# The 

Compendium*



Photo by Rudy Light
The eye is always caugbt by light, but shadows bave more to say....

- Gregory Maguire (Mirror-Mirror)

[^0]
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## Sundials for Starters - Azimuth Of Sunrise <br> Robert L. Kellogg (Potomac MD)

Archeology at Stonehenge is the cover story in the June 2008 issue of the National Geographic. Located 6 miles north of Salisbury, England on a grassy plain are the famous linteled monoliths (Fig. 1). In the United States and Canada there are interesting, but less famous, solar alignments called "Medicine Circles" and "Sun-Circles" to sight the equinoxes and solstices.


Figure1. Stonehenge

In the U.S. the largest sun-circle is the Cahokia Woodhenge located about 8 miles across the Mississippi river from St. Louis near Collinsville, Illinois. The Woodhenge is part of Cahokia Mounds Historic State Park that has been designated by UNESCO as a World Heritage Site. Within the nearly six square miles are over 120 earthwork mounds built by the Mississippian Indians. The largest is Monks’ Mound measuring 100 ( 30.5 m ) feet tall and nearly 1000 feet ( 305 m ) long. The Mississippians also constructed earthworks in Mississippi (Ocmulgee Mound) and Ohio (Serpent Mound).


Figure 2. Cahokia Woodhenge Site Map

Excavations at the Cahokia began in the 1920's, but it was Robert Hill, William Iseminger and Warren Wittry who discovered large "bathtub" pits in the early 1960's. They called them bathtub pits because of their similarity in shape to an old-time bathtub. The effect was caused by a ramp dug from the ground to the bottom of the pit, and it is associated with the job of erecting a tall wooden pole into the pit.

While many of the ordinary pits are randomly placed, the bathtub pits are uniformly placed along circular arcs. According to Michael W. Friedlander " "The first recognition of layout around circles and possible astronomical attributes was seen by Wittry"" Several circles were discovered, with the most complete circle named by the archaeologists as Circle \#2.

The western edge of the Circle \#2 archeological site (Tract 15-1A) was destroyed in the early 1960's by the construction of a highway gravel pit (Fig. 2). Only 39 pits of what would have been a 48 wood post circle remain, but their positions were done with extreme care. The posts were accurately placed within 15 inches ( 38 cm ) along a uniform circle 205 feet ( 62.5 m ) in radius.

In 1985 wooden posts were erected at the excavated bathtub pits to give a sense of the size and shape of the Cahokia Woodhenge (Fig. 3). But how were they used? Let’s become astro-archaeological detectives.


Figure 3. Cahokia Woodhenge
The north, east and south posts are nearly perfectly aligned with the cardinal points. The west post position was destroyed by the 1960's gravel pit. From the center of the circle, the azimuth of each post is almost exactly $7.5^{\circ}$. According to Friedlander, "The distance was calculated to each post-hole, yielding a mean value for the circle radius $\mathrm{R}=205.0+/-0.7 \mathrm{ft}$."; and "the actual average spacing, derived from Wittry's post-hole locations is $7.51^{\circ}+/-0.35^{\circ \prime \prime}$.

Near the center of the sun-circle Wittry found another bathtub pit. But instead of being in the exact center of the sun-circle, it was located east of center by $5 \frac{3}{4}$ feet (1.75m). Standing at this post (the "observation

[^1]post") the apparent azimuth of the circle of posts is changed ever so slightly. We'll concentrate on two posts that seem to be related to the solstices: post \#8 and post \#16, with azimuths given in Table 1 below.

As astro-archaeologists we can compare these azimuths to the sunrise summer and winter solstices and check for an alignment. We will start with the standard sundialist's equation of azimuth:

$$
\begin{equation*}
\cos (180-A z)=-\frac{\sin (l a t) \cos (z)-\sin (d e c)}{\cos (l a t) \sin (z)} \tag{1}
\end{equation*}
$$

where $A z=$ sun's azimuth measured clockwise from north
dec $=$ declination of the sun (see more below about this)

$$
\begin{aligned}
& \text { lat = site latitude } \\
& \mathrm{z} \text { = solar zenith angle }
\end{aligned}
$$

We also have a relationship between zenith angle and the sun's hour angle HA, (the longitude from local meridian) as:
or

$$
\begin{align*}
& \cos (\mathrm{z})=\sin (\text { lat }) \sin (\mathrm{dec})+\cos (\text { lat }) \cos (\text { dec }) \cos (\mathrm{HA})  \tag{2}\\
& \cos (H A)=\frac{\cos (z)-\sin (\text { lat }) \sin (\text { dec })}{\cos (l a t) \cos (d e c)} \tag{3}
\end{align*}
$$

To use these equations, we need to find the solar declination at the solstices 1000 years ago. We know that today this value (also called the earth's obliquity, " $\varepsilon$ ") is $23.4383^{\circ}$. The obliquity is the sun's declination on the solstices: summer uses $+\varepsilon$ while winter uses $-\varepsilon$. But we need the obliquity for 1000 CE. 1000 years ago the obliquity was $23.569^{\circ}$, derived from an approximate equation to account for earth precession effects:

$$
\begin{equation*}
\mathcal{E}=23^{\circ}+0.43929111+\mathrm{T}(-46.815+\mathrm{T}(-.00059+\mathrm{T}(.001813))) / 3600 \tag{4}
\end{equation*}
$$

where $\mathrm{T}=$ number of centuries from the 2000.
Ignoring refraction, the sundialist's sunrise occurs exactly on the horizon for the center of the sun. This means that the zenith angle $z=90^{\circ}$. Finally, the only other piece of information we need is Cahokia's latitude of $36.86599^{\circ}$.

We then solve equation (1) for the sun's azimuth and add that to our Table 1 set of information:

| Post | Circle Center <br> Azimuth | Observer Post <br> Azimuth | Horizon Sunrise at <br> Solstice 1000 CE |
| :---: | :---: | :---: | :---: |
| Post \#8 | $60.02^{\circ}$ | $59.12^{\circ}$ | $58.42^{\circ}$ |
| Post \#16 | $121.01^{\circ}$ | $121.77^{\circ}$ | $120.03^{\circ}$ |

Table 1 Azimuths from Center of Sun-Circle and from Observer Post
These results at first look very promising. Whether it be from the center of the sun-circle or the observer's post, the angles match that of a simple solstice sunrise within a couple of degrees.

Our euphoria at becoming the next Indiana Jones is shattered however when we realize that (1) we have ignored refraction, (2) we've used the center of the sun and may need to use the upper or bottom limb of the sun as a realistic, observable marker, and (3) we have ignored the Collinsville Bluffs that rise above the true horizon.

The bluffs are only about 120 feet ( 36.5 m ) high, but run at an angle to the sun-circle such that they are at a distance of 23800 feet $(7.25 \mathrm{~km}$ ) in the direction of post \#8 and 16750 feet ( 5.1 km ) in the direction of post \#16 [from Friedlander reporting on various investigations], giving

$$
\begin{equation*}
\text { Summer Apparent Horizon }=\text { ToDeg*ATAN }(120 / 23800)=0.29^{\circ} \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\text { Winter Apparent Horizon }=\text { ToDeg*ATAN }(120 / 16750)=0.41^{\circ} \tag{6}
\end{equation*}
$$

(Remember that many spreadsheet formulas need conversions into degrees $=>\mathrm{pi} / 180^{\circ}$ or into radians $=>$ $180^{\circ} /$ pi.)

Refraction is perhaps the hardest thing to handle. The center of the sun is usually taken to be $-0.833^{\circ}$ below the horizon at the moment of sunrise or sunset. That is, the sun has a zenith angle of $\mathrm{z}=90.833^{\circ}$. As the sun changes altitude ( $\mathrm{H}=90^{\circ} \mathrm{z}$ ) close to the horizon, the refractive index changes rapidly. The solution depends upon tracing a ray of sunlight through the atmosphere as it penetrates different altitudes of atmosphere above the earth. At sunrise or sunset we see sunlight that has gone through the stratosphere and troposphere at many different temperatures, pressures, and humidity levels ${ }^{3}$. The integration has been done for a "standard atmosphere" resulting in ${ }^{4}$ :

Case 1: solar altitude ( H in degrees) $5^{\circ}$ to $85^{\circ}$

$$
\begin{equation*}
\text { Refraction(deg) }=\left(\frac{1}{3600}\right)\left(\frac{58.1}{\tan (H)}-\frac{0.07}{\tan ^{3}(H)}+\frac{0.000086}{\tan ^{5}(H)}\right) \tag{7}
\end{equation*}
$$

Case 2: solar altitude (H in degrees) $-0.57^{\circ}$ to $5^{0}$

$$
\begin{equation*}
\operatorname{Refraction}(\mathrm{deg})=\left(\frac{1}{3600}\right)\left(1735-518.2 \cdot H+103.4 \cdot H^{2}-12.79 \cdot H^{3}+0.711 \cdot H^{4}\right) \tag{8}
\end{equation*}
$$

Case 3: solar altitude (H in degrees) <-0.575 ${ }^{\circ}$

$$
\begin{equation*}
\text { Refraction }(\operatorname{deg})=\left(\frac{1}{3600}\right)\left(\frac{-20.774}{\tan (H)}\right) \tag{9}
\end{equation*}
$$

If we include the sun's radius, we can determine when true sunrise appears over the horizon. First light appears when the sun is at an altitude of

$$
\begin{equation*}
\text { H+SunRadius }+ \text { Refraction(H+SunRadius })=0 . \tag{10}
\end{equation*}
$$

Using SunRadius $=0.5(31 / 60)^{\circ}$, and putting the refractive equations into a macro of a spreadsheet, we can iteratively solve Case 3 to find that $\mathrm{H}=-0.833^{\circ}$. But at Cahokia, the Collinsville Bluffs raise the horizon to either $0.29^{\circ}$ or $0.41^{\circ}$ near the direction of the solstices so that we must now solve:
H+SunRadius +Refraction(H+SunRadius)=BluffHeight

Table 2 Sun Altitude and Azimuth over Collinsville Bluffs at Solstice

| Alignment <br> (Solstice) | Sun True <br> Alt | Sun Apparent Alt | Sun Az | Az Error <br> From Obs <br> Post | Az Error <br> Center of <br> Circle |
| :---: | :---: | :--- | :--- | :--- | :---: |
| Summer \#8 | $-0.4845^{\circ}$ | Top limb at $0.29^{\circ}$ | $58.7454^{\circ}$ | $-0.3712^{\circ}$ | $-1.2902^{\circ}$ |
| Summer \#8 | $-0.0324^{\circ}$ | Bottom limb at $0.29^{\circ}$ | $59.2291^{\circ}$ | $0.1125^{\circ}$ | $-0.7965^{\circ}$ |
| Summer \#8 | $-0.8333^{\circ} *$ | Top limb at Horizon | $58.4159^{\circ}$ | $-0.7007^{\circ}$ | $-1.6097^{\circ}$ |
| Winter \#16 | $-0.3426^{\circ}$ | Top limb at $0.41^{\circ}$ | $120.4831^{\circ}$ | $-1.2915^{\circ}$ | $-0.5271^{\circ}$ |
| Winter \#16 | $0.1747^{\circ}$ | Btm limb at $0.41^{\circ}$ | $120.9641^{\circ}$ | $-0.8105^{\circ}$ | $-0.0461^{\circ}$ |
| Winter \#16 | $-0 . .8333^{\circ} *$ | Top limb at Horizon | $120.0316^{\circ}$ | $-1.7430^{\circ}$ | $-0.9786^{\circ}$ |

* Classic sun altitude for sunrise/sunset when sun top limb is just at true horizon

[^2]Using the same iterative strategy, we use the refraction equations to work back to the altitude angle H . The results can be determined fairly quickly. Table 2 shows what I found:
Well, Table 2 gives us lots of numbers to ponder. But there is one more parameter that we have forgotten to include: the thickness of the tree post itself. Again relying on the archaeology of Wittry and Friedlander, the pits indicated that the trees were probably 15 to 18 inches ( $30-36 \mathrm{~cm}$ ) in diameter. At a distance of 205 feet ( 62.5 m ), the arc angle subtended by a tree is therefore about $0.35^{\circ}$. Figs. 4 and 5 are scaled drawings showing summer and winter solstice sunrise near post \#8 and post \#16 as well as the sun's alignment when it just becomes a full disk over the bluffs. As they say, "One picture is worth a thousand words". Figs. 4 and 5 show that the Mississippian Indians probably used the northern edge of post \#8 and post \#16 to observe the first rays of the sun over the Collinsville Bluffs.

Included in the digital version of the Compendium is a spreadsheet that allows you to do your own calculations of sunrise over the archaeological site of Cahokia Woodhenge.


Figure 4
Summer Solstice From Observer’s Post


Figure 5
Winter Solstice From Observer's Post

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# The Discovery That Lines For Unequal Hours Are Not Straight Alessandro Gunella (Biella, Italy) 

The opinion in the gnomonic literature is widespread that the 'discovery' of the fact that the lines of the unequal hours are really curves is due to the $17^{\text {th }}$ century French mathematicians (De La Hire and others). However, this statement actually goes back at least to Christopher Clavius (a mathematician and astronomer, chairman of the Board that decided the 1582 reform of the Calendar): see Lemma XXXIX of his treatise on the Astrolabe ${ }^{5}$, and the commentary added by Clavius, that gives flavor to the statement.

Here is the translation of the text, omitting the demonstration that is long and above all not relevant today. Particularly, remember that the gnomonic projection of the circles of a sphere onto a plane gives rise to straight lines only in the case of projection of great circles.


A 16th century engraving of Christopher Clavius (1538-1612) after a painting by Francisco Villamena.
"Lemma XXXIX: In the oblique sphere, the great circles passing through the points of the unequal hours of the equator and of two opposite parallels, surely do not pass through the hour points of the intermediary parallels."
"Scholium - From what has been proven, it is clear that in the oblique sphere it could not happen that the great circles pass through the points of the unequal hours of all the parallels; that means that all the diurnal arcs of each of them could not be divided into 12 equal parts by great circles.

But all the Authors of the texts on dials are certain of it. In fact all the authors divide the diurnal arcs of Cancer or of Capricorn into 12 equal parts, finding exactly the points of the unequal hours on both tropics; through them, and through the hour points of the equinoctial line, they trace straight lines, considered as those of the unequal hours, as if they were lines pointing out the unequal hour in each moment; that is like the intersections between the plane of the dial and the great circles would pass through the points of the unequal hours of all the parallels.

This statement, I confess, has tormented me through many years, because I did not find the reason of it; I have picked brains, sending letters to many Mathematicians, Italian and not, begging them to explain to me in what way they could prove that the great circles passing through the hour points of the equator and of the two tropics would also touch the points of the unequal hours of the other included parallels between the two tropics. But I have not been able to ever get what I asked, though some among them had promised me the demonstration. But surely these people deluded themselves, because when I have devoted myself to the search of it, I concluded that it could not happen."

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[^3]
## Sheldon Moore’s Sundial Business <br> Connecticut Historical Society

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Sundial Manufactured by Sheldon Moore, Kensington, CT c. 1840.
Sheldon Moore, son of Roswell and Lovina (Phillips) Moore, was born in Southington, Connecticut, October 17, 1798. After graduation from Yale in 1818, he taught for some time in Virginia and Connecticut. He studied law, but soon relinquished his practice and devoted his full time to business interests. He was forced to retire early due to ill health, and died in Kensington, March 20, 1866. His wife, Susan Langdon Dickinson, daughter of Jesse Dickinson, was born November 20, 1809 in Kensington. They were married November 1, 1831 and had three sons and a daughter. His brother Roswell Moore carried on an extensive business in the manufacture of cement.

Sheldon was much interested in fruit culture and about 1840 commenced the manufacture of sundials. A few years ago the Society was offered a collection of papers of Sheldon and Roswell Moore. This was purchased, and in cataloguing it was noted that a portion of the letters of Sheldon Moore dealt with sundials. Recently a specimen of one of these dials was presented to the Society, which recalled the manuscript material previously acquired. It is not often that we have both the records and objects manufactured. This is a fine example of the type of materials sought for this library and also the kind of specimen particularly desired for the museum. In hopes it will inspire further gifts along these lines, we include here a resume of information discovered concerning this endeavor.

The manufacture of sundials was commenced prior to August 1840, in which month a supply was sent to Breck \& Company of Boston for sale at $20 \%$ commission. In six months they sold 18 dials, which apparently encouraged Sheldon Moore to expand his facilities.

In the Cultivator, volume 8, page 104, for June 1841, appeared the following letter:
"Messrs. Gaylord \& Tucker--- I perceive in two communications from A. Walsh, Esq. in late Nos. of the Cultivator, recommendations of a sun dial as one of the convenient and useful decorations of a garden, and inquiries made where a proper one is to be procured. The object of this is to inform your numerous readers, that when the communications met my eye, I was engaged in constructing one, partly for the very purpose for which Mr. Walsh recommends it. I expect to have a quantity finished and for sale in two or three weeks. They will be for sale in Hartford and in New York City, and if convenient, a quantity will be sent to Albany. Due notice of the persons having them for sale will be given in the Cultivator, if permited. Resp'y yours, S. MOORE. Kensington, Ct., May 21, 1841.

Willcox and Prior of New Haven, later N. Willcox, did the casting. In July 1841 they billed Moore for 36 dials:

| carting to and from railroad | .50 |
| :--- | ---: |
| pattern box | .17 |
| 36 dials | 12.00 |
| pattern | 5.00 |
| cask for packing | $\underline{.25}$ |
|  | 17.92 |

From this it is apparent that the cost of each dial was roughly fifty cents. The question of suitable metal and color arose early, for Willcox reported:
"I know of no white metal that will run smooth that will be cheaper than block tin." However they continued to experiment, for a later letter says: "I send you herewith 2 metal dials as you directed. They are not as smooth as we sometimes get of the same kind of metal. There is not the same certainty as there is in Iron. Much depends on the temperature of the metal when poured into the mould."
"Type metal would more uniformly run smooth than this, but would be too brittle. I have made these two as cheap as I can, but were I [to] make a larger quantity they might be made for 25 cents per pound."

Willcox closed with some advice: "If you fit them for patterns make the letters as beveling as you can \& also give draft to the upright piece. We had much trouble in moulding these in consequence of the glue over the surface, which I suppose got on when you put on the letters."

The bill enclosed at this time included the charge for
the two white metal dials of 2.79
25 cast iron dials 8.33

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| cask and cartage | .38 |
| :--- | ---: |
| Box for pattern | .25 |
| cartage to railroad | .$\underline{25}$ |
|  | 12.00 |

In August 1841 news of the sundial had spread to New York State, for Alexander Walsh, of Lansingburgh, acknowledged the receipt of two dials, one of which he was going to place in his garden and exhibit the other at Saratoga Springs. This gentleman is the same person alluded to previously in the columns of the Cultivator requesting a sundial.

In September, the Cultivator continued the discussion:
This instrument for measuring time has been known and in use from the earliest ages, and is a common and valued ornament in gardens, pleasure grounds and yards in Europe, but has been comparatively little known or noticed in this country. Our climate is well adapted to the use of the sun dial, the great number of our clear and beautiful days rendering it more useful here, as a correct measurer of time, than in England, where the cloudy days are more frequent. Of late, however, public attention has been attracted to their value and convenience, and we are glad to announce that a manufactory of dials, a correct and beautiful article, has been established by MR. MOORE, of Kensington, (Ct.) specimens of which may be seen at Henry Rawls \& Co's. Albany, and at Geo. c. Thorburn's, New York. In the yard or garden, they should be placed on a neat and firm pedestal, and are very ornamental. We have copied a good figure (71) of such a pedestal, from Downing's new work on Landscape Gardening, from which we extract the following just remarks respecting it: "Sun dials are among the oldest decorations for the garden and grounds, and there are scarcely any we think more suitable. They are not merely decorative, but have also a tasteful character, and may therefore be occasionally placed in distant parts of the grounds, should a favorite walk terminate there. When we meet daily in our walks for a number of years, with one of these silent monitors of the flight of time, we become in a degree attached to it, and almost look upon it as gifted with a species of intelligence, which looks out when the sunbeams smile upon the dial plate."

We cannot avoid paying, in this place, a well deserved compliment to our friend MR. WALSH, whose indefatigable labors in the cause of horticulture and correct taste are so well known, as it was his suggestion which called this manufactory of dials into existence.

In the next issue, Sheldon Moore hastened to set the record straight and give directions for the mounting of his dials:

Messrs. Gaylord \& Tucker - Will you allow me a small space in the Cultivator for next month to say a few words about the mode of setting the sun dials you so favorably noticed in your last paper, and one or two other observations regarding them.

The best and most convenient way of setting them is, after leveling the pedestal or plane on which the dial is to be placed, (being particular to get it level in an east and west direction,) then to adjust a clock or watch to the true apparent time, (either by setting it by another time piece known to be correct, or by equal altitudes or other observation of the sun,) and then at 12 o'clock, M , or at ten minutes before or after, set the dial true, and make it fast to the pedestal by screws or nails.

They can be placed by setting the stile or gnomon true north and south by a meridian line or a compass, but the foregoing method is preferable.

They are accurate for the latitude of this place, ( $41^{\circ} 36^{\prime}$ ) and will be sufficiently so for 150 or 200 miles north of this latitude, and will be entirely correct for any latitude if the dial is inclined in setting, so that the edge of the gnomon that casts the shadow will be parallel with the pole of the earth; in other words, if
when the latitude is less than that of the dial, the south side is elevated as many degrees as the latitude is less, and when the latitude is greater, the north side is raised in the same proportion.

Your remark that my manufactory of dials was called into existence by the suggestions of Mr. Walsh, is not strictly correct, as I had been engaged in preparing for making them, for some time before Mr. Walsh's articles met my eye, as will be seen by turning to my note to you in the June Cultivator; but his suggestions hastened the manufacture and I am under special obligations to him for his encouragement and his effort to make them known and introduce them into use. SHELDON MOORE. Kensington, Ct. Sept. 15, 1841.

The announcement in the Cultivator was seen by John McRae, of Fayetteville, North Carolina, who wrote September 15 requesting one. He said further: "I published some time ago an elegant map of the State 3 feet by 7 in size, handsomely engraved by Mr. Tanner of Philadelphia of which I have still a few copies on hand. If Mr. Moore is willing, I will exchange my work for his. The price of the map is ten dollars for the same amount of sun dials calculated for use in this region. ..."

During October several dealers requested shipment of new lots, for all on hand had been sold. Also, Hovey \& Company in Boston wrote requesting an agency for that vicinity. Hovey's published a horticultural periodical in which they offered to "notice in such manner as would make them sell well."

In November the following appeared in their periodical:
SUN DIAL. We are gratified to witness the introduction of the sun dial into our gardens. It is an old, but suitable ornament, and now that they can be procured at such reasonable prices, and such beautiful pedestals upon which to place them, we shall advise their general introduction into lawns and extensive flower gardens. We shall give an engraving, in a future number, of some of the pedestals made in New York, at the manufactory of Mr. Goodwin, corner of Chamber and Hudson Streets, and of Mr. Little, Chestnut Street, Philadelphia. A very neat dial plate is manufactured by S. Moore, of Connecticut, which may be had at the very low price of one dollar, and which answers every purpose. [These dials are offered for sale by Messrs. Hovey \& Co., Boston, and G. C. Thorburn, New York]

In the following August the last notice that we have located concerning the sundials appeared :
Sun Dials for Garden Ornaments.- It will be recollected that some time since (Vol. 7, p. 403) we noticed the cast iron sun dials made by Mr. S. Moore, of Connecticut. They were calculated for a northern latitude. In a late letter to us, Mr. Moore states that he is now making one for a southern latitude, $35^{\circ}$, which will answer for all places south of Mason and Dixon's line. We recommend these dials as neat ornaments to a garden, and quite as useful as ornamental.-Id.

What subsequently happened, we do not know. The correspondence ends late in 1841. No further references have been located in the Cultivator or Magazine of Horticulture. What evidence there is indicates a sale in 1840-41 of not more than 100 dials. Their cost was about fifty cents each, and they were sold at one dollar. The resulting profit is perhaps the best answer to the fate of Sheldon Moore's enterprise.

Sheldon Moore died in Kensington, Conn., March 20, 1866, aged 67 years. He was the son of Roswell and Lovina (Phillips) Moore, and was born in Southington, Conn., Oct. 17, 1798. After his graduation he taught for some time in Virginia and in Connecticut. He afterwards studied law, and was admitted to practice, but did not make it his profession. He was engaged in business from which he retired about twenty-five years ago, on account of a failure of health. (Taken from Yale Obituary Record, 1860-70, p. 202).

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## A Triple Horizontal Dial For California John Davis (Ipswich, UK)

## Introduction

There are not many clients for a horizontal sundial who have consulted an original copy of William Leybourn's 1682 book $^{1}$ on dialing before arriving to discuss their dial design. But Dr. Rudy Light, of Redwood Valley, California, had done so and also brought a copy of Leybourn's diagram of "An Horizontall Dial with its Furniture" (reproduced in Waugh ${ }^{2}$ and shown in Fig. 1) with him when he visited me in the summer of 2003. When he explained that he wanted a large scientific dial to position outside his house and that he "wasn't afraid of complications", I knew it would be an interesting project.

We started with a list of features which the dial should show. The essentials were:

* a time ring for Pacific Standard Time (or the solar version thereof)
* the equation of time
* declination lines for solstices, equinoxes and several family anniversaries
* the sun's azimuth
* the sun's declination
* the sun's altitude
* sunrise/sunset times
* right ascension of the sun.


Fig. 1. A 'conventional' horizontal dial with a full set of furniture from Leybourn (ref. 1).

An additional list of possible features included:

* local apparent time
* times of noon at various places
* a compass rose
* a moondial
* a nocturnal
* owner's and maker's names
* motto(es)
* subsidiary direct east and direct west dials on the sides of the gnomon.

It took numerous iterations of the design before we came up with the instrument shown in Fig. 2 which meets most of the requirements. I do my designs using TurboCad ${ }^{\mathrm{TM}}$ software and, such was Rudy's dedication to the project, he also installed a copy of the program so that we could correspond about the fine details of the design. The story of how the design developed and was eventually turned into bronze follows.


Figure 2a. Drawing of the dialplate of the Redwood Valley dial.


Figure 2b. Drawing of the gnomon of the Redwood Valley dial.

## A Double Horizontal Dial?

The starting point was a standard horizontal dial with a polar-pointing gnomon and a nodus, designed to fit on a rectangular dial plate like the Leybourn example. The multitude of lines produced a rather messy-looking design, though. As an alternative, the double horizontal dial ${ }^{3,4}$ seemed an attractive proposition as its stereographic projection maps the whole visible half of the celestial sphere, allowing many of the measures of the sun's position to be shown directly. At the time, I was busy researching the large double horizontal dial made in 1685 by Henry Wynne, probably for the $1^{\text {st }}$ Earl Ferrers of Staunton Harold. ${ }^{5}$ I had made several double horizontal dials before but never one for America. I showed a drawing of the Wynne dial to Rudy and he was much taken with it, so we decided that his dial should be based on, or include, a double horizontal dial. It is worth mentioning that a
replica of the Staunton Harold dial has subsequently been finely crafted by Tony Moss. ${ }^{6}$

I quickly found a snag when laying out the dial. The stereographic projection is read from the shadow of a vertical knife-edge gnomon positioned in the center of the grid. In a double horizontal dial, this vertical gnomon fits underneath the polar-pointing gnomon needed for the standard horizontal dial which is placed around the outside of the dial plate. The difficulty is to get the vertical gnomon tall enough to cast a shadow across the grid in the summer sun. In a conventional English double horizontal dial, this is achieved by putting the vertical gnomon in the center of the dial plate and offsetting the origin of the polar gnomon as far as possible towards the southern edge of the dial plate (an extreme example of this can be seen in the 'Bacon' dial'). This tends to result in a rather narrow 'tail' for the upper part of the gnomon and a high percentage of
old double horizontal dials have bent or broken gnomons. In designing a dial for California, at a much more southerly latitude (approximately $39^{\circ}$ as opposed to $51.5^{\circ}$ for London), the problem is significantly worse than in England both because the polar gnomon is at a shallower angle and because the sun can be much higher in the sky, leading to shorter shadows.

The solution which I adopted was to move the whole of the stereographic grid towards the northern edge of the dial plate, allowing a taller knife-edge to be fitted in underneath the polar gnomon. This resulted in the grid no longer completely dominating the design. The features of the grid largely follow those of a Henry Wynne design. It has declination lines at $1^{\circ}$ intervals and hour arcs for every five minutes of time. The $5^{\circ}$ and $15^{\circ}$ declination lines and the quarter- and half-hour lines have dots on them to make it easier for the eye to follow them - a feature which is easy to describe but time-consuming to do! The two arcs for the ecliptic also have dots on them; a total of 360 spaced for each degree of solar longitude. Every $10^{\text {th }}$ dot is emphasized and the zodiac sigils are placed at $30^{\circ}$ intervals. The form of the sigils is that seen on Wynne's handengraved dials: I had produced a font of his lettering styles (there are two major ones) so it seemed a nice touch to make this new dial with old-style lettering.

The declination line for the equinoxes, running across the centre of the stereographic grid, is numbered [0] to [12] and then upside down back to [24], showing the right ascension of the sun.

The date scale running around the horizon circle of the grid uses the modern Gregorian calendar, of course, rather than the Julian one on a Wynne dial. We also chose to make it run anti-clockwise, so that the outward-facing names of the months read conventionally from left to right. This is an arrangement that was not adopted on English dials until around 1710, right at the end of Wynne's career. It does make for a more practical dial than Wynne's inward-facing engraving.

An azimuth scale is placed around the outside of the date scale. It runs from $60^{\circ}$ in the southwest to $300^{\circ}$ in the southeast, with $180^{\circ}$ at the north. It might seem that this clockwise arrangement is the wrong way round, but it is designed to show the bearing of the sun directly from the shadow of the knife-edge. Old English dials normally used a $4 \times$ $0-90^{\circ}$ scale, often with the zeroes at the East and West points, but this is not very convenient today.

With a stereographic projection, the sun's altitude is measured from the origin of the grid to the point where the knife-edge shadow crosses the declination line for today's date. The scale is nonlinear, ranging from $0^{\circ}$ on the horizon circle (sunrise and sunset) to $90^{\circ}$ (zenith) at the center. It would be possible to show the altitude by a set of unequally-spaced concentric circles engraved over the grid but they would look confusing. Although one or two early $17^{\text {th }}$ century drawings show these circles, I am not aware of any real dial with them. Instead, a straight ruler-like scale was usually engraved elsewhere on the dial so that the sun's height could be measured with the aid of a pair of dividers. On some dials, provision is made for this by cutting away the foot of the knife-edge, allowing easy access to the origin of the grid. An alternative would be to provide a separate rule or alidade, pivoted about the origin and capable of being rotated to the shadow position. It is thought that some old dials had such a device although none is now extant. We decided to have a pair of alidades, mirror images of each other. The reason for having a second alidade is that the thickness of the brass casts a shadow and this can lie exactly where the measurement is being taken. With a pair of alidades, one of them can be chosen so that its fiducial edge is facing the sun. A slight extension to the length of the alidades also allows them to provide a reading on the azimuth scale.

## A Third Dial

Moving the stereographic grid towards the north of the dialplate left an empty space to the south of the center. No self-respecting $17^{\text {th }}$ century English mathematical instrument maker would leave
valuable brass un-engraved so we too sought a good use for the space. Our choice was for another dial face and thus we arrived at the triple horizontal dial of the title. This third face was given an elliptical chapter ring, both because it filled the space rather efficiently and because the different shape seemed to add something to the aesthetics. It also nicely matches the elliptical inner ring of the Leybourn design. The third dial uses the same polar-pointing gnomon as the main dial on the periphery of the plate. So that it does not simply duplicate that dial, it is delineated with the longitude offset of Redwood Valley from the standard parallel $120^{\circ} \mathrm{W}$, amounting to 12.8 minutes. Labelled "Pacific Solar Time" (rather than Pacific Standard Time), it is divided to single minutes. To avoid the spacing getting too small, this is arranged in two rings, the inner one showing the even numbers of minutes and the outer, the odd ones. This feature was used on the Thomas Heath dial at Erddig, in Wales and then adopted by George Adams Jr. in the $18^{\text {th }}$ century.

The elliptical dial also has a set of declination lines, necessitating a nodus on the gnomon. This is not a standard feature of a traditional double horizontal line. The declination lines are for the solstices and equinoxes and also for five family anniversaries, discreetly labelled with initials. The nodus (actually a pair of noduses, separated by the thickness of the gnomon - see below) also allowed a set of lines for the altitude of the sun to be added. These are pairs of semi-circles, separated by the gnomon thickness. In Leybourn's design, the altitude lines are for Equalis, Dupla, Tripla etc., depending on the length of the shadow of a vertical gnomon of unit height. But we opted for the more modern approach of altitude in $5^{\circ}$ and $10^{\circ}$ steps.

I normally draw the outline of the gnomon on the dial plate in very narrow lines, together with the centers of any bolt or dowel holes. These etch to provide a good center-mark for later drilling, ensuring proper alignment. At the 'toe' of the gnomon, the two origins of the horizontal dial delineation are marked by 2 mm diameter black
circles. When finally assembled, the tip of the gnomon should cover exactly a quarter of each circle. On old English dials, these spots are often actual holes drilled right through the dial plate though they have usually become filled with corrosion products over the centuries. It is believed that these holes were used to locate some form of rotating ruler while the dial was being delineated but the process does not seem to have been documented.

## The Outer Dial

The main horizontal dial around the periphery of the plate is very traditional. The large Roman numerals face outwards and are skewed along the hour-lines. The division to individual minutes (numbered in 10s) is on the very perimeter of the dial so that they can have maximum spacing. The half-hours are marked by fleur-de-lys though the style chosen was not the intricate design of Wynne but that of Thomas Wright who worked a generation after him. The dial naturally reads Local Apparent Time (solar time) and this point is stressed by having the location, Redwood Valley, engraved in the noon gap. Showing the times of noon at various places around the globe is another feature which Wynne frequently used (and may have actually introduced). He had a number of formats for doing this including a series of separate chapter rings but the one which we adopted is copied from the Staunton Harold dial. The names of the places are engraved in small letters in the main chapter ring with one, or sometimes two, place(s) in each half-hour space. A short line to the minute ring gives the exact time of noon, by Redwood Valley LAT, of the place. The choice of place names mixed traditional locations such as $S^{t}$ Iohns in New Found Land and Pico an Isle of $y^{e}$ Azores as used by Wynne and others such as Kalamazoo (surely not seen on a dial before?) and Wolfeboro to which Rudy had a personal connection. The font used was again Wynne's, including his use of the longs as in Frederikhab.

## The Nocturnal

Yet another feature copied from Wynne was the inclusion of a nocturnal. This is in the form of two concentric discs placed immediately to the south of the gnomon. The outer disc, designed to be fixed, is calibrated anticlockwise V-XII-VII hours, sub-divided to 5 -minutes, on a $15^{\circ}$ per hour basis. There is a small secondary mark at 12:13 labeled PST for Pacific Standard Time (the main nocturnal scale indicates local mean time). The inner disc carries a calendar scale and, superimposed on it, the names of 19 stars prominent in the Californian sky. These are positioned so that their right ascensions correspond to the dates with 12:00 at the first point of Aries. Instructions for using the nocturnal are engraved, in a slightly paraphrased version of Wynne's original script, on the dial:

The Gnomon shows you y $y^{e}$ Pole Starr.
Look southward to $y^{e}$ proper Starr which is upon the Meridian.
Turn $y^{e}$ Nocturnal dial to point that Starr name to $y^{\circ}$ figure XII.
Read the hour of the night oppofite the day of the current month.
The full list of right ascensions and declinations of the chosen stars, epoch 2000.0, are shown in two rectangular tables to the east and west of the elliptical dial.

## Other Scales and Furniture

The other features which complete the scientific part of the dial are the arc segments containing the equation of time and sunrise/sunset data. I normally use equation of time data averaged over the next 50 years for engraving on dials, aiming at a continued accuracy over the life of the dial (or its owner!). For this dial, though, Rudy wanted to keep all of the design features for a particular year, in this case the millennium of 2000. Tabulated data is easy to obtain but most almanacs show the EoT for 0 hours UT each day. This is ideal for an astronomer but less convenient for a Californian sundial, likely to be observed
around noon local time. So the data were interpolated to give appropriate values for 12:00 local mean time each day. The common English EoT display, showing a continuous date scale and the number of minutes "Watch Slow" or "Watch Fast" against it, requires a further calculation to work out the decimal day number when the EoT is an integer number of minutes. The decimal dates for half-minute EoT values (and 15-second intervals in some places) were also calculated. One further figure occurs at the maxima and minima, where the two adjacent numbers on the minutes scale are the same. Here, the extra number of seconds at the absolute max/min were calculated and inserted with a small ' $s$ ' to indicate seconds. Thus, for example, on November 2.8, the EoT is shown as 16 m 26 s (Watch slow). All this was second nature to an $18^{\text {th }}$ century mathematical instrument maker in London - no wonder they served a seven-year apprenticeship.

The same date scale also serves for the sunrise and sunset scales. The solar times of local sunrise/sunset were first calculated on a daily basis, assuming no refraction, using the standard equations. These were then converted to local mean time. These were finally used to calculate theoretical dates when the sunrise/sunset would fall at a desired time, in five-minute steps, for the scale. The result is a highly non-linear scale, completed at each end with the extreme sunrise/set times for the solstices.

The final dial furniture comprises the names of the maker and owner, the latitude and longitude, and some decorative oak leaves and acorns. We also decided to have two mottoes. Mottoes on double horizontal dials are rare but not unknown - one used by Henry Wynne was Nulla Dies Sine Linea or 'No day without a line' - an injunction not to be idle particularly appropriate for the many-lined double horizontal design. ${ }^{8}$ With a surname like 'Light', the temptation was to use something along the lines of 'Lumen in umbra, Lumen ab intus' (light in shadows, light from within) but this was resisted as too immodest. Rudy finally chose "Transit Umbra, Lux Permanet" (the shadows
pass, light remains) ${ }^{9}$ and "Qui Lucem de Tenebris Lucet in Corde" (he who [sends] light from darkness shines in his heart). ${ }^{10}$


Fig. 3. The pierced and engraved gnomon of the dial (actually a modern replica) by Thomas Tompion in Kew Gardens, the inspiration for the gnomon on the Redwood Valley dial.

## The Gnomon

The general shape of a double horizontal gnomon is largely fixed by the geometry of the dial plate. When shown the first design, with plain slab sides in the form of a solid triangle, Rudy's wife commented, quite correctly, that it looked rather 'clunky'. On his double horizontal dials, Wynne generally covered the triangular sides with engraved coats of arms or calendrical tables but this would not have broken up the much larger triangle for our dial. On his visit to England, Rudy had visited Kew Gardens where he had seen the (replica) dial by Thomas Tompion with its elegantly pierced gnomon (Fig. 3) and so we decided that we needed some decorative piercing, enhanced by engraved lines. Rudy has a longstanding passion for oak woodland conservation and has planted thousands of oak trees on his
ranch so, instead of the traditional vine or acanthus leaves, details from the local Californian black oak tree (Querus kelloggii) were to be included. Additionally, a local species of garden spider (Argiope trifasciata) was added, seen in dorsal view on the east side of the gnomon and in ventral view on the west. A smaller spider is engraved near the tip of the gnomon. As I have no artistic abilities, Rudy had a local artist draw the required design and this fact is recorded by an engraving on west side of the base of the gnomon reading "Karen Soberanis delineavit".

The choice of a nodus type for a horizontal dial is always full of compromises between visibility for all positions of the sun, non-ambiguous readings and aesthetics. This is especially the case when the gnomon is quite thick. In the past, I have used V-cuts, 'W' forms and short cylindrical crosspieces (Fig. 4). In general, the cut-out forms have the disadvantage of disappearing at noon and consuming a significant length of the style edge when the sun is low, whereas any form of addition to the style edge leads to difficulty in deciding which part of the shadow is the intended position. After some experiments, I found that putting a hole through the gnomon, just behind the style edge so that it broke through and produced a narrow slit, worked well at all times other than noon and it was also very neat. If a reading is needed at noon, a short rod can be temporarily laid across the slit. I did the experiments using a circular hole but there is no need for it to be this shape. On the actual dial, we made the hole in the shape of an acorn and had its cup engraved as part of a decorative leaf-and-acorn border running up the side of the gnomon (see Fig 4(e)).


Fig. 4. Diagram of various nodus types for a horizontal dial. (a) V-cut, (b) 'W' form, (c) cross-cylinder, (d) through-hole.


Fig. 4(e) Photograph shows the form actually implemented.

## Making the Dial

By the time the dial design was finalized, it had grown from its initial diameter of 24 " to be 27 " $(686 \mathrm{~mm})$. The dial plate was to be $5 / 16^{\prime \prime}$ ( 8.3 mm ) thick and the gnomon was a substantial 5/8" ( 16 mm ). Although originally specified as brass, this had now been changed to phosphorbronze (PB102). The old adage of "you will appreciate the quality long after you've forgotten the cost" was used to justify this Rolls-Royce material. Certainly, although $17^{\text {th }}$ and $18^{\text {th }}$ century English dials were made of alloys which were essentially brasses, many of them contain a low percentage of tin which is thought to have accounted for their surviving several centuries of weathering, at least at locations which avoided the worst of the acid rain during the Industrial Revolution. ${ }^{11}$ As well as cost, the other implication of the change from brass to bronze was one of availability. Brass is readily available in high-quality cold-rolled sheets but for bronze the larger sizes are produced by hot-rolling, resulting in an inferior surface finish. This was to lead to added difficulties during manufacture.

Normally, I do all my own photo-etching but the size of this dial put it beyond the capacity of my facilities. With the kind assistance of Tony Moss,
this work was contracted out to the same company in the Newcastle area (several hundred miles from my home) that had produced the Henry Wynne replica. This meant that the photo masks, which for in-house (literally!) use I can produce as black-on-clear plots using an inkjet printer, had to be commercially produced as large clear-on-black plots. It took some time to find a company who could do this to the required quality, working from my TurboCad files of the design. The file sizes, after converting to the appropriate printerdriver, ran to several Gigabytes and each print took several hours.

The next difficulty to overcome was that the slab of bronze for the gnomon was discovered to be bowed, being several millimeters out over its length. In-house attempts to flatten it using a 3 --ton hydraulic jack only succeeded in lifting the roof of my garage - phosphor bronze has a lot of spring! Luckily, Tony’s engineering connections had much larger presses and were able to reduce the bow to a fraction of a mm, though it still took much work with a belt sander (thanks, Tony!) to achieve an acceptable surface finish.
The gnomon required both etching (on two sides) for the oakleaf-and-spider pattern and also waterjet cutting of the piercing and outline shape. There was much discussion over the best order for these procedures. If the etching and filling of the lines was done first, the waterjet cutting would have to be aligned to these patterns and there was a chance of surface damage to them from sidespray. But if the cutting was done first, the intricate cut edges would have been almost impossible to protect from the etching so this method was not feasible. In the event, the company of Aquajet did a fine job of aligning the heavy slab of engraved bronze in their machine and very little remedial work to the fill of the etched lines was required. Once again, conversion of the CAD design into suitable form to drive the waterjet machine produced very large file sizes (spiders are not a normal engineering requirement) and an overnight run. The machine had to be run very slowly to keep the sharp angles
intact and the cuts perpendicular through the thickness of the bronze.


Fig. 5. Milling style edge on the waterjet-cut gnomon.
The two large pieces of bronze were then shipped back to my workshop for cutting and finishing. The quality of the waterjet cutting for the gnomon was such that the interior piercing required very little cleaning up with files and other hand-tools. However, the approximate $1^{\circ}$ draw on the cut meant that the outside profile did need machining to square-up the edges and this had been allowed for in sizing the outline. In principle, this was a straightforward milling machine operation but the length of the style edge meant that I had to make


Fig. 6. Milling top of the knife edge where it intersects the polar gnomon. Note the stay supporting the gnomon tail.
two passes on my rather small milling machine (Fig. 5). Tony Moss has described ${ }^{6}$ the process of machining the knife-edge of a double horizontal gnomon and this was more-or-less the procedure which I followed, though it stretched my milling machine to the limit (see Fig. 6).

The circular dial plate was roughly cut from the square sheet by bandsaw and then mounted on a rotary table on the milling machine (Fig. 7). A low-voltage motor rotated it slowly like a merry-go-round while the edges were gradually machined to the finished profile.

With the nocturnal and alidades having been completed in-house months earlier, all that now remained was to mate the gnomon to the dial plate. This was achieved with a combination of dowel pins for alignment and stability and M6 stainless steel Allen bolts for the fixings. As this was a private dial destined for relatively benign conditions, extreme precautions against vandalism were not deemed necessary. Flush-fitting brass fixings were supplied to attach the dial to the pedestal so that, as on many old dials, the only visible signs after weathering will be three pairs of tiny holes.

After some rapid photographs of the completed


Fig. 7. Machining the edge of the dial plate, mounted on a motor-driven rotary table.
dial in the rather weak February sunshine (not quite Californian) shown in Figs. 8 and 9, it was packed up into two purpose-made wooden crates for shipping across the Atlantic. A square package with the dial plate weighed 30.5 kg ( 77 lbs ) which made the delivery driver groan rather but the other package with the gnomon and the accessories was 'only' 9.5 kg ( 21 lbs ) - the piercing had left a good fraction of the original bronze slab in my scrap bin. There was a mild panic three days later when the carrier's truck delivered a single package to Redwood Valley but, to much relief, the other one turned up the following day. The dial is currently on a temporary pedestal awaiting its final purpose-designed one.

I think we met our original design brief fairly well with all the 'essential' requirements met and many of the possible extras included. The only significant omission was the moondial and these are never terribly practical anyway. I hope Henry Wynne would approve of our updating of his design.

## Acknowledgements

It is a pleasure to thank Dr Rudy Light both for commissioning the dial and his substantial input to the design; Tony Moss for much advice and practical help with the machining (not to mention divulging commercial secrets); Michael Lowne for astronomical input, and Karen Soberanis for her artistic drawings.


Fig. 8. The completed dial before shipping. Note the nocturnal to the south of the gnomon.

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Fig. 9. The dial from the north, with the two alidades pivoted underneath the knife-edge.

## Gnomonics In A Japanese Junior High School Entrance Exam <br> Barry Duell（Kawagoe，Japan）

The following problem appeared in the 2008 entrance exam for Shibuya Kyoiku Gakuen Makuhari Junior High School，Chiba City，Japan．
な测量器具を作りました。その中に，型方唯という，方位を正確に知るための器具
があります。
正方案は，水平に買かれた平らな板に同心円が等間韾に描かれたものて，中心に棒
を垂直に立てて使います。 1 つの円に対し，午前と午後の 2 回，太郋によっててきる
を正碓に知ることがてきるのてす。

間 図は正方案に描かれた同心川を表し，図中 の×は，棒の影の先端が来た位潠を示してい ます。この印をもとに南北の線を引き，北の方角を示しなさい。ただし，作図に使った線


Used with permission of Shibuya Kyoiku Gakuen Makuhari Junior High School
Basically，the problem says that around 1300 a Chinese official［Yuan Dynasty astronomer，engineer，and mathematician Guo Shoujing（1271－1368）］in charge of calendar making developed a simple technique for finding north using shadows cast by a stick set at the center of concentric circles．The tips of two shadows cast on the same day are marked with＂x＂s．What the 6th graders（12 years old）who took the test had to do was to find the simplest way to determine the $\mathrm{N}-\mathrm{S}$ meridian，and to indicate which way is north．The students were permitted to use a drawing compass．

A graphical presentation of the solution is given on the left．Dialists
 today know it as a standard（＂Indian circle＂）technique for locating the meridian．

The science teacher who made the test explained in an interview that he tries to prepare problems that students cannot easily memorize in advance．He wants students who can think－who，when confronted with a new situation，can figure out how to correctly analyze a situation．（About 30 percent of applicants correctly worked out this problem．）

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    Sightings ... in Carnegie
Steven R. Woodbury (Springfield VA)
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Last summer, driving home from Pittsburgh, we stopped in the town of Carnegie, Pennsylvania. We initially got off the Interstate to look at two domed orthodox churches. But I soon spotted a large sundial downtown on Main Street. Made by a local sculptor, the dial is made of heavy Corten steel. The equatorial arm is stamped with hours and quarter hours, labeled for both standard time and daylight savings time. The polar gnomon wire was loose, and no longer useful for telling time. In a distinctive feature, the design also includes a horizontal gnomon wire, now missing; the meridian arm is marked to indicate the months of the year.



View of Dial Against the Sky


Equatorial arm, with hour lines (standard and daylight savings time). This also shows the attachment of the missing horizontal gnomon wire.
[Send your Sightings to Steve Woodbury. Include photos and good information on the location of the dial. Highlight the interesting elements of the dial. It's that simple!]

## Steven R. Woodbury

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## A Shadow Plane Hours-Until-Sunset Dial <br> Mac Oglesby (Brattleboro VT)

This article deals with Italian hours labeled in countdown fashion (hours until sunset, or, H2SS), as shown by a shadow plane which is horizontal at sunset, then rotated around a fixed polar rod. Of course, if the plane is attached to the polar rod, the rod may be twisted for the same results. Dialists know that if you draw Babylonian or Italian hour lines on a "normal" flat dial, those lines are all straight, and they sometimes mirror each other. Less well known is that you may use a moveable plane to find these hours. Fer de Vries explained this to me in email messages sent in 2002, so, in a sense, he is responsible for this sundial.

As dialists, we commonly pretend that sunlight comes from a point source at the Sun's center, that sunbeams are all parallel as they reach the Earth, that the declination of the Sun does not change during the day, and that Earth's atmosphere does not cause refraction. These assumptions are not true, but they make designing a sundial a lot easier, and, for most dials, there is no appreciable loss of accuracy.

We have all seen simple demonstrations which use a flat vane mounted on a polar rod to simulate a


Figure 1 - polar rod with solar vane for $43^{\circ} \mathrm{N}$ at noon polar shadow plane, here defined as the plane determined by the polar rod and the Sun's center. You locate the Sun's hour angle by rotating the rod, looking for the vane's thinnest shadow. Figure 1 shows such a device for latitude $43^{\circ} \mathrm{N}$ at noon. Use your imagination to ignore the black parts of the model.


Figure 2 -- H2SS shadow plane (white disk) with solar vane at equinox sunset

Instead of considering the planes determined by a polar rod and the Sun's center (polar shadow planes), we may consider a horizontal plane fixed to the polar rod. As the polar rod is rotated, that plane, which indicates either sunrise or sunset when horizontal, shows Babylonian and Italian hours when its shadow is thinnest (when the plane, if extended, passes through the Sun's center). But here we discuss only Italian hours labeled as hours until sunset (H2SS). For the next three photos we look at the H2SS shadow plane at different times of the day, together with a polar shadow plane as a time reference.

Figure 2 shows the H2SS shadow plane (shown here as a white disk) at sunset at an equinox. The H2SS shadow plane is horizontal and the polar shadow plane points west. Naturally, at sunset the polar shadow plane would point north of west if it were summer, and south of west if winter, at least where I live.

Next, figure 3 shows the position of the H2SS shadow plane at noon, 6 hours before sunset at an equinox.
Now, in figure 4, we look back to sunrise, 12 hours before sunset, again at an equinox.
Although I do not show the entire solstice sunset back to sunrise sequence, Figure 4a shows the H2SS shadow plane at sunset at or near the summer solstice.

We may use an equatorial scale and a fixed pointer to read the hours remaining until sunset. Figure 5 shows a model for latitude $43^{\circ} \mathrm{N}$ where the H2SS shadow plane (here, not a circular disk) is formed by slicing a cylinder whose axis is polar. The polar rod and the pointer are fixed - it is the scale that moves


Figure 3 -- H2SS shadow plane and solar vane at noon on an equinox


Figure 4a -- H2SS shadow plane and solar vane at sunset on summer solstice


Figure 4 -- H2SS shadow plane and solar vane at sunrise on an equinox
as the cylinder rotates. The H2SS shadow plane, which has a shallow circular cutout to help identify precisely when the plane is just being grazed by sunbeams, is horizontal at sunset. Here, the dial shows about 1 hour and 30 minutes until sunset. Of course, the hour marks are $15^{\circ}$ apart.

To use this dial, it must first be positioned with the polar rod (here, a wood dowel) aligned at the north celestial pole. The H2SS plane is horizontal at sunset. Then the cylinder is rotated until the H2SS plane is just grazed by sunbeams. Note that during spring and summer, when the Sun may be visible for more than 12 hours a day, this H2SS dial may offer two different readings, one of which will obviously be false.


Figure 5 - Shadow plane H2SS dial for latitude $43^{\circ} \mathrm{N}$ shows 1.5 hours to sunset.


Figure 6 - Adjustable H2SS shadow plane dial for latitudes $0^{\circ}$ $66^{\circ} \mathrm{N}$

This dial could also have a scale showing hours since sunrise, but, since the focus here is on H2SS, this dial face doesn't include that scale. On a personal note, I find that multiple scales, as well as sundials with two or more types of hour lines, are often confusing to read.

Figure 6 shows an adjustable model, good for $0^{\circ}$ to $66^{\circ} \mathrm{N}$. Just adjust the polar rod to the latitude and the H2SS shadow plane to be horizontal at sunset. Here, the dial shows 2.5 hours until sunset. This dial could have an hours since sunrise scale instead of, or in addition to, the H2SS scale shown. By simply relabeling the scale, a similar dial could be used at southern latitudes.

If we adopt a dual scale approach, we can design a "universal" Hours to Sunset (H2SS) dial, usable in latitudes $66^{\circ} \mathrm{S}$ to $66^{\circ} \mathrm{N}$, as in Figure 7. With the base horizontal, the thin cylinder's axis must be set for latitude (here, $43^{\circ}$ ) and be aimed at the celestial pole. The H2SS plane has a hole to aid alignment with sunbeams, and is set to horizontal at sunset, zero on either scale. Rotate the cylinder until sunbeams just graze the H2SS plane, and use the black (lower) numerals for hours until sunset and the red (upper) numerals for hours since sunrise. [In the Southern hemisphere, read the red numerals for H2SS and black for hours since sunrise.] However, stay alert! The dial below (figure 7), while it shows 4.5 hours until sunset, does not show 19.5 hours since sunrise. There's not that much daylight at $43^{\circ} \mathrm{N}$, even in summer. The second dial (figure 8) shows 1 hour since sunrise, but not 23 hours to sunset.


Figure 7. 4.5 hours until sunset - not 19.5 hours since sunrise.


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[Editor's Note: Mac Oglesby was awarded the Sawyer Dialing Prize in 2007. He used the proceeds of the prize to construct an Hours-UntilSunset sundial for the municipal center in his hometown.

In the next issue of The Compendium, we will have a brief article describing that dial and its installation. In the meantime, readers may wish to see a video of the dial's installation day at:
http://ru.youtube.com/watch?v $=0 \mathrm{og} 0 \mathrm{Xn} 25 \mathrm{XQg}$.]

Figure 8. 1 hour since sunrise - not 23 hours to sunset

## Slope Of The Summer Solstice Curve At Large Hour Angles Ortwin Feustel (Glashütten, Germany)

On close examination of vertical north, respectively south, sundials it is noticeable that the summer solstice line becomes nearly linear after sunrise and before sunset respectively in the early morning and in the afternoon. An approximate formula will be derived for calculating the sundial's geographical latitude based only on the slope angle of the solstice line's linear section. The slope angle must be measured on a dial face. Hence it is possible to do a quick check whether the line goes with the sundial location's latitude. Furthermore, the consequences of errors in measurement will be analyzed. Four examples supplement the topic.

Parametric presentation for lines of declination - [2], [3]
The symbols used in the following relations mean:

| $F D_{d a t}$ | small sphere position on a polar axis gnomon: distance between a point $D$ on a polar axis gnomon and the <br> slating gnomon's foot point $F ;$ point $D$ corresponds with the tip of an adequate vertical gnomon |
| :---: | :--- |
| $F E$ | shadow length: distance between gnomon's foot point $F$ and the small sphere's shadow position $E$ on a dial <br> face; position $E$ indicates date on a dial face; $E$ corresponds with the shadow tip's position of an adequate <br> vertical gnomon |
| $\varepsilon$ | shadow angle: angle between the polar axis gnomon and the gnomon's shadow on a dial face |
| $\delta$ | sun's declination |
| $\varphi$ | geographical latitude |
| $Z$ | time line angle: angle that a time line makes with noon |

The shadow length is given by (1) $F E=F D_{d a t} \frac{\cos \varepsilon}{\cos (\delta \mp \varepsilon)}$.
The shadow angle depends on geographical latitude and time line angle (2) $\varepsilon=\mp \arccos (\sin \varphi \cdot \cos z)$.
The parametric equations
(3) $x_{E}= \pm F E \cdot \sin z= \pm F D_{d a t} \frac{\cos \delta \sin z}{\cos (\delta \mp \varepsilon)}= \pm F D_{d a t} \frac{\sin z}{\cos \varepsilon \pm \tan \delta \sin \varepsilon} \quad$ and
(4) $y_{E}= \pm F E \cdot \cos z= \pm F D_{\text {dat }} \frac{\cos \delta \cos z}{\cos (\delta \mp \varepsilon)}= \pm F D_{\text {dat }} \frac{\cos z}{\cos \varepsilon \pm \tan \delta \sin \varepsilon}$
describe in Cartesian coordinates each point of a sundial's declination line. The upper signs in relations (1) to (4) correspond with the north sundial and the lower signs with the south sundial.

## Slope angle of north sundial's declination line

With regard to (2) the coordinates (3) and (4) take the form
(5) $x_{E N}=F D_{d a t} \frac{\sin z}{\sin \varphi \cos z+\tan \delta \sqrt{1-\sin ^{2} \varphi \cos ^{2} z}}=F D_{d a t} \frac{u}{v}$,
(6) $y_{E N}=F D_{\text {dat }} \frac{\cos z}{\sin \varphi \cos z+\tan \delta \sqrt{1-\sin ^{2} \varphi \cos ^{2} z}}=F D_{d a t} \frac{w}{v}$
for the shadow position $E$ on a north dial face.
Differentiation of (5) and (6) with respect to the time line angle $z$ yields
(7) $\frac{d x_{E N}}{d z}=F D_{d a t} \frac{\sin \varphi+\frac{\tan \delta \cos ^{2} \varphi \cos z}{\sqrt{1-\sin ^{2} \varphi \cos ^{2} z}}}{v^{2}}, \quad$ (8) $\frac{d y_{E N}}{d z}=-F D_{d a t} \frac{\frac{\tan \delta \sin z}{\sqrt{1-\sin ^{2} \varphi \cos ^{2} z}}}{v^{2}}$.
(8) divided by (7) leads to the sought-after differential quotient
(9) $\frac{d y_{E N}}{d x_{E N}}=\frac{\tan \delta \sin z}{-\sin \varphi \sqrt{1-\sin ^{2} \varphi \cos ^{2} z}-\tan \delta \cos ^{2} \varphi \cos z}$ and therefore the slope angle
(10) $N=\tan ^{-1}\left(\frac{\tan \delta \sin z}{-\sin \varphi \sqrt{1-\sin ^{2} \varphi \cos ^{2} z}-\tan \delta \cos ^{2} \varphi \cos z}\right)$ of the tangent line on the declination line.

In view of given values for slope angle $N$, sun's declination $\delta$ and time line angle $z$, an iteration method e.g. available with EXCEL - can be used to calculate the geographical latitude $\varphi$ in demand.

Approximate formula for calculating a north sundial's latitude
Setting $z=90^{\circ}$ in (10) we obtain the slope angle (11) $N=\tan ^{-1}\left(-\frac{\tan \delta_{S}}{\sin \varphi}\right)$ of the tangent to the summer solstice curve at the point directly east or west of the foot of the gnomon (corresponding to $z=90^{\circ}$ ). And with this we have the sought geographical latitude (12) $\varphi=\sin ^{-1}\left(\frac{\tan \delta_{S}}{-\tan N}\right)$.
Assuming in one's mind's eye a slope triangle at the summer solstice - e.g. on a wall sundial - one can take from it the slope required for (12): (13) $\frac{\Delta y}{\Delta x}=\tan N$.

## $\underline{\text { Slope angle of south sundial's declination line }}$

Using the same procedure as (5) to (11) for the declination line of a south sundial, the following slope angle is obtained (14) $S=\tan ^{-1}\left(\frac{\tan \delta \sin z}{\sin \varphi \sqrt{1-\sin ^{2} \varphi \cos ^{2} z}-\tan \delta \cos ^{2} \varphi \cos z}\right)$. In this case, too, it is practical to use an iteration method for calculating the geographical latitude $\varphi$ in demand.

Approximate formula for calculating a south sundial's latitude
Setting $z=90^{\circ}$ in (14) we similarly obtain the slope angle of the tangent to the summer solstice curve (15) $S=\tan ^{-1}\left(\frac{\tan \delta_{S}}{\sin \varphi}\right)$ at a point directly east or west of the foot of the gnomon. From this we have the desired latitude (16) $\varphi=\sin ^{-1}\left(\frac{\tan \delta_{S}}{\tan S}\right)$.

## Error estimation



Fig. 1: Graphical representation of (17) with $\delta_{S}=23.4393^{\circ}$ (standard epoch Y2000.0).


Fig. 2: Graphical representation of error estimation factor $F$ from formula (19) depending on angle $G$; sun's declination $\delta_{S}=23.4393^{\circ}$.

Using the absolute value of the angle's tangent $|\tan G|$ for both sundial types results from (12) and (16):
(17) $\varphi=\sin ^{-1}\left(\frac{\tan \delta_{S}}{|\tan G|}\right)$. The graphical representation of equation (17) with fixed quantity $\delta_{S}=23.4393^{\circ}$ is shown in figure 1. $\delta_{S}$ is the declination's value at the
time Y2000.0 (standard epoch, [1]).
Now, with help of the differential quotient (18) $\frac{d \varphi}{d G}=\frac{-2 \tan \delta_{S}}{|\sin (2 G)| \sqrt{\tan ^{2} G-\tan ^{2} \delta_{S}}}$ and the transition from differential quantities to difference quantities we obtain

$$
\text { (19) } \Delta \varphi=\frac{-2 \tan \delta_{S}}{|\sin (2 G)| \sqrt{\tan ^{2} G-\tan ^{2} \delta_{S}}}( \pm \Delta G)=F( \pm \Delta G) \text {. }
$$

Relation (19) may be used to estimate the consequences of errors in measurement or construction faults for geographical latitude values calculated with formula (17). Figure 2 makes clear the error estimation factor $F$ 's strong dependence on the angle $G$.

## Examples

## North sundial 1

A wall sundial has a summer solstice line's slope $\frac{\Delta y}{\Delta x}=-0.74545$. Assuming $\delta_{S}=24^{\circ}$ (rounded value), what is the geographical latitude for which this dial face is made? The error in measurement should be $f= \pm 2 \%$. Inserting (13) in relations (17) and (19) yields
(20) $\varphi=\sin ^{-1}\left(\frac{\tan \delta_{S}}{\left|\frac{\Delta y}{\Delta x}\right|}\right)$,
(21) $F=\frac{-2 \tan \delta_{S}}{\left|\sin \left(2 \tan ^{-1} \frac{\Delta y}{\Delta x}\right)\right| \sqrt{\left(\frac{\Delta y}{\Delta x}\right)^{2}-\tan ^{2} \delta_{S}}}$.

Slope tolerances $\pm f$ lead to the following angle tolerances
(22) $\Delta G_{p l u s}=\tan ^{-1}\left(\frac{\Delta y}{\Delta x}(1+f)\right)-\tan ^{-1}\left(\frac{\Delta y}{\Delta x}\right)$,
(23) $\Delta G_{\text {minus }}=\tan ^{-1}\left(\frac{\Delta y}{\Delta x}(1-f)\right)-\tan ^{-1}\left(\frac{\Delta y}{\Delta x}\right)$.

Calculation results:

| Formula | $(20)$ | $(21)$ | $(22)$ | $(23)$ | $(19)$ | (19) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Quantity | $\varphi$ | $F$ | $\Delta G_{\text {plus }}$ | $\Delta G_{\text {minus }}$ | $\Delta \varphi$ | $\Delta \varphi$ |
| Value | $36.674^{\circ}$ | -1.554 | $0.545^{\circ}$ | $-0.553^{\circ}$ | $-0.847^{\circ}$ | $0.859^{\circ}$ |

Hence the geographical latitude's range calculated is $35.827^{\circ} \ldots . .36 .674^{\circ} . .37 .533^{\circ}$.

## North sundial 2

A wall sundial has a summer solstice line with slope $\frac{\Delta y}{\Delta x}=-0.74545$. The sun's declination shall follow from (24) $\delta_{S}=23.4394^{\circ}-0.01300^{\circ} T ; T$ means time in Julian centuries from Y2000.0 (standard epoch) [1]. In case of $T=-21$ and $f= \pm 2 \%$, what is the geographical latitude for which this dial face is made?

Calculation results:

| Formula | $(24)$ | $(20)$ | $(21)$ | $(22)$ | $(23)$ | $(19)$ | $(19)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Quantity | $\delta_{S}$ | $\varphi$ | $F$ | $\Delta G_{\text {plus }}$ | $\Delta G_{\text {minus }}$ | $\Delta \varphi$ | $\Delta \varphi$ |
| Value | $23.712^{\circ}$ | $36.101^{\circ}$ | -1.522 | $0.545^{\circ}$ | $-0.553^{\circ}$ | $-0.829^{\circ}$ | $0.842^{\circ}$ |

Hence the geographical latitude's range calculated is $35.272^{\circ}$...36.101 ${ }^{\circ} . .36 .925^{\circ}$.

## North sundial 3

An ancient north sundial, which has been engraved on a stone wall about 100 B.C, shows that the slope of the tangent to the summer solstice line directly east or west of the foot of the gnomon equals the geographical latitude of the location. What is the location's latitude?

With $G=\varphi$ we obtain from (17) the formula
$\varphi=\cos ^{-1}\left(\frac{-\tan \delta_{S}}{2}+\sqrt{\left(\frac{\tan \delta_{S}}{2}\right)^{2}+1}\right)$ for the geographical latitude in demand.

Calculation results:

| Formula | $(24)$ | $(25)$ |
| :---: | :---: | :---: |
| Quantity | $\delta_{S}$ | $\varphi$ |
| Value | $23.712^{\circ}$ | $36.465^{\circ}$ |

Geographical latitudes of Athens - [4]

| Source | Hipparchos | Ptolemaios <br> (Geography) | Ptolemaios <br> (Astronomy) | Plinius | Vitruvius | Reality |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value | ca. $37^{\circ}$ | $37 ; 15^{\circ}$ | $36^{\circ}$ | $37.30^{\circ}$ | $36.87^{\circ}$ | $37 ; 58^{\circ}$ |

South sundial

Using a goniometer, one measures on a modern metal engraved sundial face ( $0.83 \mathrm{~m} \times 0.69 \mathrm{~m}$ ) an angle $G=29^{\circ}$ (the angle which the tangent line when $\mathrm{z}=90^{\circ}$ makes with the axes of abscissas). Assuming $\delta_{S}=23.439^{\circ}$ and an error in measurement $\Delta G= \pm 0.5^{\circ}$, what is the geographical latitude for which this dial face is made?

Calculation results:

| Formula | $(20)$ | $(21)$ | $(19)$ | $(19)$ |
| :---: | :---: | :---: | :---: | :---: |
| Quantity | $\varphi$ | $F$ | $\Delta \varphi(\Delta G=0,5)$ | $\Delta \varphi(\Delta G=-0,5)$ |
| Value | $51.457^{\circ}$ | -2.96 | $-1.48^{\circ}$ | $1.48^{\circ}$ |

Hence the geographical latitude’s range calculated is $49.977^{\circ} \ldots 51.457^{\circ} \ldots 52.937^{\circ}$. The real latitude is $\varphi=50.2^{\circ}$ 。

## Concluding remarks

Applying approximation formulas (17) or (20) one should be aware that increasing latitude values decreasing slope angle values - are accompanied by the fact that decreasing changes in slope angle correspond with increasing changes in latitude: see figure 1. It means that towards increasing latitudes the measurement of angles must be done with special care. In this latitude's range the factor $F$ in the equation (19) can reach considerable values: see figure 2.

Furthermore one must proceed carefully in view of the tangent line's "placement" on a dial face if only a relatively short section of the summer solstice line exists. Then the further solstice line's course (hyperbola!) has to be interpolated. The south sundial example shows that the limiting angle has been measured too low.

The calculation results for the latitude values of north sundials 1 and 2 (antique Athenian sundials) disclose the consequences of declination values which were varied slightly. Nevertheless both results match the antique latitudes of Athens as mentioned above.

On the whole, the method presented surely offers a possibility to check the geographical latitude for which the dial in question was most likely made, also quickly on the spot with help of a pocket calculator.

## Literature

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Quiz Answer: Ancient Sundial Location<br>Posed by Ortwin Feustel (Glashütten, Germany)

The zodiacal curves for the solstices of vertical north and south sundials approximate to straight lines near sunrise and sunset. Getting the slope angle of such a straight line (limiting angle of a declination line) from a sundial's face - for instance with help of a slope triangle - one can check among other things whether the sundial design matches the location's geographical latitude.

An ancient vertical north sundial, which had been engraved on a stone wall about 100 B.C., shows that the limiting angle of the declination line for the summer solstice equals the geographical latitude of the location. What is the location's latitude?

## Solution

This quiz lends itself to more than one answer. It asks you to find a dial whose latitude equals the slope of the tangent to its summer solstice curve, which, on this dial comes close to being a straight line. However, the curve is not exactly a straight line, so the answer we obtain will depend on where on the curve we select the tangent to measure.

The answer intended by Ortwin Feustel, who posed the question, is given as the example North Sundial 3 in his article in this issue: Given a solar declination in 100 B.C. of $23.712^{\circ}$, the desired latitude (obtained from his equation 25 ) is $36.465^{\circ}$. This results from considering the tangent to the summer solstice curve at the point directly east or west of the foot of the gnomon. Ortwin selects this point because it is on a portion of the curve close to the gnomon and likely to have survived the ravages of time, and because it corresponds to setting $\mathrm{z}=90^{\circ}$ in his equation 10 for the slope $N$ of the tangent to the solstice curve:
$N=\tan ^{-1}\left(\frac{\tan \delta \sin z}{-\sin \varphi \sqrt{1-\sin ^{2} \varphi \cos ^{2} z}-\tan \delta \cos ^{2} \varphi \cos z}\right)$. With the given value for z , this equation
yields $\varphi=\sin ^{-1}\left(\frac{\tan \delta_{S}}{|\tan N|}\right)$. By setting $\varphi=N$, we get $\varphi=\cos ^{-1}\left(\frac{-\tan \delta_{S}}{2}+\sqrt{\left(\frac{\tan \delta_{S}}{2}\right)^{2}+1}\right)$ after some
algebra, and thereby the values Ortwin offers as solution, corresponding closely to the North sundial on the Tower of Winds in Athens - the very dial that inspired the quiz.

However, for this particular dial, assuming a summer solstice declination of $23.712^{\circ}$, the slope of the tangent to the solstice curve varies from $42.4805^{\circ}$ when the sunbeams first graze the dial in the afternoon to $30.3686^{\circ}$ when the sun sets. This slope changes most slowly in the early afternoon when we are dealing with a portion of the hyperbolic solstice curve that comes very close to its asymptotic line. And, if this portion of the solstice line is still extant on the ancient dial, it may lead to less measurement error if we evaluate the slope of the tangent on this portion of the curve.

This latter approach is indeed equivalent to the strategy adopted by Rolf Wieland, Alessandro Gunella and Herb Ramp in their quiz answers. If we consider this portion of the hyperbolic solstice curve, we can calculate the slope $\left(\sin \varphi \sqrt{\cot ^{2} \delta-\cot ^{2} \varphi}\right)^{-1}$ of the asymptote as a proxy for the curve's own slope. If we set this slope equal to $\tan \varphi$, a bit of algebraic manipulation gives us the condition $\sin ^{2} \varphi=\sin \delta$. With the solar declination equal to $23.712^{\circ}$, we obtain the alternative answer $\varphi=39.357^{\circ}$. To check this result, we can use Ortwin's equation 10 to obtain the slope of the tangent when the sunbeams first graze the dialface. This occurs when $t=\cos ^{-1}(\tan \delta / \tan \varphi)=57.62^{\circ}, \quad z=\tan ^{-1}(\tan t \cos \varphi)=-129.356^{\circ}$. Fitting these values into Ortwin's equation gives a slope value of $39.356^{\circ}$. - Editor

## A Glimpse of Alsace <br> Fred Sawyer (Glastonbury CT)

One of the things I have often said to myself is that what the world really needs is more opportunities for sundial tourism!

The last time I said it, my lament was followed by the sudden realization that there were such opportunities available that I had neglected. Indeed, for several years the British Sundial Society has annually sponsored week-long sundial safaris to interesting sites all over Europe. These tours, originally led by David Young, and now run by Mike Cowham and his wife Valerie, have in past years visited such locations as Austria, Italy, and Scotland; and the plan for September of this year was for a trip to the Alsace region in Northeastern France. I decided it was time to join the tour.

On September 5, a luxury tour bus picked up 42 of us either at the airport or train station in Basel, Switzerland, and then transported us to the hotel that would serve as homebase in Bollwiller, France. Members of the group came from England, Wales, the U.S., Canada, Denmark, Ireland and Germany - all interested in seeing the sundials of Alsace.


The Astrologer Sundial in Strasbourg


Sundial on the church in Soultz
We began on Saturday with an entire day in Strasbourg, the largest city in the region. Following the obligatory stop for coffee/tea, we had a guided walking tour of the city and of the magnificent cathedral. Construction of this cathedral began in the $12^{\text {th }}$ century and spanned several centuries. The building itself has approximately 14 sundials; in our meanderings, we were able to find about 8 . The highlights were the 'Astrologer dial', one of the oldest polar gnomon dials in Europe; the meridian line inside the church adjacent to the famous $19^{\text {th }}$ century astronomical clock, both designed by Jean-Baptiste Schwilgué; and the trio of large dials designed by the $16^{\text {th }}$ century mathematician Conrad Dasypodius to regulate the cathedral's original astronomical clock. The Dasypodius dials are no longer easy to read, but they indicate local apparent, babylonian and italian hours. (The local sundial society in Alsace is named the Dasypodius Society.) The day was filled out with walks, river rides, or tourist trams around the city to see the sights.

On Sunday, we began in Soultz, where the church tower has an impressive $18^{\text {th }}$ century painted sundial recently renovated. We then traveled through vineyard country (actually that's most of Alsace) and visited the Gsell Cellars in Orschwihr for a tour of the winery and a wine tasting. The premium bottles of wine available for purchase all carried a label showing the sundial we could see on the front of their building. Our next stop was Guebwiller, where we found several dials, including a massive new painted dial covering the entire side of a three-story house, and a clock and dial combination at the top of the Church of St. Leger.

We then went on to Gueberschwihr and visited the convent of St. Marc. Here we were viewed two identical sundials painted on either side of the door to the main building.


Convent of St. Marc, Gueberschwihr
We finished the day with a visit to Rouffach to view the Eclipse Dial - a large painted dial whose face records astronomical positions of the sun, moon, and earth during the eclipse of 16 August 1617. Before returning to our hotel, we were treated to a display of sundials created by members of the Dasypodius Society.


A modern dial in Guebwiller

Most of Monday was spent touring around the picturesque town of Colmar, full of very old homes. As we stood in front of one $16^{\text {th }}$ century house that features on a lot of postcards, we were amazed to turn around to find that directly across the street was a house from the $14^{\text {th }}$ century! We had a good deal of time to tour on our own - I toured the town (with Geoff and Graham) in a horsedrawn buggy. All of this centered on the massive cathedral that had several sundials painted on, engraved in, or attached to its walls.


Eclipse dial in Rouffach - with tree shadow


A modern dial in medieval Eguisheim

We then went on to the $13^{\text {th }}$ century fortified village of Eguisheim. After enjoying the ambience of the village, we searched around the outskirts to find one of the more modern dials of the week, on the wall of a private residence - complete with analemmic hour lines.

Tuesday began with an early ride to Riquewihr, a well-preserved medieval town. Here we saw spy windows, flying martins, portcullis \& drawbridge, and 3 sundials, the most interesting of which was a painted wall dial in la Cour des Nobles; it had been nicely restored sometime in the 70's by Rene Rohr.


Bergheim dial, with notations for times of sunrise and sunset
defensive wall; today it is an active place of pilgrimage, honoring the patron saint of Alsace. Here on a terrace overlooking the entire valley 2500 feet below, we found an $18^{\text {th }}$ century stone polyhedral dial with 24 facets - all but one of them in good working order with properly placed gnomons. Four of the faces show antique, hebraic, babylonic and italic hours; the remaining 20 show the time at different locations around the world.

After a quick driveby of a forlorn cube dial stuck in the middle of a roundabout in Dorlisheim, we ended the day with a walk around Molsheim. On the former Jesuit Gothic church we found 5 (modern) dials scratched into the stone walls - but none of them has a gnomon. On the south transept, the easiest of these to see includes the inscription per DeCeM Lineas soL reLegIt Vias which refers to the miracle of the dial of Achaz, with the sun turning back 10 steps; it also includes a chronogram for 1758 (summing the uppercase letters as Roman numerals). Another scratched horizontal dial was found on a window sill of the church tower after a climb of some 72 steps.

Our trip continued through stork country replete with homes and churches carrying wire frames to encourage the nesting of storks (cicognes). Next, in Bergheim we saw a beautifully painted dial (restored in 1977) somewhat reminiscent of the Queen's College dial in Cambridge, complete with declination lines for the zodiac and months, and notations of times of sunrise and sunset throughout the year.

The highlight of the day was a trip to the convent at the top of Mont Ste-Odile, a wooded plateau surrounded by a prehistoric


Polyhedral dial at Mont-St.-Odile


Dial in the cour des Chartreux, Molsheim
Finally, in the cour des Chartreux, on the wall of a museum that began its existence as a convent, we were treated to a 4 square meter wall dial declining somewhat east of south. The design dates from 1703 but the dial has been repainted a number of times since then and is in good working order today.


Der Sonnenuhr Mann at the Freiburg Münster

On Wednesday, we crossed the border into Germany to visit Freiburg-im-Brezgau. The Münster (cathedral) here has 5 dials, the three most prominent are lined up vertically - a 13th century Sonnenuhr Mann on top, a modern dial in the middle, and an analemmic meridian line below.


After lunch we were welcomed at the Augustinermuseum - currently closed for construction, but opened exclusively for our tour and a special display of a subset of their collection of portable sundials. This special attention resulted in several of us being able to handle these historic dials.

After returning to Bollwiller, we enjoyed yet another of the huge meals we had throughout the week and celebrated with the BSS cabaret, at which everyone was invited to perform - reciting, singing - or whatever. A grand time was had by all.

A selection of dials at the Augustinermuseum

Finally, on Thursday we returned to Basel and visited the Historical Museum - an 18th century, 50 room house dedicated to domestic life in Basel in the 18th and 19th centuries. In addition to a large array of general historical displays, the museum had an entire room dedicated to portable sundials.


A sampling of the sundial room at the Historical Museum in Basel
The week was superb. We saw about 50 sundials 'in the wild' and another 150 or so in the relative captivity of a museum. Before returning to the U.S, I was then fortunate enough to be able to spend another 3 days back in England with Martin and Janet Jenkins, who shared their lovely home and the gnomonic highlights of Devonshire with me. All in all, a wonderful vacation.

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## Quiz: Nicole’s Reflections <br> Fred Sawyer (Glastonbury CT)

Nicole enjoyed spending her vacations in this cottage. She and her siblings had inherited it from her grandparents. Her favorite spot was the large sunroom - it held many memories. The rectangular room had a long, (roughly) southwestern back wall, 24 ft . wide and 8 ft . high with two windows. Grampa had set a small horizontal mirror on the sill of one of the windows ( 3 ft . above the floor) and then had drawn hourlines for a reflected sundial on the flat ceiling and side walls. She remembered having a date every afternoon with Grampa to see "The Great Fall" - that's what Grampa called it. They watched the spot of sunlight creep across the ceiling until it reached the 4 pm (local apparent time) hour line. At that time, each and every day, the spot seemed to give in to the pull of gravity and start its long fall down the side wall. Throughout the summer, Great Fall began at 4pm; and Nicole and Grampa were there to watch it!

Last year she and her family refreshed the paint on the hourlines in this special room. She gave her daughter the same assignment that Grampa had given her when she was in grammar school and the dial was first laid out. She remembered that Grampa made a mark where the 11am line hit the top of the side wall. He then gave her the marking pencil and told her to mark the 10am line where it hit the top of the side wall, measuring exactly halfway from the back wall to his 11am mark. It was important to Nicole that her daughter could repeat her same contribution to the original dial. Silly thoughts? Perhaps. But these memories were important to Nicole. They contributed as much to the room's warmth as the sunbeams that still streamed through the windows.

Questions: What is the latitude of the cottage, the exact orientation of the room, and the position of the mirror on the windowsill?

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The Tove's Nest....


#### Abstract

NASS Webmaster Retires Bob Terwilliger, a founding member of NASS, created our website (http://sundials.org) ten years ago. Since that time he has volunteered as webmaster and worked diligently to grow the awareness of NASS and our website in the dialing community. Bob has now decided to retire, and the NASS board hereby wishes to express its true appreciation for all his efforts over the last decade! Without the NASS website, we would not have the membership that we do today. Thanks Bob! (The website is currently under professional website management, overseen by Bob Kellogg.)


## Sundial Project at Simon Fraser University Len Berggren berggren@sfu.ca

[Ed. note - The Math Dept. at SFU is holding \$1000CN for NASS - money resulting from our conference in Vancouver a few years ago. On behalf of the NASS membership, the Board has offered this sum as a donation to the University for use in a dialing project. Len Berggren has been looking into this possibility.]

Hi, Fred. Good news! I have very recently talked to Prof. Howard Trottier of our Physics Department who has raised a considerable sum of money towards the construction of an astronomical observatory on the SFU campus, and he is enthusiastic about the idea of one or two dials being built on the site of the observatory (a split-analemma analemmatic dial and a vertical declining dial) as part of the observatory complex. Then, two days ago, I had a conversation with the Faculty of Science Development Officer, Anna duBois, who is working with Prof. Trottier on the fundraising for the astronomical observatory. Howard had spoken to her about the sundials and she was glad to hear from me about NASS's generous offer. So it now appears that at least one, and likely two, dials will be included in a fund-raising component for the sundials within the observatory project. They are trying to get the rest of the money in place for the observatory within the coming year and have it built by the end of 2010.

## Error Correction

Thanks to Herb Ramp for noticing a typographical error in an equation in Bob Kellogg's spreadsheet included with the June issue of The Compendium. The spreadsheet calculates the declination and azimuth of the sun for various days of the year. Herb correctly pointed out that the value 0.98560028 had been mistyped - thus leading to some errors in the resulting declinations and azimuths. A corrected spreadsheet is included with this issue of The Compendium. Figure 3 of Bob’s article should have been:


Tony Moss on the BBC
Mike Shaw
jmikeshaw@ntlworld.com
There's a nice little video about Tony Moss and the Fontburn reservoir sundial on the BBC web site. See: http://news.bbc.co.uk/1/hi/england/7701026.stm.

Loss of a NASS friend Fred Sawyer fwsawyer@aya.yale.edu
Smithsonian Curator David Shayt died on Nov. 4 of myeloma in Gaithersburg MD. David served on early NASS boards, was a planner and host for our 1995 (first) conference held at the Smithsonian, and gave us a tour of the Latitude Observatory in Gaithersburg on our 2007 conference.

Drawing an Ellipse Fred Sawyer fwsawyer@aya.yale.edu
While touring together in Alsace, Don Petrie asked me to elaborate on a comment Bob Kellogg made in his last Sundials for Starters column. In describing the layout of the ellipse for an analemmatic sundial, Bob wrote: "... the hour points were determined using calculated distances from the two foci. This method is easier than trying to measure the perpendicular distances in $x$ and $y$. Two tape measures are used, with two volunteers holding an end at each focus." What Don wanted was an expression for the distances of any given hour point from the two foci of the ellipse. If we let the length of the semi-major axis of the ellipse be $k$, then the foci are at distances $\pm k \cos \varphi$ from the origin (to the east and west), and the distances from these foci to the hour point for hour angle $t$ are $k \sqrt{(\sin t \mp \cos \varphi)^{2}+\cos ^{2} t \sin ^{2} \varphi}$.

A Note On Analemmatic Sundials Brian Albinson brianalbinson@shaw.ca
I have found, even among some gnomonists, that the full implication of using the analemma [to correct for mean time] on analemmatic walk-on dials is not quite clear. I have been told that the use of a single analemma is completely wrong. But within school hours the error is less than the error inherent in using a personal shadow. In addition, the use of the double analemma seems to be largely ignored. I now use only the double analemma format for any school or public walk-on dial. Quite apart from the high overall accuracy it adds an element of interest. I am hoping to persuade the Ministry of Education in BC to make a walk-on dial standard playground equipment for the 1500 schools in British Columbia. See my 'Teacher's Notes", included as a digital bonus item.

An Interesting Photographic Development Fred Sawyer fwsawyer@aya.yale.edu
http://www.newscientist.com/article/dn16048-photos-with-shifting-shadows-come-to-life.html

## Digital Bonus

The digital issue of The Compendium comes with 7 bonus items.

1. The spreadsheet SunAzimuthOnSolstice.xls supplements Bob Kellogg's Sundials for Starters column. It allows you to do your own calculations of sunrise over archaeological sites.
2. SunDecAzElevShadow.xls is a corrected version of the spreadsheet distributed earlier with the June issue of The Compendium. The spreadsheet calculates the declination and azimuth of the sun for various days of the year.

3-4. TeachersNotes.pdf is the set of notes Brian Albinson distributes as background on the analemmatic dials with analemmic declination curves that he hopes to introduce as standard equipment in the schools of British Columbia. ConstructingDoubleAnalemmaWalk-onDials.pdf is his instruction manual.

5-7. SundialTour.pdf is the set of notes distributed by Don Snyder for our St. Louis sundial tour. KatePond2008.pdf and SundialsSouth.pdf are the sets of slides used by Kate Pond and Mike Isaacs, respectively, in delivering their talks at the annual conference.


Dual Scales on a H2SS Dial - Here is my original H2SS shadow plane model with a temporary paper scale showing both hours until sunset and hours since sunrise. It is placed on a simple heliodon alongside a normal horizontal sundial. The date is summer solstice and the "sunbeams" just graze the shadow plane - out of sight in this photo. The hours since sunrise reading is about correct for the 3pm time shown on the normal dial, but the hours until sunset scale should read about 4.5 hours, not 13.5. On those dates when the Sun is above the horizon for more than 12 hours, the H2SS dial may give two different results, but, if you know the approximate time of day, you will quickly decide which result is correct. - See Mac Oglesby's article in this issue.


[^0]:    * Compendium... "giving the sense and substance of the topic witbin small compass." In dialing, a compendium is a single instrument incorporating a variety of dial types and ancillary fools.

[^1]:    ${ }^{1}$ Friedlander, Michael W. "The Cahokia Sun-Circles", The Wisconsin Archeologist, Vol. 88(1), pp.78-90, 2007.
    ${ }^{2}$ Wittry, Warren "An American Woodhenge", Cranbrook Institute of Science Newsletters, Vol. 33(9), pp. 102-107, 1964 Bloomfield Hills, Michigan. Reprinted in Explorations into Cahokia Archaeology, Bulletin 7, Illinois Archaeological Survey, 1969.

[^2]:    ${ }^{3}$ Néda, Z. and S. Volkán "Flatness of the Setting Sun", http://web.ift.uib.no/~neda/sunset/index.html 2008
    ${ }^{4}$ NOAA, http://www.srrb.noaa.gov/highlights/sunrise/calcdetails.html 2008

[^3]:    5 The text has been drawn from the following edition of the work of Clavius: CHRISTOPHORI CLAVII BAMBERGENSIS E SOCIETATE IESU OPERUM MATHEMATICORUM- TOMUS TERTIUS Complectens COMMENTARIUM IN SPHAERAM IOANNIS DE SACRO BOSCO \& ASTROLABIUM- Moguntiae- anno MDCXI (1611) - Lemma 39 is on page 65 of the text on the Astrolabe.

