Short Run Cost Functions

In the short run, one or more inputs are fixed, so the firm chooses the variable inputs to minimize the cost of producing a given amount of output.

With several variable inputs, the procedure is the same as long run cost minimization. For example, if we have $f(K, L, Land)$ and Land is fixed, we solve the cost minimization problem to find the demand for capital and labor, conditional on input prices and x, $K^*(w, r, x)$ and $L^*(w, r, x)$. Then we evaluate the cost of K, L, and Land to get the total cost function.

With one variable input, things are quite a bit easier, since there is no substitutability between inputs.

Suppose that we have a fixed amount of capital, $\overline{K}$. Then the production function can be interpreted as a function of L only. For example, if we have $f(K, L) = K^{\alpha}L^{\beta}$, then the short run production function is $f(L; \overline{K}) = \overline{K}^{\alpha}L^{\beta}$.

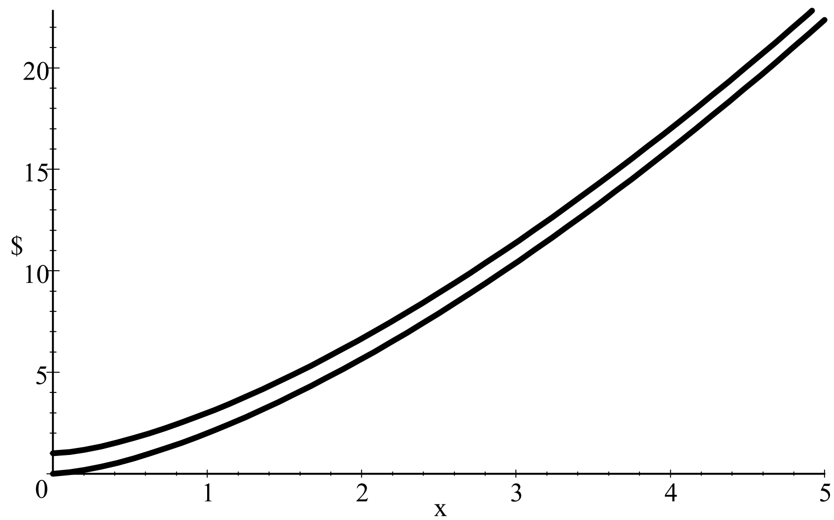
To find the conditional labor demand, we invert the short run production function by solving $x = f(L; \overline{K})$ for L. This gives us $L(x; \overline{K})$, which does not depend on input prices, since this amount of labor is *required* in order to produce x units of output.

Then the short run total cost function is given by

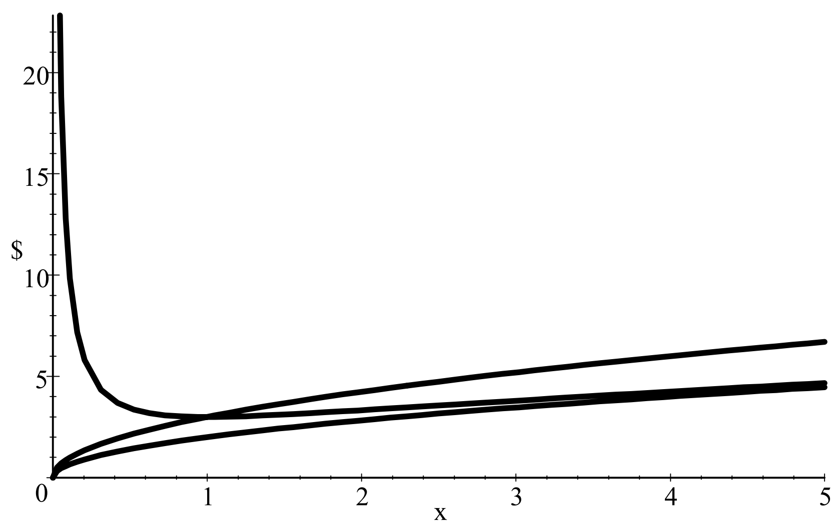
$$SRTC(x; \overline{K}, w, r) = wL(x; \overline{K}) + r\overline{K}.$$

We can also define the following:

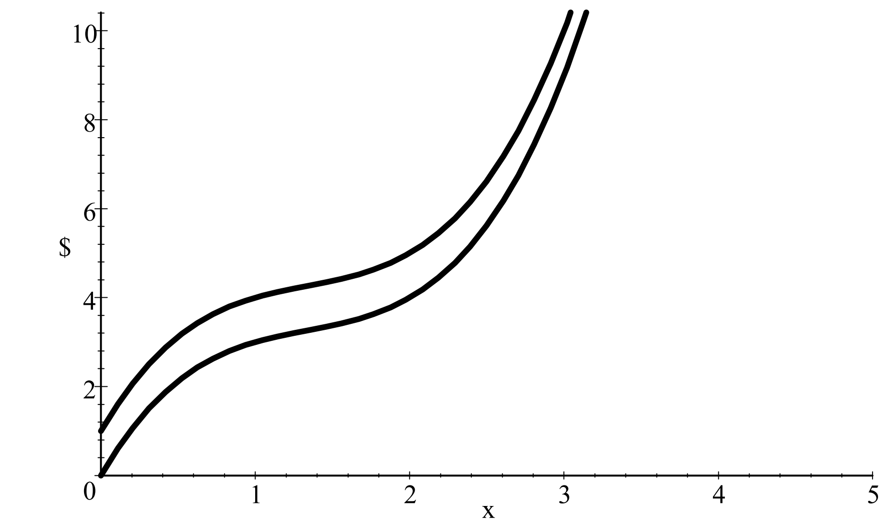
$$
\begin{aligned}
SRTC(x;\overline{K},w,r) &= wL(x;\overline{K}) + r\overline{K} \\
SRVC &= wL(x;\overline{K}) \\
FC &= r\overline{K} \\
SRATC &= \frac{wL(x;\overline{K})}{x} + \frac{r\overline{K}}{x} \\
SRAVC &= \frac{wL(x;\overline{K})}{x} \\
AFC &= \frac{r\overline{K}}{x} \\
SRMC &= \frac{d(SRTC)}{dx} \\
&= \frac{d(SRVC)}{dx} = w\frac{dL(x;\overline{K})}{dx}
\end{aligned}
$$



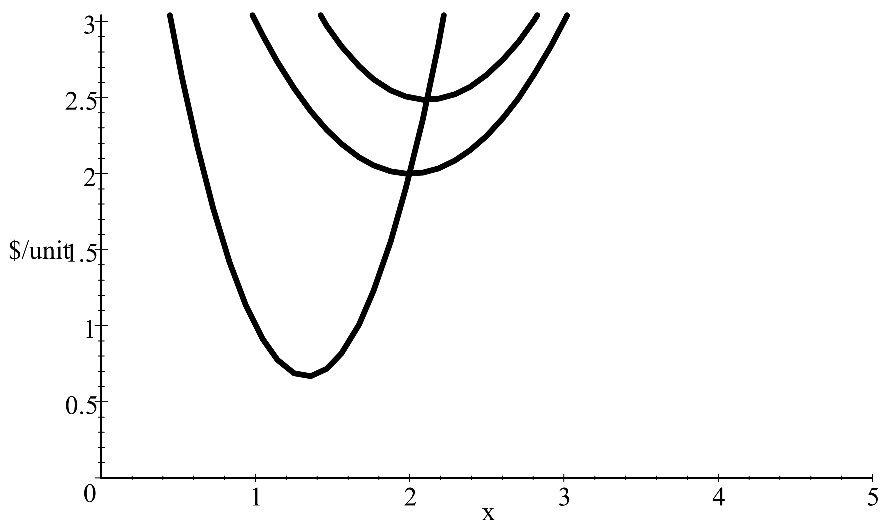
SRTC and SRVC: Cobb Douglas



SRTC, SRVC, and SRMC: Cobb Douglas



SRTC and SRVC (S-shaped)



SRATC, SRAVC and SRMC (U-shaped)

Because of diminishing marginal returns and the presence of a fixed input,

1. SRMC eventually becomes upward sloping

$$SRMC = w\frac{dL(x;\overline{K})}{dx} = \frac{w}{\frac{df(L;\overline{K})}{dL}} = \frac{w}{MP_L}.$$

2. SRATC and SRAVC eventually become upward sloping.

3. SRATC is U-shaped. (Remember, SRATC is very large for small x, because of fixed costs.)

example: $x = L^{2/3}K^{1/3}$ (with $w = 2, r = 1, \overline{K} = 1$)

solving for L, we first plug in $\overline{K} = 1$

$$x = L^{2/3}.$$

Now take both sides to the 3/2 power:

$$L = x^{3/2}.$$

Therefore, the short run total cost function is

$$SRTC = 2x^{3/2} + 1 \tag{1}$$

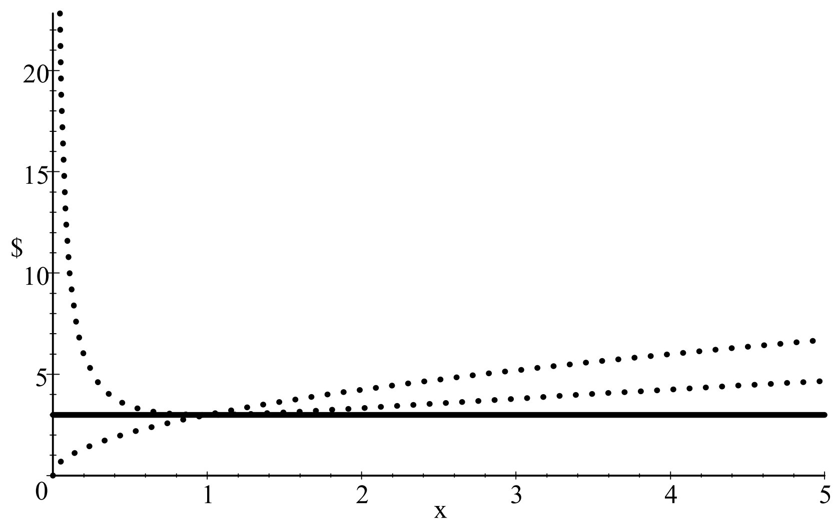
From equation (1), we have: $SRVC = 2x^{3/2}, FC = 1, SRATC = 2x^{1/2} + 1/x, SRAVC = 2x^{1/2}, AFC = 1/x, SRMC = 3x^{1/2}$.

The Relationship Between Short Run and Long Run Cost Curves

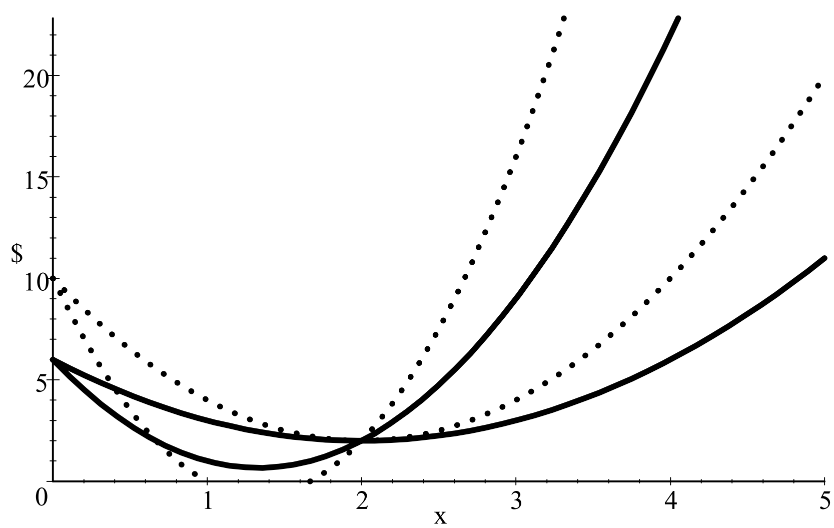
LRATC must be less than SRATC, because in the long run, all inputs are variable. You can always choose $K = \overline{K}$ and have average cost equal to SRATC, but choosing a different K (when K is variable) might yield lower costs.

Choosing $K = \overline{K}$ will be optimal for some level of x (when K is variable), so for that x, LRATC=SRATC. For other values of x, a different K will be optimal, so LRATC $<$ SRATC.

As we vary $\overline{K}$, we trace out a different SRATC curve. LRATC is the *lower envelope* of all the SRATC curves, as we vary $\overline{K}$.



Short Run and Long Run: Cobb Douglas



SR vs. LR: U-shaped LRATC