# Bonds, Bond Prices, Interest Rates, and the Risk and Term Structure of Interest Rates <br> ECON 40364: Monetary Theory \& Policy 

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## Readings

- Text:
- Mishkin Ch. 4, Mishkin Ch. 5 pg. 85-100, Mishkin Ch. 6
- Other:
- Poole (2005): "Understanding the Term Structure of Interest Rates"
- Bernanke (2016), "What Tools Does the Fed Have Left? Part 2: Targeting Longer-Term Interest Rates"


## Bonds

- We will generically refer to a "bond" as a debt instrument where a borrower promises to pay the holder of the bond (the lender) interest plus principal at some known date
- There are many different types of bonds. Differ according to:
- Details of how bond is paid off
- Time to maturity
- Default risk
- The yield to maturity is a measure of the interest rate on the bond, although the interest rate is often not explicitly laid out. Will use terms interest rate and yield interchangeably
- Want to understand how interest rates are determined and how and why they vary across different characteristics of bonds


## Present Value

- Present discounted value (PDV): a dollar in the future is worth less than a dollar in the present
- You "discount" future payouts relative to the present because you could put money "in the bank" in the present and earn interest
- For a future cash flow (CF), how many dollars would be equivalent to you today:

$$
P V_{t}=\frac{C F_{t+n}}{\left(1+i_{t}\right)\left(1+i_{t+1}\right)\left(1+i_{t+2}\right) \cdots \times\left(1+i_{t+n-1}\right)}
$$

- Here $t$ is the "present," $t+n$ is the future ( $n$ periods away), and $i_{t}, i_{t+1}, \ldots$ are the one period interest rates between periods
- If $i_{t}=i_{t+1}=\cdots=i_{t+n-1}=i$, then formula reduces to:

$$
P V_{t}=\frac{C F_{t+n}}{(1+i)^{n}}
$$

## Present Value: Example I

- Suppose you are promised $\$ 10$ in period $t+1$
- You could put $\$ 1$ in bank in period $t$ and earn $i_{t}=0.05$ in interest between $t$ and $t+1$
- How many dollars would you need in the present to have $\$ 10$ in the future?

$$
\begin{aligned}
\left(1+i_{t}\right) P V_{t} & =C F_{t+1} \\
P V_{t} & =\frac{C F_{t+1}}{1+i_{t}} \\
& =\frac{10}{1.05}=9.5238
\end{aligned}
$$

## Present Value: Example II

- Suppose you are promised $\$ 10$ in period $t+3$
- You could put $\$ 1$ in bank in period $t$ and earn $i_{t}=0.05$ in interest between $t$ and $t+1$
- You expect to be able to earn interest of $i_{t+1}=0.07$ between $t+1$ and $t+2$ and $i_{t+2}=0.03$ between $t+2$ and $t+3$
- If you put $\$ 1$ in bank in period $t$ and kept it there (re-investing any interest income), you would have $\left(1+i_{t}\right)\left(1+i_{t+1}\right)\left(1+i_{t+2}\right)$ dollars in $t+3$. Hence, the present value of $\$ 10$ three periods from now is:

$$
P V_{t}\left(1+i_{t}\right)\left(1+i_{t+1}\right)\left(1+i_{t+2}\right)=C F_{t+3}
$$

$$
\begin{aligned}
P V_{t} & =\frac{C F_{t+3}}{\left(1+i_{t}\right)\left(1+i_{t+1}\right)\left(1+i_{t+2}\right)} \\
& =\frac{10}{(1.05)(1.07)(1.03)}=8.6415
\end{aligned}
$$

## Present Value and the Price of an Asset

- A financial asset is something which entitles the holder to periodic payments (cash flows)
- The classical theory of asset prices is that the price of an asset is equal to the present discounted value of all future cash flows
- A bond is an asset: it entitles you to periodic cash flows. A stock is another kind of financial asset
- Price is just the present discounted value of cash flows
- The yield or interest rate on an asset is the interest rate you use to discount those future cash flows


## Different Types of Bonds

- The following are different types of bonds/debt instruments depending on the nature of how they are paid back:

1. Simple loan: you borrow $X$ dollars and agree to pay back $(1+i) X$ dollars at some specified date (interest plus principal) (e.g. commercial loan)
2. Fixed payment loan: you borrow $X$ dollars and agree to pay back the same amount each period (e.g. month) for a specified period of time. "Full amortization" (e.g. fixed rate mortgage)
3. Coupon bond: you borrow $X$ dollars and agree to pay back fixed coupon payments, $C$, each period (e.g. year) for a specified period of time (e.g. 10 years), at which time you pay off the "face value" of the bond (e.g. Treasury Bond)
4. Discount bond: you borrow $X$ dollars and agree to pay back $Y$ dollars after a specified period of time with no payments in the intervening periods. Typically, $Y>X$, so the bond sells "at a discount" (e.g. Treasury Bill)

- Interest rate is not explicit for coupon or discount bonds


## Yield to Maturity

- The yield to maturity (YTM) is the (fixed) interest rate that equates the PDV of cash flows with the price of the bond in the present
- This measures the return (expressed at an annualized rate) that would be earned on holding a bond if it is held until maturity
- The YTM does not necessarily correspond to the return if the bond is not held until maturity (i.e. if you sell a bond before its maturity date)
- The YTM is another way of conveying the price of a bond, taking future cash flows as given


## YTM on a Simple Loan

- The YTM on a simple loan is just the contractual interest rate
- For a one period loan, the YTM is the same thing as the return
- Let $P$ be the price of the loan, CF the payout after one year, and $i$ the interest rate. Then:

$$
P=\frac{C F}{1+i}
$$

- Or:

$$
1+i=\frac{C F}{P}
$$

- Or:

$$
i=\frac{C F-P}{P}
$$

- Where $\frac{C F-P}{P}$ is the return (or rate of return) on the loan


## Price and Yield on a Simple Loan



## YTM on a Discount Bond

- The YTM on a discount bond is similar to a simple loan, just with different maturities
- In particular, for a face value of $F$, maturity of $n$, and price of $P$, the YTM satisfies:

$$
P=\frac{F}{(1+i)^{n}}
$$

- Or:

$$
1+i=\left(\frac{F}{P}\right)^{\frac{1}{n}}
$$

## Price and Yield on a Discount Bond



## YTM on a Coupon Bond

- Suppose a bond has a face value of $\$ 100$ and a maturity of three years
- It pays coupon payments of $\$ 10$ in years $t+1, t+2$, and $t+3$ (the coupon rate in this example is 10 percent)
- The face value is paid off after period $t+3$
- The period $t$ price of the bond is $\$ 100$
- The YTM solves:

$$
100=\frac{10}{1+i}+\frac{10}{(1+i)^{2}}+\frac{10}{(1+i)^{3}}+\frac{100}{(1+i)^{3}}
$$

- Which works out to $i=0.1$
- More generally, for an $n$ period maturity:

$$
P=\sum_{j=1}^{n} \frac{C}{(1+i)^{j}}+\frac{F V}{(1+i)^{n}}
$$

## Price and Yield on a Coupon Bond



## Perpetuities

- Perpetuities (also called "consols") are like coupon bonds, except they have no maturity date
- Here, the relationship between price, yield, and coupon payments works out cleanly and is given by:

$$
i=\frac{C}{P}
$$

- For a coupon bond with a sufficiently long maturity, this is a reasonable approximation to the bond's YTM (because the PDV of the face value after many years is close to zero)
- This expression is also sometimes called the current yield as an approximation to the YTM on a coupon bond


## Price and Yield on a Perpetuity



## Observations

- Several observations are noteworthy from the previous slides:

1. The bond price and yield are negatively related. This is true for all types of bonds. Bond prices and interest rates move in opposite directions
2. For discount bonds, we would not expect price to be greater than face value - this would imply a negative yield
3. For a coupon bond, when the bond is priced at face value, the yield to maturity equals the coupon rate
4. For a coupon bond, when the bond is priced less than face value, the YTM is greater than the coupon rate (and vice-versa)

## Yields (Interest Rates) and Returns

- Returns and yields (interest rates) are in general not the same thing
- Rate of return: cash flow plus new security price, divided by current price
- Useful way to think about it (terminology here is related to equities): "dividend rate plus capital gain," where capital gain is the change in the security's price
- The return on a coupon bond held from $t$ to $t+1$ is:

$$
R=\frac{C+P_{t+1}-P_{t}}{P_{t}}
$$

- Or:

$$
R=\underbrace{\frac{C}{P_{t}}}_{\text {Current Yield }}+\underbrace{\frac{P_{t+1}-P_{t}}{P_{t}}}_{\text {Capital Gain }}
$$

- Return will differ from current yield (approximation to YTM) if bond prices fluctuates in unexpected ways


## Discount Bond

- Suppose that you hold a discount bond with face value $\$ 1000$, a maturity of 30 years, and a current yield to maturity of 10 percent
- The current price of this bond is $\frac{1000}{1.1^{30}}=57.31$.
- Suppose that interest rates stay the same after a year. Then the bond has a price of $\frac{1000}{1.1^{29}}=63.04$
- Since there is no coupon payment, your one year holding period return (holding period refers to length of time you hold the security) is just the capital gain:

$$
R=\frac{P_{t+1}-P_{t}}{P_{t}}=\frac{63.04-57.31}{57.31}=0.10
$$

- If interest rates do not change, then the return and the yield to maturity are the same thing


## Interest Rate Risk

- Continue with the same setup
- But now suppose that interest rates go up to 15 percent in period $t+1$ and are expected to remain there
- Then the price of the bond in period $t+1$ will be: $\frac{1000}{1.15^{29}}=17.37$
- Your return is then:

$$
R=\frac{P_{t+1}-P_{t}}{P_{t}}=\frac{17.37-57.31}{57.31}=-0.69
$$

- On a discount bond, an increase in interest rates exposes you to large capital loss


## Return and Time to Maturity, Coupon Bond



## Observations

- Return and initial YTM are equal if the holding period is the same as time to maturity ( 1 period). The capital gain is simply the face value (which is fixed) minus the initial price
- Increase in interest rates results in returns being less than initial yield
- Reverse is true
- Return is more affected by interest rate change the longer is the time to maturity
- If you hold the bond until maturity, your return is locked in at initial YTM
- Concept of return is relevant even if you do not sell the bond and realize the capital loss. There is an opportunity cost - if interest rates rise, had you not locked yourself in on a long maturity bond you could have purchased a bond in the future with a higher yield
- Longer maturity bonds are therefore riskier than short maturity bonds


## Determinants of Bond Prices (and interest rates)

- What determines bond prices and interest rates?
- Supply and demand!
- Though there are many different kinds of bonds and many different kinds of issuers of bonds, think about a world in which there is just one type of bond (and just one type of interest rate)
- For simplicity, think of this as a discount bound
- Remember: bond prices and interest rates move opposite one another


## Portfolio Choice

- Our theory of demand of a bond (or any asset) is that demand is based on the following factors:

1. Wealth: assets are normal goods, so the more wealth, the more you want to hold at every price
2. Expected returns: you hold assets to earn returns. The higher the expected return, the more of it you demand
3. Risk: assume agents are risk averse. Holding expected return constant, you would prefer a less risky return. The more risky an asset is, the less of it you demand
4. Liquidity: refers to the ease with which you can sell an asset. The more liquid it is, the more attractive it is to hold (it's easier to sell if you need to raise cash in a pinch)

## Demand for Bonds

- How does the demand for bonds vary with the price of bonds?
- As the price goes down, the interest rate goes up
- Therefore, holding everything else fixed, the expected return on holding a bond goes up as the price falls
- Therefore, demand slopes down
- Demand will shift (change in quantity demanded for a given price) with changes in other factors


## Bond Demand



## Supply of Bonds

- Issuers of bonds are borrowers. You are issuing a bond to raise funds in the present to be paid back in the future
- Recall that bond prices move opposite interest rates
- As the bond price increases, the yield decreases
- Therefore, at a higher bond price, the cost of borrowing is lower
- So there will be more supply of bonds at a higher price supply slopes up
- Changes in other factors, holding price fixed, will cause the supply curve to shift


## Bond Supply



## Bond Market Equilibrium



## Equilibrium

- The intersection of supply and demand determines the equilibrium price and quantity of bonds
- By determining price, the equilibrium effectively determines the interest rate, which is inversely related to price
- An alternative way to think about equilibrium is the market for loanable funds
- The loanable funds diagram puts the interest rate (rather than bond price) on the vertical axis, and essentially swaps who demands and who supplies:
- In the previous setup, savers demand bonds whereas borrowers supply bonds
- In the loanable funds setup, savers supply funds whereas borrowers demand funds
- We call the supply curve the supply of savings, and the demand curve the demand for investment. In equilibrium, we must have $S=I$


## Loanable Funds Diagram



## An Increase in Risk



## An Increase in Liquidity



## An Increase in Expected Future Interest Rates



## An Increase in Government Budget Deficit



## Different Bond Characteristics

- Bonds with the same cash flow details (e.g. discount bonds vs. coupon bonds vs. perpetuities) nevertheless often have very different yields
- Why is this?
- Aside from details about cash flows, bonds differ principally on two dimensions:

1. Default risk
2. Time to maturity

## Default Risk

- Default occurs when the borrower decides to not make good on a promise to pay back all or some of his/her outstanding debts
- We think of federal government bonds as being (essentially) default-free: since government can always "print" money, should not explicitly default (though monetization of debt is effectively form of default)
- Corporate (and state and local government) bonds do have some default risk
- Rating agencies: Aaa is highest rated, then Bs, then Cs measure credit risk of lenders
- Risk premium: difference (i.e. spread) between yield on relatively more risky debt (e.g. Aaa corporate debt) and less risk debt (e.g. government debt), assuming same time to maturity


## Yields on Different Bonds

Annual
Yield (\%)


## Observations

- We see more or less exactly the pattern we would expect from our demand/supply analysis - risker bonds have higher yields
- Obvious exception: municipal bonds
- Why? Interest income on these bonds is exempt from federal taxes, which makes their expected returns higher, and therefore drives up price (and drives down yield)
- Important: interesting time variation in spreads (see next couple of slides)


FRED


## Countercyclical Default Risk

- It stands to reason that default risk ought to be high when economy is in recession
- When default risk is high, we might expect a "flight to safety": reduces demand for risky bonds and increases demand for riskless bonds, which moves credit spread up
- This is consistent with what we see in the previous slides: credit spreads tend to rise during recessions


## Flight to Safety

Corporate Bonds
Government Bonds



## Yields During Great Recession

## Yields and the Great Recession



## Term Structure of Interest Rates

- Bonds with otherwise identical cash flows and risk characteristics can have different times to maturity (or just maturities, for short)
- How do yields vary with time to maturity for a bond with otherwise identical characteristics?
- A plot of yields on bonds against the time to maturity is known as a yield curve

Interest Rate
(\% annual rate)



## Observations

- The following observations can be made from previous two pictures

1. Yields on bonds of different maturities tend to move together
2. Yield curves are upward-sloping most of the time

- The slope of the yield curve is often predictive of recession. Flat or downward-sloping ("inverted") yield curves often precede recessions

3. When short term interest rates are low, yield curves are more likely to be upward-sloping

## Yield Curves Prior to Recent Recessions



## Theories of the Yield Curve

- We would like to understand the term structure of interest rates:

1. Expectations hypothesis
2. Segmented markets
3. Liquidity premium theory

- The liquidity premium theory essentially combines (1) and (2)


## Expectations Hypothesis

- Expectations hypothesis: the yield on a long maturity bond is the average of the expected yields on shorter maturity bonds
- For example, suppose you consider 1 and 2 year bonds
- The yield on a 1 year bond is 4 percent; you expect the yield on a 1 year bond one year from now to be 6 percent
- Then the yield on a two year bond ought to be 5 percent $(0.5 \times(4+6)=5)$
- Why? If you buy a two 1 year bonds in succession, your yield over the two year period is (approximately, ignoring compounding) 10 percent $-(1+0.04) \times(1+0.06)-1=0.1024 \approx 0.10$
- If the 2 and 1 year bonds are perfect substitutes, the yield on the two year bond has to be the same: $(1+i)^{2}=0.1 \Rightarrow i \approx 0.05$
- Demand and supply: if you expect future short yields to rise, this lowers expected profitability of long term bonds in the present, reducing demand, driving down price, and driving up yield: current long term yield tells you something about expected future short term yields


## Simple Theory Behind the Expectations Hypothesis

- A household lives for three periods: $t, t+1$, and $t+2$. Consumes ( $C$ ) and earns income ( $Y$ ). Lifetime utility:

$$
U=\ln C_{t}+\beta \ln C_{t+1}+\beta^{2} \ln C_{t+2}
$$

- In period $t$, can save in either a one period bond, $B_{1, t}$, or a two period bond, $B_{2, t}$. Normalize prices in $t$ to 1 , with yields (interest rates) of $i_{1, t}$ and $i_{2, t}$ (could equivalently make these discount bonds with future prices of 1 and current prices less than 1 with yields implicit rather than explicit)
- In period $t+1$, can save in a one period discount bond, $B_{1, t+1}$, with yield of $i_{1, t+1}$. Sequence of budget constraints:

$$
\begin{aligned}
C_{t}+B_{1, t}+B_{2, t} & =Y_{t} \\
C_{t+1}+B_{1, t+1} & =Y_{t+1}+\left(1+i_{1, t}\right) B_{1, t} \\
C_{t+2} & =Y_{t+2}+\left(1+i_{1, t+1}\right) B_{1, t+1}+\left(1+i_{2, t}\right)^{2} B_{2, t}
\end{aligned}
$$

## Optimality Conditions

- The optimality conditions can be written as a sequence of Euler equations:

$$
\begin{aligned}
\frac{1}{C_{t}} & =\beta \frac{1}{C_{t+1}}\left(1+i_{1, t}\right) \\
\frac{1}{C_{t+1}} & =\beta \frac{1}{C_{t+2}}\left(1+i_{1, t+1}\right) \\
\frac{1}{C_{t}} & =\beta^{2} \frac{1}{C_{t+2}}\left(1+i_{2, t}\right)^{2}
\end{aligned}
$$

- If you combine these, you get:

$$
\left(1+i_{1, t}\right)\left(1+i_{1, t+1}\right)=\left(1+i_{2, t}\right)^{2}
$$

- Which is approximately:

$$
i_{2, t} \approx \frac{1}{2}\left[i_{1, t}+i_{1, t+1}\right]
$$

## More Generally

- For a bond with a maturity of $n$ periods, the expectations theory tells us:

$$
i_{n, t}=\frac{i_{t}+i_{t+1}^{e}+\cdots+i_{t+n-1}^{e}}{n}
$$

- Yields after period $t$ have e superscripts to denote that they are expectations
- Observing $i_{n, t}$ and $i_{t}$ in period $t$, this relationship can be used to infer market expectations of future short term interest rates ("forward rates"). In particular:

$$
\begin{aligned}
& i_{t+1}^{e}=2 i_{2, t}-i_{t} \\
& i_{t+2}^{e}=3 i_{3, t}-2 i_{2, t} \\
& \quad \vdots \\
& i_{t+h}^{e}=(h+1) i_{h+1, t}-h i_{h, t}
\end{aligned}
$$

## Implied 1 Year Ahead Forward Rate



56 / 68

## Implied 2 Year Ahead Forward Rate



## Evaluating the Expectations Hypothesis

- Expectations hypothesis can help make sense of several facts:

1. Long and short term rates tend to move together. If current short term rates are low and people expect them to stay low for a while (interest rates are quite persistent), then long term yields will also be low
2. Why a flattening/inversion of yield curve can predict recessions. If people expect economy to go into recession, they will expect the Fed to lower short term interest rates in the future. If short term interest rates are expected to fall, then long term rates will fall relative to current short term rates, and the yield curve will flatten.

- Where the expectations hypothesis fails: why are yield curves almost always upward-sloping? If interest rates are mean-reverting, then the average yield curve ought to be flat, not upward-sloping


## Segmented Markets Hypothesis

- The Expectations Hypothesis assumes that bonds of different maturities are perfect substitutes, which forces expected returns to be equal across bonds of differing maturities
- Segmented Markets is the polar extreme: bonds of different maturities aren't substitutes at all
- If this is the case, there is no reason for returns to be equalized
- Furthermore, since longer maturity bonds are subject to more interest rate risk, there will in general be more demand for short term bonds than long term bonds
- This means that the price of short term bonds will be high relative to long term bonds, meaning that long term bonds will have higher yields
- Hence, segmented markets can explain why yield curves slope up most of the time


## Liquidity Premium Theory

- Segmented markets hypothesis cannot explain why interest rates of different maturities tend to move together, or account for why you can predict future short term rates from long term yields
- Liquidity premium theory: essentially combines expectations hypothesis with market segmentation
- Bonds of different maturities are substitutes, but not perfect substitutes: savers demand higher yields on longer term bonds because of their heightened risk
- Relationship between long and short rates:

$$
i_{n, t}=\underbrace{\frac{i_{t}+i_{t+1}^{e}+\ldots i_{t+n-1}^{e}}{n}}_{\text {Expectations Hypothesis }}+\underbrace{I_{n, t}}_{\text {Liquidity Premium }}
$$

- The liquidity premium, $I_{n, t}$, is increasing in the time to maturity



## A First Look at Quantitative Easing and Unconventional Monetary Policy

- The interest rates relevant for most investment and consumption decisions are long term (e.g. mortgage) and risky (e.g. Baa corporate bond)
- Conventional monetary policy targets short term and riskless interest rates (e.g. Fed Funds Rate)
- Out theory of of bond pricing helps us understand the connection between the these different kinds of interest rates


## Monetary Policy in Normal Times

- Think of a world where the liquidity premium theory holds (with a fixed liquidity premium)
- A central bank lowers (raises) short term interest rates and is expected to keep these low (high) for some time
- This ought to also lower (raise) longer term rates to the extent to which longer term rates are the average of expected shorter term rates
- Holding risk factors constant, substitutability between bonds means that riskier yields also ought to fall (increase)
- Simulation:
- Consider 1 period, 10 year, 20 year, and 30 year bonds
- Fixed liquidity premia of 1,2 , and 3
- Short term rate is cut from 4 to 3 persistently
- Simulate yields for 30 quarters (one-fourth of a year)


## Theoretical Simulation



## Monetary Loosening and Tightening in Historical Episodes



## Monetary Loosening and Tightening in Historical Episodes



## Term Structure Puzzle?

- If you look at last two pictures, in the 1990s other rates tracked the FFR reasonably closely, but this falls apart in the 2000s
- This has been referred to as the term structure puzzle and is discussed in Poole (2005)
- From perspective of expectations hypothesis, would expect longer maturity, riskier rates to move along with shorter maturity rates. Not what we see in 2000s
- But two key issues:
- Forecastability: if people expected the Fed to raise rates starting in 2004/2005 from the perspective of 2002/2003, this would have been incorporated into long rates before the FFR started to move. Most people think Fed policy has become more forecastable
- Persistence: behavior of long rates depends on how persistent changes in short rates are expected to


## Unconventional Monetary Policy

- In a world where Fed Funds Rate is at or very near zero, conventional monetary loosening isn't on the table
- Unconventional monetary policy:

1. Quantitative Easing (or Large Scale Asset Purchases): purchases of longer maturity government debt or risky private sector debt. Idea: raise demand for this debt, raise price, and lower yield. See here
2. Forward Guidance: promises to keep future short term interest rates low. Idea is to work through expectations hypothesis and to lower long term yields immediately. See here

- Ben Bernanke: "The problem with quantitative easing is that it works in practice but not in theory"
- Under expectations hypothesis, cannot affect long term yields without impacting path of short term yields
- Must either impact liquidity premium or be a form of forward guidance

