many pens are green as are blue. If one writing utensil is to be selected at random from the box, what is the probability that the utensil is a blue pen?

To solve this problem, you must find the probability of selecting a pen and the probability that the pen is blue. You already know the probability of selecting a pen (three-fourths), so determine the chances of pulling a blue pen:

$$\begin{array}{l} \text{Green : Blue} \\ 5 : 1 \qquad \text{Pens + Pencils = Denominator} \rightarrow 5 + 1 = 6 \\ \\ \frac{5}{6}, \frac{1}{6} \qquad \frac{5}{6} \text{ of the pens are green, } \frac{1}{6} \text{ of the pens are blue} \end{array}$$

The probability of two events occurring is the product of each individual event occurring:

$$\frac{3}{4} \times \frac{1}{6} = \frac{3}{24} = \frac{1}{8} \qquad \text{The probability of selecting a blue pen is } \frac{1}{8}.$$

Have you noticed that all of the probability questions we have covered were grid-in questions? Probability questions have a high probability of appearing in the Student-Produced Response section because their answers are in fraction form.

Another ratio and probability question involves adding to or subtracting from the total number of events. The following grid-in question is a good example:

17. There are 20 apples and 15 oranges in a bin. If only apples are to be subtracted from the bin so that the probability of randomly drawing an apple becomes $\frac{2}{5}$, how many apples must be subtracted from the bin?

A certain amount of apples will be subtracted from both the favorable and total possible outcomes in order to create the fraction two-fifths. If x is the number of apples subtracted, write an equation and solve for x :

$$\frac{\text{favorable} - x}{\text{possible} - x} = \frac{2}{5} \rightarrow \frac{20 - x}{35 - x} = \frac{2}{5} \rightarrow 5(20 - x) = 2(35 - x) \rightarrow$$

$$100 - 5x = 70 - 2x \rightarrow 30 = 3x \rightarrow 10 = x$$

Ten apples must be subtracted from the bin in order for the probability to be two-fifths. This question is very easy to solve, but difficult for many students to set up. By understanding ratios and probability, you can earn a point most students will omit or answer incorrectly.

As seen in the previous question, probability may also involve algebra. Some questions make you find a set of sums or products to determine probability:

16. A number is randomly selected from the set $\{-8, -4, 0, 4, 8\}$. What is the probability that the number is a member of the solution set of both $x + 4 > -3$ and $5x - 6 < 9$?

Solve both inequalities:

$$\begin{array}{ll} x + 4 > -3 & 5x - 6 < 9 \\ x > -7 & 5x < 15 \\ & x < 3 \end{array}$$

The value of x must be less than 3 but greater than -7 ($-7 < x < 3$). How many answer choices satisfy this requirement? Just two: -4 and 0 . All other answer choices only satisfy one of the inequalities.

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{2 \text{ favorable numbers}}{5 \text{ possible numbers}} \rightarrow \frac{2}{5}$$

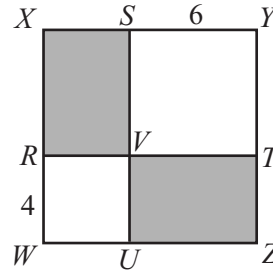
The probability of selecting -4 or 0 at random is two-fifths.

Confidence Quotation

“Vision is perhaps our greatest strength. It has kept us alive to the power and continuity of thought throughout the centuries, it makes us peer into the future and lends shape to the unknown.”

—Li Ka-shing, one of the wealthiest businessmen in the world

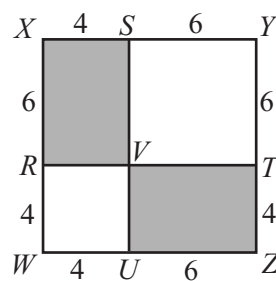
As mentioned in the Required Knowledge and Skill Set, probability is often applied to geometry questions. There are two types of questions to watch for the on SAT. The first type offers you a way to find the exact area of the figure and the exact area of the shaded region:



Can you **SIZE UP** the figure and estimate an answer? It looks like the shaded region takes up about half of the figure. Choices (B), (C), and (D) would be good guesses if you were running out of time or did not remember how to solve the question.

19. In the figure above, $WXYZ$ is a square, as are $SYTV$ and $RVUW$. If a point is selected at random from $WXYZ$, what is the probability that the point is in one of the shaded regions?
- (A) $\frac{2}{5}$
 - (B) $\frac{15}{32}$
 - (C) $\frac{12}{25}$
 - (D) $\frac{1}{2}$
 - (E) $\frac{3}{5}$

The figure gives you the side length of each square, which allows you to find the length and width of each shaded rectangle. **DIAGRAM** the question to find the area of each shaded region:



The area of $XSVR$:

$$\text{Area} = \ell w \rightarrow (6)(4) \rightarrow 24$$

The area of $VTZU$:

$$\text{Area} = \ell w \rightarrow (6)(4) \rightarrow 24$$

The area of $WXYZ$:

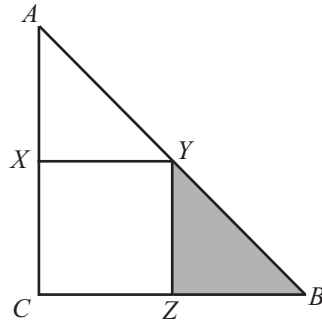
$$\text{Area} = \ell w \rightarrow (10)(10) \rightarrow 100$$

In order to find the probability of a geometric question, use the formula:

$$\text{Geometric Probability} = \frac{\text{shaded area}}{\text{total possible area}} \rightarrow \frac{24 + 24}{100} \rightarrow \frac{48}{100} \rightarrow \frac{12}{25}$$

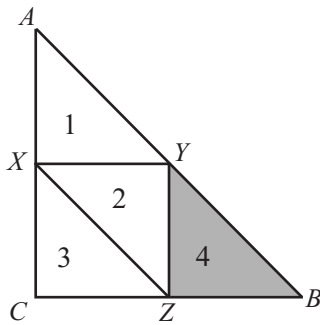
The correct answer is (C).

The other type of geometry question does not provide the measurements to find the exact area of the figure or the area of the shaded region.



8. In $\triangle ABC$ above, $XYZC$ is a square and $CZ = ZB$. If a point is randomly selected from triangle ABC , what is the probability that the selected point is in the shaded region?

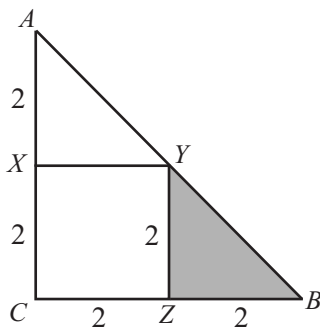
There are two ways to solve this question. The most efficient way is to understand that the shaded region is one-fourth of the entire area. To see this, draw the diagonal of the square:



There are four congruent triangles in ABC . One of those triangles is shaded, so the probability is one-fourth:

$$\frac{\text{shaded area}}{\text{total possible area}} \rightarrow \frac{1}{4}$$

If visualizing geometric figures is difficult for you, use the other solution method: SUPPLY a number for the base of the triangle, and use the information in the question to find other measurements. You can SUPPLY the length of CB as 5, 10, or 1000, and the fraction of the area that is shaded will always be the same. For the ease of calculations, let's make $CB = 4$:



The area of the YBZ :

$$\text{Area} = \frac{1}{2} bh \rightarrow \frac{1}{2} (2)(2) \rightarrow 2$$

The area of the ABC :

$$\text{Area} = \frac{1}{2} bh \rightarrow \frac{1}{2} (4)(4) \rightarrow 8$$

When in doubt, SUPPLY numbers. It's a great last-resort strategy!

$$\text{Geometric Probability} = \frac{\text{shaded area}}{\text{total possible area}} \rightarrow \frac{2}{8} \rightarrow \frac{1}{4}$$

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

Probability Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 458.

1. There are 1096 marbles in a bag. One of the marbles is to be randomly chosen from the bag. If the probability that a red marble will be selected is $\frac{5}{8}$, how many red marbles are in the bag?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

3. A negative even integer x is randomly chosen from the negative integers greater than or equal to -20 . What is the probability that $2x + 10 > -10$?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

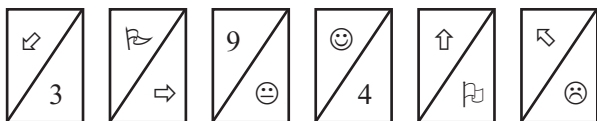
NUMBER OF CARS AT WALKER MOTORS

	Coupe	Sedan
Mileage 20,000 or less	10	30
Mileage over 20,000	30	50

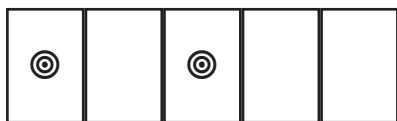
2. The table above shows the number of used cars on the lot of Walker Motors. They have been classified by their mileage and style (coupe or sedan). If a sedan is to be randomly selected, what is the probability that the car's mileage is 20,000 miles or less?
- (A) $\frac{1}{4}$
- (B) $\frac{3}{8}$
- (C) $\frac{1}{2}$
- (D) $\frac{5}{8}$
- (E) $\frac{3}{4}$

Probability Problem Set

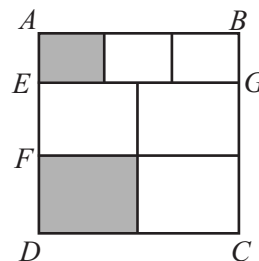
Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 458.



4. The six cards above are laid face down on a table. If one is to be picked at random, which of the following types of cards has the greatest probability of being chosen?
- (A) A card with an arrow
 - (B) A card with a flag
 - (C) A card with a number
 - (D) A card with a face
 - (E) A card with both an arrow and a flag



5. The figure above shows five lockers. Five students, including Jud and Remy, will be randomly assigned one of the lockers, one student per locker. What is the probability that Jud and Remy will each be given a locker marked by a bullseye?
- (A) $\frac{1}{25}$
 - (B) $\frac{1}{20}$
 - (C) $\frac{2}{25}$
 - (D) $\frac{1}{10}$
 - (E) $\frac{1}{5}$



6. In the figure above, $ABCD$ is a square with an area of 576. Line segment AE is one-fourth of AD , and $ABGE$ is divided into three equal rectangles. Line segment FD is one-half of ED , and $EGCD$ is divided into 4 equal rectangles. If a point is randomly chosen from $ABCD$, what is the probability that the point will be from a shaded region?
- (A) $\frac{1}{12}$
 - (B) $\frac{3}{16}$
 - (C) $\frac{11}{24}$
 - (D) $\frac{7}{36}$
 - (E) $\frac{13}{48}$

Sequences

Frequency Guide: 3

Terms in an arithmetic sequence increase by a constant difference.

MEMORY MARKER:

You should add this formula to your knowledge bank if you are good at memorizing formulas.

MEMORY MARKER:

There is a slight chance you will need to know how to find the sum of a specific number of the first terms in an arithmetic sequence.

The College Board uses sequence questions to test critical reasoning skills.

Required Knowledge and Skill Set

1. A sequence is a patterned list of numbers. Three types of series are tested on the SAT. In an arithmetic series, each term increases by a constant value. Consider the following sequence:

$$3, 7, 11, 15, 19, \dots$$

In this arithmetic sequence, 4 is added to each term to create the following term.

Most SAT sequence questions can be solved without using a formula. However, a rare sequence question can benefit from using sequence formulas. For this reason, we recommend that test takers looking to maximize their scores memorize the different formulas for sequences.

The formula for finding any term of an arithmetic sequence uses several variables:

$$a_n = a_1 + (n - 1)d$$

Where:

a_1 = the first term

n = the number of terms

d = constant difference

Returning to the sequence (3, 7, 11, 15, 19, ...), you can use the formula to find any term in the sequence. For example, if you wanted to know the tenth term of the sequence, plug values into the formula:

$$\begin{aligned} a_n &= a_1 + (n - 1)d \\ a_{10} &= 3 + (10 - 1)4 \rightarrow 3 + (9)4 \rightarrow 3 + 36 \rightarrow 39 \end{aligned}$$

The tenth term of the sequence is 39.

You can also use a formula to find the sum of the first n terms of an arithmetic sequence. Again, the chances of having to use this formula on the SAT are very low, as there is likely a more logical way to solve the question, but we want you to be prepared for any test question:

$$\text{Sum of the first } n \text{ terms in an arithmetic sequence} = n \frac{a_1 + a_n}{2}$$

To find the sum of the first 10 terms of the sequence above, plug values into the formula:

$$\text{Sum of the first 10 terms} = (10) \frac{3 + 39}{2} \rightarrow (10) \frac{42}{2} \rightarrow \frac{420}{2} \rightarrow 210$$

2. In a geometric sequence, each term increases by a constant ratio:

4, 8, 16, 32, 64,

In this geometric sequence, 2 is multiplied by each term to create the following term.

You can find any term in a geometric sequence by using the following formula:

$$a_n = a_1 \times r^{n-1}$$

Where:

a_1 = the first term

n = the number of terms

r = constant ratio

To find the tenth term in the sequence above (4, 8, 16, 32, 64, ...), plug values into the formula:

$$a_n = a_1 \times r^{n-1}$$

$$a_{10} = 4 \times 2^{(10-1)} \rightarrow 4 + 2^9 \rightarrow 4 + 512 \rightarrow 516$$

To solve this question, you had to find the value of 2^9 . However, in the chapter on Algebra, we said that you would never be asked to find the value of a number raised to an exponent higher than 5 or 6. This is because not all students have access to a calculator, and the test must be fair to all test takers. The chances of using this formula on the SAT are very low. Whether you choose to memorize this formula should depend on your ability and your goal. Students who have a hard time memorizing formulas can omit both the arithmetic sequence formulas and the geometric sequence formulas from their required study because it is highly unlikely that you will need these formulas. But students who are strong in math, and have all of the other formulas memorized may choose to add these formulas to their arsenals on the off chance that they are needed.

Students who choose not to memorize these formulas should still review this section. You will see sequences on the SAT, and on the following pages we will provide you with solution methods that do not require formulas.

Just as you can find the sum of the first n terms in an arithmetic sequence, you can find the sum of the first n terms in a geometric formula:

Sum of the first n terms in a geometric sequence: $\frac{a_1(1-r^n)}{1-r}$

To find the sum of the first 6 terms above, plug in the missing values:

$$\text{Sum of the first 6 terms} = \frac{4(1-2^6)}{1-2} \rightarrow \frac{4(1-64)}{-1} \rightarrow \frac{4(-63)}{-1} \rightarrow 252$$

Terms in a geometric sequence increase by a constant ratio.

MEMORY MARKER:

You may choose to memorize the formula for finding the n th term of a geometric sequence.

If you choose not to memorize the formulas for arithmetic and geometric sequence, be sure to carefully read the next section, "Application on the SAT."

MEMORY MARKER:

This is another formula that likely will not be needed, as there are other ways to solve these questions.

3. The majority of sequences on the SAT are not arithmetic or geometric sequences. They are sequences, though, because they have a pattern:

$-4, 2, -3, -4, 2, -3, -4, \dots$ *Repeats the terms $-4, 2, -3$*
 $1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, \dots$ *Adds and then subtracts 1*
 $1, 1, 2, 3, 5, 8, 13, 21, \dots$ *Terms are the sum of the two previous terms*

These types of sequences are much more likely to appear on the SAT because there are no formulas required to solve for them.

When given a sequence that is not an arithmetic or geometric sequence, there are two ways to solve SAT questions.

- 1.) If you are asked to find the eighth term or less, simply follow the pattern provided by the question to compute all terms in the sequence.

For example, in the sequence 5, 8, 14, ..., the first term is 5. Each term after the first is obtained by doubling the previous number and then subtracting 2. What is the sixth term of the sequence?

Because the question asks for a relatively low-numbered term, calculate using the information in the text. Label each term to avoid careless errors:

1st		2nd		3rd		4th		5th		6th
5		8		14		26		50		98
<small>$5 \times 2 - 2$</small>		<small>$8 \times 2 - 2$</small>		<small>$14 \times 2 - 2$</small>		<small>$26 \times 2 - 2$</small>		<small>$50 \times 2 - 2$</small>		

- 2.) If you are asked to find a term greater than the eighth term, establish the pattern and then use multiples to find the answer.

For example, in the sequence 2, -6, 8, ..., the first term is 2 and the first three terms repeat continuously. What is the 41st term in the sequence?

You could write out all 41 numbers, but the solution is time-consuming and inefficient. A better solution is to establish the pattern. In this problem, no calculations are required. The pattern is simply 2, -6, 8, 2, -6, 8, 2, ...

1st		2nd		3rd		4th		5th		6th		7th		8th		9th
2		-6		8		2		-6		8		2		-6		8

There are three numbers in the pattern. Therefore, all terms that are multiples of three are 8. The 3rd term, the 6th term, and the 9th term are all 8. The 12th term, 15th term, 18th term, etc. are also all 8. What multiple of 3 is close to 41?

37th		38th		39th		40th		41st		42nd		43rd		44th		45th
2		-6		8		2		-6		8		2		-6		8

$3 \times 13 = 39$ The 39th term is 8, the 40th term is 2, and the 41st term is -6.

The number of the term requested determines the solution method used.

≧ARITHMETRICK≦
Use multiples to find a large-numbered term!

Application on the SAT

Most sequence questions ask you to find a specific term. The easiest sequence questions ask you to find a low-numbered term:

5. The first number is 3 in a sequence of numbers. Each term after the first is 5 less than 3 times the preceding number. What is the fifth number in the sequence?
- (A) -595
 (B) 43
 (C) 123
 (D) 443
 (E) 1407

As discussed on the previous page, because the requested term is less than the eighth term, label each term and solve using the information in the text:

1st	2nd	3rd	4th	5th
3	4	7	16	43
<small>$3 \times 3 - 5$</small>	<small>$4 \times 3 - 5$</small>	<small>$7 \times 3 - 5$</small>	<small>$16 \times 3 - 5$</small>	

The correct answer is (B).

Medium and Hard level questions ask you to find a much higher-numbered term using a repeating sequence.

$$0.\overline{42659} = 0.4265942659\dots$$

16. In the repeating decimal above, the digits 42659 repeat. What digit is in the 1001st place to the right of the decimal point?
- (A) 4
 (B) 2
 (C) 6
 (D) 5
 (E) 9

Every term that is a multiple of five is 9:

1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
4	2	6	5	9	4	2	6	5	9

Because 1000 is a multiple of 5 ($5 \times 200 = 1000$), the 1000th term is 9. Therefore, the 1001st term is 4:

996th	997th	998th	999th	1000th	1001st
4	2	6	5	9	4

The correct answer is (A).

Confidence Quotation

“Man is what he believes.”
 —Anton Chekhov,
 Russian short-story
 writer

Don't let the decimal fool you. This is a sequence question with repeating terms.

In all of the hundreds of tests PowerScore experts have analyzed, questions like this have only appeared two or three times.

The most difficult and most infrequent term questions use arithmetic or geometric sequences:

19. The first number is -5 in a sequence of numbers. Each term after the first is 4 more than the preceding term. What is the 99th number in the sequence?
- (A) 385
 (B) 387
 (C) 390
 (D) 391
 (E) 395

The most efficient way to solve this question is to use the arithmetic sequence formula, as the sequence increases by a constant value:

$$a_n = a_1 + (n - 1)d$$

Where:

- $a_1 = -5$ (the first term)
 $n = 99$ (the number of terms)
 $d = 4$ (constant difference)

$$a_{99} = -5 + (99 - 1)4 \rightarrow -5 + (98)4 \rightarrow -5 + 392 \rightarrow 387$$

The correct answer is (B).

If you fail to memorize the formula, you can still solve this question. Look at the first few numbers in the sequence:

1st	2nd	3rd	4th	5th	6th
-5	-1	3	7	11	15
<small>$-5+4$</small>	<small>$-1+4$</small>	<small>$3+4$</small>	<small>$7+4$</small>	<small>$11+4$</small>	

This solution method is not as efficient as memorizing the formula for finding a term in an arithmetic sequence. However, it demonstrates that the formulas are not required to solve the question.

Do you see a pattern using multiples? All of the terms are one less than a multiple of 4. Therefore, the answer must be one less than a multiple of 4. This eliminates two answer choices, (A) and (C), because 385 and 390 are not one less than a multiple of 4. Now that you have eliminated an answer choice, you are free to make a guess. But further study will lead you to the exact answer.

Since you are dealing with multiples of 4, multiply each term number by 4:

$1 \times 4 = 4$	$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$	$6 \times 4 = 24$
-5	-1	3	7	11	15

What is the difference between the term number and the value of the term? For all six terms, the difference is 9. Look at the sixth term: $24 - 15 = 9$. The same is true for the second term: $8 - -1 = 9$. Apply this knowledge to find the 99th term:

$$99 \times 4 = 396 \rightarrow 396 - 9 = 387 \quad \text{The correct answer is (B).}$$

Most sequence questions involving the sum deal with a sequence in which some or all of the numbers cancel out each other. Consider an example:

$$1, 3, -3, \dots$$

17. In the sequence above, the first term is 1. Each even-numbered term is 2 more than the previous term and each odd-numbered term, after the first, is -1 times the previous term. For example, the second term is $1 + 2$ and the third term is 3×-1 . What is the sum of the first 40 terms of this sequence?
- (A) 0
 (B) 3
 (C) 9
 (D) 21
 (E) 27

This question cannot be solved using the arithmetic or geometric series sum formulas, as the sequence does not have a constant difference or constant ratio. Calculate the other terms of the sequence until you establish a pattern:

	Even	Odd	Even	Odd	Even	Odd	Even	Odd
1st	2nd	3rd	4th	5th	6th	7th	8th	9th
1	3	-3	-1	1	3	-3	-1	1
	$1+2$	3×-1	$-3+2$	-1×-1	$1+2$	3×-1	$-3+2$	-1×-1

The pattern repeats every fourth term. Find the sum of these four terms:

$$1 + 3 + -3 + -1 = 0$$

The values cancel each other out. The sum of all terms that are a multiple of 4 will be 0. Consider the sum of the first 8 terms:

$$1 + 3 + -3 + -1 + 1 + 3 + -3 + -1 = 0$$

Since the 40th term is a multiple of 4, all 40 terms will cancel each other out and the sum will be 0. Answer choice (A) is correct.

What if the question asked for the sum of the first 42 terms? Well, 42 is 2 more than a multiple of 4. Study a smaller group of terms. Since the sixth term is 2 more than a multiple of 4, find the sum of the first six terms:

$$1 + 3 + -3 + -1 + 1 + 3 = 4$$

All terms that are 2 more than a multiple of 4 will have a sum of 4. So the sum of the first 42 terms is 4.

≧ARITHMETRICK≦

In questions about the sum of sequences, the terms usually cancel each other out.

The final type of sequence questions on the SAT uses expressions. You may be asked to find the expression or perform an operation given the expression. The first group, finding the expression, is the easiest:

11. In a sequence, the first term is 6. Each term after the first is 4 more than the previous term. Of the following, which is an expression for the n th term of the sequence for any positive integer n ?
- (A) $4n$
 - (B) $4n + 1$
 - (C) $4n + 2$
 - (D) $5n + 1$
 - (E) $5n + 2$

Find the first few terms of the sequence:

$n = 1$ st	<u>2nd</u>	<u>3rd</u>	<u>4th</u>
6	10	14	18
<small>$6 + 4$</small>	<small>$10 + 4$</small>	<small>$14 + 4$</small>	

When given an expression for terms, either in the question or the answer choices, run terms through the expression to compute the term's value.

Select one of the terms to test the answer choices. For this example, we chose the 2nd term, 10. Run $n = 2$ through each answer choice to find the one (or more) that results in 10:

- (A) $4n$ $4(2) = 8$ No
- (B) $4n + 1$ $4(2) + 1 = 9$ No
- (C) $4n + 2$ $4(2) + 2 = 10$ ✓
- (D) $5n$ $5(2) = 10$ ✓
- (E) $5n + 1$ $5(2) + 1 = 11$ No

Both (C) and (D) work for the second term. To eliminate one, try another term. For the 3rd term, $n = 3$ and the result is 14:

- (C) $4n + 2$ $4(3) + 2 = 14$ ✓
- (D) $5n$ $5(3) = 15$ No

Only choice (C) works with all terms in the sequence.

Some questions will give an expression for finding terms. They may ask you to compare two terms in the sequence. Consider a grid-in question:

16. The formula $n^2 - 3n$ gives the n th term of a sequence. How much larger is the 12th term of the sequence than the 5th term?

This Hard level question is really quite easy. First, find the 5th term and 12th term:

$$\begin{array}{l} n = 12 \quad n^2 - 3n \rightarrow 12^2 - (3)(12) \rightarrow 144 - 36 \rightarrow 108 \\ n = 5 \quad n^2 - 3n \rightarrow 5^2 - (3)(5) \rightarrow 25 - 15 \rightarrow 10 \end{array}$$

Now find the difference: $108 - 10 = 98$. The correct answer is 98.

Sequences Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 460.

$t, 5t, \dots$

1. In the sequence above, the first term is t . Each term after the first is 5 times the preceding term and the sum of the first four terms is 936. What is the value of t ?

	/	/	
○	○	○	○
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

2. In a sequence of positive integers, the ratio of each term to the term immediately following it is 1 to 4. What is the ratio of the 2nd term to the 5th term?
- (A) 1 to 16
 (B) 1 to 32
 (C) 1 to 64
 (D) 1 to 128
 (E) 1 to 256

$-3, -1, 0, 1, 5$

3. In the sequence above, the first 5 numbers repeat continuously. What is the sum of the first 30 numbers of this sequence?

	/	/	
○	○	○	○
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

4. In the sequence above, the first term is -4 . Each even-numbered term is -1 times the previous and each odd-numbered term, after the first, is 4 less than the previous term. For example, the second term is -4×-1 and the third term is $4 - 4$. What is the 45th term of the sequence?
- (A) -8
 (B) -4
 (C) 0
 (D) 4
 (E) 8

Frequency Guide: 1

MEMORY MARKER:

The formula for overlapping groups is rarely needed, but the wise test taker will be prepared. Don't forget that there are over 100 free math flash cards available in the book owner's web site at www.powerscore.com/satmathbible.

Overlapping Groups

A rare question involves overlapping group members. Although these questions are very easy to solve, they are intimidating to students who have never seen them before.

Required Knowledge and Skill Set

1. The formula for solving these questions involves simple operations:

$$\text{Total} = \text{Group A} + \text{Group B} + \text{Neither Group} - \text{Both Groups}$$

Let's look at some sample questions from the SAT to see how this formula applies.

Application on the SAT

There are two types of overlapping group questions that may appear on the SAT. The first—which appears most often on the SAT—gives you the total population. Let's examine a grid-in question:

14. There are 30 characters in a video game. Each character can only run, only jump, or both run and jump. If 22 of the characters can run and 15 of the characters can jump, how many characters can both run and jump?

If the members of Group A are runners and the members of Group B are jumpers, set up the equation using the values in the question:

$$\begin{aligned} \text{Total} &= \text{Group A} + \text{Group B} + \text{Neither Group} - \text{Both Groups} \\ \text{Total} &= \text{Run} + \text{Jump} + \text{Neither Run nor Jump} - \text{Both Run and Jump} \\ 30 &= 22 + 15 + 0 - \text{Both Run and Jump} \end{aligned}$$

Because all of the characters can either run or jump, there are no members in “Neither Group.” This is usually the case on the SAT.

Since you are trying to find the number of characters that can both run and jump, set up the equation to solve for “Both Groups:”

$$\begin{aligned} 30 &= 22 + 15 + 0 - \text{Both Run and Jump} \\ 30 &= 37 - \text{Both Run and Jump} \\ -7 &= - \text{Both Run and Jump} \\ 7 &= \text{Both Run and Jump} \end{aligned}$$

There are 7 characters in the video game that can both run and jump.

The other type of question that may appear on the SAT asks for the value of the total population:

16. At a summer camp, 60 children take sailing class, 25 children take pottery class, and 100 children do not take either sailing class or pottery class. If 12 children take both sailing class and pottery class, how many total children are at the summer camp?
- (A) 27
 (B) 147
 (C) 173
 (D) 185
 (E) 197

Again, use the formula to plug in values from the question. Make “sailing” Group A, and “pottery” Group B:

$$\begin{aligned} \text{Total} &= \text{Group A} + \text{Group B} + \text{Neither Group} - \text{Both Groups} \\ \text{Total} &= \text{Sail} + \text{Pottery} + \text{Neither Sail Nor Pottery} - \text{Both Sail and Pottery} \\ \text{Total} &= 60 + 25 + 100 - 12 \\ \text{Total} &= 185 - 12 \\ \text{Total} &= 173 \end{aligned}$$

Easy, right? It is hard to believe that these questions have such a high difficulty level, but most students have never been exposed to overlapping group questions prior to taking the SAT. You would be wise to memorize the formula for overlapping groups.

Confidence Quotation

“There are admirable potentialities in every human being. Believe in your strength and your youth. Learn to repeat endlessly to yourself, ‘It all depends on me.’”

—André Gide,
 French author and Nobel Peace Prize recipient for literature

Overlapping Groups Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers are on page 462.

1. On a game show, there are 100 sealed boxes. Each box contains dollar bills only, coins only, or both dollar bills and coins. If 76 of the boxes contain dollar bills and 52 of the boxes contain coins, how many contain both dollar bills and coins?

	1	1	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

2. At a clothing store, 35 shirts have stripes, 12 shirts have polka dots, and 5 shirts have both stripes and polka dots. If 63 shirts have neither stripes nor polka dots, how many total shirts are in the clothing store?

	1	1	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Logical Reasoning

Logical Reasoning questions ask you to draw conclusions based on information given about particular situations or problems.

Frequency Guide: 1

Required Knowledge and Skill Set

1. Formal logic is not tested on the SAT.
2. Nearly all Logical Reasoning questions have an Easy difficulty level, and can be solved by simply studying the text in the questions.

Application on the SAT

On the SAT, one type of Logical Reasoning question involves a text-only format. These present a scenario and ask you to draw a conclusion:

5. The senior class is ranked by grade point average. There is an equal number of girls and boys in the senior class, but more girls than boys are in the top 50% of the class. Which of the following must be true?
 - (A) The highest ranked senior is a girl.
 - (B) The lowest ranked senior is a boy.
 - (C) There are more girls than boys in the senior class.
 - (D) There are at least 30 girls in the top 50% of the class.
 - (E) There are more boys than girls in the bottom 50% of the class.

For Logical Reasoning questions, work through each condition or each answer choice to prove or disprove its validity.

For these questions, work through each answer choice to determine its validity. It may help to think of an example or use a small sample to study the problem. For example, imagine a senior class of just six students. Three must be boys and three must be girls. In order for there to be more girls in the top 50%, two girls must be in the top three spots.

For answer choice (A), the boy could be in the first spot, so this answer is not true:



For answer choice (B), refer to the diagram above. A girl can be in the last spot.

Choice (C) is not true because the question tells you “there are an equal number of girls and boys in the senior class.”

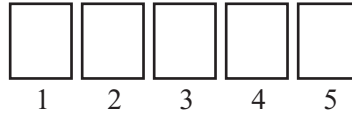
Answer choice (D) is also not true. In our sample, there are only 3 girls in the senior class.

Answer choice (E) must be true. If there are more girls in top half of the class, there must be more boys in the bottom half of the class.

Confidence Quotation

“Act as if it were impossible to fail.”
—Dorothea Brande,
author and editor

Another type of Logical Reasoning question involves diagrams. Usually these diagrams contain four to six boxes, and conditions about each box are given in the question:



5. In the figure above, 5 lockers are to be assigned to 5 students, Amy, Barb, Cal, Dan, and Ed. Amy and Barb are girls, and Cal, Dan and Ed are boys. A different student must be assigned to each locker and the following conditions must be met:

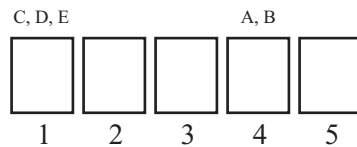
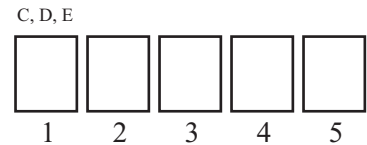
- Locker 1 is assigned to a boy.
- Locker 4 is assigned to a girl.
- Cal is assigned to an even-numbered locker and Barb is assigned to an odd-numbered locker.
- Ed is assigned to Locker 5.

Which student is assigned to Locker 3?

- (A) Amy
- (B) Barb
- (C) Cal
- (D) Dan
- (E) Ed

Use the figure to DIAGRAM the question. Use the first letter of each student’s name to show where they may be assigned.

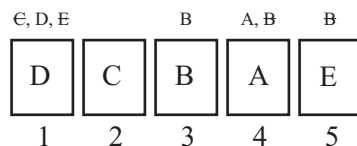
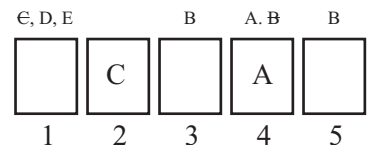
The first condition says that a boy must be in Locker 1. Place Cal, Dan, and Ed above Locker 1.



The second bullet calls for a girl in Locker 4. Put Amy and Barb above Locker 4.

As each member is eliminated from a box, cross out their name or symbol.

The third condition puts Cal in an even-numbered locker. He cannot be assigned to 4, so he must be in 2. It also states that Barb is in an odd numbered locker. This means Amy is in 4 and Barb is in 3 or 5.



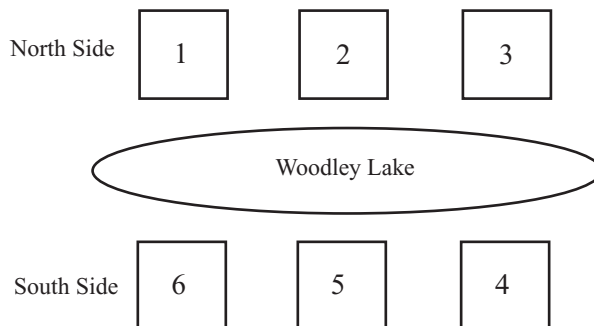
The last bullet puts Ed in Locker 5 . This leaves Dan in Locker 1 and Barb in Locker 3. The correct answer is (B).

Logical Reasoning Problem Set

Solve the following multiple-choice questions by selecting the best answer from the five answer choices. For grid-in questions, write your answer in the grids and completely mark the corresponding ovals. Answers begin on page 462.

1. Raj just bought a pet from a pet store that only sells birds and snakes. Of the following, which must be true?

- (A) The pet is a bird.
- (B) The pet is a snake.
- (C) The pet is not a yellow bird.
- (D) The pet is not a brown dog.
- (E) The pet is not a snake with fangs.



2. In the diagram above, six cabins are shown on Woodley Lake. Three of the cabins are on the north side of the lake, and three cabins are on the south side, each one directly across from another cabin, as shown above. Five people—Ron, Sue, Tom, Val, and Will—are each assigned to one of the cabins given the following conditions:

- One cabin will remain unoccupied.
- Tom and Val will be assigned to cabins on the north side of the lake. Val's cabin is next to Tom's cabin but no other cabin.
- Sue will be in cabin 5
- Ron and Will will be in cabins on opposite sides of the lake, directly across from each other.

If Val is assigned to cabin 1, who among the following could be assigned to cabin 3?

- I. Ron
 - II. Tom
 - III. Will
- (A) I only
 - (B) III only
 - (C) I and III only
 - (D) II and III only
 - (E) I, II, and III

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY ANSWER KEY

Data Analysis Problem Set—Pages 406-407

1. (B) Easy

This is a proportion question, as the values in the table are proportional:

$$\begin{array}{l} \text{Gallons:} \\ \text{Liters:} \end{array} \quad \frac{2}{7.6} = \frac{x}{30.4}$$

APPROXIMATE CONVERSIONS

Number of Gallons	2	4	x
Number of Liters	7.6	15.2	30.4

Cross multiply:

$$2(30.4) = 7.6x$$

$$60.8 = 7.6x$$

$$8 = x \quad \text{The correct answer is (B).}$$

2. (D) Medium

Many students get caught on this problem because they only look at one “slice” of the pie graph. However, there are two slices that show data about people less than 6 feet tall:

$h \leq 5.5$	15%
$5.6 \leq h \leq 6.0$	50%
Total:	65%

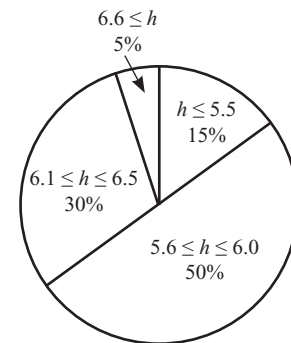
Now TRANSLATE:

65% of 760 seniors are 6 feet tall or less

$$0.65 \times 760 =$$

$$0.65 \times 760 = 494 \quad \text{Choice (D) is correct.}$$

Height of Seniors

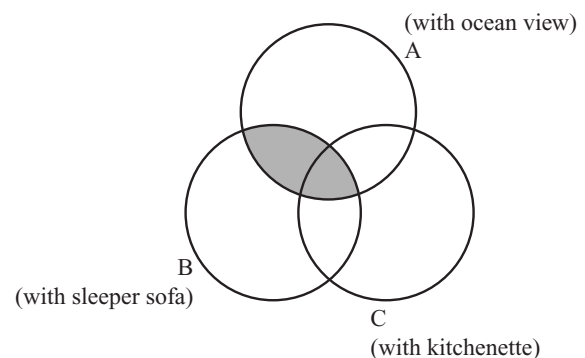


3. (C) Medium

The shaded area combines Circle A and Circle B, so it includes condominiums with an ocean view and a sleeper sofa. This eliminates (D) and (E).

There is a small part of Circle C in the shaded area, so it includes some rooms with a kitchenette. The correct answer is (C).

CONDOMINIUMS



4. (C) Medium

Begin by finding the number of customers in Kentucky and Ohio in 2000:

$$\text{Kentucky: } 100 (\times 1000)$$

$$\text{Ohio: } 400 (\times 1000)$$

Because the actual numbers are so large, it is easier to manipulate them in their abbreviated form.

Now TRANSLATE:

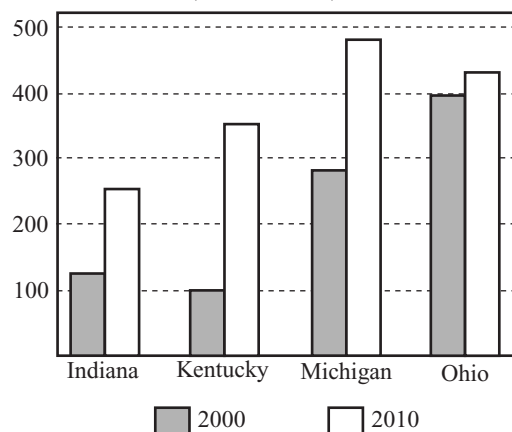
The number of customers in Kentucky was what percent of the number of customers in Ohio?

$$100 = \frac{x}{100} \times 400$$

$$10,000 = x \times 400$$

$$25 = x \quad \text{The correct answer is (C).}$$

HIGH SPEED INTERNET CUSTOMERS
(in thousands)



5. (E) Hard

First, find the approximate total number of customers in 2000 and 2010:

$$2000: \text{ Indiana (125) + Kentucky (100) + Michigan (275) + Ohio (400) = Total (900)}$$

$$2010: \text{ Indiana (250) + Kentucky (350) + Michigan (475) + Ohio (425) = Total (1500)}$$

Now find the difference to learn the increase:

$$1500 - 900 = 600$$

Finally, TRANSLATE:

600 is what percent of 900?

$$600 = \frac{x}{100} \times 900$$

$$60,000 = x \times 900$$

$$60,000 = x \times 900$$

$$66.67 = x$$

The closest answer is choice (E).

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

6. (C) Hard

Find the value of $x + y$:

$$176 + x + 97 + y = 500$$

$$x + 97 + y = 324$$

$$x + y = 227$$

In order for x to have its greatest possible value, y must have its least possible value. The smallest positive integer is 1:

$$x + 1 = 227$$

$$x = 226 \quad \text{Choice (C) is correct.}$$

Color	Number of People Choosing Color
Blue	176
Green	x
Red	97
Yellow	y

Average, Median, and Mode Problem Set—Pages 416-417

1. (D) Medium

You can find the average and median of each set, or use your knowledge of averages to eliminate several answer choices and choose the correct choice. This is the most efficient solution.

You can eliminate choice (A), because in a patterned list of consecutive numbers, the average and median are the same. The average of set (A) is 5 and the median is 5.

You can also eliminate (B). Because the sum has increased by 1 from set (A), the average is going to increase, too. The average will be greater than 5 and the median is 5.

Some students will also eliminate (C). The lowest number is 2 less than 5, and the greatest number is 2 more than 5. The other three numbers are 5, so the average is 5, as is the median.

Astute students can pick (D) without doing any calculations. The average in (A) was 5. Choice (D) replaced the 3 in set (A) with a -2 . Therefore, the average will now be less than 5. The median is 5. This is the correct answer.

A more time-consuming solution is to find the average and median of each set:

- (A) Average = 5, Median = 5
- (B) Average = 5.2, Median = 5
- (C) Average = 5, Median = 5
- (D) Average = 4, Median = 5
- (E) Average = 5, Median = 5

2. (C) Medium

The median of the set is c . Which answer choice is equal to c ? Since the integers increase by 2, $b + 2 = c$.

3. 200 Medium

Remember, problems with the words “consecutive” and “sum” indicates a disguised average problem. Start by finding the average of the last four numbers:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \rightarrow \frac{232}{4} \rightarrow 58$$

The average of the four consecutive odd integers is 58. The median is also 58. Therefore, the two integers in the second and third spot in the list are 57 and 59.

$$_, _, _, _ \rightarrow _, 57, 59, _ \rightarrow 55, 57, 59, 61$$

Knowing the last four numbers allows you to find the first four numbers:

$$_, _, _, _, 55, 57, 59, 61 \rightarrow \boxed{47, 49, 51, 53}, 55, 57, 59, 61$$

What is the sum of the first four numbers?

$$47 + 49 + 51 + 53 = 200$$

4. (A) Medium

Each answer choice represents the sum; divide each by 3. If an odd integer results, try adding the next lowest odd integer and next highest odd integer to the result to see if the three add up to the answer choice.

$$(A) 153 \div 3 = 51 \quad 49 + 51 + 53 = 153 \checkmark$$

Choice (A) is correct.

$$(B) 154 \div 3 = 51.33 \text{ (Not an integer)}$$

$$(C) 155 \div 3 = 51.67 \text{ (Not an integer)}$$

$$(D) 156 \div 3 = 52 \text{ (Not odd)}$$

$$(E) 157 \div 3 = 52.33 \text{ (Not an integer)}$$

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

5. 0 Hard

Do you remember the rule that says that in a patterned list of consecutive numbers, the average and median are the same? If so, you should be able to solve this question without any calculations. Because Mario cooks two more pounds each day, the list of 7 numbers has a consecutive pattern; each number is 2 more than the previous number. Therefore, the average and median are the same. Therefore, their difference is 0. If the average is 10, the median is 10 ($10 - 10 = 0$). If the average is 1000, the median is 1000 ($1000 - 1000 = 0$).

If you did not remember this rule, you can solve the question the hard way:

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
x	$x + 2$	$x + 4$	$x + 6$	$x + 8$	$x + 10$	$x + 12$

Note that Day 4 is the median, as there are three days to the right and three to the left. If you find the value of x , you can find the value of both the average and median.

In 7 days, Mario cooks 98 pounds. Find x :

$$\begin{aligned}x + (x + 2) + (x + 4) + (x + 6) + (x + 8) + (x + 10) + (x + 12) &= 98 \\7x + 42 &= 98 \\7x &= 56 \\x &= 8\end{aligned}$$

The average is 14:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \rightarrow \frac{98}{7} \rightarrow 14$$

Now use the value of x to find the median. As noted previously, the median is Day 4, $x + 6$:

$$\begin{aligned}x + 6 \\x = 8 \\8 + 6 = 14\end{aligned}$$

The average and the median are both 14. Therefore, their difference is 0 ($14 - 14 = 0$).

6. 140 Hard

Use the average formula to find the sum of the five integers:

$$\frac{\text{sum}}{\# \text{ of \#s}} = \text{average} \rightarrow \frac{\text{sum}}{5} = 30 \rightarrow \text{sum} = 150$$

The five numbers are all different, and their sum is 150. In order for one to be as large as possible, the other four must be as small as possible:

$$\boxed{?} + \boxed{?} + \boxed{?} + \boxed{?} + \boxed{?} = 150 \rightarrow \boxed{1} + \boxed{2} + \boxed{3} + \boxed{4} + \boxed{?} = 150 \rightarrow 10 + ? = 150$$

$$? = 140$$

The most common wrong answer is 146 because students fail to read that the integers are different.

7. 8 Hard

Find the sum in order to find x :

$$\frac{x + 2x + (2x + 6) + (3x - 1) + (4x - 8)}{5} = 9 \rightarrow 12x - 3 = 45 \rightarrow 12x = 48 \rightarrow x = 4$$

To find the mode, you must find each of the five numbers using $x = 4$:

$$\begin{aligned} x &= 4 \\ 2x &= 8 \\ 2x + 6 &= 8 + 6 = 14 \\ 3x - 1 &= 12 - 1 = 11 \\ 4x - 8 &= 16 - 8 = 8 \end{aligned}$$

$$\{4, 8, 8, 11, 14\}$$

The mode, the number that appears most often in the set is 8.

Counting Problems Problem Set—Pages 425-426

1. (A) Easy

You must find the product of all 10 combinations:

$$\begin{array}{llll} 3 \times 4 = 12 & 4 \times 5 = 20 & 5 \times 8 = 40 \checkmark & 8 \times 9 = 72 \checkmark \\ 3 \times 5 = 15 & 4 \times 8 = 32 \checkmark & 5 \times 9 = 45 \checkmark & \\ 3 \times 8 = 24 & 4 \times 9 = 36 \checkmark & & \\ 3 \times 9 = 27 & & & \end{array}$$

Five of the pairs have a product greater than 30, so answer choice (A) is correct.

2. 120 Medium

This is a simple combination problem, so multiply each of the elements:

$$6 \text{ sandwiches} \times 4 \text{ sides} \times 5 \text{ desserts} = 120 \text{ combinations}$$

3. (C) Medium

Because this combination repeats the condiments, you must count each combination of mustard (M), ketchup (K), and horseradish (H).

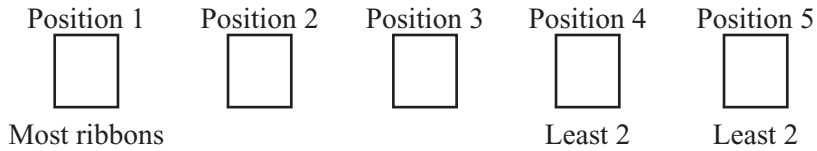
$$\begin{array}{lll} M & M-K & M-K-H \\ K & K-H & \\ H & M-H & \end{array}$$

There are seven possible combinations.

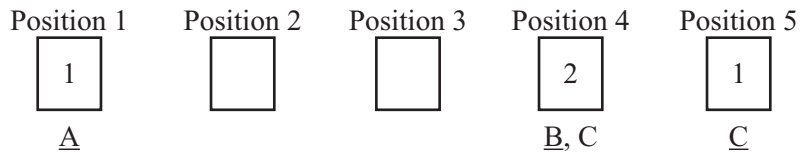
DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

4. 4 Medium

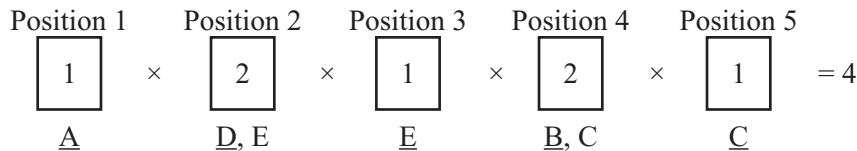
Set up five blank cards, one for each position in the row:



Then start with the restrictions:



Complete the cards without restrictions and then multiply all of the cards:



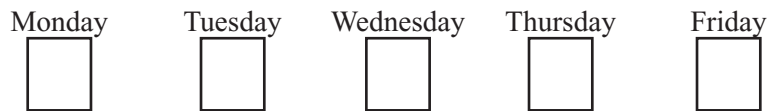
This is a very complicated permutation, but it has a medium difficulty level because it can be easily counted. Consider the dog with the most ribbons as M, the two dogs with the least as L_1 and L_2 , and the other two dogs as 4 and 5:

M-4-5- L_1 - L_2 M-4-5- L_2 - L_1
 M-5-4- L_1 - L_2 M-5-4- L_2 - L_1

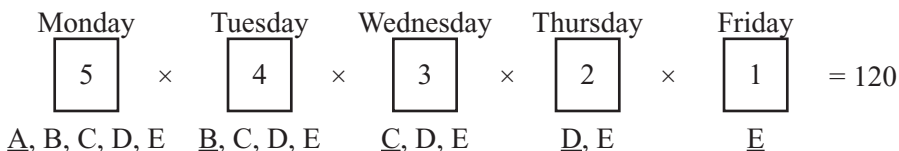
There are four possible arrangements.

5. 120 Hard

Set up five blank cards, one for each day of the work week:



Name the meetings A, B, C, D, and E:



There are 120 possible ways to schedule the meetings.

6. (C) Hard

You must understand the problem before you can solve it:

$$2 + 4 + 6 + 8 \dots + 100 = x$$

$$1 + 2 + 3 + 4 \dots + 50 = y$$

You can count out all 50 numbers for each equation, but this is a lengthy, inefficient solution. Instead, study a small group of numbers. We are going to look at the sum of the first five numbers from each set, rather than all fifty numbers:

$$2 + 4 + 6 + 8 + 10 = x \qquad 30 = x$$

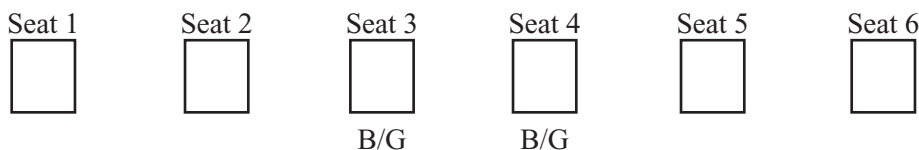
$$1 + 2 + 3 + 4 + 5 = y \qquad 15 = y$$

What is x in terms of y ? The value of x is twice as large as y : $x = 2y$

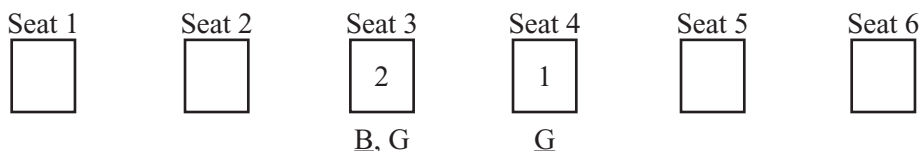
This is true no matter how many numbers we study from the series, as long as you use the same number of elements in x and y . The correct answer is (C).

7. 48 Hard

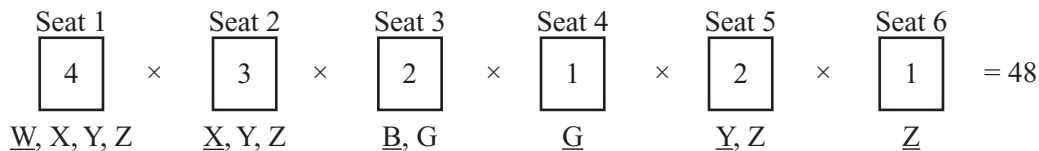
Set up six blank cards, each one representing a seat at the table:



Start with your restrictions first. The restriction says that the bride and groom must be in the center seats:



And then place the other four members of the bridal party (W, X, Y, Z). Multiply each of the cards to find the number of seating arrangements:



There are 48 possible seating arrangements.

Probability Problem Set—Pages 434-435

1. 685 Easy

Use the probability formula to set up an equation solving for the red marbles:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{5}{8} = \frac{\text{red marbles}}{1096} \rightarrow (5)(1096) = (8)(\text{red marbles}) \rightarrow$$

$$5480 = (8)(\text{red marbles}) \rightarrow 685 = \text{red marbles}$$

2. (B) Medium

Probability questions are often used with Data Analysis questions. This question earns a Medium difficulty level because test takers use the wrong information in the table. Since the question is about sedans, you can ignore the column concerning coupes. There are a total of 80 sedans on the lot ($30 + 50 = 80$), and 30 of those have 20,000 miles or less:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{30}{80} \rightarrow \frac{3}{8}$$

3. $\frac{2}{5}$ or .4 Medium

There are 10 possible numbers: $-2, -4, -6, -8, -10, -12, -14, -16, -18,$ and -20 . Start plugging them into the inequality:

$$-2: \quad 2x + 10 > -10 \rightarrow 2(-2) + 10 > -10 \rightarrow -4 + 10 > -10 \rightarrow 6 > -10 \quad \checkmark$$

$$-4: \quad 2x + 10 > -10 \rightarrow 2(-4) + 10 > -10 \rightarrow -8 + 10 > -10 \rightarrow 2 > -10 \quad \checkmark$$

$$-6: \quad 2x + 10 > -10 \rightarrow 2(-6) + 10 > -10 \rightarrow -12 + 10 > -10 \rightarrow -2 > -10 \quad \checkmark$$

$$-8: \quad 2x + 10 > -10 \rightarrow 2(-8) + 10 > -10 \rightarrow -16 + 10 > -10 \rightarrow -6 > -10 \quad \checkmark$$

$$-10: \quad 2x + 10 > -10 \rightarrow 2(-10) + 10 > -10 \rightarrow -20 + 10 > -10 \rightarrow -10 > -10 \quad \text{No}$$

All of the other possible negative even integers will produce results that are smaller than -10 . Four of the possible integers worked:

$$\text{Probability} = \frac{\text{favorable}}{\text{possible}} \rightarrow \frac{4}{10} \rightarrow \frac{2}{5}$$

4. (A) Medium

Find the probability of each answer choice to determine the one with the greatest possibility:

(A) A card with an arrow. There are 4 arrows on 6 cards. $(\frac{4}{6})$.

(B) A card with a flag. There are 2 flags on 6 cards $(\frac{2}{6})$.

(C) A card with a number. There are 3 numbers on 6 cards $(\frac{3}{6})$.

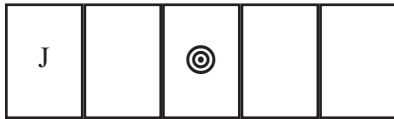
(D) A card with a face. There are 3 faces on 6 cards $(\frac{3}{6})$.

(E) A card with both an arrow and a flag. There are 2 arrow/flags on 6 cards $(\frac{2}{6})$.

The greatest probability is choice (A).

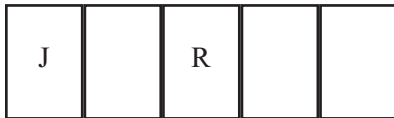
5. (D) Hard

This question has two events, which makes it the hardest difficulty level. Start with the first locker. There are 5 total lockers, but only 2 favorable students (Jud and Remy) for the first one with a bulls-eye:



$$\begin{array}{l} \text{J or R} \\ \frac{2}{5} \end{array}$$

If Jud is assigned the first locker, that leaves 4 total lockers, and only one favorable student (Remy) for the one with a bulls-eye:



$$\begin{array}{l} \text{R} \\ \frac{1}{4} \end{array}$$

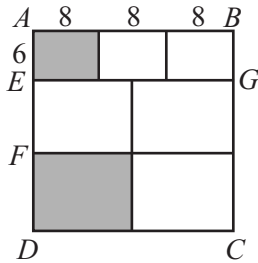
To find the probability of both events happening, find the product of the two individual probabilities:

$$\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

DATA ANALYSIS, STATISTICS, AND PROBABILITY MASTERY

6. (E) Hard

If the area of ABCD is 576, then each side is 24 (Area = side² so $576 = 24^2$). Use the side length and the information from the text to DIAGRAM the question. Start with the information about the three small rectangles:



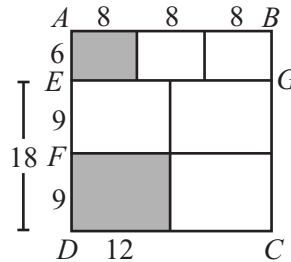
If AE is one-fourth of AD , then AE , the width of the small rectangle, is one-fourth of 24, or 6.

If $ABGE$ is divided into three equal rectangles, then the length of each small rectangle is $24 \div 3$, or 8. Therefore, the area of the small shaded region is $6 \times 8 = 48$.

Now find the area of the larger shaded rectangle:

If FD is one-half of ED , then FD , the width of the large rectangle, is one-half of 18, or 9.

If $EGCD$ is divided into four equal rectangles, then the length of each large rectangle is $24 \div 2$, or 12. Therefore, the area of the small shaded region is $9 \times 12 = 108$.



The combined area of the two shaded regions is 156 ($48 + 108 = 156$). So the probability of selecting an area from the shaded region is

$$\text{Geometric Probability} = \frac{\text{shaded area}}{\text{total possible area}} = \frac{156}{576} = \frac{13}{48}$$

The correct answer is (E).

Sequences Problem Set—Page 443

1. 6 Medium

Find the first four terms of the sequence:

1st	2nd	3rd	4th
t	$5t$	$25t$	$125t$
<small>$t \times 5$</small>	<small>$5t \times 5$</small>	<small>$25t \times 5$</small>	

Solve for t by setting the sum of the terms equal to 936:

$$1t + 5t + 25t + 125t = 936 \rightarrow 156t = 936 \rightarrow t = 6$$

2. (C) Medium

Since no numbers are given, SUPPLY them:

<u>1st</u>	<u>2nd</u>	<u>3rd</u>	<u>4th</u>	<u>5th</u>
1	4	16	64	256
1×4	4×4	16×4	64×4	

What is the ratio between the 2nd and 5th terms?

$$4 : 256 \rightarrow 2 : 128 \rightarrow 1 : 64$$

The correct answer is (C).

3. 12 Medium

Find the sum of the first 5 numbers:

$$-3 + -1 + 0 + 1 + 5 = 2$$

For every 5 numbers, the sum increases by 2. The sum of the first 10 numbers is 4:

$$-3 + -1 + 0 + 1 + 5 + -3 + -1 + 0 + 1 + 5 = 4$$

Since there numbers repeat 6 times between the 1st and 30th term (30 terms \div 5 repeating terms = 6), multiply 6 times the sum of the first 5 numbers:

$$6 \times 2 = 12$$

4. (B) Hard

Calculate the terms of the sequence until you find a pattern:

	Even	Odd	Even	Odd	Even	Odd	Even	Odd
<u>1st</u>	<u>2nd</u>	<u>3rd</u>	<u>4th</u>	<u>5th</u>	<u>6th</u>	<u>7th</u>	<u>8th</u>	<u>9th</u>
-4	4	0	0	-4	4	0	0	-4
-4×-1	$4 - 4$	0×-1	$0 - 4$	-4×-1	$4 - 4$	0×-1	$0 - 4$	

The pattern repeats after every 4 numbers. Therefore, multiples of 4 will help us find our answer. All terms that are multiples of 4 have a value of 0. What multiple of 4 is close to 45? You can use $4 \times 10 = 40$ or $4 \times 11 = 44$:

	Even	Odd	Even	Odd	Even	Odd	Even	Odd
<u>37th</u>	<u>38th</u>	<u>39th</u>	<u>40th</u>	<u>41st</u>	<u>42nd</u>	<u>43rd</u>	<u>44th</u>	<u>45th</u>
-4	4	0	0	-4	4	0	0	-4
-4×-1	$4 - 4$	0×-1	$0 - 4$	-4×-1	$4 - 4$	0×-1	$0 - 4$	

The correct answer is (B).

Overlapping Groups Problem Set—Page 446

1. 28 Medium

Use the formula for overlapping groups:

$$\begin{aligned}\text{Total} &= \text{Group A} + \text{Group B} + \text{Neither Group} - \text{Both Groups} \\ \text{Total} &= \text{Bills} + \text{Coins} + \text{Neither Bills nor Coins} - \text{Both Bills and Coins} \\ 100 &= 76 + 52 + 0 - \text{Both Bills and Coins} \\ 100 &= 128 - \text{Both Bills and Coins} \\ -28 &= - \text{Both Bills and Coins} \\ 28 &= \text{Both Bills and Coins}\end{aligned}$$

2. 105 Hard

Use the formula for overlapping groups:

$$\begin{aligned}\text{Total} &= \text{Group A} + \text{Group B} + \text{Neither Group} - \text{Both Groups} \\ \text{Total} &= \text{Stripes} + \text{Dots} + \text{Neither Stripes nor Dots} - \text{Both Stripes and Dots} \\ \text{Total} &= 35 + 12 + 63 - 5 \\ \text{Total} &= 110 - 5 \\ \text{Total} &= 105\end{aligned}$$

Logical Reasoning Problem Set—Page 449

1. (D) Easy

The pet is either a bird or a snake. Evaluate each answer choice:

Choice (A) is incorrect because the pet could be a snake.

Similarly, choice (B) is wrong because the pet could be a bird.

Choice (C) is wrong because it could be a yellow bird. The color of the bird is not relevant.

Choice (D) is correct. The store does not sell dogs, so the pet Raj bought cannot be a dog.

Choice (E) is wrong because it could be a snake. Whether that snake has fangs is not relevant.

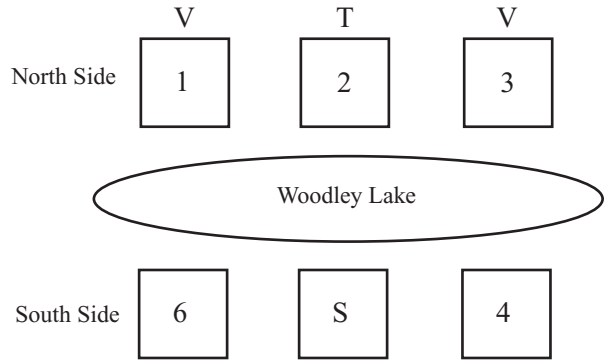
2. (C) Medium

DIAGRAM the question with each condition:

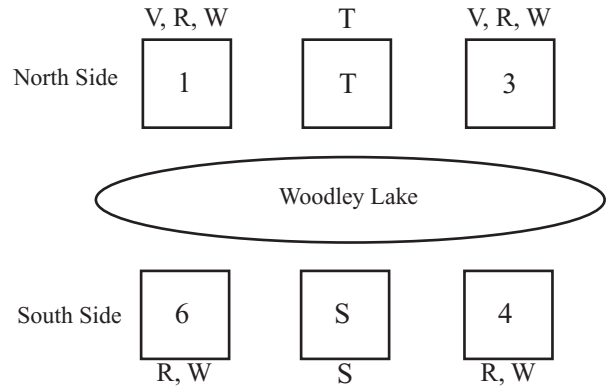
The first condition states that one cabin will remain unoccupied.

The second states that Tom and Val will be assigned to the north side. Val only has one cabin next to her, so she must be on the two ends. Since Tom is her only neighbor, he must be in the center cabin, cabin 2.

The third bullet places Sue in cabin 5.



The final condition states that Ron and Will live directly across from each other. Since cabin 2 and 5 are occupied, they must be in either 1 and 6 or 3 and 4.



The question then tells us that Val is assigned to cabin 1. This removes Val from the possibility list for cabin 3, leaving only Ron and Will. Answer choice (C) is correct.

