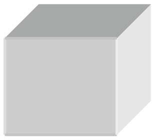
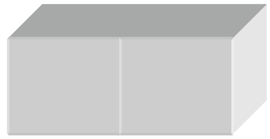
# Cognitive Demand Defined

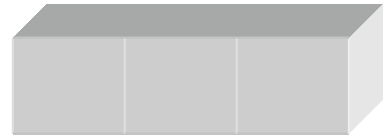
# Participant Handouts

## Elementary Mathematics

Los Angeles Unified School District







# Paint the Building

- On Monday, a painter had to paint a building that was shaped like a cube. When she read the label on the can of paint, she realized one can of paint would cover one face of the building. She had to paint all four sides **and** the roof of the building.
- On Tuesday, she had to paint the building next door. It was the size of two of the first buildings put together.
- On Wednesday, she had to paint the third building on the block. It was the size of three of the cubic buildings put together.
- On Thursday, she had to paint yet another building that was, of course, like four of the cubic buildings put together.

Your job is to figure out how many gallons of paint she would need each day. Continue this pattern up to ten cubic units put together? Use the T chart to help you. Create a formula to help you figure out how many gallons of paint it would take to paint a building 23 cubic units long.

The formula is___________________. It would take ___________ gallons of paint to cover a building 23 cubic units long.

HO # 2

HO # 3

HO # 3b

# Identifying Cognitive Demand: Task Analysis Guide

| Lower-Level Cognitive Demands | | Higher Level Cognitive Demands | |
|---|---|---|---|
| **Memorization Tasks** | **Procedures Without Connections Tasks** | **Procedures With Connections Tasks** | **Doing Mathematics Tasks** |
| • Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.<br>• Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.<br>• Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.<br>• Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced. | • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.<br>• Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.<br>• Have no connection to the concepts or meaning that underlie the procedure being used.<br>• Are focused on producing correct answers rather than developing mathematical understanding.<br>• Require no explanations, or explanations that focus solely on describing the procedure that was used. | • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.<br>• Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.<br>• Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.<br>• Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding. | • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).<br>• Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships.<br>• Demands self-monitoring or self-regulation of one's own cognitive processes.<br>• Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.<br>• Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.<br>• Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required. |

# Thinking Through a Lesson Protocol

## Considering and Addressing Student Misconceptions and Errors

Los Angeles Unified School District
Elementary Mathematics
Participant Handouts
Fourth Grade
2007-2008

# Overview of Activities

- Examine Thinking Through a Lesson Protocol (TTLP)
- Engage in Fourth Grade Lesson considering components of the TTLP that the facilitator demonstrates
- Debrief the lesson with the TTLP as a frame for discussion
- Anticipate student misconceptions
- Review concept lesson and consider how student misconceptions are addressed

# Thinking Through a Lesson Protocol

The main purpose of the *Thinking Through a Lesson Protocol* is to prompt you in thinking deeply about a specific lesson that you will be teaching that is based on a cognitively challenging mathematical task.

| SET-UP<br>*Selecting and setting up a mathematical task* | EXPLORE<br>*Supporting students' exploration of the task* | SHARE, DISCUSS, AND ANALYZE<br>*Sharing and discussing the task* |
|---|---|---|
| ▪ What are your mathematical goals for the lesson (i.e., what is it that you want students to know and understand about mathematics as a result of this lesson)?<br><br>▪ In what ways does the task build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?<br><br>▪ What are all the ways the task can be solved?<br>- Which of these methods do you think your students will use?<br>- What misconceptions might students have?<br>- What errors might students make?<br><br>▪ What are your expectations for students as they work on and complete this task?<br>- What resources or tools will students have to use in their work?<br>- How will the students work – independently, in small groups, or in pairs – to explore this task?<br>- How long will they work individually or in small groups/pairs? Will students be partnered in a specific way? If so, in what way?<br>- How will students record and report their work?<br><br>▪ How will you introduce students to the activity so as not to reduce the demands of the task?<br><br>▪ What will you hear that lets you know students understand the task? | ▪ As students are working independently or in small groups:<br>- What questions will you ask to focus their thinking?<br>- What will you see or hear that lets you know how students are thinking about the mathematical ideas?<br>- What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations?<br>- What questions will you ask to advance students' understanding of the mathematical ideas?<br>- What questions will you ask to encourage students to share their thinking with others or to assess their understanding of their peer's ideas?<br><br>▪ How will you ensure that students remain engaged in the task?<br>- What will you do if a student does not know how to begin to solve the task?<br>- What will you do if a student finishes the task almost immediately and becomes bored or disruptive?<br>- What will you do if students focus on non-mathematical aspects of the activity (e.g., spend most of their time making beautiful poster of their work)? | ▪ How will you orchestrate the class discussion so that you accomplish your mathematical goals? Specifically:<br>- Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?<br>- In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?<br>- What specific questions will you ask so that students will:<br>▪ make sense of the mathematical ideas that you want them to learn?<br>▪ expand on, debate, and question the solutions being shared?<br>▪ make connections between the different strategies that are presented?<br>▪ look for patterns?<br>▪ begin to form generalizations?<br><br>▪ What will you see or hear that lets you know that students in the class understand the mathematical ideas that you intended for them to learn?<br><br>▪ What will you do tomorrow that will build on this lesson? |

# Connecting to the Big Idea, Concepts and Skills for Quarter 2

## Fourth Grade Quarterly Concept Organizer

**<u>Number Relationships and Algebraic Reasoning</u>**
**Arithmetic and algebra are guided by properties of operations and equivalence.**

Commutative, associative, and distributive properties are inherent in the algorithms for operations of rational numbers.

- •Add, subtract, multiply, and divide whole numbers.
- •Show relationships between operations.
- •Solve problems involving addition, subtraction, multiplication, and division.
- •Estimate reasonableness.
- •Evaluate and use expressions with parentheses.

One of the concepts of algebraic reasoning is balance.

- •Show that equals added or multiplied by equals are equal.

**<u>Data Analysis</u>**
**Data can be interpreted from organized visual representations.**

Data can be collected, classified, displayed, and analyzed.

- •Create survey questions and collect data.
- •Identify mode, median, and outliers.
- •Interpret and share data.

# Cookie Containers

**Fourth Grade: Quarter 2**

Mrs. Baker's Cookies is making their cookies bigger. Each container holds 8 cookies.

1. If they make 264 cookies in one day, how many containers will they need each day? Explain how you know.

2. Now suppose they make the cookies even bigger and only 5 cookies will fit in each container. If they still make 264 cookies in one day, how many containers will they need each day? Explain how you know.

# <u>Explore</u>

## What do you expect your students to <u>do</u> as they engage in the lesson?

**What misconceptions might surface for students as they engage with this task?**

# Questioning: A Tool for Surfacing Errors and Misconceptions

Read the concept lesson for Quarter 2, Grade 4.

- As you read, mark places within the lesson where student misconceptions are addressed.

- Once you are finished, discuss how the ways in which student misconceptions are addressed scaffold and/or support the learning for the students.

- Choose and recorder and reporter and be prepared to share at least two ideas from your discussion.

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Los Angeles Unified School District
Elementary Mathematics
Participant Handouts
Fifth Grade
2007-2008

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| ▪ What are your mathematical goals for the lesson (i.e., what is it that you want students to know and understand about mathematics as a result of this lesson)?<br><br>▪ In what ways does the task build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?<br><br>▪ What are all the ways the task can be solved?<br>- Which of these methods do you think your students will use?<br>- What misconceptions might students have?<br>- What errors might students make?<br><br>▪ What are your expectations for students as they work on and complete this task?<br>- What resources or tools will students have to use in their work?<br>- How will the students work – independently, in small groups, or in pairs – to explore this task?<br>- How long will they work individually or in small groups/pairs? Will students be partnered in a specific way? If so, in what way?<br>- How will students record and report their work?<br><br>▪ How will you introduce students to the activity so as not to reduce the demands of the task?<br><br>▪ What will you hear that lets you know students understand the task? | ▪ As students are working independently or in small groups:<br>- What questions will you ask to focus their thinking?<br>- What will you see or hear that lets you know how students are thinking about the mathematical ideas?<br>- What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations?<br>- What questions will you ask to advance students' understanding of the mathematical ideas?<br>- What questions will you ask to encourage students to share their thinking with others or to assess their understanding of their peer's ideas?<br><br>▪ How will you ensure that students remain engaged in the task?<br>- What will you do if a student does not know how to begin to solve the task?<br>- What will you do if a student finishes the task almost immediately and becomes bored or disruptive?<br>- What will you do if students focus on non-mathematical aspects of the activity (e.g., spend most of their time making beautiful poster of their work)? | ▪ How will you orchestrate the class discussion so that you accomplish your mathematical goals? Specifically:<br>- Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?<br>- In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?<br>- What specific questions will you ask so that students will:<br>▪ make sense of the mathematical ideas that you want them to learn?<br>▪ expand on, debate, and question the solutions being shared?<br>▪ make connections between the different strategies that are presented?<br>▪ look for patterns?<br>▪ begin to form generalizations?<br><br>▪ What will you see or hear that lets you know that students in the class understand the mathematical ideas that you intended for them to learn?<br><br>▪ What will you do tomorrow that will build on this lesson? |

# Connecting to the Big Idea, Concepts and Skills for Quarter 2

## Fifth Grade Quarterly Concept Organizer

**Number Relationships and Algebraic Reasoning**
**Arithmetic and algebra are guided by equivalence and properties of operations.**

Arithmetic operations are represented by both models and algorithms for fractions, decimals, and integers.

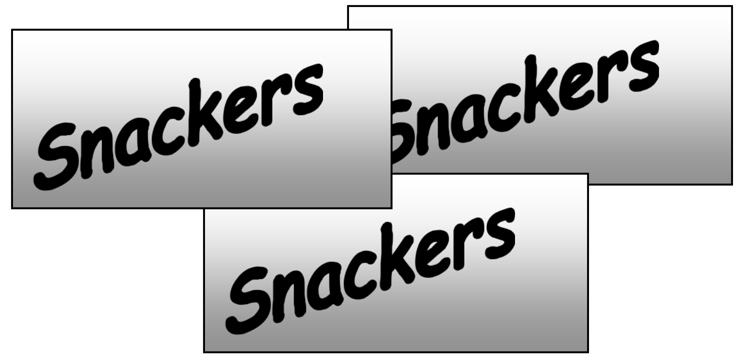
Represent and identify positive and negative integers on a number line.
Add with negative integers and subtract a positive integer from a negative integer.
Solve problems involving addition, subtraction, multiplication, and division of fractions accurately and represent in simplest form.
Recognize equivalent fractions and solve problems involving fractions with like and unlike denominations.

**Data Analysis**
**Data can be interpreted from organized visual representations.**

Data is collected, sorted and/or classified, and analyzed visually and numerically depending upon the problem situation.

Understand and explain the concepts of mean, median, and mode.
Compute and compare mean, median, and mode to show that they may differ.
Use graphic organizers, including histograms and circle graphs, and explain which type of graph(s) is appropriate for various data sets.
Determine the best choice of visual representations based on the type of data.
Use fractions and percentages to compare data sets of different sizes.
Identify, graph, and write ordered pairs of data from a graph and interpret meaning of data.

# Candy Bar Capers




You and your friends, Marcus and Tamra, each have a "Snackers" candy bar.

- Marcus has eaten $\frac{1}{2}$ of his Snackers bar.
- Tamra has eaten $\frac{3}{4}$ of her Snackers bar.
- You have eaten $^{5}/_{8}$ of your Snackers bar.

Marcus claims that if you put the leftover parts of the 3 Snackers bars together, it would be more than a whole Snackers bar. Tamra disagrees. Which of your friends is correct? Use numbers and pictures or diagrams to explain how you know.

# Explore

## What do you expect your students to do as they engage in the lesson?

**What misconceptions might surface for students as they engage with this task?**

## Questioning: A Tool for Surfacing Errors and Misconceptions

Read the concept lesson for Quarter 2, Grade 5.

- As you read, mark places within the lesson where student misconceptions are addressed.

- Once you are finished, discuss how the ways in which student misconceptions are addressed scaffold and/or support the learning for the students.

- Choose and recorder and reporter and be prepared to share at least two ideas from your discussion.

Los Angeles Unified School District

# Elementary Mathematics

# Fourth and Fifth Grade

# Quarter 2 Concept Lesson

## Classroom Discourse and Asking Questions

2007-2008

**Asking Questions**

For students to have deeper mathematical understanding, the process of asking questions reveals what they truly understand about procedures and problem solving in mathematics. By asking open-ended, thought-provoking questions, teachers:

- engage and guide students' thinking to deeper levels;
- gain insight into what students understand and the depth of that understanding;
- engage and guide the class in deeper mathematical thinking about the concepts; and
- place greater emphasis on mathematical thinking and reasoning.

Because these questions cannot be answered effectively with single-word responses, students should be asked to formulate more elaborate responses to promote learning. Some strategies for maintaining a low-stress, low-anxiety environment in the classroom in order to facilitate learning through open-ended questioning are:

- anticipating the questions students will likely ask;
- organizing students in ways that allow them to interact more freely with one another;
- using wait time to allow students more time to process the questions and develop multiple responses; and
- honoring all responses and remaining neutral in order to allow students opportunities to validate each others' responses.

*References:*


Chapin, S., O'Connor, C., & Anderson, N. C. (2003). *Classroom discussions: Using math talk to help students learn, Grades 1-6*. Sausalito, CA: Math Solutions Publications.

LAUSD Grades 4 and 5 Intervention Kits.

Manouchehri, A. and Lapp, D. A. (2003) Unveiling student understanding: The role of questioning in instruction. *Mathematics Teacher*. 96(8), 562 – 566.

Sullivan, P. and Lilburn, P. (2002). *Good questions for math teaching: Why we ask them and what to ask [K-6]*. Sausalito, CA: Math Solutions Publications.

# Thinking Through a Lesson Protocol

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## SET-UP

*Selecting and setting up a mathematical task*

- What are your mathematical goals for the lesson (i.e., what is it that you want students to know and understand about mathematics as a result of this lesson)?
- In what ways does the task build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?
- What are all the ways the task can be solved?
  - Which of these methods do you think your students will use?
  - What misconceptions might students have?
  - What errors might students make?
- What are your expectations for students as they work on and complete this task?
  - What resources or tools will students have to use in their work?
  - How will the students work – independently, in small groups, or in pairs – to explore this task?
  - How long will they work individually or in small groups/pairs? Will students be partnered in a specific way? If so, in what way?
  - How will students record and report their work?
- How will you introduce students to the activity so as not to reduce the demands of the task?
- What will you hear that lets you know students understand the task?

## EXPLORE

*Supporting students' exploration of the task*

- As students are working independently or in small groups:
  - What questions will you ask to focus their thinking?
  - What will you see or hear that lets you know how students are thinking about the mathematical ideas?
  - What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations?
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## SHARE, DISCUSS, AND ANALYZE

*Sharing and discussing the task*

- How will you orchestrate the class discussion so that you accomplish your mathematical goals? Specifically:
  - Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?
  - In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?
  - What specific questions will you ask so that students will:
    - make sense of the mathematical ideas that you want them to learn?
    - expand on, debate, and question the solutions being shared?
    - make connections between the different strategies that are presented?
    - look for patterns?
    - begin to form generalizations?
- What will you see or hear that lets you know that students in the class understand the mathematical ideas that you intended for them to learn?
- What will you do tomorrow that will build on this lesson?

# Question Types

| Types and Purpose of Questions | Focusing Thinking | Assessing Thinking | Advancing Thinking |
|---|---|---|---|
| **What it does** | ▪ Talks about issues outside of math in order to enable links to be made with mathematics.<br>▪ Helps students to focus on key elements or aspects of the situation in order to enable problem-solving. | ▪ Ask students to articulate, elaborate, or clarify ideas.<br>▪ Enables correct mathematical language to be used to talk about them<br>▪ Rehearses known facts/ procedures. Enables students to state facts/procedures.<br>▪ Requires immediate answer. | ▪ Extends the situation under discussion to other situations where similar ideas may be used.<br>▪ Makes links between mathematical ideas and representations.<br>▪ Points to relationships among mathematical ideas and mathematics and other areas of study/life.<br>▪ Points to underlying mathematical relationships and meanings. |
| **What it sounds like** | ❖ What is the problem asking you?<br>❖ What is important about this?<br>❖ What games have you played where you used…?<br>❖ What is a…? (reference to context of problem) | ❖ How could you record what you just told me?<br>❖ How could you use a … to help you record what is happening?<br>❖ How did you get your answer?<br>❖ How do you know you are correct?<br>❖ What is this called?<br>❖ How would you use an equation to record what you just told me? | ❖ How would this work with other numbers?<br>❖ How do you know whether or not this pattern always works?<br>❖ In what other situations could you apply this?<br>❖ How are … and … related?<br>❖ What other patterns do you see?<br>❖ Where else have we used this? |

What are you trying to figure out?

What does the 8 mean in this problem?

About how many containers do you think they will need?

Why are you subtracting 8 each time?

How can you record what you are doing?

How are you going to find out how many containers are needed?

How will you know when you have found the number of containers needed?

Explain how you kept track of what you were doing.

What other ways could we find the numbers of containers?

How can you keep track of how many groups of 8 you used?

Now suppose the company makes the cookies even bigger and only 5 cookies will fit in each container. How could you find how many containers they will need every day?

Los Angeles Unified School District

# Elementary Mathematics

## Grades 4 and 5

“Algebra and Functions”

2007-2008

## Howard Gardner's nine intelligences:

1. **Verbal-Linguistic** - Learning through language, the ability to use language to communicate, like writers and TV announcers.
2. **Logical-Mathematical** - Learning through orderly processes, like scientists, mathematicians and detectives.
3. **Visual-Spatial** - Learning through manipulating mental images or building models, like artists, architects and sailors.
4. **Bodily Kinesthetic** - Using one's body to solve problems or communicate, like dancers, athletes, surgeons, and craftspeople. May learn best through simulations, role-play, and actual experience.
5. **Musical** - Learning through rhythm, dance, and melodies.
6. **Interpersonal** - Ability to understand and interact well with others, like teachers, actors, or politicians.
7. **Intrapersonal** - Ability to understand oneself through reflection and to manage one's thoughts and feelings, like psychotherapists and philosophers.
8. **Naturalist** - Learning through recognizing patterns in nature, classifying and interacting with the flora and fauna of the natural environment, like biologists and ecologists.
9. **Existential** - Talent for grappling with big questions like the meaning of life and death, as well as sensitivity to spiritual dimensions. (Other researchers suggest that Spiritual Intelligence may be a separate category).

# Guiding Questions: Reading Part 2b

1. Under which circumstances would one of these representations be more appropriate than another?

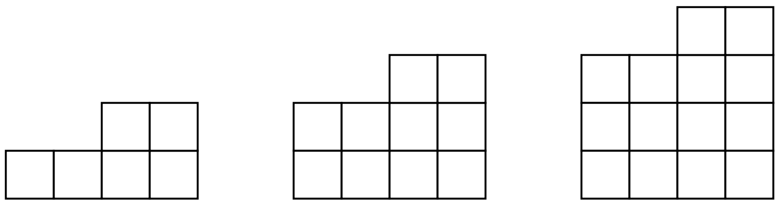
2. What questions will you ask to address students who do not see a particular representation, considering the diverse learners in our classrooms (ELs, SELs, GATE, and students with disabilities)?

3. What questions will you ask to make connections between each representation?

# "Piles of Tiles"

*Adapted from* *<u>Lessons for Algebraic Thinking Grades 3-5,</u>* *pages 197-221*

Look at the piles of tiles below.



**Pile 1** **Pile 2** **Pile 3** **Pile 4?**

1. Draw or use your tiles to show how you would build the next pile.

2. What *words* could describe how the number of tiles changes when you build a new pile?

3. How might you *create a table* that shows the functional relationship between the Pile Number and the Number of Tiles in that pile? Use the back of this paper.

4. How many tiles will you need for the $10^{th}$ pile? The $100^{th}$ pile? How many tiles will you need for any Pile Number?

5. How might you *graph* the functional relationship between the Pile Number and the Number of Tiles in that pile? Use the graph paper on HO #4.

6. What *expression* could you use to describe the functional relationship between the Pile Number and the Number of Tiles in that pile?

# Graph Paper

# Identifying Cognitive Demand: Task Analysis Guide

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| | |
|---|---|
| $x + 2 = y$ | $y = x + 1$ |
| $(3, 10)$ | $(3, 9)$ |

| | |
|---|---|
| $x + 4 = y$ | $y = x + 3$ |
| $(4, 6)$ | $(3, 11)$ |

| | |
|---|---|
| $x + 6 = y$ | $y = x + 5$ |
| $(4, 10)$ | $(4, 8)$ |

| | |
|---|---|
| $x + 8 = y$ | $y = x + 7$ |
| $(6, 10)$ | $(4, 12)$ |

$2x - 1 = y$

$y = 2x$

$(1, 3)$

$(1, 2)$

| | |
|---|---|
| $2x + 1 = y$ | $y = 2x - 2$ |
| $(1, 6)$ | $(1, 5)$ |

$2x + 3 = y$

$y = 2x + 2$

$(1, 8)$

$(1, 7)$

| | |
|---|---|
| $2x + 5 = y$ | $y = 2x + 4$ |
| $(2, 3)$ | $(1, 9)$ |

$2x + 7 = y$

$y = 2x + 6$

$(2, 5)$

$(2, 4)$

$3x - 1 = y$

$y = 3x$

$(2, 7)$

$(2, 6)$

| | |
|---|---|
| $3x + 1 = y$ | $y = 3x - 2$ |
| $(2, 9)$ | $(2, 8)$ |

$4x - 1 = y$

$y = 3x + 2$

$(2, 11)$

$(2, 10)$

$4x + 3 = y$

$y = 4x + 2$

$(3, 6)$

$(3, 5)$

Preparation:

1. You will need to create one set of quarter-sheet “cards” for each room.

2. Make copies of the cards on bright yellow paper or tag.

3. Using a paper cutter, cut each of the previous 13 pages into quarters, to create 52 quarter-sheet “cards”.

4. Distribute the cards at the tables on the day of the PD, before teachers arrive. Try to have a good mix of expressions and ordered pairs at each table.

Directions:

1. Have teachers or students find the yellow quarter-sheets of paper.

2. Have teachers or students find either their matching expression or matching ordered pair. Once they pair up, have them share how they know how they match. Unpaired matches should come to the front of the room.

3. Allow about 5 minutes for pairs to match up and share. Ask those at the front of the room to first see if they can find matches at the front of the room, and if not, have them wait at the front. After 5 minutes, signal everyone to return to their seats, keeping the quarter-sheets that weren’t matched.

4. Depending on time, have the group either share an expression that would go with an unmatched ordered pair, or an ordered pair that would go with an unmatched expression.

5. Debrief the activity with the following questions.

    - ***What were some strategies that you used to find your match?***
    [Anticipated possible choices that might work, looked at many cards to see if my card worked with any of them, ignored matches that were already made in order to narrow my choices, went to the front of the room and waited to see if someone came up with a card that matched mine]

    - ***What do you have to know and be able to do in order to participate in an activity such as this?***
    [Be able to work with numbers mentally, understand that the first number of the ordered pair replaces the $x$ in the expression, understand that the second number of the ordered pair represents the result of the expression, understand that the number next to $x$ (the coefficient of $x$) needs to be multiplied to $x$]

# Table Representations

Juan secures a summer job to walk dogs in his neighborhood. The neighborhood association pays him a \$5 starting bonus and \$2 for every dog he walks. How much money does he earn for any number of dogs walked?

Serena is paid \$25 a session to tutor Jhalisa, her best friend's niece. In order to tutor Jhalisa she has to buy a mathematics text, which costs \$15. How much money does she earn for any number of sessions that she tutors?

Monique is 5 years older than Marcus. If we know Monique's age, how can we determine Marcus's age?

# Language Expressions

Juan secures a summer job to walk dogs in his neighborhood. The neighborhood association pays him a \$5 starting bonus and \$2 for every dog he walks. How much money does he earn for any number of dogs walked?

Serena is paid \$25 a session to tutor Jhalisa, her best friend's niece. In order to tutor Jhalisa she has to buy a mathematics text, which costs \$15. How much money does she earn for any number of sessions that she tutors?

Monique is 5 years older than Marcus. If we know Monique's age, how can we determine Marcus' age?

# Graphical Representations

Juan secures a summer job to walk dogs in his neighborhood. The neighborhood association pays him a $5 starting bonus and $2 for every dog he walks. How much money does he earn for any number of dogs walked?

Serena is paid $25 a session to tutor Jhalisa, her best friend’s niece. In order to tutor Jhalisa she has to buy a mathematics text, which costs $15. How much money does she earn for any number of sessions that she tutors?

Monique is 5 years older than Marcus. If we know Monique’s age, how can we determine Marcus’s age?

# Equations

Juan secures a summer job to walk dogs in his neighborhood. The neighborhood association pays him a $5 starting bonus and $2 for every dog he walks. How much money does he earn for any number of dogs walked?

Serena is paid $25 a session to tutor Jhalisa, her best friend’s niece. In order to tutor Jhalisa she has to buy a mathematics text, which costs $15. How much money does she earn for any number of sessions that she tutors?

Monique is 5 years older than Marcus. If we know Monique’s age, how can we determine Marcus’s age?

Directions:

1. Make enough copies so that each table has one.
2. Ideally you will have 4 groups of 6-8 people. Some groups may have to come together to make a larger group. You can have participants count off 1-2-3-4.
3. Each table chooses a leader. That leader reads the assigned representation and then each of the contextual representations.
4. The team has to devise a skit, demonstration or visual to illustrate one of the three contextual representations through the lens of their assigned representation.

Los Angeles Unified School District

# Elementary Mathematics

# Fourth Grade

# Quarter 3 Concept Lesson

## Question Types
## Share, Discuss, and Analyze
## 2007-2008

# Outcomes

- Experience the Quarter 3 Concept Lesson as a Learner
- Select and Order Student Work that We Might Use to Generate a Mathematically-Rich Discussion
- Consider how the Concept Lessons address the needs of our Diverse Learners

# Thinking Through a Lesson Protocol

The main purpose of the *Thinking Through a Lesson Protocol* is to prompt you in thinking deeply about a specific lesson that you will be teaching that is based on a cognitively challenging mathematical task.

| SET-UP<br>*Selecting and setting up a mathematical task* | EXPLORE<br>*Supporting students' exploration of the task* | SHARE, DISCUSS, AND ANALYZE<br>*Sharing and discussing the task* |
|---|---|---|
| ▪ What are your mathematical goals for the lesson (i.e., what is it that you want students to know and understand about mathematics as a result of this lesson)?<br><br>▪ In what ways does the task build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?<br><br>▪ What are all the ways the task can be solved?<br>- Which of these methods do you think your students will use?<br>- What misconceptions might students have?<br>- What errors might students make?<br><br>▪ What are your expectations for students as they work on and complete this task?<br>- What resources or tools will students have to use in their work?<br>- How will the students work – independently, in small groups, or in pairs – to explore this task?<br>- How long will they work individually or in small groups/pairs? Will students be partnered in a specific way? If so, in what way?<br>- How will students record and report their work?<br><br>▪ How will you introduce students to the activity so as not to reduce the demands of the task?<br><br>▪ What will you hear that lets you know students understand the task? | ▪ As students are working independently or in small groups:<br>- What questions will you ask to focus their thinking?<br>- What will you see or hear that lets you know how students are thinking about the mathematical ideas?<br>- What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations?<br>- What questions will you ask to advance students' understanding of the mathematical ideas?<br>- What questions will you ask to encourage students to share their thinking with others or to assess their understanding of their peer's ideas?<br><br>▪ How will you ensure that students remain engaged in the task?<br>- What will you do if a student does not know how to begin to solve the task?<br>- What will you do if a student finishes the task almost immediately and becomes bored or disruptive?<br>- What will you do if students focus on non-mathematical aspects of the activity (e.g., spend most of their time making beautiful poster of their work)? | ▪ How will you orchestrate the class discussion so that you accomplish your mathematical goals? Specifically:<br>- Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?<br>- In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?<br>- What specific questions will you ask so that students will:<br>▪ make sense of the mathematical ideas that you want them to learn?<br>▪ expand on, debate, and question the solutions being shared?<br>▪ make connections between the different strategies that are presented?<br>▪ look for patterns?<br>▪ begin to form generalizations?<br><br>▪ What will you see or hear that lets you know that students in the class understand the mathematical ideas that you intended for them to learn?<br><br>▪ What will you do tomorrow that will build on this lesson? |

# Connecting to the Big Idea, Concepts and Skills for Quarter 3

## Fourth Grade Quarterly Concept Organizer

**Algebraic Reasoning**

Problem situations can be represented as algebraic expressions and equations, as variables, and as charts and graphs.

Algebraic expressions are used to represent problem situations.

- Use variables.
- Use and interpret formulas.
- Understand the functional relationship within equations such as $y = 3x$.

Functions can be expressed with words, symbols, tables, and graphs.

- Use coordinate grids.
- Graph ordered pairs and lines.
- Find the distance between two points on a coordinate grid.

# Question Types

| Types and Purpose of Questions | Focusing Thinking | Assessing Thinking | Advancing Thinking |
|---|---|---|---|
| **What it does** | ▪ Talks about issues outside of math in order to enable links to be made with mathematics.<br>▪ Helps students to focus on key elements or aspects of the situation in order to enable problem-solving. | ▪ Ask students to articulate, elaborate, or clarify ideas.<br>▪ Enables correct mathematical language to be used to talk about them<br>▪ Rehearses known facts/ procedures. Enables students to state facts/procedures.<br>▪ Requires immediate answer. | ▪ Extends the situation under discussion to other situations where similar ideas may be used.<br>▪ Makes links between mathematical ideas and representations.<br>▪ Points to relationships among mathematical ideas and mathematics and other areas of study/life.<br>▪ Points to underlying mathematical relationships and meanings. |
| **What it sounds like** | ❖ What is the problem asking you?<br>❖ What is important about this?<br>❖ What games have you played where you used…?<br>❖ What is a…? (reference to context of problem) | ❖ How could you record what you just told me?<br>❖ How could you use a … to help you record what is happening?<br>❖ How did you get your answer?<br>❖ How do you know you are correct?<br>❖ What is this called?<br>❖ How would you use an equation to record what you just told me? | ❖ How would this work with other numbers?<br>❖ How do you know whether or not this pattern always works?<br>❖ In what other situations could you apply this?<br>❖ How are … and … related?<br>❖ What other patterns do you see?<br>❖ Where else have we used this? |

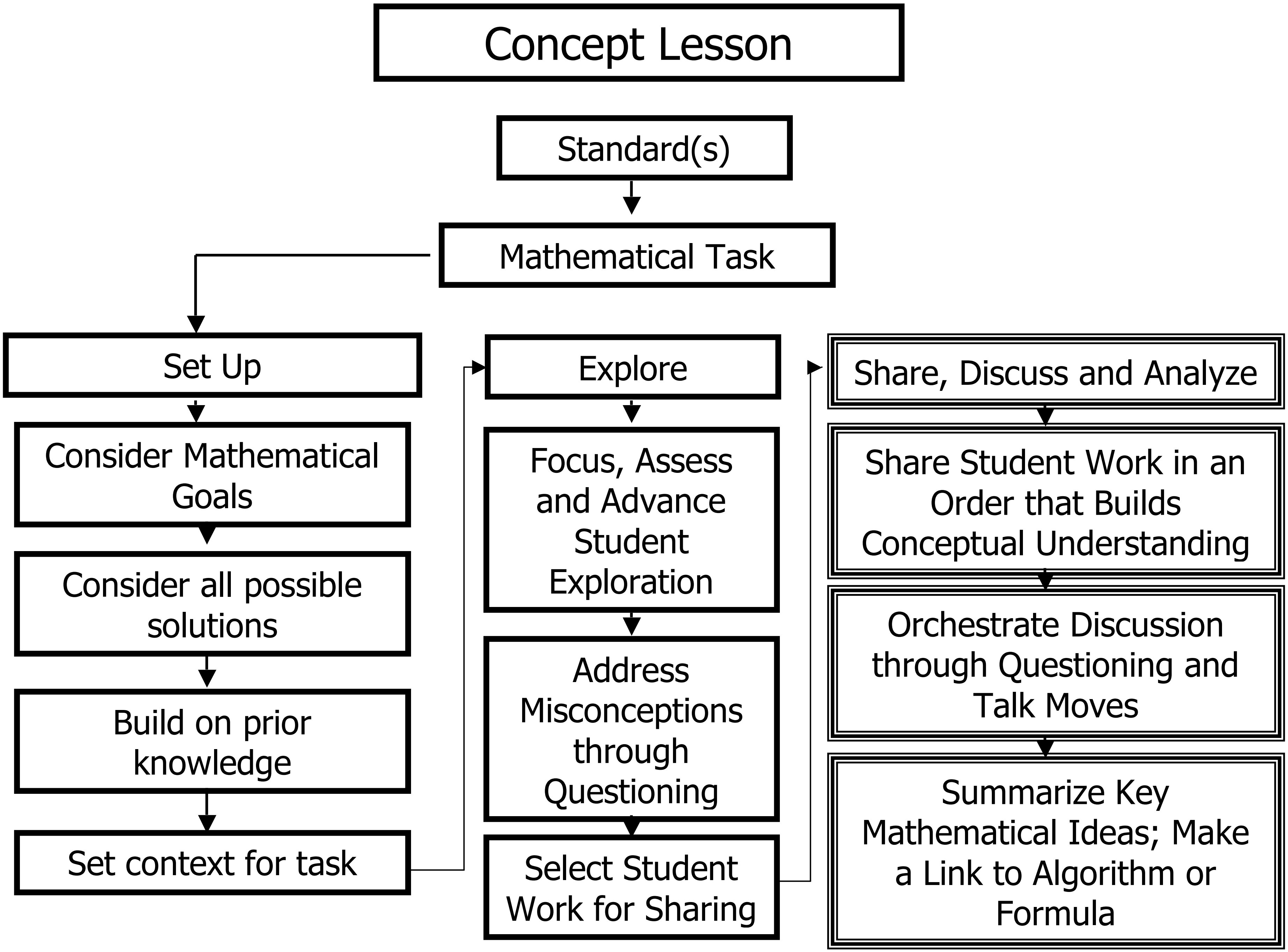
# Share, Discuss, and Analyze

## What will you see or hear that lets you know students are developing understanding of the concept? What questions will you need to ask to build mathematical understanding?

| What key mathematical ideas do you want the students to understand as a result of this lesson? | |
|---|---|
| How will these solutions and the order in which you share them help you facilitate a discussion to get at the key mathematical ideas? | |
| What questions could you ask to help students connect solutions? | How will you have the students summarize the mathematical ideas of the lesson? |

# Addressing Diverse Learners

What instructional strategies are embedded in the concept lessons and how are the needs of diverse learners (ELs, SELs, GATE students and students with disabilities) addressed?


Concept Lesson
Standard(s)
Mathematical Task
Set Up
Consider Mathematical Goals
Consider all possible solutions
Build on prior knowledge
Set context for task
Explore
Focus, Assess and Advance Student Exploration
Address Misconceptions through Questioning
Select Student Work for Sharing
Share, Discuss and Analyze
Share Student Work in an Order that Builds Conceptual Understanding
Orchestrate Discussion through Questioning and Talk Moves
Summarize Key Mathematical Ideas; Make a Link to Algorithm or Formula

Los Angeles Unified School District

# Elementary Mathematics

# Fourth and Fifth Grade

# Quarter 3 Concept Lesson

## Question Types

## Share, Discuss, and Analyze

## 2007-2008

# Outcomes

- Experience the Quarter 3 Concept Lesson as a Learner

- Select and Order Student Work that We Might Use to Generate a Mathematically-Rich Discussion

- Consider how the Concept Lessons address the needs of our Diverse Learners

# Thinking Through a Lesson Protocol

The main purpose of the *Thinking Through a Lesson Protocol* is to prompt you in thinking deeply about a specific lesson that you will be teaching that is based on a cognitively challenging mathematical task.

## SET-UP

*Selecting and setting up a mathematical task*

- What are your mathematical goals for the lesson (i.e., what is it that you want students to know and understand about mathematics as a result of this lesson)?
- In what ways does the task build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?
- What are all the ways the task can be solved?
  - Which of these methods do you think your students will use?
  - What misconceptions might students have?
  - What errors might students make?
- What are your expectations for students as they work on and complete this task?
  - What resources or tools will students have to use in their work?
  - How will the students work – independently, in small groups, or in pairs – to explore this task?
  - How long will they work individually or in small groups/pairs? Will students be partnered in a specific way? If so, in what way?
  - How will students record and report their work?
- How will you introduce students to the activity so as not to reduce the demands of the task?
- What will you hear that lets you know students understand the task?

## EXPLORE

*Supporting students' exploration of the task*

- As students are working independently or in small groups:
  - What questions will you ask to focus their thinking?
  - What will you see or hear that lets you know how students are thinking about the mathematical ideas?
  - What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations?
  - What questions will you ask to advance students' understanding of the mathematical ideas?
  - What questions will you ask to encourage students to share their thinking with others or to assess their understanding of their peer's ideas?
- How will you ensure that students remain engaged in the task?
  - What will you do if a student does not know how to begin to solve the task?
  - What will you do if a student finishes the task almost immediately and becomes bored or disruptive?
  - What will you do if students focus on non-mathematical aspects of the activity (e.g., spend most of their time making beautiful poster of their work)?

## SHARE, DISCUSS, AND ANALYZE

*Sharing and discussing the task*

- How will you orchestrate the class discussion so that you accomplish your mathematical goals? Specifically:
  - Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?
  - In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas that are the focus of your lesson?
  - What specific questions will you ask so that students will:
    - make sense of the mathematical ideas that you want them to learn?
    - expand on, debate, and question the solutions being shared?
    - make connections between the different strategies that are presented?
    - look for patterns?
    - begin to form generalizations?
- What will you see or hear that lets you know that students in the class understand the mathematical ideas that you intended for them to learn?
- What will you do tomorrow that will build on this lesson?

Connecting to the Big Idea, Concepts and Skills for Quarter 3

# Fifth Grade Quarterly Concept Organizer

**Algebraic Reasoning**

Equations, expressions, and variables are mathematical models used to represent real situations.

Linear relationships are presented in multiple ways.

- Write and evaluate simple algebraic expressions using one variable.
- Use the distributive property in equations and expressions with variables.
- Identify and graph ordered pairs in the four quadrants.
- Graph ordered pairs of integers on a grid based on a linear equation.

# Question Types

| Types and Purpose of Questions | Focusing Thinking | Assessing Thinking | Advancing Thinking |
|---|---|---|---|
| **What it does** | ▪ Talks about issues outside of math in order to enable links to be made with mathematics.<br>▪ Helps students to focus on key elements or aspects of the situation in order to enable problem-solving. | ▪ Ask students to articulate, elaborate, or clarify ideas.<br>▪ Enables correct mathematical language to be used to talk about them<br>▪ Rehearses known facts/ procedures. Enables students to state facts/procedures.<br>▪ Requires immediate answer. | ▪ Extends the situation under discussion to other situations where similar ideas may be used.<br>▪ Makes links between mathematical ideas and representations.<br>▪ Points to relationships among mathematical ideas and mathematics and other areas of study/life.<br>▪ Points to underlying mathematical relationships and meanings. |
| **What it sounds like** | ❖ What is the problem asking you?<br>❖ What is important about this?<br>❖ What games have you played where you used…?<br>❖ What is a…? (reference to context of problem) | ❖ How could you record what you just told me?<br>❖ How could you use a … to help you record what is happening?<br>❖ How did you get your answer?<br>❖ How do you know you are correct?<br>❖ What is this called?<br>❖ How would you use an equation to record what you just told me? | ❖ How would this work with other numbers?<br>❖ How do you know whether or not this pattern always works?<br>❖ In what other situations could you apply this?<br>❖ How are … and … related?<br>❖ What other patterns do you see?<br>❖ Where else have we used this? |

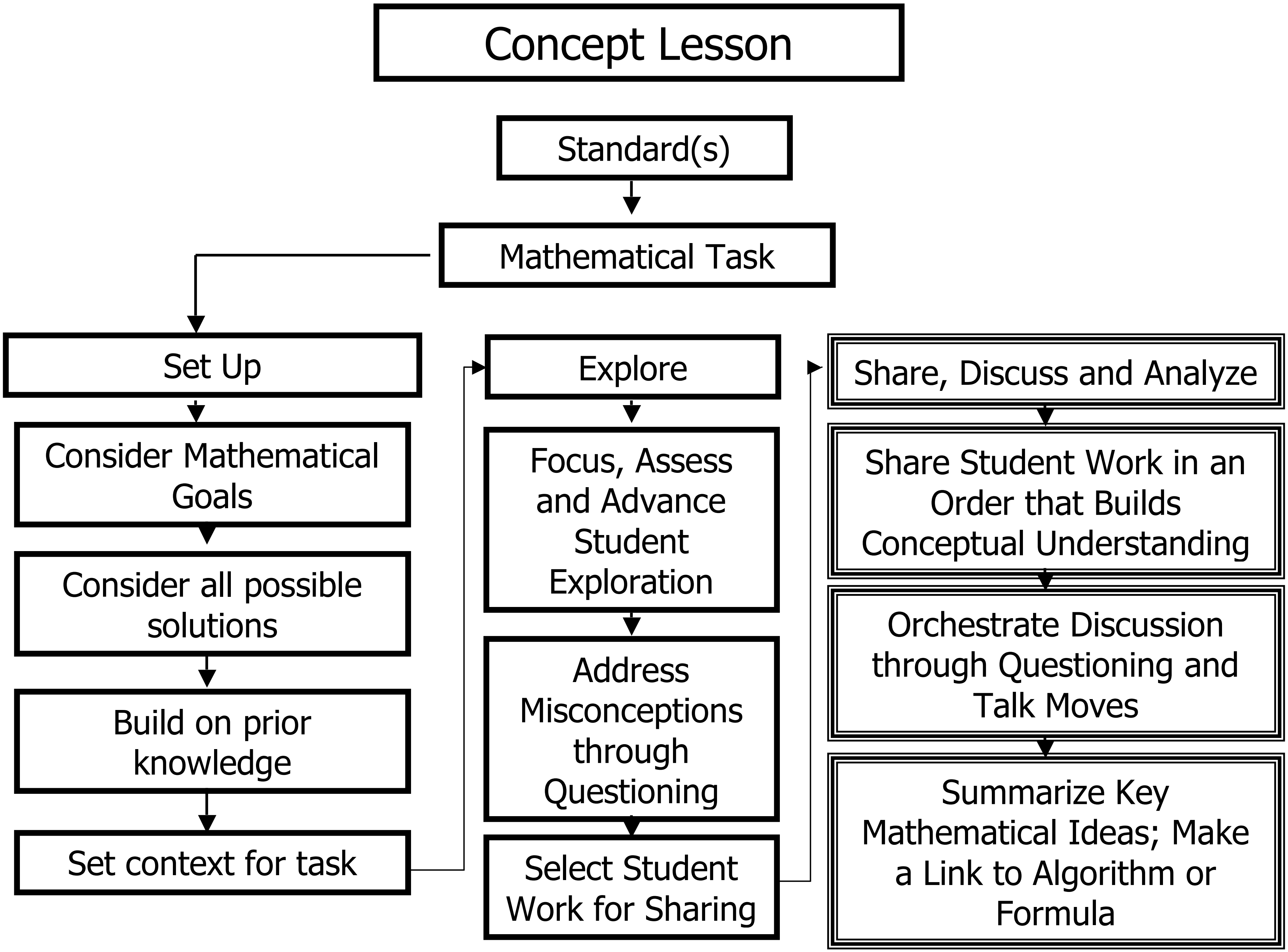
# Share, Discuss, and Analyze

## What will you see or hear that lets you know students are developing understanding of the concept? What questions will you need to ask to build mathematical understanding?

| What key mathematical ideas do you want the students to understand as a result of this lesson? | |
|---|---|
| How will these solutions and the order in which you share them help you facilitate a discussion to get at the key mathematical ideas? | |
| What questions could you ask to help students connect solutions? | How will you have the students summarize the mathematical ideas of the lesson? |

# Addressing Diverse Learners

What instructional strategies are embedded in the concept lessons and how are the needs of diverse learners (ELs, SELs, GATE students and students with disabilities) addressed?


Concept Lesson
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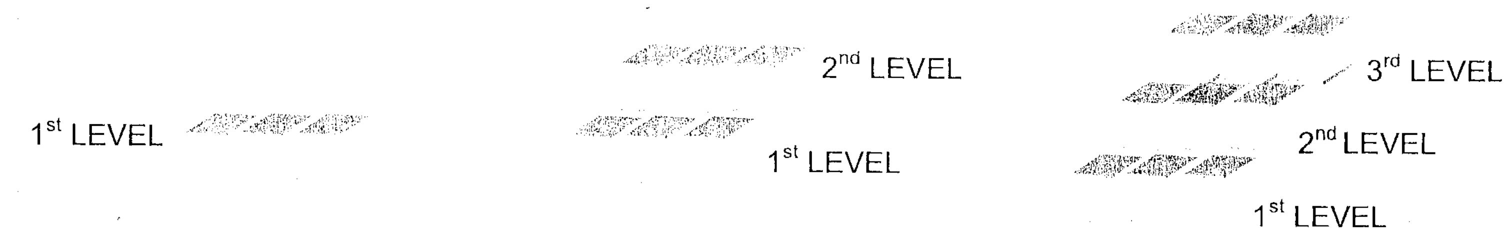
Focus, Assess and Advance Student Exploration
Address Misconceptions through Questioning
Select Student Work for Sharing
Share, Discuss and Analyze
Share Student Work in an Order that Builds Conceptual Understanding
Orchestrate Discussion through Questioning and Talk Moves
Summarize Key Mathematical Ideas; Make a Link to Algorithm or Formula

SAMPLE A

## Stacking Blocks
## $4^{th}$ Grade Lesson
## Quarter 3

Edgar's little sister is playing with blocks. He notices a pattern as she adds a level.

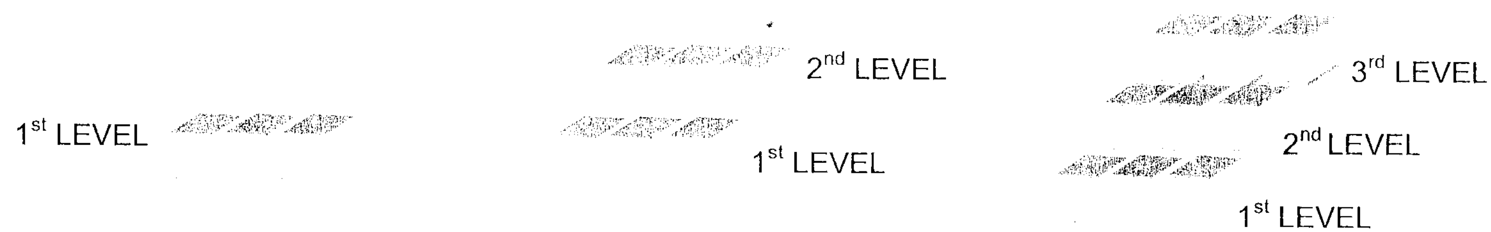



1. If the pattern continues, how many blocks would be needed to build the $5^{th}$ level? How many blocks would be needed to build the $10^{th}$ level?
   → 15 blocks
   → 30 blocks
2. Write an equation to describe the number of blocks that would be needed to build level "x".
   $Y = 3x$
3. Construct a graph on a coordinate grid that shows the relationship between the level number and the number of blocks in that level.

SAMPLE B

# Stacking Blocks
## 4th Grade Lesson
## Quarter 3

Edgar's little sister is playing with blocks. He notices a pattern as she adds a level.




1. If the pattern continues, how many blocks would be needed to build the 5th level? How many blocks would be needed to build the 10th level?

   3, 6, 9, 12, 15, 18, 21, 24, 27, 30 ← level 10

   level 5 →

2. Write an equation to describe the number of blocks that would be needed to build level "x". level 1: $1 \times 3 = 3$ level 5: $5 \times 3 = 15$ level 10: $10 \times 3 = 30$

   Multiply the level times 3 to find the # of blocks in all.

3. Construct a graph on a coordinate grid that shows the relationship between the level number and the number of blocks in that level.

If x is the level and y is the total number of blocks, then:

$3x = y$

or

$y = 3x$

Sample C

| level | blocks |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |
| 5 | 15 |
| 6 | 18 |
| 7 | 21 |
| 8 | 24 |
| 9 | 27 |
| 10 | 30 |

+3 +3 +3 +3 +3 +3 +3 +3 +3

30 blocks for the 10th level

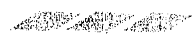
Sample D

**Stacking Blocks**
**4th Grade Lesson**
**Quarter 3**

| x level | L×3 | y # of blocks |
|---|---|---|
| 1 | 1×3 | 3 |
| 2 | 2×3 | 6 |
| 3 | 3×3 | 9 |
| 4 | 4×3 | 12 |
| ⋮ | | |
| 10 | | 30 |

$y = 3x$

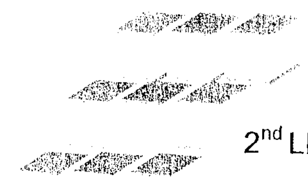
Edgar's little sister is playing with blocks. He notices a pattern as she adds a level.

1st LEVEL 

3

 2nd LEVEL
1st LEVEL

6

 3rd LEVEL
2nd LEVEL
1st LEVEL

9

1. If the pattern continues, how many blocks would be needed to build the 5th level? How many blocks would be needed to build the 10th level?

3, 6, 9, 12, 15, . . . . 30

2. Write an equation to describe the number of blocks that would be needed to build level "x".

$3x = y$

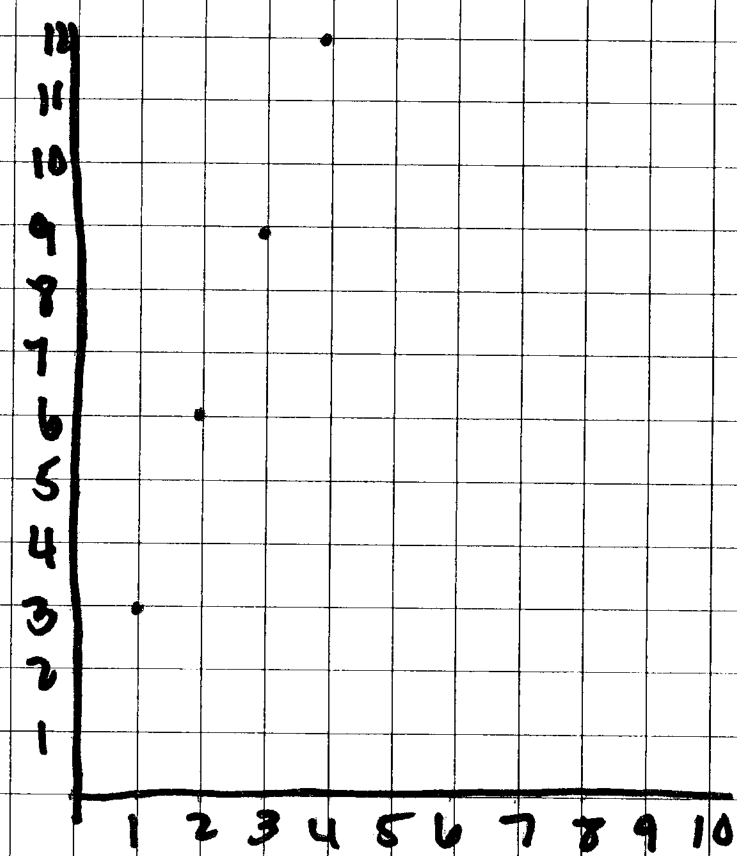
3. Construct a graph on a coordinate grid that shows the relationship between the level number and the number of blocks in that level.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Level | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | | | | |
| # of blocks | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | | | | |

Sample E

| x Level | y Blocks |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |
| 5 | 15 |
| 6 | 18 |
| 7 | 21 |
| 8 | 24 |
| 9 | 27 |
| 10 | 30 |

You keep adding 3
so $y = x + 3$



Sample F

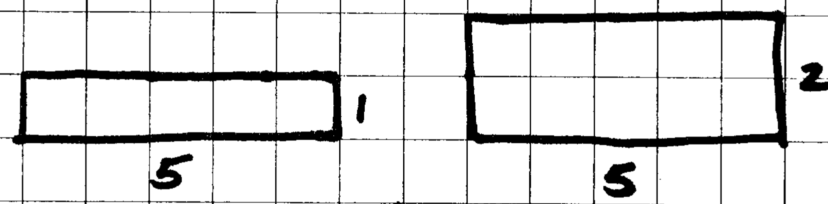
| x | y |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |
| 5 | 15 |
| 10 | 30 |

Equation increases by 3

$3x = y$

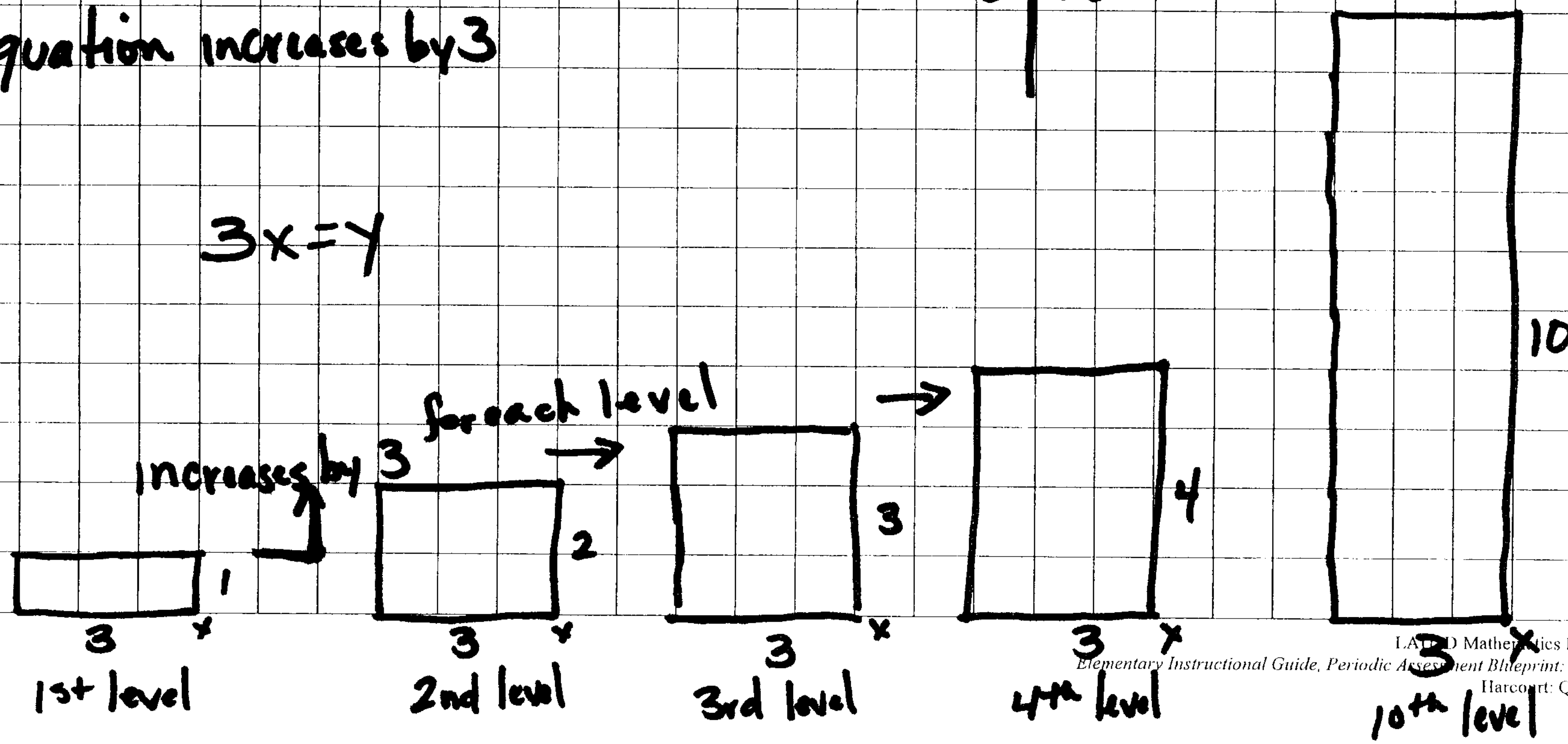
same process

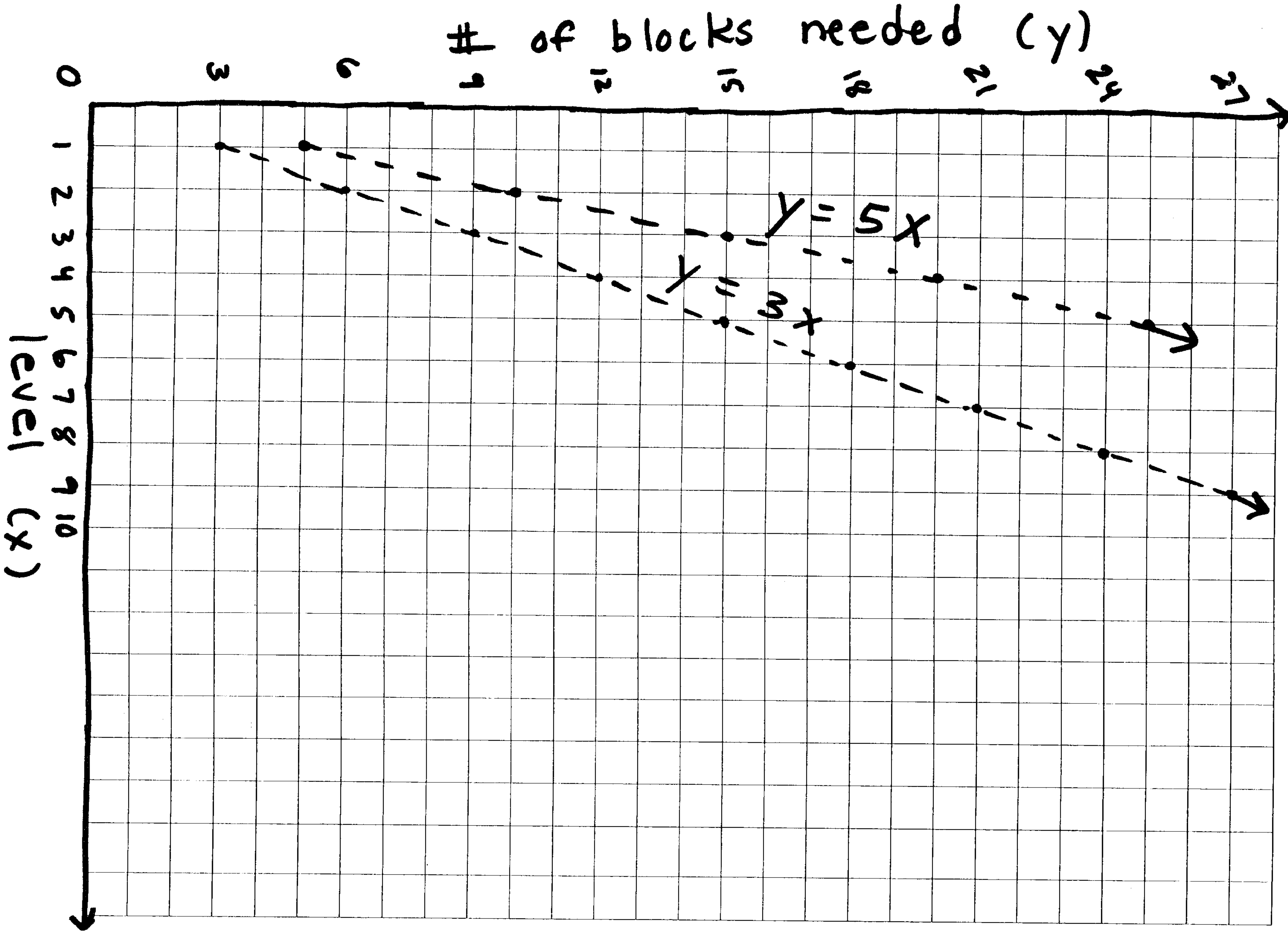
$5x = y$




| x | y |
|---|---|
| 1 | 5 |
| 2 | 10 |
| 3 | 15 |

increases by 3 for each level

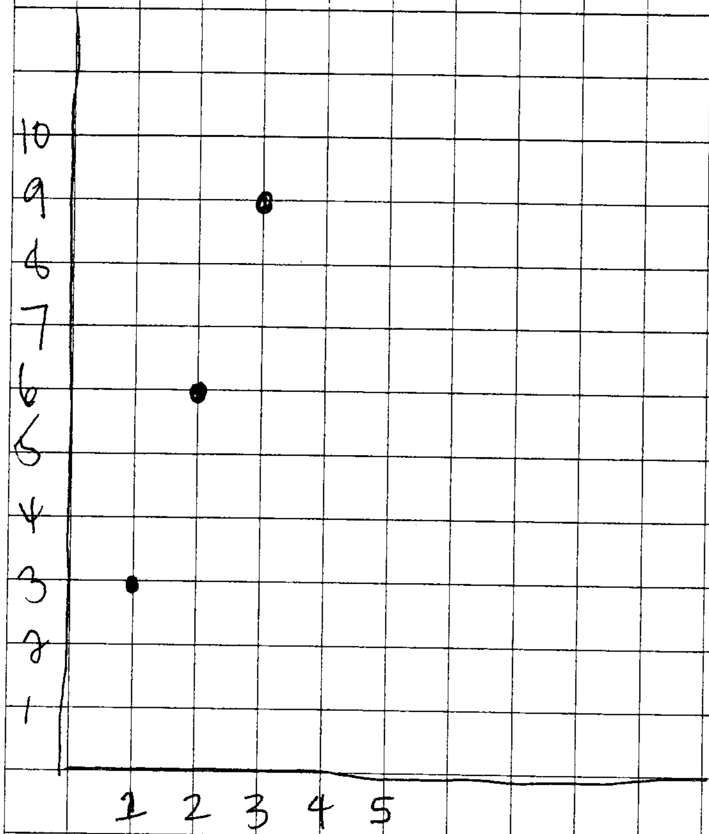




# of blocks needed (y)
0
3
6
9
12
15
18
21
24
27
1
2
3
4
5
6
7
8
9
10
level (x)
y = 5x
y = 3x


Sample 6

Sample H




3 6 9 12 15

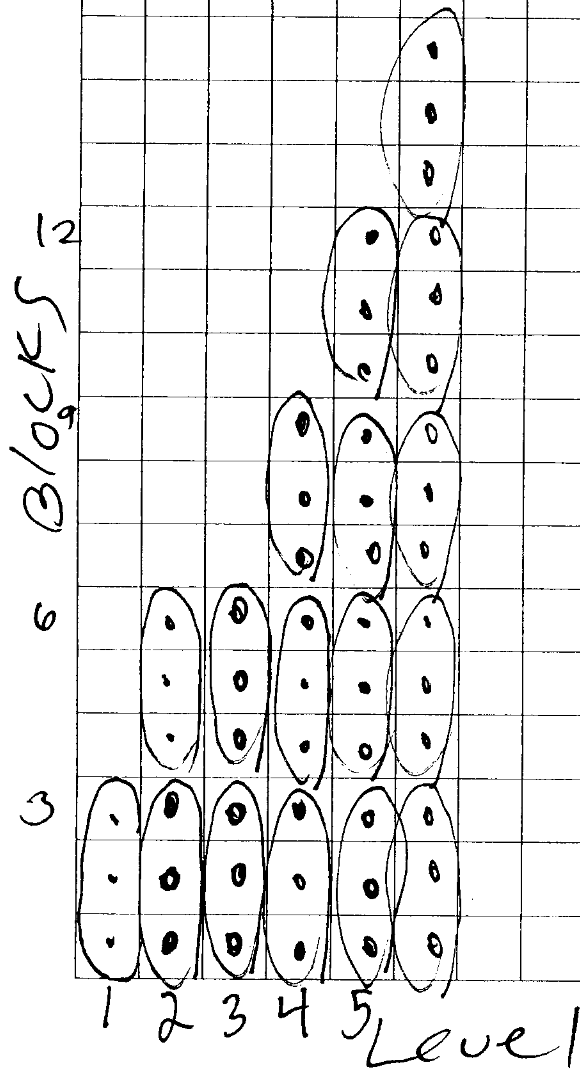
x = level

3x = y

| x | 1 | 2 | 3 | 4 | 5 | 10 |
|---|---|---|---|---|---|---|
| y | 3 | 6 | 9 | 12 | 15 | 30 |

# Sample I

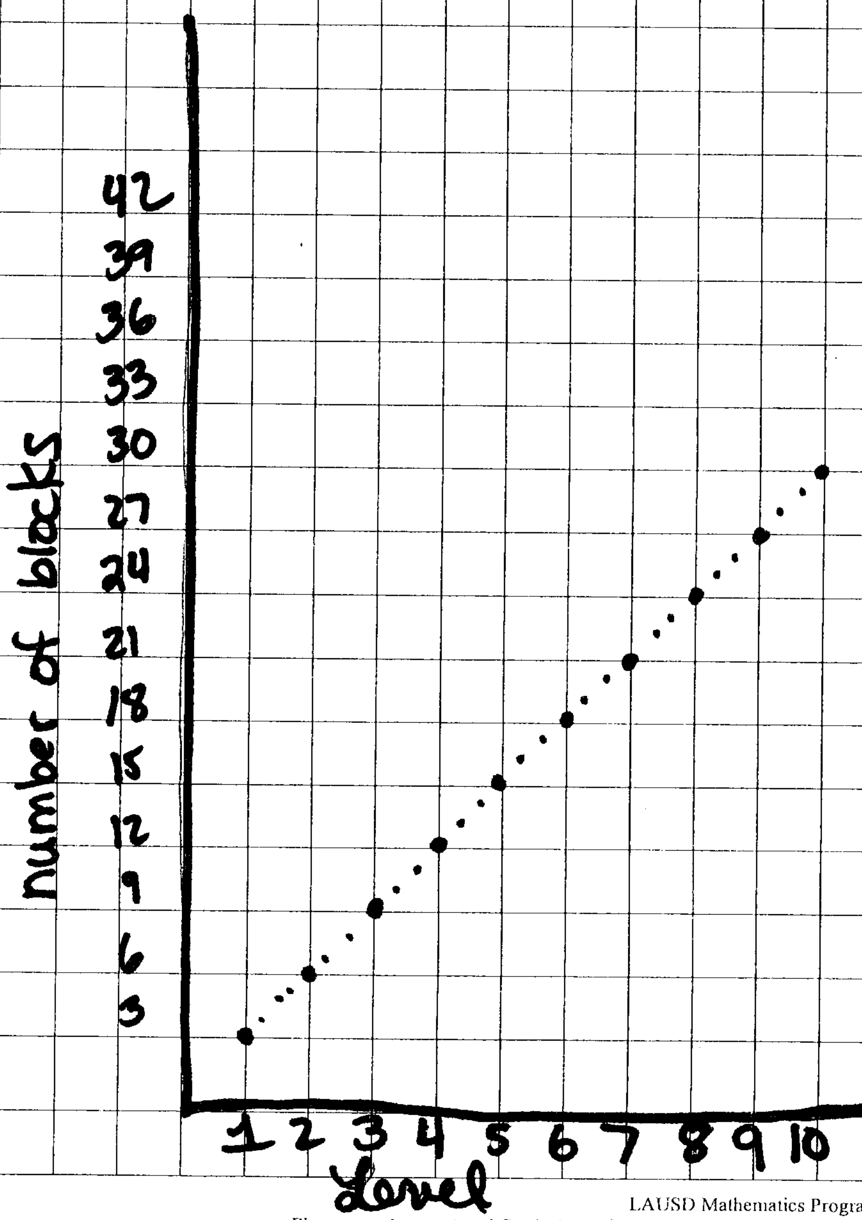
blocks = ⋮      (⋮) = level



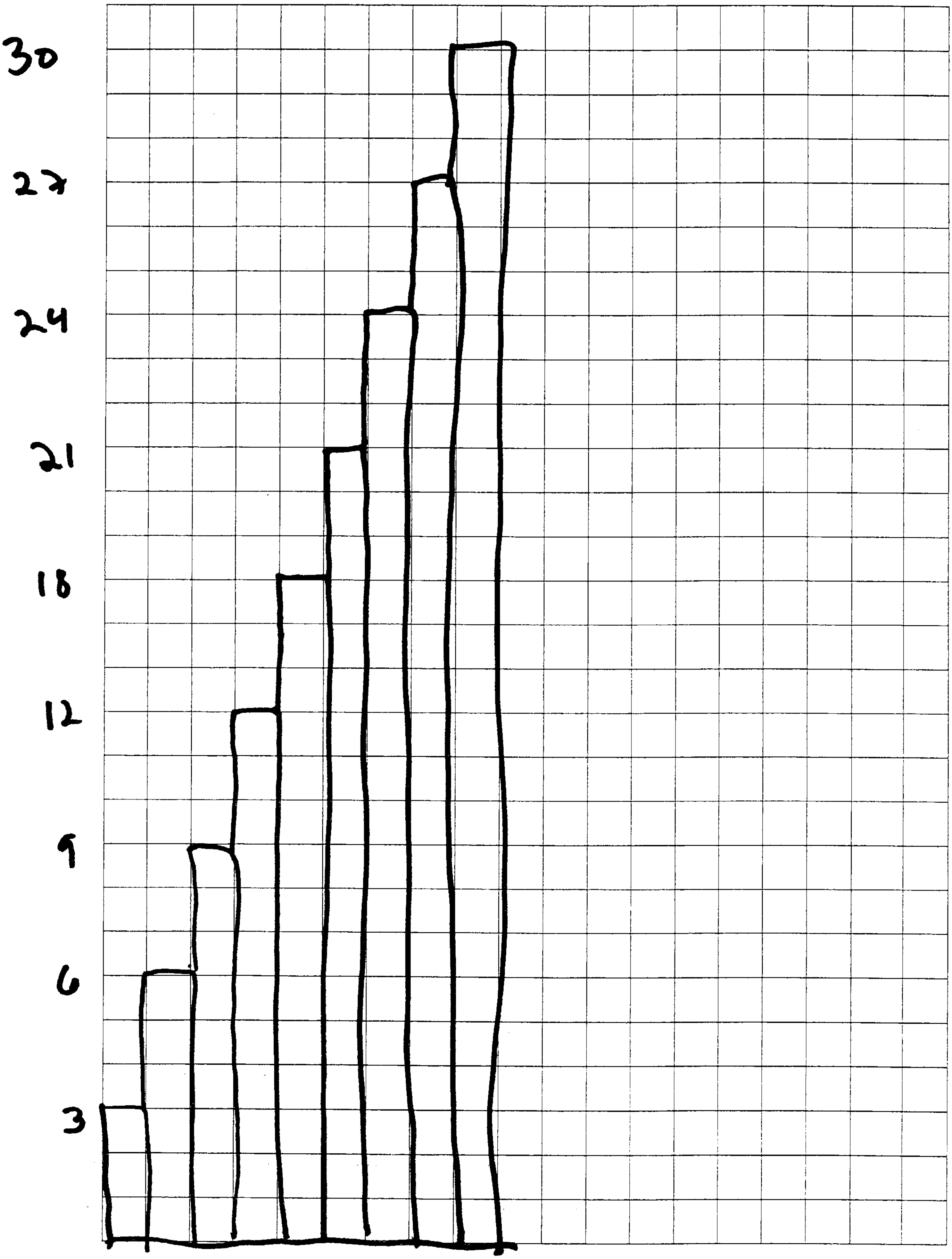

| level | Blocks |
|---|---|
| 10 | 30 |
| 9 | 27 |
| 8 | 24 |
| 7 | 21 |
| 6 | 18 |
| 5 | 15 |
| 4 | 12 |
| 3 | 9 |
| 2 | 6 |
| 1 | 3 |

Sample J

1st level = 3    3
2nd level = 6    3+3
3rd level = 9    3+3+3
4th level = 3+3+3+3 = 12
5th level = 3+3+3+3 = 15
6th level = 6x3 = 18
7th level = 7x3 = 21
⋮
10th level = 10x3 = 30
⋮
Xth level = Xx3 = 3X

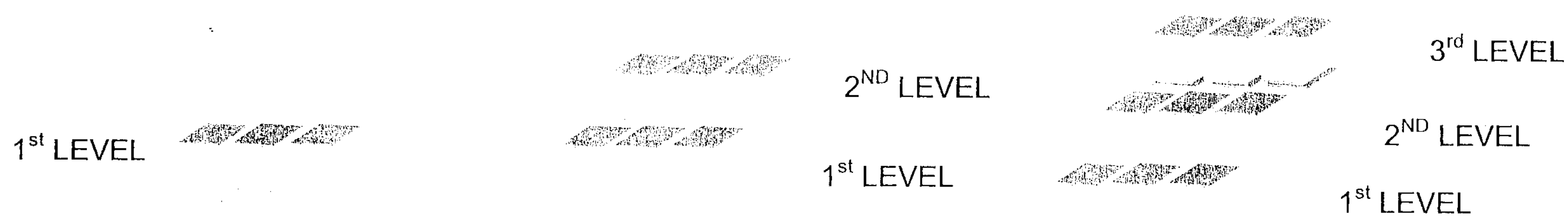


Sample K



Sample L

**Stacking Blocks**
**Part 2**
**4th Grade Lesson**
**Quarter 3**




4. How does your equation show that the number of blocks increases by 3 for each level? How does your graph show that the number of blocks increases by 3 for each level? The 3 is constant in the equation. Each time the x increases by 1, the y increases by 3. (the points move over 1 and up 3 for each point.)

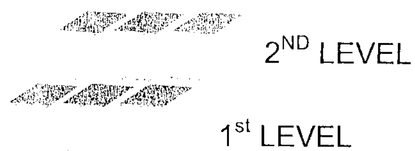
5. Describe how the equation and graph would change if each level were 5 blocks wide. What if each level was only one block wide?

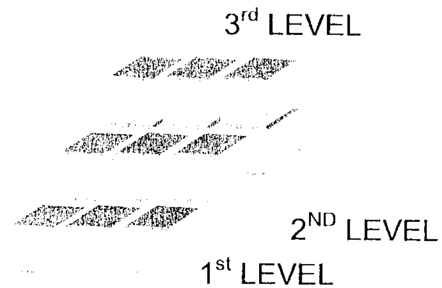
The constant would be 5 instead of 3:

$y = 5x$ The points would rise 5 each time on the y axis, rather than 3.

Sample M

# Stacking Blocks




1st level 3 blocks
2 level 6 blocks
3 level 9 blocks
4 level 12 blocks
5 level 15 blocks
6 level 18 blocks
7 level 21 blocks
8 level 24 blocks
9 level 27 blocks
10 level 30 blocks

1st LEVEL

Sample N

$$\overset{15}{(5\times3)} + \overset{12}{(4\times3)} + \overset{9}{(3\times3)} + \overset{6}{(2\times3)} + \overset{3}{3} = 45$$

| Blocks | 3 | 9 | 18 | 30 | 45 |
|---|---|---|---|---|---|
| Level | 1 | 2 | 3 | 4 | 5 |

Since level $5 \times 2 = 10$ then
# blocks $45 \times 2 = 90$