9.5 Amortization

Objectives

1. Calculate the payment to pay off an amortized loan.
2. Construct an amortization schedule.
3. Find the present value of an annuity.
4. Calculate the unpaid balance on a loan.

Congratulations! You just bought a new home—it's lovely—and in a good neighborhood. Only 360 more payments and it's all yours. When you make such a large purchase, you usually have to take out a loan that you repay in monthly payments. The process of paying off a loan (plus interest) by making a series of regular, equal payments is called **amortization**, and such a loan is called an **amortized loan**.

If you were to make such a purchase, one of the first questions you might ask is, "What are my monthly payments?" Of course, the lender can answer this question, but you may find it interesting to learn the mathematics involved with paying off a mortgage so that you can answer that question yourself.

KEY POINT

Paying off a loan with regular payments is called amortization.

Amortization

Assume that you have purchased a new car and after your down payment, you borrowed \$10,000 from a bank to pay for the car. Also assume that you have agreed to pay off this loan by making equal monthly payments for 4 years. Let's look at this transaction from two points of view:

Banker's point of view: Instead of thinking about your payments, the banker might think of this transaction as a future value problem in which she is making a \$10,000 loan to you now and compounding the interest monthly for 4 years. At the end of 4 years, she expects to be paid the full amount due. Recall from Section 9.2 that this future value is

$$A = P\left(1 + \frac{r}{m}\right)^n.$$

Your point of view: For the time being, you could also ignore the question of monthly payments and choose to pay the banker in full with one payment at the end of 4 years. In order to have this money available, you could make monthly payments into a sinking fund to have the amount A available in 4 years. As you saw in Section 9.4, the formula for doing this is

$$A = R\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}.$$

Thus, to find your monthly payment, we will set the amount the banker expects to receive equal to the amount that you will save in the sinking fund and then solve for R.

FORMULA FOR FINDING PAYMENTS ON AN AMORTIZED LOAN

Assume that you borrow an amount P, which you will repay by taking out an amortized loan. You will make m periodic payments per year for n total payments and the annual interest rate is r. Then, you can find your payment by solving for R in the equation

$$P\left(1 + \frac{r}{m}\right)^n = R\left(\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right).^*$$

Do not let this equation intimidate you. You have done the calculation on the left side many times in Section 9.2 and the computation on the right side in Section 9.4. Once you find these two numbers, you do a simple division to solve for R, as you will see in Example 1.

EXAMPLE 1 Determining the Payments on an Amortized Loan

Assume that you have taken out an amortized loan for \$10,000 to buy a new car. The yearly interest rate is 18% and you have agreed to pay off the loan in 4 years. What is your monthly payment?

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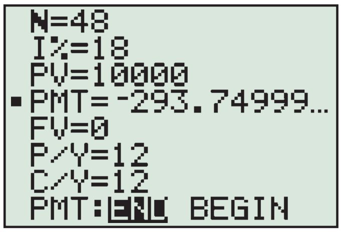
*Certainly we could do the necessary algebra to solve this equation for R. Then we could use this new formula for solving problems to find the monthly payments for amortized loans. We chose not to do this because our philosophy is to minimize the number of formulas that you have to memorize to solve the problems in this chapter. We will round payments on a loan *up* to the next cent.



TI-83 calculator confirms our computations in Example 1.

SOLUTION: We will use the preceding equation. The values of the variables in this equation are

$$P = 10{,}000$$
$$n = 12 \times 4 = 48$$
$$\frac{r}{m} = \frac{18\%}{12} = 0.015$$

We must solve for R in the equation

$$\underbrace{10{,}000}_{\text{amount of loan}}(1 + \overbrace{0.015}^{\text{monthly interest rate}})^{\overbrace{48}^{\text{number of payments}}} = R\left[\frac{(1 + 0.015)^{48} - 1}{0.015}\right].$$

If we calculate the numerical expressions on both sides of this equation as we did in Sections 9.2 and 9.4, we get

$$20{,}434.78289 = R(69.56521929).$$

Therefore, your monthly payment is

$$R = \frac{20{,}434.78289}{69.56521929} \approx \$293.75.$$

Now try Exercises 3 to 10. 18

Quiz Yourself 18

What would your payments be in Example 1 if you agree to pay off the loan in 5 years?

Amortization Schedules

Payments that a borrower makes on an amortized loan partly pay off the principal and partly pay interest on the outstanding principal. As the principal is reduced, each successive payment pays more toward principal and less toward interest. A list showing payment-by-payment how much is going to principal and interest is called an **amortization schedule**. We illustrate such a schedule in Example 2.

EXAMPLE 2 Constructing an Amortization Schedule

To expand your business selling collectibles on the Internet, you need a loan of \$5,000. Your banker loans you the money at a 12% annual interest rate, which you agree to pay back in three equal monthly installments of \$1,700.12.* Construct an amortization schedule for this loan.

SOLUTION: At the end of the first month, you have borrowed \$5,000 for 1 month at a 1% monthly interest rate. So using the simple interest formula, the interest that you owe the bank is

$$\overset{P}{\$5{,}000} \times \overset{r}{0.01} \times \overset{t}{1} = \overset{I}{\$50}.$$

Your payment is \$1,700.12; therefore, \$50 pays the interest, and the rest, \$1,700.12 − \$50 = \$1,650.12, is applied to the principal.

For the second month, you are now borrowing \$5,000 − \$1,650.12 = \$3,349.88 at 1% monthly interest. We complete the computations for the payments on this loan in Table 9.4.

Payment Number	Amount of Payment	Interest Payment	Applied to Principal	Balance
				\$5,000.00
1	\$1,700.12	\$50.00	\$1,650.12	\$3,349.88
2	\$1,700.12	\$33.50	\$1,666.62	\$1,683.26
3	\$1,700.12	\$16.83	\$1,683.29	−\$0.03

TABLE 9.4 An amortization schedule.

*We used the method from Example 1 to calculate the exact payment to be \$1,700.110557. Because we increase this ever so slightly to \$1,700.12, after the third payment we have overpaid by \$0.03.

As expected, we ended with a negative balance because the payment of $1,700.12 is a fraction of a cent larger than it needs to be. In an actual banking situation, the bank would adjust the final payment so that the final balance is exactly $0.00.

Example 3 illustrates how discouraging it can be when you make your first payment on a mortgage for a house and realize how little of your payment goes toward paying the principal.

EXAMPLE 3 *Constructing an Amortization Schedule*

Assume that you have saved money for a down payment on your dream house, but you still need to borrow $120,000 from your bank to complete the deal. The bank offers you a 30-year mortgage at an annual rate of 7%. The monthly payment is $798.37. Construct an amortization schedule for the first three payments on this loan.

SOLUTION: We compute Table 9.5* as we did Table 9.4 in Example 2.

Payment Number	Amount of Payment	Interest Payment	Applied to Principal	Balance
				$120,000.00
1	$798.37	$700.00	$98.37	$119,901.63
2	$798.37	$699.43	$98.94	$119,802.69
3	$798.37	$698.85	$99.52	$119,703.17

TABLE 9.5 Making an amortization schedule for a lengthy mortgage.

Quiz Yourself 19

Compute the fourth line of Table 9.5.

You see that for such a lengthy amortized loan, the early payments are mostly interest. Fortunately, because the debt is being reduced, each month a little more of the payment goes toward principal and a little less toward interest.

Now try Exercises 11 to 14. 19

HIGHLIGHT

Between the Numbers—Can They Really Do That to You?

How would you feel if you took out a $200,000 mortgage for a house, faithfully made all of your payments on time, and at the end of 1 year owed $201,118? Incredibly, this can actually happen if you have an adjustable rate mortgage, or ARM. Some ARMs allow you to make payments that *do not even cover the interest* on the loan, so the amount you owe increases even though you make your payments on time.

ARMs can have other very serious problems for the consumer. With an ARM, it is possible to start with a low interest rate, say 4%, and with yearly increases after several years *your interest rate could be much higher.* Mortgage lenders use an index, often tied to government securities, to determine how much to increase your interest rate. There are many different types of ARMs—some limit the rate increase from year to year, and others limit the maximum rate that can be charged. However, even with these limits, your monthly payments in an ARM could increase from $900 to $1,400 over a 3-year period, causing you great financial distress.

The *Consumer Handbook on Adjustable Rate Mortgages*, available from the Federal Reserve Board, is an excellent guide to ARMs and contains numerous examples, cautions, and a worksheet to help you make sensible decisions regarding mortgages.

*If you verify these computations by hand, your answers may differ slightly from ours due to a difference in the way we are rounding off our intermediate calculations.

HIGHLIGHT

Using a Spreadsheet to Make an Amortization Schedule

A spreadsheet can create an amortization schedule in the blink of an eye. The following is a spreadsheet that calculates the schedule for an amortized loan for \$10,000 with 60 monthly payments of \$202.77. We first show the spreadsheet displaying the formulas in the cells of the spreadsheet.

	A	B	C	D	E	F
1	End of Month	Payment	Interest	Principal	Balance	
2	0	\$202.77			\$10,000	
3	1	\$202.77	E2*0.08/12	B3 – C3	E2 – D3	
4	2	\$202.77	E3*0.08/12	B4 – C4	E3 – D4	
5	3	\$202.77	E4*0.08/12	B5 – C5	E4 – D5	
6	4	\$202.77	E5*0.08/12	B6 – C6	E5 – D6	
7	5	\$202.77	.	.	.	
8	6	\$202.77	.	.	.	
9	7	\$202.77	.	.	.	

Here is the same spreadsheet when the formulas in the spreadsheet are evaluated.

	A	B	C	D	E	F
1	End of Month	Payment	Interest	Principal	Balance	
2	0	\$202.77			\$10,000.00	
3	1	\$202.77	\$66.67	\$136.10	\$9,863.90	
4	2	\$202.77	\$65.76	\$137.01	\$9,726.89	
5	3	\$202.77	\$64.85	\$137.92	\$9,588.96	
6	4	\$202.77	\$63.93	\$138.84	\$9,450.12	
7	5	\$202.77	.	.		.
8	6	\$202.77	.	.		.
9	7	\$202.77	.	.		.

To generate a new schedule for a mortgage, all we have to do is change the formulas in several cells and the entire spreadsheet will be recalculated.

KEY POINT

We use the formula for finding the size of monthly payments to determine the present value of an annuity.

Finding the Present Value of an Annuity

When buying a car, your budget determines the size of the monthly payments you can afford, and that determines how much you can pay for the car you buy. Assume that you can afford car payments of \$200 per month for 4 years and your bank will grant you a car loan at an annual rate of 12%. We can think of this as a future value of an annuity problem where R is 200, $\frac{r}{m}$ is 1%, and n is 48 months. We know from Section 9.4 that the future value of this annuity is

$$A = 200\left[\frac{(1 + 0.01)^{48} - 1}{0.01}\right] = \$12{,}244.52.$$

This result does not mean that now you can afford a \$12,000 car! This amount is the *future value* of your annuity, not what that amount of money would be worth in the *present*.

DEFINITION If we know the monthly payment, the interest rate, and the number of payments, then the amount we can borrow is called the **present value of the annuity**.

We can find the present value of an annuity by setting the expression for the future value of an account using compound interest equal to the expression for finding the future value of an annuity and solving for the present value P.

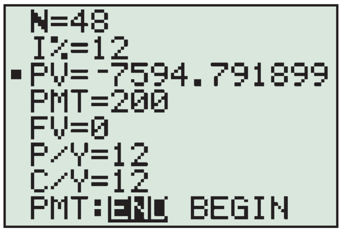
FINDING THE PRESENT VALUE OF AN ANNUITY Assume that you are making m periodic payments per year for n total payments into an annuity that pays an annual interest rate of r. Also assume that each of your payments is R. Then to find the present value of your annuity, solve for P in the equation

$$P\left(1+\frac{r}{m}\right)^n = R\left(\frac{\left(1+\frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right).$$

Again, you have done the computations on both sides of this equation many times in Sections 9.2 and 9.4.

EXAMPLE 4 *Determining the Price You Can Afford for a Car*

If you can afford to spend \$200 each month on car payments and the bank offers you a 4-year car loan with an annual rate of 12%, what is the present value of this annuity?



TI-83 screen confirms our computations in Example 4.

SOLUTION: To solve this problem, we can use the formula for finding payments on an amortized loan:

$$P\left(1+\frac{r}{m}\right)^n = R\left(\frac{\left(1+\frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right). \qquad (1)$$

We know $R = 200$, $\frac{r}{m} = 1\% = 0.01$, and $n = 48$ months. If we substitute these for the variables in equation (1), we get

$$P(1+0.01)^{48} = 200\left[\frac{(1+0.01)^{48}-1}{0.01}\right]. \qquad (2)$$

Calculating the numerical expressions on both sides of equation (2) gives us

$$P(1.612226078) = 12{,}244.52155.$$

Now dividing both sides of this equation by 1.612226078, we find

$$P = \left[\frac{12.244.52155}{1.612226078}\right] \approx \$7.594.79.$$

You may find this answer surprising, but the mathematics of this problem are clear. If you can only afford payments of \$200 per month, then you can only afford to finance a car loan for about \$7,600!

Now try Exercises 25 to 30. ✵ 20

Quiz Yourself 20

Redo Example 4, but now assume that you can afford payments of \$250 per month.

Finding the Unpaid Balance of a Loan

KEY POINT

To refinance a loan, we must know the unpaid balance on the loan.

During times when interest rates are high, people are forced to borrow money at these high rates if they want to buy a car or a house on credit. If interest rates decline, then it is wise to consider paying off the remaining debt on the first loan by taking out a second loan at a lower interest rate. This procedure is called **refinancing** the loan. To understand refinancing, we must be able to compute how much debt remains on a loan after a certain number of payments have been made.

EXAMPLE 5 Finding the Unpaid Balance on a Loan

a) Assume that you take out a 30-year mortgage for \$100,000 at an annual interest rate of 9%. If, after 10 years, interest rates drop and you want to refinance, how much remains to be paid on your mortgage?

b) If you can refinance your mortgage for the remaining 20 years at an annual interest rate of 7.2%, what will your monthly payments be?

c) How much will you save in interest in 20 years by paying the lower rate?

SOLUTION:

a) Doing the same kind of calculations as we did in Example 1, we find that the monthly payment is \$804.63.

Your monthly payment was based on the assumption that you would be paying the loan for 30 years. Therefore, after 10 years, you will not have accumulated enough in your annuity to pay off the banker. This means that the amount you owe, $P\left(1+\frac{r}{m}\right)^n$, must be larger than the amount that you have accumulated in your sinking fund,

$$R\left(\frac{\left(1+\frac{r}{m}\right)^n-1}{\frac{r}{m}}\right).$$

The unpaid balance U on the loan is therefore

$$U = \underbrace{P\left(1+\frac{r}{m}\right)^n}_{\text{what you owe}} - R\underbrace{\left(\frac{\left(1+\frac{r}{m}\right)^n-1}{\frac{r}{m}}\right)}_{\text{what you have accumulated}}{}^{*}. \tag{3}$$

It is important to recognize in equation (3) that only 10 years have elapsed, so $n = 12 \times 10 = 120$, not 360.

We can now substitute the values $P = \$100{,}000$, $\frac{r}{m} = 0.09/12 = 0.0075$, $n = 120$, and $R = \$804.63$ in equation (3).

$$U = 100{,}000(1+0.0075)^{\overset{\text{10 years}}{120}} - 804.63\left[\frac{(1+0.0075)^{\overset{\text{10 years}}{120}}-1}{0.0075}\right] = \$89{,}428.32.$$

Therefore, you still owe \$89,428.32 on this mortgage.

b) Because you have a balance of \$89,428.32 on your mortgage, in effect you are now taking out a new mortgage for this amount at an annual interest rate of 7.2% for the remaining 20 years. In this case, $P = \$89{,}428.32$, $n = 12 \times 20 = 240$, and $\frac{r}{m} = 0.072/12 = 0.006$.

We now solve for the monthly payment R in the equation

$$\underbrace{89{,}428.32}_{\text{amount borrowed for second loan}}(1+\underbrace{0.006}_{\text{new monthly interest rate}})^{\overset{\text{20 years}}{240}} = R\left[\frac{(1+0.006)^{240}-1}{0.006}\right]$$

*The unpaid balance appears in the balance column of an amortization table.

If we calculate the numerical expressions on both sides of this equation as we did in Sections 9.2 and 9.4, we get

$$375{,}829.1355 = R(533.7623389).$$

Therefore, your new monthly payment is

$$R = \frac{375{,}829.1355}{533.7623389} \approx \$704.12.$$

Therefore, by refinancing, you are able to reduce your monthly mortgage payment by $\$804.63 - \$704.12 = \$100.51$ per month.

c) If you make monthly payments on the unpaid balance for 20 years at the old interest rate, your total payments will be $240 \times \$804.63 = \$193{,}111.20$. If you make monthly payments for 20 years at the new interest rate, you will pay $240 \times \$704.12 = \$168{,}988.80$. The difference

$$\$193{,}111.20 - \$168{,}988.80 = \$24{,}122.40^*$$

is the amount that you save in interest over 20 years at the reduced rate.

Now try Exercises 31 to 36.

Often, when refinancing a mortgage you must pay a refinancing fee. The refinancing fee is often calculated as a percentage of the balance remaining on the mortgage. Suppose in Example 5 that you had to pay a 2% refinancing fee. Two percent of \$89,428.32 is \$1,788.57. You would gain this fee back in 18 months with the reduced payments on the loan. In this case, it is clear that after 18 months, you would benefit from the refinancing.

Exercises 9.5

Looking Back†

These exercises follow the general outline of the topics presented in this section and will give you a good overview of the material that you have just studied.

1. What does the left side of the first equation in Example 1 represent? What does the right side represent?

2. After reading the Highlight on adjustable rate mortgages, name two possible dangers of ARMs.

Sharpening Your Skills

Solve the equation

$$P\left(1+\frac{r}{m}\right)^n = R\left(\frac{\left(1+\frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right)$$

for R *to find the monthly payment necessary to pay off the loan. You are given the loan amount, the annual interest rate, and the length of the loan.*

3. Amount, \$5,000; rate, 10%; time, 4 years

4. Amount, \$6,000; rate, 8%; time, 3 years

5. Amount, \$8,000; rate, 7.5%; time, 4 years

6. Amount, \$10,000; rate, 8.4%; time, 4 years

7. Amount, \$12,500; rate, 8.25%; time, 4 years

8. Amount, \$10,500; rate, 9.75%; time, 4 years

9. Amount, \$1,900; rate, 8%; time, 18 months

10. Amount, \$1,050; rate, 6.5%; time, 15 months

In Exercises 11–14, complete the first three lines of an amortization schedule for each loan. Your answer should look like Table 9.4.

11. The loan described in Exercise 3

12. The loan described in Exercise 5

13. The loan described in Exercise 7

14. The loan described in Exercise 9

Applying What You've Learned

15. Paying off a mortgage. Assume that you have taken out a 30-year mortgage for \$100,000 at an annual rate of 7%.

a. Construct the first three lines of an amortization schedule for this mortgage.

b. Assume that you have decided to pay an extra \$100 per month to pay off the mortgage more quickly. Find the first three lines of your payment schedule under this assumption.

c. What is the difference in interest that you will pay on the mortgage during the fourth month if you pay the extra \$100 per month versus paying only the required payment?

*Note that you can find this amount more quickly by multiplying the difference in mortgage payments, \$100.51, by 240 months.

†Before doing these exercises, you may find it useful to review the note *How to Succeed at Mathematics* on page xix.

16. **Paying off a mortgage.** Repeat Exercise 15, but assume that the mortgage is a 20-year mortgage for $80,000 and the annual rate is 8%.

In Exercises 17–20, a) the monthly payment for each amortized loan and b) the total interest paid on the loan. Assume that all interest rates are annual rates.

17. **Paying off a boat.** Wilfredo bought a new boat for $13,500. He paid $2,000 for the down payment and financed the rest for 4 years at an interest rate of 7.2%.

18. **Paying off a car.** Beatrice bought a new car for $14,800. She received $3,500 as a trade-in on her old car and took out a 4 year loan at 8.4% to pay the rest.

19. **Paying off a consumer debt.** Franklin's new skis cost $350. After his down payment of $75, he financed the remainder at 18% for 5 months.

20. **Paying off a consumer debt.** Richard's used motorcycle cost $3,500. He paid $1,100 down and financed the rest at 8.5% for 2 years.

*In Exercises 21–24, assume that all mortgages are 30-year, adjustable rate mortgages. In each situation, use the additional information given to calculate the monthly payment on the mortgage a) in year one and b) in year five.**

21. $P = \$200{,}000$; beginning interest rate, 4%; rate increases 2% per year

22. $P = \$180{,}000$; beginning interest rate, 3.5%; rate increases 7% by year five

23. $P = \$220{,}000$; beginning interest rate, 4.4%; rate increases 2%, then 2%, then 1%, then 1.8%

24. $P = \$160{,}000$; beginning interest rate, 3.6%; rate increases 2%, then 1.6%, then 1.8%, then 2%

In Exercises 25–30, find the present value of each annuity. Assume that all rates are annual rates.

25. **The value of a lottery prize.** Marcus has won a $1,000,000 state lottery. He can take his prize as either 20 yearly payments of $50,000 or a lump sum of $425,000. Which is the better option? Assume an interest rate of 10%.

26. **The value of a lottery prize.** Belinda has won a $3,400,000 lottery. She can take her prize as either 20 yearly payments of $170,000 or a lump sum of $1,500,000. Which is the better option? Assume an interest rate of 10%.

27. **Present value of a car.** If Addison can afford car payments of $350 per month for 4 years, what is the price of a car that she can afford now? Assume an interest rate of 10.8%.

28. **Present value of a car.** If Pete can afford car payments of $250 per month for 5 years, what is the price of a car that he can afford now? Assume an interest rate of 9.6%.

29. **Planning for retirement.** Shane has a retirement plan with an insurance company. He can choose to be paid either $350 per month for 20 years, or he can receive a lump sum of $40,000. Which is the better option? Assume an interest rate of 9%.

30. **Planning for retirement.** Nico has a retirement plan with an investment company. She can choose to be paid either $400 per month for 10 years, or she can receive a lump sum of $30,000. Which is the better option? Assume an interest rate of 9%.

In Exercises 31–36, find the unpaid balance on each loan.

31. **Paying off a loan early.** You have taken an amortized loan at 8.5% for 5 years to pay off your new car, which cost $12,000. After 3 years, you decide to pay off the loan.

32. **Paying off a loan early.** In order to pay for new scuba equipment, you took an amortized loan for $1,800 at 11%, which you agreed to repay in 3 years. After 18 payments, you decide to pay off the loan.

33. **Paying off a mortgage early.** The MacGuffs took out a 30-year mortgage for $120,000 on their vacation home at an annual interest rate of 7%. They decide to refinance the mortgage after 8 years.

34. **Paying off a mortgage early.** In order to modernize their restaurant, the Buccos took out a 25-year mortgage for $135,000 at an annual interest rate of 6%. They decide to refinance the mortgage after 10 years.

35. **Paying off a loan early.** Garrett took out an amortized loan for $8,000 for 2 years at 8% to finish culinary school. After 12 payments, he decided to pay off the loan.

36. **Paying off a loan early.** Sheila borrowed $14,000 to invest in her floral shop. She took out an amortized loan at 6% for 5 years. After making payments for 1 year, she decided to pay off the loan.

*To keep these exercises simple, during year five, use the same values for P and n as you used in year one. Technically, to get a more exact answer, you should take into account that by year five, some of the principal would have been paid off and also that only 26 years of payments remain. Ignoring these does not affect the spirit of the exercise because during the first few years, most of your mortgage payments are going towards interest.

In Exercises 37–42, you are given the amount of an amortized loan P, *the annual interest rate* r, *the number of payments of the loan* n, *and the monthly payment* R. *After the specified number of months, the borrower decides to refinance at the new interest rate for the remaining length of the loan.*

a) What is the new monthly payment?

b) How much does the borrower save on interest?

37. $P = \$10{,}000$; $r = 8\%$; $n = 48$; $R = \$244.13$; refinance after 24 months at an interest rate of 6.5%.

38. $P = \$20{,}000$; $r = 9\%$; $n = 48$; $R = \$497.71$; refinance after 12 months at an interest rate of 7%.

39. $P = \$100{,}000$; $r = 8\%$; $n = 240$; $R = \$836.45$; refinance after 60 months at an interest rate of 7%.

40. $P = \$100{,}000$; $r = 9.5\%$; $n = 360$; $R = \$840.86$; refinance after 120 months at an interest rate of 7.5%.

41. $P = \$40{,}000$; $r = 8\%$; $n = 48$; $R = \$976.52$; refinance after 24 months at an interest rate of 7.5%.

42. $P = \$50{,}000$; $r = 10\%$; $n = 60$; $R = \$1{,}062.36$; refinance after 24 months at an interest rate of 9.5%.

Communicating Mathematics

43. In the formula for finding the payments on an amortized loan, where have you seen the expressions on the left and right sides of the equation before?

44. What does Example 5 show you with regard to the benefit of refinancing a loan?

45. What do the words *amortize* and *mortality* have in common? In light of the Three-Way Principle, how does this help you understand the word *amortization*?

46. Why is the interest payment on the mortgage decreasing in Table 9.4?

Using Technology to Investigate Mathematics

47. Ask your instructor for tutorials and spreadsheets to perform the amortization computations that we did in this section. Duplicate some of the results that we obtained in our examples.*

48. Find mortgage calculators on the Internet and use them to reproduce some of the examples that we explained in this section. Report on your findings.

For Extra Credit

49. In deciding to refinance at a lower interest rate, how does it affect your payments if you decide to refinance early in the loan period versus later in the loan period? For example, for a 60-month loan, would the new payments be larger, smaller, or the same if you refinance after 12 months instead of 36 months? Make up numerical examples to answer this question. Explain your answer.

50. Some mortgage agreements allow the borrower to make payments that are larger than what is required. Because this extra money goes toward reducing principal, increasing your payments may allow you to pay off the mortgage many years earlier, thus saving a great amount of interest. Assume you take out a 30-year amortized loan at 8% for \$100,000 and your monthly payments will be \$733.77. Suppose that instead of making the specified payment, you increase it by \$100 to \$833.77. How much do you save on interest over the life of the loan if you make the larger payment?