Astronomy Lab - Dimensional Analysis and Significant Figures

OBJECTIVES:
Use dimensional analysis to convert a number and its units to a number with different units, e.g., convert 60 mi/h to km/s using dimensional analysis.
Identify the significant figures in a given number.
Perform scientific calculations using the correct number of significant figures.

ASSUMPTIONS:
For this material, it is assumed you know how to round off numbers (we review some), multiply fractions, cancel within fractions, and use a calculator for basic calculations. If you need to practice doing this, then see the links below.

For tutorials on:	See
Rounding	http://www.khanacademy.org/video/rounding-whole-numbers-1?playlist=Developmental+Math AND http://www.khanacademy.org/video/rounding-whole-numbers-2?playlist=Developmental+Math AND http://www.khanacademy.org/video/rounding-whole-numbers-3?playlist=Developmental+Math
Multiplying and canceling fractions	http://www.khanacademy.org/video/multiplying-fractions?playlist=Developmental+Math AND http://www.khanacademy.org/video/multiplying-mixed-numbers?playlist=Developmental+Math

BEFORE YOU COME TO LAB - PREPARATION:

- At the top of the next right hand page in your lab notebook, enter the title "Dimensional Analysis".
- Enter the title and the page number in the Table of Contents.
- Copy or cut and paste the OBJECTIVES for this exercise into your lab notebook under the title.
- Write "PREPARATION" under the OBJECTIVES then write or cut and paste the "SUMMARY OF STEPS TO USE DIMENSIONAL ANALYSIS" shown below.
- Go to the next page in your lab notebook. Then cut and paste the table of USEFUL CONVERSION FACTORS into your notebook from page 2.
- Next, write or cut and paste the "RULES FOR SIGNIFICANT FIGURES" from page 2.
- Next, write or cut and paste the "RULES FOR ROUNDING NUMBERS" from page 2.
- Do this problem to the best of your ability and show your work. **What is 29 mi/h in km/h?** Show how you change the dimensions from initial units to the wanted units.

SUMMARY OF STEPS TO USE DIMENSIONAL ANALYSIS:
Use these steps to convert from one set of units to another:

1. **Start with the number and its units that you want to convert.**
2. **Write them as a fraction.**
3. **Put parentheses around them and multiply with another set of parentheses.**
4. **Fill in the open parentheses with a useful conversion factor so that UNWANTED UNITS CANCEL.**
5. **Do the math including checking for significant figures.**

SOME USEFUL CONVERSION FACTORS:

- Here are some useful conversion factors. You may always ask for conversion factors but it is very helpful to have these in front of you when working on conversion problems. Feel free to add others.
- Put the table below in your notebook. On this page in your lab notebook, add a piece of tap folded over to make a tab so this page is easy to find.

USEFUL CONVERSION FACTORS

Length/Distance	Time	Volume	Mass/Weight
1 ft = 12 in	1 min = 60 s	1 oz ~ 30. mL	1 kg ~ 2.2 lb
1 yd = 3 ft	1 h = 60 min	1 gal = 4 qt	1 lb = 16 oz
1 in ~ 2.54 cm	1 h = 3600 s	1 qt = 2 pt	
1 mi = 5280 ft	1 year ~ 365.25 days	1 pt = 2 cups	
1 AU ~ 93 000 000 mi		1 cup = 8 oz	
1 km = 1000 m		1 L ~ 1.06 qt	
1 mi ~ 1.6 km Or 1 mi ~ 1.609 km			

IMPORTANT Note: = means the conversion factor is exact (unlimited SF)
~ means the conversion factor is approximate & the SF need to be counted (count SF for the non-1 number).

RULES FOR SIGNIFICANT FIGURES (SF):

1. **Count from left to right starting at the first non-zero digit.**
2. **If there is a decimal, count all digits from the non-zero digit as significant.**
3. **If there is no decimal, stop counting at the last non-zero digit.**
4. **If a number is written in scientific notation, apply the rules to the factor multiplying the power of ten (not to X 10^n).**

RULES FOR ROUNDING NUMBERS

1. **Decide which is the last digit to keep**
2. **Leave it the same if the next digit is less than 5 (this is called rounding down)**
3. **But increase it by 1 if the next digit is 5 or more (this is called rounding up)**

PART 1: SIGNIFICANT FIGURES (SF)

HOW MANY DIGITS SHOULD YOU KEEP IN YOUR ANSWER and
WHY BOTHER WITH SIGNIFICANT FIGURES?

Some numbers are exact and some numbers are only approximate. Measurements are always approximate. In science we need to be aware of how accurate our measurements and our results are. For example, if you measure your body temperature with a thermometer, you have exactly 1 thermometer but your temperature may be 98.5 which is approximate. It is absurd to report your temperature as 98.526391 because you didn't measure that accurately.

Significant figures also help in calculations. For example, divide 2 by 3 on a calculator and you get something like 0.666666667. In science 0.67 may be good enough and here's why. Suppose you measure that you ran 37.5 meters in 6.4 seconds. You only know the distance to 3 digits of accuracy and you only know the time to 2 digits of accuracy. To get your average speed, divide 37.5 m by 6.4 s (37.5 m / 6.4 s). Your calculator reads 5.859375 meters per second. But you do not know your speed to that many digits of accuracy. So be honest and report your result only as accurately as you know it (5.9 meters per second -- more on that later). Besides, don't write down all those digits if you don't need them. So why bother with significant figures? Because we want to be honest with how accurate you measured and to save time and effort.

WHEN IS A DIGIT COUNTED AS A SIGNIFICANT FIGURE?
You already put the **RULES FOR SIGNIFICANT FIGURES** in your lab notebook, so now you get to put them into practice.

EXAMPLE SET 1 Make sure you understand these & add them to your notebook.

Ex: 235. Start at 2. Count to the 5. Decimal shown so all three digits are significant. Three sig. figs.

Ex: 1.287 Start at 1. Count to the 7. Decimal shown so all four digits are significant. Four sig. figs.

Ex: .16 Start at the 1. Count to the 6. Decimal shown so both digits are significant. Two sig. figs.

Ex: 3.21×10^{6} Apply only to the 3.21. Start at the 3. Count to the 1. Decimal shown so all three digits are significant. Three sig. figs.

Ex: 6.500×10^{-4} Apply only to the 6.500. Start at 6. Decimal shown so count all four digits as significant. Four sig. figs.

Ex: 3×10^{8} Apply only to the 3. One sig. fig.

Ex: 3.0200×10^{-3} Start at the three. There is a decimal so count every digit. Five sig. figs.

Ex: 0.00345 Start at 3 (first non-zero digit) and count to the 5. Three sig. figs.

Ex: 34000 Start at the 3. No decimal shown so stop at the 4 (last non-zero digit). Two sig. figs.

Ex: 34000. Start at the 3. Decimal shown so all five digits are significant. Five sig. figs.

Ex: 300 000 Start at the 3. No decimal so stop counting at the 3 (last non zero digit). One sig. fig.

PROBLEM SET 1 – SIGNIFICANT FIGURES PRACTICE

How many significant figures are in each of the following? Now you try doing these. You can write your number of significant figures on the right of each box inside the table.

1) 521.9 m	2) 503 h	3) 0.30986 s
4) 0.000 000 91 m/s	5) 93,000,000 mi	6) 9.1×10^{-31} kg
7) 5.4030×10^{12} Hz	8) 0.00405 km/h	9) 3.0×10^{8} m/s
10) 0.0060070 min	11) 0.0344 µm	12) 89,310 AU
13) 204.50 nm	14) 5.260×10^{-5} m	15) 7.70×10^{-6} s
16) 2700. LY	17) 2700 LY	18) 2700.0 LY
19) 0.002700 ms	20) 0.035 m	21) 500 mi

ANSWER SET 1

(Don't peak! Cover these up.)

1) 4	2) 3	3) 5
4) 2	5) 2	6) 2
7) 5	8) 3	9) 2
10) 5	11) 3	12) 4
13) 5	14) 4	15) 3
16) 4	17) 2	18) 5
19) 4	20) 2	21) 1

PART 2: EXACT NUMBERS

Some numbers are exact and have unlimited number of significant figures. Measurements are always approximate so count significant figures. Some whole numbers are exact. Some conversion factors have both exact and approximate numbers (whenever ~ is used). If the conversion factor shows ~ then it is approximate and you must count significant figures. If the conversion factors are in the same set of units (inches to feet or m to cm, for example) they are exact but if they switch sets of units (miles to meters for example), they are not exact.

EXAMPLE SET 2

Ex. I measured my height as 1.8 m (2 sig. figs.)

Ex. There are three exit doors in the lab. Exactly 3 so unlimited sig. figs. Some whole numbers are exact.

Ex. 2 times the radius of a circle equals its diameter. That 2 is an exact number (it is the whole number two) and so it has an unlimited number of significant figures.

Ex. 1 min = 60 seconds Exact therefore both the 1 and the 60 have unlimited number of sig. figs.

Ex. 1 meter = 100 centimeters. Again exact so unlimited number of sig. figs.

Ex. 1 kg ~ 2.2 lbs is an approximation. In that case, the 1 in 1 kg is a whole number with unlimited number of significant figures but the 2.2 is approximate and has two significant figures.

Ex. I measure the voltage as 110 V. 110 is whole number but it is a measurement. 2 sig. figs. Sometimes whole numbers are not exact for the case where it is measured.

PROBLEMS SET 2 – SIGNIFICANT FIGURES & EXACT NUMBERS

How many significant figures are in each of the following numbers? You can write your answer in the box for each of these.

1) My weight is 61 kg	2) The 60 in the 60 miles per hour on a speedometer.	3) The 60 in 60 minutes equals 1 hour. (exact)
4) The three in "Her hybrid car used only 3 gallons of gas for that trip."	5) The 2 in the area of a triangle where area = (base)(height)/2	6) The 1 in $3m = \frac{3m}{1}$
7) The 3 in the average of 6.2, 5.7, and 8.4 where $average = \frac{6.2+5.7+8.4}{3}$	8) The 2 in "He is about 2 meters tall."	9) The 5280 in 1 mi = 5280 ft (exact)
10) The 1.609 in 1 mi ~ 1.609 km (approximate)	11) The 2.54 in 1 in ~ 2.54 cm (approximate)	12) The 1 in 1 mi ~ 1.609 km

ANSWER SET 2

1) 2	2) 1	3) unlimited
4) 1	5) unlimited	6) unlimited
7) unlimited	8) 1	9) unlimited
10) 4	11) 3	12) unlimited – Even though this is an approximation, we are saying exactly 1 mi is approximately 1.609 km. Treat the 1 like a whole number.

PART 3: ROUNDING

What is ROUNDING?
Rounding means reducing the digits in a number while trying to keep its value similar. The result is less accurate, but easier to use. You have **RULES FOR ROUNDING NUMBERS** at the beginning of this lab.

EXAMPLE SET 3

Example 1: 73 rounded to the nearest ten is 70, because 73 is closer to 70 than to 80.

Example 2: Round 32.355923 to 2 significant figures.
Solution: 32|.355923 becomes 32. Or 32

Example 3: Round 32.355923 to 5 significant figures.
Solution: 32.355|923 becomes 32.356

Example 4: Round 32.355923 to 4 significant figures.
Solution: 32.35|5923 becomes 32.36

Example 5: Round 179,283 to 2 significant figures.
Solution: 17|9,283 becomes 180,000 (not 18)

Example 6: Round 683.28 to 1 significant figure.
Solution: 6|83.28 becomes 700 (not 7)

Example 7: Round 1,499 to 3 significant figures.
Solution: 1,49|9 becomes 1,500 (not 150) keep going (this is 2 significant figures)…
You have to put it in scientific notation to express 3 sig figs. Ans: 1.50×10^3

PROBLEM SET 3 – ROUNDING & SIGNIFICANT FIGURES

Round each number to the requested number of significant figures.

1) 24.4568 kg to 5 significant figures	2) 24.4568 kg to 4 significant figures
3) 24.4568 kg to 3 significant figures	4) 24.4568 kg to 2 significant figures
5) 24.4568 kg to 1 significant figure	6) 0.05535 m to 3 significant figures
7) 0.05535 m to 2 significant figures	8) 0.05535 m to 1significant figure
9) 14,535.45 mi/h to 4 significant figures	10) 14,535.45 mi/h to 3 significant figures
11) 14,535.45 mi/h to 2 significant figures	12) 14535.45 mi/h to 1 significant figure
13) 29.73 km/s to 1 significant figure	14) 0.000 6083 s to 2 significant figures
15) 30.2 km/s to 2 significant figures	16) 30.2 km/s to 1 significant figure
17) 16,002 to 4 significant figures	18) 0.000 500 381 to 3 significant figures

ANSWER SET 3

1) 24.457 kg	2) 24.46 kg
3) 24.5 kg	4) 24 kg
5) 20 kg (no decimal point)	6) 0.0554 m
7) 0.055 m	8) 0.06 m
9) 14,540 mi/h (no decimal)	10) 14,500 mi/h
11) 15,000 mi/h	12) 10,000 mi/h
13) 30 km/s (No decimal)	14) 0.000 61 s
15) 30. Km/s (Keep decimal) or 3.0 x 10 km/s	16) 30 km/s (No decimal)
17) 1.600×10^{4}	18) 0.000 500 or 5.00×10^{-4}

PART 4: CALCULATIONS INVOLVING SIGNIFICANT FIGURES

When you multiply or divide, keep the ***least number of significant figures*.**
For calculations involving multiple steps, wait until you are all finished to round off.

EXAMPLE SET 4

Example 1 – Calculation including sig. figs.

Calculate this on your calculator $\frac{(3.83)(1.402)}{1300}$

The calculator shows 0.004130508 .
Looking at each number: 3.83 has 3 sig. figs., 1.402 has 4 sig. figs, and 1300 has 2 sig. figs.
KEEP THE LEAST NUMBER of significant figures (2 from the 1300).
Answer: 0.0041 .

Example 2- Calculation including sig. figs.

Average these two numbers: 31.8 ft/s and 43.25 ft/s (Don't round yet)
Add 31.8 + 43.25 = 75.05 / 2 = 37.525
Your calculator shows: 37.525
Now go back through all the numbers in the calculation to check for sig. figs. When you are averaging, the number that is on the bottom and divides the whole sum, is an exact number. The 2 here is exact.
KEEP THE LEAST NUMBER OF SIG. FIGS. (3 from the 31.8 ft/s)
Answer: 37. 5 ft/s

Example 3 – Calculation including sig. figs.

Average 6.2 and 5.5 (Keep extra digits for now because we aren't at the end yet.)
Multiply that answer by 5.328
Divide that answer by 178
Divide that answer by 524.628
Multiply that answer by 186,000
Divide that answer by 0.86
Now go back through all the steps to check for the LEAST number of significant figures. Here is a list of the numbers to consider and their significant figures. Do you agree?

Number(s)	Sig. Fig.
6.2 mm and 5.5	2
2 (in the average)	Unlimited
5.328	4
178	3
524.628	6
186,000	3
0.86	2

The least of these is 2 significant figures, so your final answer is 72.

PROBLEM SET 4 – CALCULATIONS & SIGNIFICANT FIGURES

Perform the following calculations. Report your results with the correct number of significant figures. You can write your answers in the boxes below.

1) 48.2 mi / 123.8 s	2) (48.2 m)(123.8 m)
3) (5.2 ft)(6.78 ft)	4) $\frac{0.04nm}{430.725nm}$ (3.00 X 10^5 km/s)
5) $\left(\frac{0.26\,\text{mm}}{4812.63\,\text{mm}}\right)$ (3.00 X 10^5 km/s)	6) Average 3.32 nm and 2.65 nm Then divide by 248.3 nm Then multiply by 3.0 X 10^5 km/s Then divide by 0.739

ANSWER SET 4

1) 0.389 mi/s	2) 5970 m^2
3) 35 ft^2	4) 30 km/s
7) 16 km/s	8) 4900 km/s

PART 5: CALCULATIONS WITH DIMENSIONAL ANALYSIS & SIGNIFICANT FIGURES

You already put the **SUMMARY OF STEPS TO USE DIMENSIONAL ANALYSIS** in your lab notebook, so now you get to put them into practice. Setup the problems using those steps and show your work. You need to show your work to get any partial credit. Depending on which conversion factors you use, you can get different answers and I can't tell if you did the problem correctly unless you show your work.

EXAMPLE SET 5

Example 1: Convert 3.28 miles to km.

1. Start with what you want to convert. Answer: 3.28 mi (Note: Units included)

2. Write it as a fraction. Answer: $\frac{3.28mi}{1}$

3. Put parentheses around it and multiply with another set of parentheses.

$$\left(\frac{3.28mi}{1}\right)\left(\quad\right)$$

4. Fill in the open parentheses with a useful conversion factor SO THAT MI CANCELS.

$$\left(\frac{3.28mi}{1}\right)\left(\frac{1.6km}{1mi}\right)$$ (MILES CANCEL)

5. Do the math.

The mi cancels giving 5.248 km but your lowest number of significant figures is 2. Answer is 5.2 km

Example 2: Convert 280 ft/min to ft/s

1. Start with what you want to convert. 280 ft/min

2. Write it as a fraction. $\frac{280\,ft}{\min}$

3. Put parentheses around it and multiply with another set of parentheses.

$$\left(\frac{280\,ft}{\min}\right)\left(\quad\right)$$

4. Fill in the other parentheses with a useful conversion factor that cancels unwanted units.

$$\left(\frac{280\,ft}{\min}\right)\left(\frac{1\min}{60s}\right)$$ (MIN CANCELS)

5. Do the math.

The min cancels giving 4.6666666 ft/s but your lowest number of significant figures is the measurement you started with which has 2 sig. figs. Answer is 4.7 ft/s

Example 3: Convert 63.287 mi/h to ft/s

Combining steps 1, 2 and 3: $\left(\frac{63.287mi}{h}\right)\left(\quad\right)$

4. Fill in the parentheses with a useful conversion factor.

Here we note from the problem that both mi and h are unwanted units and must cancel.

Use one set of parentheses to deal with *mi* and another set of parentheses to handle *h*.

Example 3 continued:

$$\left(\frac{63.287mi}{h}\right)\left(\frac{5280\,ft}{1mi}\right)\left(\frac{1h}{3600s}\right)$$ (mi CANCELS, h CANCELS)

5. Do the math.

The *mi* cancels and the *h* cancels giving 92.82093333 ft/s. The lowest number of significant figures is 5 from the measurement you were given. Answer is 92.821 ft/s

Example 4: Convert 54 mi/h to km/h

$$\left(\frac{54\,mi}{h}\right)\left(\frac{1.609km}{1mi}\right) = 87 \text{ km/h}$$

Example 5: Convert 35.2 km to mi

$$\left(\frac{35.2km}{1}\right)\left(\frac{1mi}{1.6km}\right) = 22 \text{ mi}$$

Example 6: Convert 18.2 km/s to mi/min

$$\left(\frac{18.2km}{s}\right)\left(\frac{1mi}{1.609km}\right)\left(\frac{60s}{1min}\right) = 679 \text{ mi/min}$$

Or $$\left(\frac{18.2km}{s}\right)\left(\frac{1mi}{1.6km}\right)\left(\frac{60s}{1min}\right) = 680 \text{ mi/min}$$

PROBLEM SET 5 – DIMENSIONAL ANALYSIS & SIGNIFICANT FIGURES

Show your work so you can get partial credit. Setup your problems using parentheses like the examples. Follow the **SUMMARY OF STEPS TO USE DIMENSIONAL ANALYSIS.**

1) Convert 1.5 km to mi	2) Convert 34.2 mi to km
3) Convert 153.2 mi/h to km/h	4) Convert 13.8 mi/h to ft/s (don't use mph)
5) Convert 16.8 h to min	6) Convert 93.8 km/s to km/min
7) Convert 15.3 m/s to km/h	8) Convert 0.098 km/h to km/s
9) Convert 13.8 m/s to km/h	10) Convert 4.01 X 10^4 km to mi
11) 83 ft/s to mi/h (don't use mph)	12) 3.00 X 10^5 km/s to m/s
13) 3.00 X 10^5 km/s to mi/s	**14) 3.00 X 10^5 km/s to mi/h**
15) 75 mi/h to km/h	16a) 40 miles per hour to ft/s 16b) 40.0 mi/h to ft/s
17) 186,000 mi/s to km/s	**18) 186,000 mi/s to km/h**
19) Speed of rocket 25 000 mi/h to mi/min	20) Speed of rocket 25,000 mi/h to mi/s
21) Car speed 45 mi/h to m/s	22) Earth's orbital speed 30. km/s to mi/s

ANSWER SET 5

Your answers may differ slightly depending on which conversion factors you used.

1. $\left(\frac{1.5km}{1}\right)\left(\frac{1mi}{1.609km}\right)$ = 0.93 mi OR $\left(\frac{1.5km}{1}\right)\left(\frac{1mi}{1.6km}\right)$ = 0.94 mi

2. $\left(\frac{34.2mi}{1}\right)\left(\frac{1.609km}{1mi}\right)$ = 55.0 km OR $\left(\frac{34.2mi}{1}\right)\left(\frac{1.6km}{1mi}\right)$ = 55km

3. $\left(\frac{153.2mi}{h}\right)\left(\frac{1.609km}{1mi}\right) = 246.5$ km/h OR $\left(\frac{153.2mi}{h}\right)\left(\frac{1.6km}{1mi}\right)$ = 250 km/h

4. $\left(\frac{13.8mi}{h}\right)\left(\frac{5280\,ft}{1mi}\right)\left(\frac{1h}{3600s}\right)$ = 20.2 ft/s

5. $\left(\frac{16.8h}{1}\right)\left(\frac{60\,\text{min}}{1h}\right)$ = 1,008 = 1.01 x 10^3 min

6. $\left(\frac{93.8km}{s}\right)\left(\frac{60s}{1\text{min}}\right)$ = 5,630 km/min

7. $\left(\frac{15.3m}{s}\right)\left(\frac{1km}{1000m}\right)\left(\frac{3600s}{1h}\right)$ = 55.1 km/h

8. $\left(\frac{.098km}{h}\right)\left(\frac{1h}{3600s}\right)$ = 0.000027 km/s OR 2.7 x 10^{-5} km/s

9. $\left(\frac{13.8m}{s}\right)\left(\frac{1km}{1000m}\right)\left(\frac{3600s}{1h}\right)$ = 49.7 km/h

10. $\left(\frac{4.01*10^4km}{1}\right)\left(\frac{1mi}{1.6km}\right)$ = 25,100 mi OR 2.51 x 10^4 mi OR

$\left(\frac{4.01*10^4km}{1}\right)\left(\frac{1mi}{1.609km}\right)$ = 24,900 mi OR 2.49 x 10^4 mi

11) 57 mi/h	12) 3.00 X 10^8 m/s
13) 190,000 mi/s or 1.9 X 10^5 mi/s (1.6km) 186,000 mi/s or 1.86 X 10^5 mi/s (1.609km)	14) 6.8 X 10^8 mi/h (using 1 mi = 1.6 km) 6.71 X 10^8 mi/h (using 1 mi = 1.609 km)
15) 120 km/h	16a) 60 ft/s 16b) 58.7 ft/s
17) 3.0 X 10^5 km/s (using 1 mi = 1.6 km) 2.99 X 10^5 km/s (using 1 mi = 1.609 km)	18) 1.1 X 10^9 km/h (using 1 mi = 1.6 km) 1.08 X 10^9 km/h (using 1 mi = 1.609 km)
19) 420 mi/min	20) 6.9 mi/s
21) 20. m/s	22) 19 mi/s

MORE RESOURCES, EXAMPLES AND PRACTICE

http://www.khanacademy.org Search for "unit conversion" and watch videos in the Converting Units section.
http://education-portal.com Search for "unit conversion" and watch the lessons.
http://www.chem.tamu.edu/class/fyp/mathrev/mr-da.html Math skills review on dimensional analysis.
http://www.alysion.org/dimensional/fun.htm Fun with dimensional analysis.
http://mathforum.org/library/drmath/sets/select/dm_unit_convert.html Examples and practice.
http://sciencespot.net/Media/metriccnvsn.pdf Practice problems