Tutorial: Using Excel for Linear Optimization Problems

Part 1: Organize Your Information

There are three categories of information needed for solving an optimization problem in Excel: an *Objective Function*, *Decision Variables*, and *Constraints*. It is simplest to organize these on paper before you start working with the spreadsheet. For tutorial purposes we will follow an example, "The Cargo Problem", from start to finish:

A shipping company has the capacity to move 100 tons of cargo per day by air. The company charges $250/ton for air freight. Besides the weight constraint, the company can only move 50,000 ft^3 of cargo per day because of limited volume of aircraft storage compartments. The following amounts of cargo are available for shipping each day:

Cargo	Weight (tons)	Volume (ft^3/ton)
1	30	550
2	40	800
3	50	400

Maximize the profit for the shipping company.

➤ Set up this problem:

Objective Function: Profit = 250*(Cargo1 + Cargo2+ Cargo3) (*$/week*)
Decision Variables:

Cargo 1 (*weight in tons*)
Cargo 2
Cargo 3

Constraints:

Weight: Cargo1 + Cargo2 + Cargo3 ≤ 100
Volume: 550*Cargo1 + 800*Cargo2 +400*Cargo3 ≤ 50000
Amount 1:   Cargo1 ≤ 30
Amount 2:   Cargo2 ≤ 40
Amount 3:   Cargo3 ≤ 50
*Do not forget to include the trivial constraints!
Trivial 1:   Cargo1 ≥ 0
Trivial 2:   Cargo2 ≥ 0
Trivial 3:   Cargo3 ≥ 0

Part 2: Set Up the Problem in the Excel Spreadsheet

Solver is an Add-in for Microsoft Excel which is typically not enabled during the initial installation of Excel. If 'Solver' does not appear on the 'Tools' menu in Excel, then you need to enable it as follows:

> ➢ Select the 'Tools' menu in Excel, and then choose 'Add-ins'. Check the box titled 'Solver Add-ins' and then click 'OK'. 'Solver' should then appear as an item on the Tools menu.

In order to enter your data and work with it in the most straightforward way it is best to stay organized by using consistent labels and columns. Refer to the following figure as one example for setting up this problem in a spreadsheet.

> ➢ Enter the names of the *Objective Function*, *Decision Variables* and *Constraints* into cells in column A—the more descriptive the labels are, the easier it is to keep track of the relationships among the variables and constraints.
> ➢ For the *Decision Variables*, you can enter a 0 into each of the corresponding cells in column B or you can leave them blank—Solver will calculate the optimum values for these variables.
> ➢ In the cell in column B corresponding to the *Objective Function*, enter the formula determined above, where the names of each of the *Decision Variables* (e.g. Cargo1, Cargo2, and Cargo3) are replaced with their corresponding cell locations (here, B9, B10, and B11). Refer to the following example.
> ➢ For each *Constraint*, enter the formula for the left hand side of the inequality/equation in the cell in column B, and enter the *Constraint* value into the corresponding cell in column C.
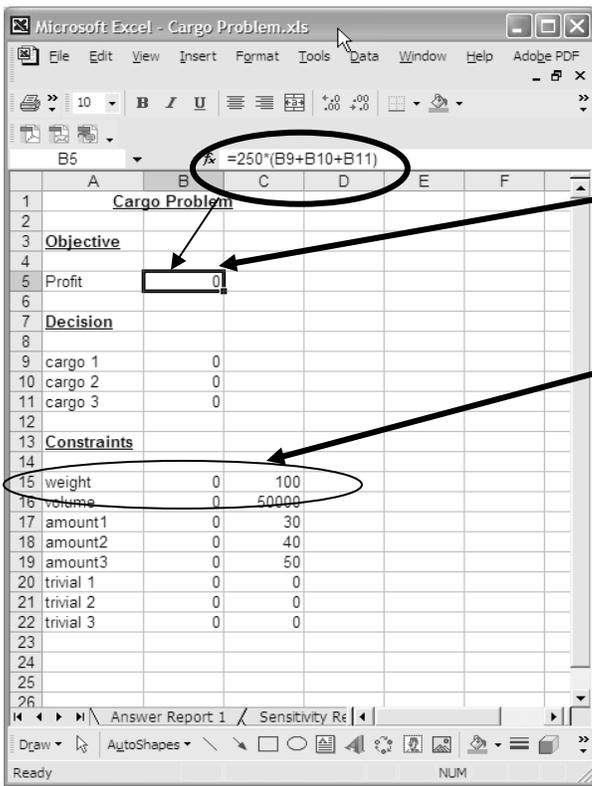
Column A	Column B
Profit	= 250*(B9 + B10 + B11)

Variable Name	Formula	Constraint Value
weight	= B9 + B10 + B11	= 100

The final spreadsheet should look as follows when you are ready to run Solver:
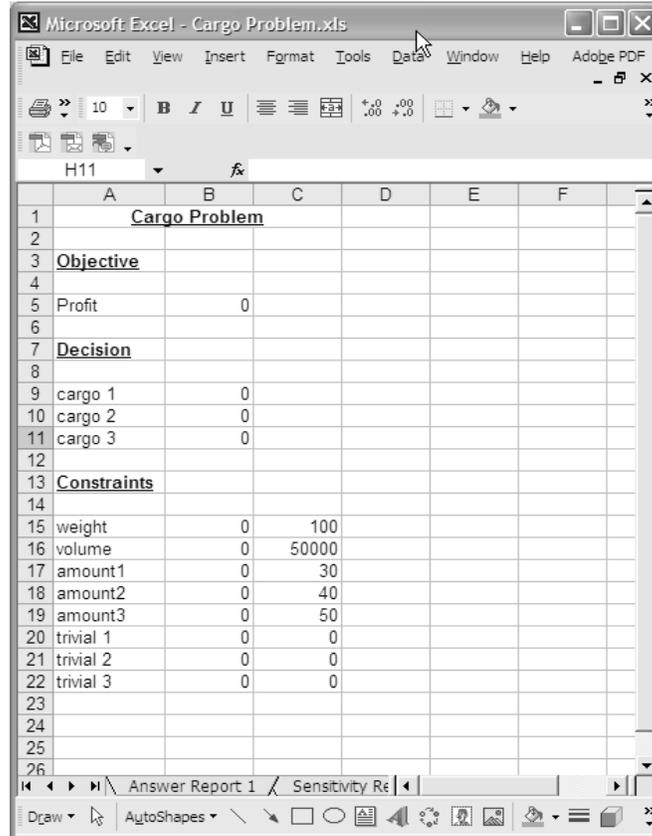
Part 3: Running Solver

> Under "Tools" in the menu bar select "Solver". The following dialog box will appear:

> In 'Set Target Cell' enter the cell corresponding to your *Objective Function* and choose the appropriate outcome under 'Equal To:' (maximize, minimize, or set to a specific value). *NOTE*: *You can enter cells into any of these target boxes by selecting them on the spreadsheet using the cursor; it's easier than typing in cell addresses.*

➢ If you know that your function is linear you can check the 'Linear Solution' box under 'Options' to speed up the solver process. For most problems this is not necessary.

➢ Select the cells in column B corresponding to all of your *Decision Variables* in the 'By Changing Cells:' target box.

➢ To set up the *Constraints*, select 'Add' under 'Subject to the Constraints:' The following dialog box opens:

Enter the cell location (in column B) for each *Constraint* into the left hand box ('Cell Reference:') and the cell location (in column C) of the *Constraint* value into the right hand box. Use the pull-down menu in the middle to select the appropriate inequality relation. Continue to click the "Add" button until all of your *Constraints* are entered, then select 'OK'. *NOTE: You can later change or delete these Constraints from the Solver dialog box if necessary.*

➢ Click OK. This will return you to the Solver dialog box. When all the targets and constraints have been entered, select 'Solve'. The following dialog box will appear:
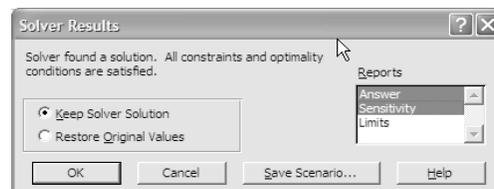
➢ At this point you have the option to generate any of three reports: Answer, Sensitivity, and Limits. These are inserted as new workbooks in the Excel file. Select any you wish to have included. If Solver was able to find a solution, it will notify you that all constraints and optimality conditions were satisfied. You may elect to Keep the Solver Solution or Restore the Original Values. NOTE: If you choose to generate an Answer Report, you can still select 'Restore Original Values' to continue working with the unaltered spreadsheet; the results from Solver will be saved in the Answer Report.

➢ Click 'OK'. You are then returned to the spreadsheet.

➢ *HINT: The last set of target and constraints used in the Solver dialog box will be saved with each worksheet in the Excel file, and will be copied with the worksheet if you need to duplicate it. So you can repeatedly run Solver in the sheet to investigate the effects of changing Constraint values or relationships.*

Part 4: Interpreting the Results

Until you become familiar with each report and the information it holds you may wish to generate all possible reports to ensure you do not miss any required information. The following is an example of the answer report generated for this Cargo Problem.

Answer Report

The *Answer Report* identifies the names, the corresponding cells, the initial and optimal (final) values of the Target (i.e. the *Objective Function*), and the values of Adjustable Cells (i.e. the *Decision Variables*) that produced it. In the *Constraint* section the status column indicates if a *Constraint* was binding or not. For non-binding *Constraint*s, the slack column shows how far from the *Constraint* your results are.

Microsoft Excel 10.0 Answer Report
Worksheet: [Book2]Sheet1
Report Created: 11/10/2004 6:01:18 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$B$5	Profit	0	24218.75

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$9	cargo 1	0	30
$B$10	cargo 2	0	16.875
$B$11	cargo 3	0	50

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$15	weight	96.875	$B$15<=$C$15	Not Binding	3.125
$B$16	volume	50000	$B$16<=$C$16	Binding	0
$B$17	amount1	30	$B$17<=$C$17	Binding	0
$B$18	amount2	16.875	$B$18<=$C$18	Not Binding	23.125
$B$19	amount3	50	$B$19<=$C$19	Binding	0
$B$20	trivial 1	30	$B$20>=$C$20	Not Binding	30
$B$21	trivial 2	16.875	$B$21>=$C$21	Not Binding	16.875
$B$22	trivial 3	50	$B$22>=$C$22	Not Binding	50

Example: The answer report indicates that the value for total weight of cargo (cell B15) in the optimal solution is 96.875 lbs, which is 3.125 lbs below the 100 lb constraint.

Sensitivity Report

The *Sensitivity Report* gives the final optimal values of both the *Decision Variable*s and the *Constraints*, along with the Lagrange multiplier (i.e. shadow prices) of the binding *Constraint*s.

Adjustable Cells

Cell	Name	Final Value	Reduced Gradient
$B$9	cargo 1	30	0
$B$10	cargo 2	16.875	0
$B$11	cargo 3	50	0

*Constraint*s

Cell	Name	Final Value	Lagrange Multiplier
$B$15	weight	96.875	0
$B$16	volume	50000	0.3125
$B$17	amount1	30	78.125
$B$18	amount2	16.875	0
$B$19	amount3	50	125
$B$20	trivial 1	30	0
$B$21	trivial 2	16.875	0
$B$22	trivial 3	50	0

Example: The shadow price for Volume is $0.31 which tells us that our optimal profit will increase by 31 cents for each additional cubic foot of space we can manage to secure. In other words, the value of 1 cubic foot of shipping space is $0.31 profit.

Limits Report

The *Limits Report* holds some interesting information. It has the upper and lower limits of the decision variables given all the constraints of the problem. In the Cargo problem we have lower limits of zero for each decision variable indicating that we can have no cargo shipped, the upper limits are the optimal values for the function. The target result for each lower limit tells us what value the objective function will take on if that particular constraint drops to the lower limit and all other constraints remain at the optimal value. The target result for the maximum column tells us the profit that is realized if all decision variables are at their optimal values.

Cell	Target Name	Value
$B$5	Profit	24218.75

Cell	Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
$B$9	cargo 1	30	0	16718.75	30	24218.75
$B$10	cargo 2	16.875	0	20000	16.875	24218.75
$B$11	cargo 3	50	0	11718.75	50	24218.75

Example: In the cargo problem if the amount of cargo 1 drops to zero and we still ship 50 tons of cargo 3 and 16.875 tons of cargo 2 we realize a profit of $16 718.75.