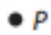
Ch. 1.1 Pg. 5

The most basic figures in geometry are *undefined terms*, which cannot be defined using other figures. The terms *point*, *line*, and *plane* are undefined terms. Although they do not have formal definitions, they can be described as shown in the table.

In geometry, the word *between* is another undefined term, but its meaning is understood from its use in everyday language. You can use undefined terms as building blocks to write definitions for defined terms, as shown in the table.

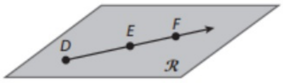
Undefined Terms		
Term	**Geometric Figure**	**Ways to Name the Figure**
A **point** is a specific location. It has no dimension and is represented by a dot.		point *P*
A **line** is a connected straight path. It has no thickness and it continues forever in both directions.		line ℓ, line *AB*, line *BA*, $\overleftrightarrow{AB}$, or $\overleftrightarrow{BA}$
A **plane** is a flat surface. It has no thickness and it extends forever in all directions.		plane $\mathcal{R}$ or plane *XYZ*

Defined Terms		
Term	**Geometric Figure**	**Ways to Name the Figure**
A **line segment** (or *segment*) is a portion of a line consisting of two points (called **endpoints**) and all points between them.		segment *CD*, segment *DC*, $\overline{CD}$, or $\overline{DC}$
A **ray** is a portion of a line that starts at a point (the *endpoint*) and continues forever in one direction.		ray *PQ* or $\overrightarrow{PQ}$

The Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) on the coordinate plane is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Use the figure for Exercises 17 and 18.



17. Name two different rays in the figure.

$\overrightarrow{DE}$ (or $\overrightarrow{DF}$) and $\overrightarrow{EF}$

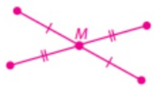
18. Name three different segments in the figure.

$\overline{DE}$ (or $\overline{ED}$), $\overline{EF}$ (or $\overline{FE}$), and $\overline{DF}$ (or $\overline{FD}$)

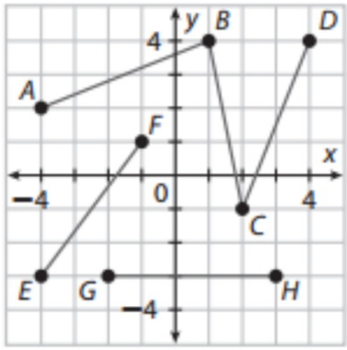
Sketch each figure.

19. two rays that form a straight line and that intersect at point P

20. two line segments that both have a midpoint at point M



Determine whether the given segments have the same length. Justify your answer.



9. $\overline{AB}$ and $\overline{BC}$

$A(-4, 2), B(1, 4), C(2, -1)$

$$AB = \sqrt{(1-(-4))^2 + (4-2)^2} = \sqrt{29}$$

$$BC = \sqrt{(2-1)^2 + (-1-(4))^2} = \sqrt{26}$$

$AB \neq BC$, so $\overline{AB}$ and $\overline{BC}$ do not have the same length.

10. $\overline{EF}$ and $\overline{GH}$

$E(-4, -3), F(-1, 1), G(-2, -3), H(3, -3)$

$$EF = \sqrt{(-1-(-4))^2 + (1-(-3))^2} = 5$$

$$GH = \sqrt{(3-(-2))^2 + (-3-(-3))^2} = 5$$

$EF = GH = 5$, so $\overline{EF}$ and $\overline{GH}$ have the same length.

11. $\overline{AB}$ and $\overline{CD}$

$A(-4, 2), B(1, 4), C(2, -1), D(4, 4)$

$$AB = \sqrt{(1-(-4))^2 + (4-2)^2} = \sqrt{29}$$

$$CD = \sqrt{(4-2)^2 + (4-(-1))^2} = \sqrt{29}$$

So, $AB = CD = \sqrt{29}$. Therefore, $\overline{AB}$ and $\overline{CD}$ have the same length.

12. $\overline{BC}$ and $\overline{EF}$

$B(1, 4), C(2, -1), E(-4, -3), F(-1, 1)$

$$BC = \sqrt{(2-1)^2 + (-1-(-4))^2} = \sqrt{26}$$

$$EF = \sqrt{(-1-(-4))^2 + (1-(-3))^2} = 5$$

So, $BC \neq EF$. Therefore, $\overline{BC}$ and $\overline{EF}$ do not have the same length.

Show that each statement is true.

13. If $\overline{DE}$ has endpoints $D(-1, 6)$ and $E(3, -2)$, then the midpoint M of $\overline{DE}$ lies in Quadrant I.

$$M\left(\frac{-1+3}{2}, \frac{6+(-2)}{2}\right) = M(1, 2)$$

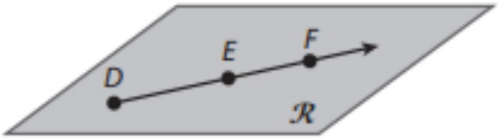
So M lies in Quadrant I, since the x- and y-coordinates are both positive.

14. If $\overline{ST}$ has endpoints $S(-6, -1)$ and $T(0, 1)$, then the midpoint M of $\overline{ST}$ lies in on the x-axis.

$$M\left(\frac{-6+0}{2}, \frac{-1+1}{2}\right) = M(-3, 0)$$

So M lies on the x-axis, since the y-coordinate is 0.

Use the figure for Exercises 17 and 18.



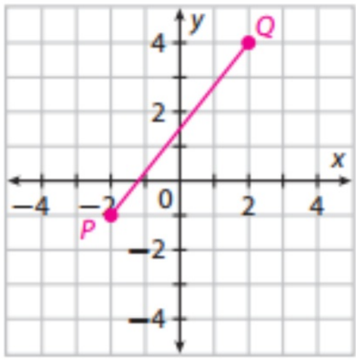
17. Name two different rays in the figure.

$\overrightarrow{DE}$ (or $\overrightarrow{DF}$) and $\overrightarrow{EF}$

18. Name three different segments in the figure.

$\overline{DE}$ (or $\overline{ED}$), $\overline{EF}$ (or $\overline{FE}$), and $\overline{DF}$ (or $\overline{FD}$)

22. Draw the segment PQ with endpoints $P(-2, -1)$ and $Q(2, 4)$ on the coordinate plane. Then find the length and midpoint of $\overline{PQ}$.



$$PQ = \sqrt{(2-(-2))^2 + (4-(-1))^2} = \sqrt{41}$$

$$M\left(\frac{-2+2}{2}, \frac{-1+4}{2}\right) = M(0, 1.5)$$

23. **Multi-Step** The sign shows distances from a rest stop to the exits for different towns along a straight section of highway. The state department of transportation is planning to build a new exit to Freestone at the midpoint of the exits for Roseville and Edgewood. When the new exit is built, what will be the distance from the exit for Midtown to the exit for Freestone?

The distance from the Roseville to Edgewood exits is $59 - 35 = 24$ mi, so the distance from the Roseville to Freestone exits will be $\frac{1}{2} \cdot 24 = 12$ mi. The distance from the Midtown to Roseville exits is $35 - 17 = 18$ mi, so the distance from the Midtown to Freestone exits will be $18 + 12 = 30$ mi.

24. On a town map, each unit of the coordinate plane represents 1 mile. Three branches of a bank are located at $A(-3, 1)$, $B(2, 3)$, and $C(4, -1)$. A bank employee drives from Branch A to Branch B and then drives halfway to Branch C before getting stuck in traffic. What is the minimum total distance the employee may have driven before getting stuck in traffic? Round to the nearest tenth of a mile.

The minimum total distance occurs when the employee drives along a straight line from A to B and from B to the midpoint of $\overline{BC}$.

The midpoint N of $\overline{BC}$ is $N(3, 1)$.

$AB = \sqrt{29}$, $BN = \sqrt{5}$, $AB + BN = \sqrt{29} + \sqrt{5} \approx 7.6$.

The minimum total distance the employee may have driven is 7.6 miles.

25. A city planner designs a park that is a quadrilateral with vertices at $J(-3, 1)$, $K(1, 3)$, $L(5, -1)$, and $M(-1, -3)$. There is an entrance to the park at the midpoint of each side of the park. A straight path connects each entrance to the entrance on the opposite side. Assuming each unit of the coordinate plane represents 10 meters, what is the total length of the paths to the nearest meter?

Midpoint P of $\overline{JK}$ is $P(-1, 2)$,
midpoint Q of $\overline{KL}$ is $Q(3, 1)$,
midpoint R of $\overline{LM}$ is $R(2, -2)$
midpoint S of $\overline{MJ}$ is $S(-2, -1)$.

The paths are $\overline{PR}$ and $\overline{SQ}$.
$PR = \sqrt{25} = 5$
$SQ = \sqrt{29}$

The total length is $\sqrt{29} + 5 \approx 10.39$, which represents 103.9 meters.
The total length of the paths is approximately 104 meters.

26. Communicate Mathematical Ideas A video game designer places an anthill at the origin of a coordinate plane. A red ant leaves the anthill and moves along a straight line to $(1, 1)$, while a black ant leaves the anthill and moves along a straight line to $(-1, -1)$. Next, the red ant moves to $(2, 2)$, while the black ant moves to $(-2, -2)$. Then the red ant moves to $(3, 3)$, while the black ant moves to $(-3, -3)$, and so on. Explain why the red ant and the black ant are always the same distance from the anthill.

At any given moment, the red ant's coordinates may be written as (a, a) where $a > 0$. The red ant's distance from the anthill is $\sqrt{(a-0)^2 + (a-0)^2} = \sqrt{2a^2} = a\sqrt{2}$. The black ant's coordinates may be written as $(-a, -a)$ and the black ant's distance from the anthill is $\sqrt{(-a-0)^2 + (-a-0)^2} = \sqrt{2a^2} = a\sqrt{2}$. This shows both ants are always $a\sqrt{2}$ units from the anthill.

27. Which of the following points are more than 5 units from the point $P(-2, -2)$? Select all that apply.

A. $A(1, 2)$ **$AP = 5$, so AP is not greater than 5.**

B. $B(3, -1)$ **$BP \approx 5.1$, so BP is greater than 5.**

C. $C(2, -4)$ **$CP \approx 4.5$, so CP is not greater than 5.**

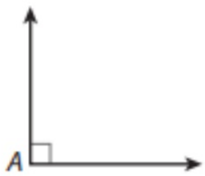
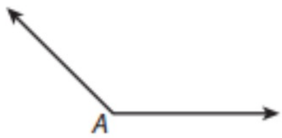
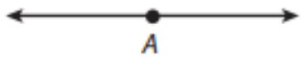
D. $D(-6, -6)$ **$DP \approx 5.7$, so DP is greater than 5.**

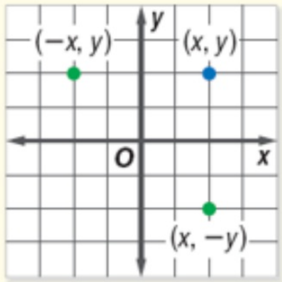
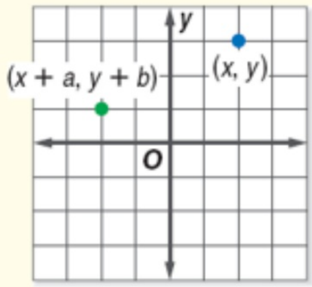
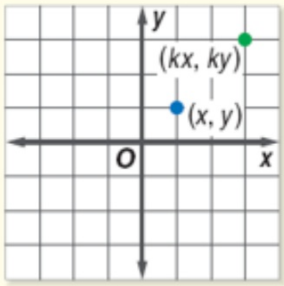
E. $E(-5, 1)$ **$EP \approx 4.2$, so EP is not greater than 5.**

Answer: B, D

Ch. 1.2

Classifying Angles

Acute Angle	Right Angle	Obtuse Angle	Straight Angle
			
$0° < m\angle A < 90°$	$m\angle A = 90°$	$90° < m\angle A < 180°$	$m\angle A = 180°$

Reflection	To reflect a point over the x-axis, multiply the y-coordinate by -1. To reflect a point over the y-axis, multiply the x-coordinate by -1.	reflection over x-axis: $(x, y) \rightarrow (x, -y)$ reflection over y-axis: $(x, y) \rightarrow (-x, y)$	
Translation	To translate a point by an ordered pair (a, b), add a to the x-coordinate and b to the y-coordinate.	$(x, y) \rightarrow (x + a, y + b)$	
Dilation	To dilate a figure by a scale factor k, multiply both coordinates by k. If $k > 1$, the figure is enlarged. If $0 < k < 1$, the figure is reduced.	$(x, y) \rightarrow (kx, ky)$ fraction	

Rotation	To rotate a figure 90° *counterclockwise* about the origin, switch the coordinates of each point and then multiply the new first coordinate by −1. To rotate a figure 180° about the origin, multiply both coordinates of each point by −1.	90° rotation: $(x, y) \rightarrow (-y, x)$ **switch & mult. by -1** 180° rotation: $(x, y) \rightarrow (-x, -y)$ **Mult. both by -1**