## CS/EE 260M

Homework 3 Solutions

1. (MK 2-16)

Simplify the following Boolean functions by means of a four-variable map:
(a) $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(1,5,9,12,13,15)$
(b) $\mathrm{F}(\mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z})=\sum m(1,3,9,11,12,13,14,15)$
(c) $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(0,2,4,5,6,7,8,10,13,15)$
(a) $\mathbf{F}(\mathbf{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(1,5,9,12,13,15)$


$$
\mathbf{F}=\mathbf{A B}\left(\mathbf{C}^{\prime}+\mathbf{D}\right)+\mathbf{C}^{\prime} \mathbf{D}
$$

(b) $\mathbf{F}(\mathbf{W}, \mathbf{X}, Y, Z)=\sum m(\mathbf{1}, \mathbf{3}, \mathbf{9}, \mathbf{1 1}, \mathbf{1 2}, \mathbf{1 3}, \mathbf{1 4}, 15)$

(c) $\mathbf{F}(\mathbf{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(0,2,4,5,6,7,8,10,13,15)$

2. (MK 2-20)

Simplify the following Boolean functions by finding all prime implicants and essential prime implicants and applying the selection rule:
(a) $\mathrm{F}(\mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z})=\sum m(1,5,6,7,11,12,13,15)$
(b) $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(1,3,4,5,7,8,9,12)$
(c) $\mathrm{F}(\mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z})=\sum m(0,1,2,5,6,7,8,9,10,13,14,15)$
(a) $\mathbf{F}(\mathbf{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z})=\sum m(\mathbf{1 , 5 , 6 , 7 , 1 1 , 1 2 , 1 3 , 1 5 )}$

prime implicants: $\mathbf{X Z}, \mathbf{W X Y} \mathbf{Y}^{\prime}, \mathbf{W}^{\prime} \mathbf{X Y}, \mathbf{W}^{\prime} \mathbf{Y}^{\prime} \mathbf{Z}, \mathbf{W Y Z}$ all are essential, so

$$
\mathbf{F}=\mathbf{X} \mathbf{Z}+\mathbf{W} \mathbf{X} \mathbf{Y}^{\prime}+\mathbf{W}^{\prime} \mathbf{X} \mathbf{Y}+\mathbf{W}^{\prime} \mathbf{Y}^{\prime} \mathbf{Z}+\mathbf{W} \mathbf{Y} \mathbf{Z}
$$

(b) $\mathbf{F}(\mathbf{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(\mathbf{1 , 3 , 4 , 5 , 7 , 8 , 9 , 1 2})$

|  |  |  | CD |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  |  | 00 | 01 | 11 | 10 |

prime implicants: $\mathbf{A}^{\prime} \mathbf{D}, \mathbf{A}^{\prime} \mathbf{B C}^{\prime}, \mathbf{B C}^{\prime} \mathbf{D}^{\prime}, \mathbf{A C}^{\prime} \mathbf{D}^{\prime}, \mathbf{A B}^{\prime} \mathbf{C}^{\prime}$ essential: $\mathbf{A}^{\prime} \mathbf{D}, \mathrm{AB}^{\prime} \mathbf{C}^{\prime}$
so select $B C^{\prime} D^{\prime}$ to complete cover
$\mathbf{F}=\mathbf{A}^{\prime} \mathbf{D}+\mathbf{A} \mathbf{B}^{\prime} \mathbf{C}^{\prime}+\mathbf{B} \mathbf{C}^{\prime} \mathbf{D}^{\prime}$
(c) $\mathbf{F}(\mathbf{W}, \mathbf{X}, \mathbf{Y}, \mathbf{Z})=\sum m(\mathbf{0}, \mathbf{1 , 2 , 5 , 6 , 7 , 8 , 9 , 1 0 , 1 3 , 1 4 , 1 5 )}$

prime implicants: $\mathbf{X Z}, \mathbf{X Y}, \mathbf{Y Z}^{\prime}, \mathbf{Y}^{\prime} \mathbf{Z}, \mathbf{X}^{\prime} \mathbf{Z}^{\prime}, \mathbf{X}^{\prime} \mathbf{Y}^{\prime}$
essential: none

$$
\mathbf{F}=\mathbf{X Z}+\mathbf{Y} \mathbf{Z}^{\prime}+\mathbf{X}^{\prime} \mathbf{Y}^{\prime}
$$

3. (MK 2-23)

Simplify the follwoing functions into (1) sum-of-products and (2) product.
(a) $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(2,3,5,7,8,10,12,13)$
(b) $\mathrm{F}(\mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z})=\Pi M(2,10,13)$
(a) $\mathbf{F}(\mathbf{A}, \mathbf{B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(\mathbf{2}, 3,5,7,8,10,12,13)$

|  | CD |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 00 | 00 | 01 | 11 | 10 |  |
| 00 | 0 | 0 | 1 | 1 |  |
| 01 | 0 | 1 | 1 | 0 |  |
| 11 | 1 | 1 | 0 | 0 |  |
| 10 | 1 | 0 | 0 | 1 |  |

$$
\begin{aligned}
& \text { 1) } \mathbf{F}=\mathbf{A}^{\prime} \mathbf{B}^{\prime} \mathbf{C}+\mathbf{A}^{\prime} \mathbf{B D}+\mathbf{A B} \mathbf{C}^{\prime}+\mathbf{A} \mathbf{B}^{\prime} \mathbf{D}^{\prime} \\
& \text { 2) } \mathbf{F}^{\prime}=\mathbf{A}^{\prime} \mathbf{B}^{\prime} \mathbf{C}^{\prime}+\mathbf{A}^{\prime} \mathbf{B} \mathbf{D}^{\prime}+\mathbf{A B C}+\mathbf{A} \mathbf{B}^{\prime} \mathbf{D} \\
& \mathbf{F}=(\mathbf{A}+\mathbf{B}+\mathbf{C})\left(\mathbf{A}+\mathbf{B}^{\prime}+\mathbf{D}\right)\left(\mathbf{A}^{\prime}+\mathbf{B}^{\prime}+\mathbf{C}^{\prime}\right)\left(\mathbf{A}^{\prime}+\mathbf{B}+\mathbf{D}^{\prime}\right)
\end{aligned}
$$

(b) $\mathbf{F}(\mathbf{W}, \mathbf{X}, \mathbf{Y}, \mathbf{Z})=\Pi M(\mathbf{2}, \mathbf{1 0}, \mathbf{1 3})$

|  | Y |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |

$$
\begin{aligned}
& \text { 1) } \mathbf{F}=\mathbf{X} \mathbf{Z}^{\prime}+\mathbf{X}^{\prime} \mathbf{Y}^{\prime}+\mathbf{Y Z}+\mathbf{W}^{\prime} \mathbf{Y}^{\prime} \\
& \text { 2) } \mathbf{F}^{\prime}=\mathbf{X}^{\prime} \mathbf{Y} \mathbf{Z}^{\prime}+\mathbf{W} \mathbf{X} \mathbf{Y}^{\prime} \mathbf{Z} \\
& \mathbf{F}=\left(\mathbf{X}+\mathbf{Y}^{\prime}+\mathbf{Z}\right)\left(\mathbf{W}^{\prime}+\mathbf{X}^{\prime}+\mathbf{Y}+\mathbf{Z}^{\prime}\right)
\end{aligned}
$$

4. (MK 2-24)

Simplify the following Boolean functions $F$ together with the don't-care conditions $d$ :
(a) $\mathrm{F}(\mathrm{X}, \mathrm{Y}, \mathrm{Z})=\sum m(0,1,2,4,5), d(\mathrm{X}, \mathrm{Y}, \mathrm{Z})=\sum \mathrm{m}(3,6,7)$
(b) $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(0,6,8,13,14), d(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(2,4,10)$
(c) $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(1,3,5,7,9,15), d(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(4,6,12,13)$
(a) $\mathbf{F}(\mathbf{X}, \mathrm{Y}, \mathrm{Z})=\sum m(0,1,2,4,5), d(\mathbf{X}, Y, Z)=\sum \mathrm{m}(\mathbf{3 , 6 , 7})$

(b) $\mathbf{F}(\mathbf{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(0,6,8,13,14), d(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(\mathbf{2}, 4,10)$


$$
\mathbf{F}=\mathbf{C D}^{\prime}+\mathbf{B}^{\prime} \mathbf{D}^{\prime}+\mathbf{A B C} \mathbf{C}^{\prime} \mathbf{D}
$$

(c) $\mathbf{F}(\mathbf{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\sum m(1,3,5,7,9,15), d(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})=\sum \mathrm{m}(4,6,12,13)$

| CD |  |  |  |  | $\mathbf{F}=\mathbf{C}^{\prime} \mathbf{D}+\mathbf{A}^{\prime} \mathbf{D}+\mathbf{B D}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 00 |  | 01 | 11 | 10 |  |
| 00 |  | 1 | 1 |  |  |
| ${ }_{\text {AB }}{ }^{1}$ | x | 1 | 1) | x |  |
| 11 | x | x | 1. |  |  |
| 10 |  | 1 |  |  |  |

5. (MK 2-27)

Simplify each of the following expressions, and implement them with NAND gates. Assume that both true and complement versions of the input variables are available.
(a) $\mathrm{WX}^{\prime}+\mathrm{WXZ}+\mathrm{W}^{\prime} \mathrm{Y}^{\prime} \mathrm{Z}^{\prime}+\mathrm{W}^{\prime} \mathrm{XY}^{\prime}+\mathrm{WXZ}^{\prime}$ (b) $\mathrm{XZ}+\mathrm{XYZ}^{\prime}+\mathrm{WX}^{\prime} \mathrm{Y}^{\prime}$
(a) $\mathbf{W} \mathbf{X}^{\prime}+\mathbf{W} \mathbf{X Z}+\mathbf{W}^{\prime} \mathbf{Y}^{\prime} \mathbf{Z}^{\prime}+\mathbf{W}^{\prime} \mathbf{X} \mathbf{Y}^{\prime}+\mathbf{W} \mathbf{X} \mathbf{Z}^{\prime}$


$$
\mathbf{F}=\mathbf{X} \mathbf{Y}^{\prime}+\mathbf{W}+\mathbf{Y}^{\prime} \mathbf{Z}^{\prime}+\mathbf{X}^{\prime} \mathbf{Z}^{\prime}
$$


(b) $\mathbf{X Z}+\mathbf{X Y Z} \mathbf{Z}^{\prime}+\mathbf{W} \mathbf{X}^{\prime} \mathbf{Y}^{\prime}$

|  | YZ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 |
| 00 |  |  |  |  |
| WX 01 |  | 1 | 1 | 1 |
| 11 |  | 1 | 1 | 1. |
| 10 | 1 | 1. |  |  |

$$
\mathbf{F}=\mathbf{X Z}+\mathbf{X Y}+\mathbf{W} \mathbf{X}^{\prime} \mathbf{Y}^{\prime}
$$


6. (MK 2-29)

Draw the NAND logic diagram for each of the following questions using a multiple-level NAND circuit:
(a) $\mathrm{W}(\mathrm{X}+\mathrm{Y}+\mathrm{Z})+\mathrm{XYZ}$
(b) $\left(\mathrm{A}^{\prime} \mathrm{B}+\mathrm{CD}^{\prime}\right) \mathrm{E}+\mathrm{BD}^{\prime}(\mathrm{A}+\mathrm{B})$
(a) $\mathbf{W}(\mathbf{X}+\mathbf{Y}+\mathbf{Z})+\mathbf{X Y Z}$

(b) $\left(\mathbf{A}^{\prime} \mathbf{B}+\mathbf{C D}^{\prime}\right) \mathbf{E}+\mathbf{B D}^{\prime}(\mathbf{A}+\mathbf{B})$

7. (MK 2-32)

Convert the AND/OR/NOT logic diagram in Figure 2-46 to a) a NAND logic diagram and b) a NOR logic diagram
a) a NAND logic diagram

b) a NOR logic diagram

8.

When A, B or C changes, the output can change after 3 ns . This is the minimum.
Changes in C or D can cause output changes after 6 ns . This is the maximum.

