## Section 3-4, Present Value of an Annuity; Amortization

The present value of an account paying a certain amount of interest (compounded periodically) into which regular periodic deposits are made can be found by solving the compound interest formula for $P$.

Present Value of an Ordinary Annuity

$$
P V=P M T \frac{1-(1+i)^{-n}}{i}
$$

where
$P M T=$ periodic payment
$i=$ rate per period
$n=$ number of periods
$P V=$ present value of all payments
Q1 (\#22, page 167). American General offers a 7-year ordinary annuity with a guaranteed rate of $6.35 \%$ compounded annually. How much should you pay for one of these annuities if you want to receive payments of $\$ 10,000$ annually over the 7year period?

Q2 (\#30, page 167). You want to purchase an automobile for $\$ 28,500$. The dealer offers you $0 \%$ financing for 60 months or a $\$ 6,000$ rebate. You can obtain $6.2 \%$ financing for 60 months at the local bank. Which option should you choose? Explain.

Amortizing a debt means that the debt is retired in a given length of time by equal periodic payments that include compound interest ("Mort" means "death.").

Q3 (\#34, page 168). Construct the amortization schedule for a $\$ 10,000$ debt that is to be amortized in six equal quarterly payments at $2.6 \%$ interest per quarter on the unpaid balance.

| Payment <br> number | Payment | Interest | Unpaid balance <br> reduction | Unpaid balance |
| :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  | $\$ 10,000.00$ |
| 1 |  |  |  |  |
| 2 |  | 177.74 | $1,643.84$ | $5,192.40$ |
| 3 |  | 135.00 | $1,686.58$ | $3,505.82$ |
| 4 |  | 91.15 | $1,730.43$ | $1,775.39$ |
| 5 |  | $\$ 6.16$ | $1,775.39$ | 0.00 |
| 6 |  | $\$ 29.45$ | $\$ 10,000.00$ |  |
| Totals | $\$ 10,929.45$ |  |  |  |

The unpaid balance of a loan with $n$ remaining payments is the present value of that annuity.

Q4 (\#36, page 168). A man establishes an annuity for retirement by depositing \$50,000 into an account that pays $7.2 \%$ compounded monthly. Equal monthly withdrawals will be made each month for 5 years, at which time the account will have a zero balance. Each year taxes must be paid on the interest earned by the account during that year. How much interest was earned during the first year? [Hint: The amount in the account at the end of the first year is the present value of a 4 -year annuity.]

The equity in a home = (current net market value) - (unpaid loan balance).
Q5 (\#50, page 169, slightly modified). A person purchased a house 10 years ago for $\$ 100,000$. The house was financed by paying $20 \%$ down and signing a 30 -year mortgage at $9.6 \%$ on the unpaid balance. Equal monthly payments were made to amortize the loan over a 30 -year period. The owner now (after the $120^{\text {th }}$ payment) wishes to refinance the house because of a need for additional cash. If the appraised value of the house is $\$ 136,000$, what is the owner's equity in the house? If the loan company agrees to a new 30 -year mortgage of $80 \%$ of the new appraised value of the house, how much cash (to the nearest dollar) will the owner receive after repaying the balance of the original mortgage?

