Section 5.5 Exponential and Logarithmic Equations

To Solve an Exponential Equation Using Natural Logarithms

- 1. Isolate the exponential expression.
- 2. Take the natural logarithm on both sides of the equation.
- 3. Solve for the variable.

Note: The base that is used when taking the logarithm on both sides can be any base at all. If you use a base other than 10 or e, then you can apply the change-of-base formula to show that the solutions are the same no matter which base you use.

| Example 1: Solve. | |
|-------------------|--------------------|
| a. $4^x = 64$ | b. $3^{2x-1} = 27$ |

c.
$$5^{3x} = 25\sqrt{5}$$
 d. $2^{x-1} = \frac{1}{16}$

e.
$$5^x = 17$$
 f. $3^x = 19$

g. $7^x = 4$

h. $11^x = -121$

i. $3e^{5x-1} + 3 = 1980$

To Solve Logarithmic Equations

1. Isolate the logarithmic terms on one side of the equation. If necessary, use the properties of logarithms to write multiple logarithms as a single logarithm.

2. Rewrite the equation in its equivalent exponential form or use the base in the logarithm equation and raise that base to each side of the logarithmic equation.

Note: You must always check the proposed solutions of a logarithmic equation. Exclude from this set, solutions that produce the logarithm of a negative number or 0.

Example 2: Solve. a. $\log_{16} x = \frac{1}{4}$ b. $\ln x = -2$ c. $\log_2(4x+1) - 5 = 0$

d. $\log_6(x+5) + \log_6 x = 2$

e. $\log_{6}(2x+1) = \log_{6}(x-5)$

f. $\log_8(x+1) - \log_8(x-2) = 2\log_8 3$

g. $13 + 11\log_{13}(13x) = 35$