Mathematics Paper 2
Grade 12
Preliminary Examination
2017

DURATION:	**180 min**	**EXAMINER:**	**R. Obermeyer**
MARKS:	**150**	**MODERATOR:**	**A. Janisch**
Date:	**15 September 2017**	**External Moderator: I. Atteridge**	

INSTRUCTIONS:

▪ **See overleaf for Instructions.**
▪ **This paper consists of 25 pages (including cover) and an information sheet.**

NAME: _____

ASSESSMENT					
Question	**Level Tested**	**Topic**	**Time Allocation**	**Possible mark**	**Actual mark**
SECTION A					
1	1 – 4	Analytical Geometry	22 mins	**18**	
2	1 – 4	Trigonometry Graphs	10 mins	**8**	
3	1 – 4	Trigonometry	28 mins	**23**	
4	1 – 4	Euclidean Geometry	16 mins	**13**	
5	1 – 4	Euclidean Geometry	11 mins	**9**	
6	1 – 4	Statistics	16 mins	**13**	
SECTION B					
7	1 – 4	Analytical Geometry	26 mins	**22**	
8	1 – 4	Statistics	12 mins	**10**	
9	1 – 4	Trigonometry	10 mins	**8**	
10	1 – 4	Measurement	6 mins	**5**	
11	1 – 4	Euclidean Geometry	19 mins	**16**	
12	1 – 4	Euclidean Geometry	6 mins	**5**	
			TOTAL:	**150**	
			PERCENTAGE:		

Teacher's Signature: _____

Controller's Signature: _____

Moderator's Signature: _____

<u>Instructions</u>

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 25 pages (including the cover page) and an Information Sheet of 2 pages. Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to *<u>one</u>* decimal digit where necessary, unless otherwise stated.
8. Ensure that your calculator is in **DEGREE** mode.
9. It is in your own interest to write legibly and to present your work neatly.

SECTION A

Question 1

In the figure below, $PQRS$ is a quadrilateral with $P(4; 10)$; $Q(1; 1)$; $R(4; 0)$ and $S(8; 2)$.
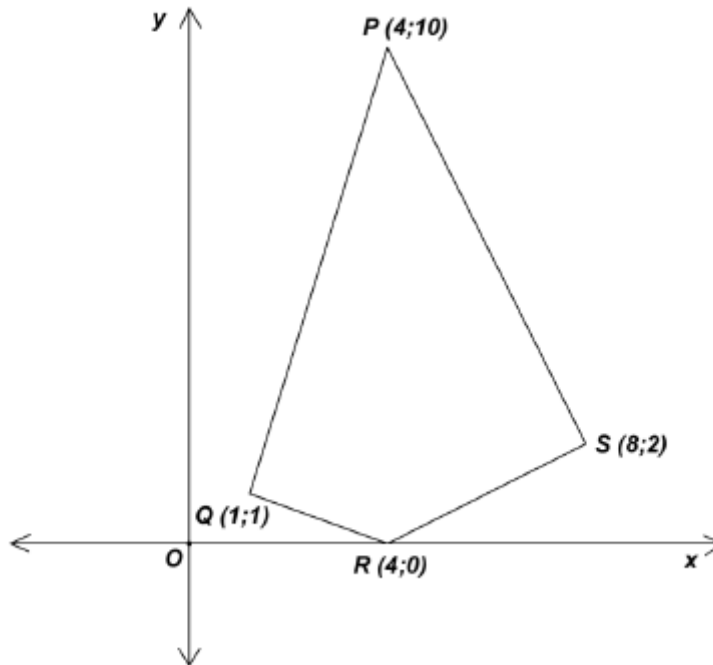
a. Determine the gradient of QR and RS. (4)

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b. Determine the length of PS in simplest surd form. (2)

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c. Show that $PQ \perp QR$. (2)

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d. A is a point in the first quadrant. Determine the coordinates of A to make $PARQ$ a rectangle. (2)

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e. Prove that $PQRS$ is a cyclic quadrilateral. (3)

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f. Determine the equation of the circle passing through P; Q; R and S. (5)

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[18]

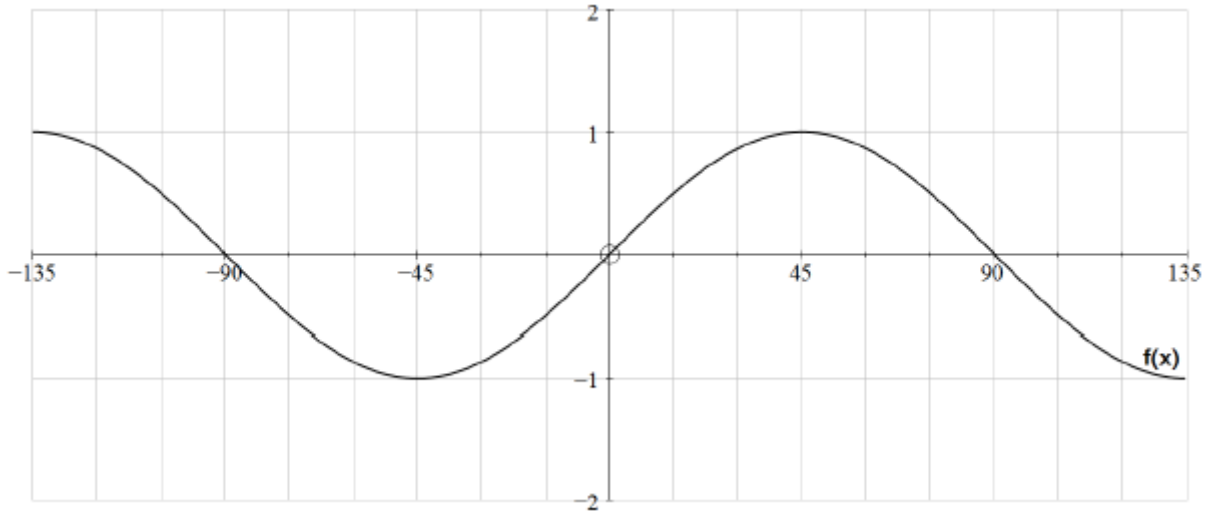## Question 2

Given: $f(x) = \sin 2x$ for $x \in [-135°; 135°]$

a. Sketch $g(x) = -\tan(x + 45°)$ for the given domain on the same axis as $f(x)$. (4)

b. Find the value of $f(90°) - g(0°)$. (2)

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c. If the equation $y = a\sin(x + p)$ represents the graph of $f(x)$ reflected over the $x-$axis and shifted $60°$ to the right, find the values of a and p. (2)

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[8]

Question 3

a. Simplify the following without the use of a calculator:
    $\cos 25° . \sin 55° - \cos 65° . \sin 35°$ (5)

b. Prove that: $\dfrac{1-\cos 2x}{\sin 2x} = \tan x$ (4)

c. Determine the general solution of: $\sin x . \cos x = \dfrac{1}{3}$ (5)
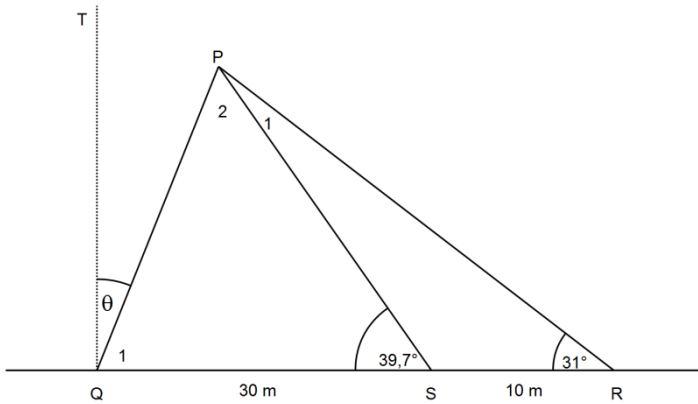
d. In the photograph below, a bridge is supported by a tower which is not perpendicular to the ground.

In the diagram, the tower PQ is shown and stays (steel ropes), PR and PS, help keep the tower stable. (There are also other cables). From base Q of the tower a vertical line QT is shown. $QS = 30m$, $SR = 10m$, $\hat{R} = 31°$ and $\hat{S} = 39,7°$.

1. Why is $\hat{P}_1 = 8,7°$? (1)

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2. Show that $PS = 34,05\ m$. (2)

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3. Determine the length of the tower PQ correct to one decimal place. (2)

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4.   Now find the angle marked θ, the inclination of the tower from the vertical.   (4)

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[23]

All statements must have reasons in Question 4 – 5

Question 4

a. Use the diagram below to prove the theorem that states:
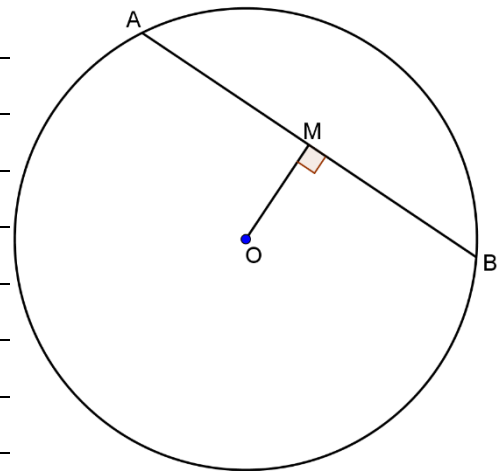   The line from the centre of a circle perpendicular to a chord bisects the chord.   (6)
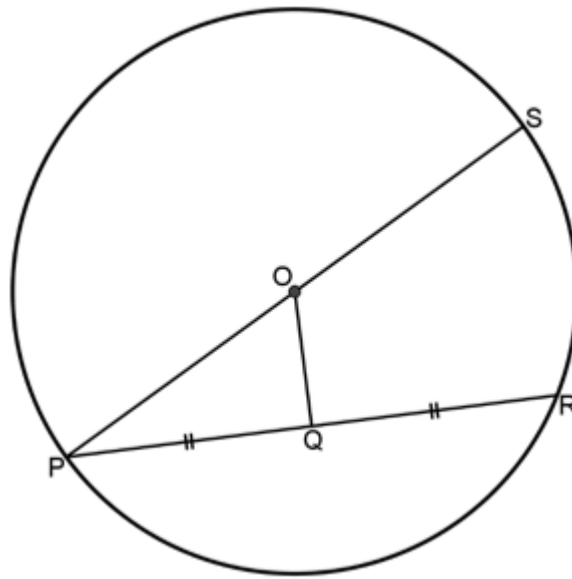
b. In the diagram below, O is the centre of the circle, Q is the mid-point of PR. $OQ = 20$mm and $PR = 40$mm

1. Determine, stating reasons, the length of SR. (3)

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2. Hence, or otherwise, determine the length of the diameter PS. (4)

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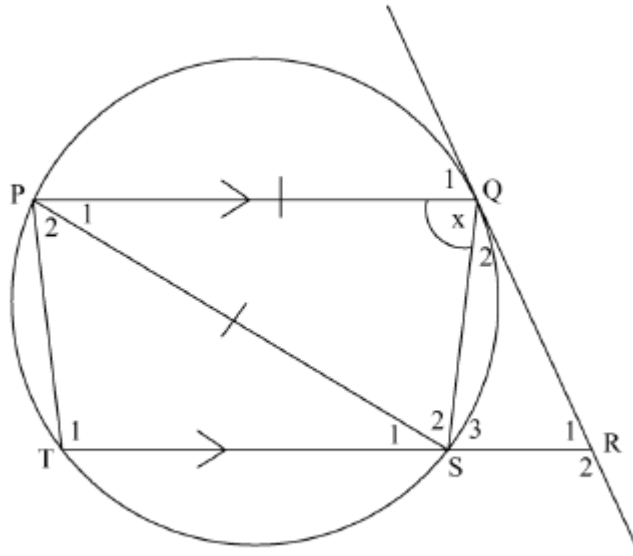**[13]**

Question 5

$PQST$ is a cyclic quadrilateral. QR is a tangent to the circle at Q. TR is a straight line. $PQ//TR$, $PQ = PS$ and $P\hat{Q}S = x$.

a. Complete the given table: (5)

STATEMENT	REASON
$\hat{S}_2 = x$	
... $= x$	Alternate $<'$ s; $PQ//TR$
$\hat{P}_{1+2} = x$	
$\hat{Q}_1 = x$	
... $= x$	Corresponding $<'$ s; $PQ//TR$

b.   Prove that $PQRT$ is a parallelogram.   (4)

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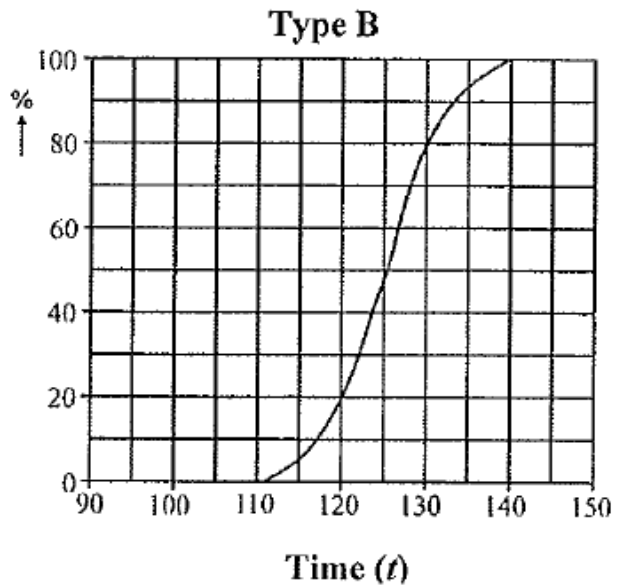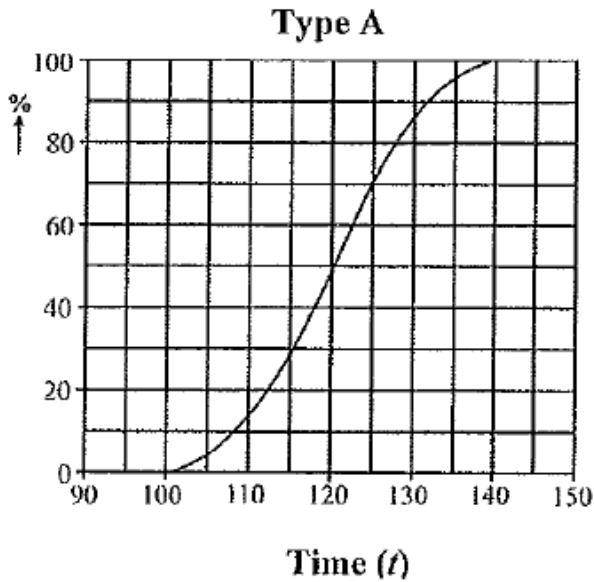
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Question 6

a. The battery life of two different makes of laptops are compared. The following cumulative frequency curves about the battery lifespan of two types of laptops are given below. The graphs indicate the percentage of batteries that die after t minutes of usage.

Type A

Type B

1. Give the median battery lifespan of each type of laptop.   (2)

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2. Which of the two types has a range which is more than 30? Show some working out.   (1)

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3. Which of the two types has an interquartile range which is less than 10? Show your working out.   (1)

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4. Using the information above, give TWO reasons why Type B laptop should be chosen over Type A laptop.   (2)

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b. A large company employs 7 salespersons. The commission that each salesperson earned (in rands) in a certain month is shown below.

3900 5700 10600 13600 15100 15800 17100

1. Calculate the mean of the data. (1)

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2. Calculate the standard deviation of the data. (2)

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3. The company rates the staff according to the amount of commission earned. A salesperson whose commission is more than one standards deviation above the mean receives a rating of "good". How many salespersons will receive a rating of "good" for that month? Substantiate your answer. (4)

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[13]

SECTION B

Question 7

The circle with centre A and equation $(x - 2)^2 + (y + 2)^2 = 4$ is given.

    a.   Write down the co-ordinates of the centre of the circle, A, and the radius of the circle. (3)

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    b.   Determine the equation of the tangent to circle A at the point $T\left(\frac{2}{5}; -\frac{4}{5}\right)$. (5)

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c. A second circle with centre B has the equation $x^2 + y^2 + 4x - 2y + k = 0$.
Determine the value of k for which the two circles with centres A and B will touch
each other externally at one point.   (8)

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d.   Assume that the radius of circle B is 3 units and circle B touches circle A externally. Circle A and B have a common tangent that touches the circles at D and C respectively. Determine the area of the trapezium $ABCD$ made by this common tangent.   (6)

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Question 8

The scores (out of 50) obtained for a project by the learners of a Grade 12 Mathematics class are shown in the frequency polygon below.

Frequency polygon for project scores
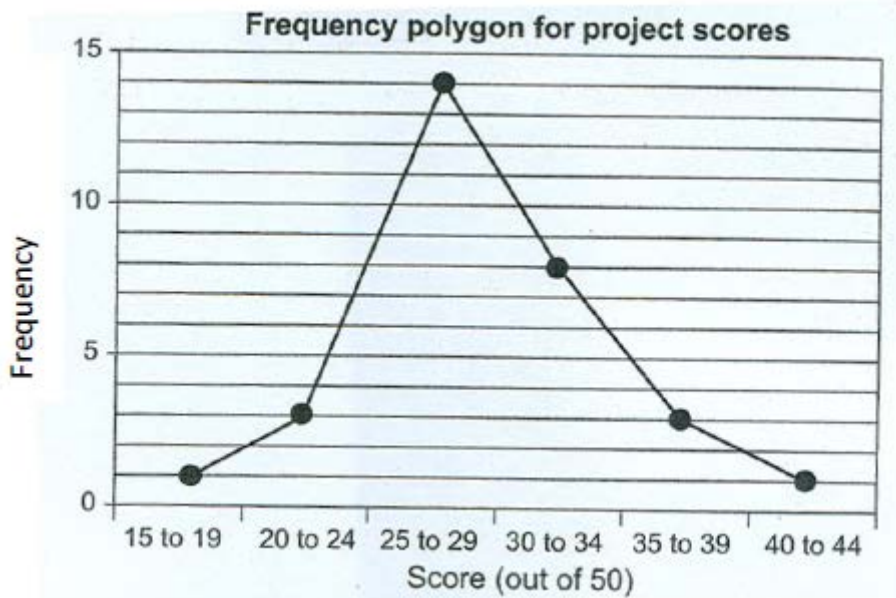
a. How many learners are there in the class? (1)

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b. Describe the distribution: normal, positively skewed or negatively skewed. (1)

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c. Use the graph paper to sketch the ogive for this information. Indicate clearly where the first quartile, median and third quartile can be read off. (8)

SCORE	CUMULATIVE FREQUENCY
$15 < x \leq 20$	
$20 < x \leq 25$	
$25 < x \leq 30$	
$30 < x \leq 35$	
$35 < x \leq 40$	
$40 < x \leq 45$	

[10]

Question 9

a. If $\tan 15° = \frac{a}{b}$ and $a^2 + b^2 = c^2$, prove without a calculator, that $\frac{2ab}{c^2} = \frac{1}{2}$. (4)

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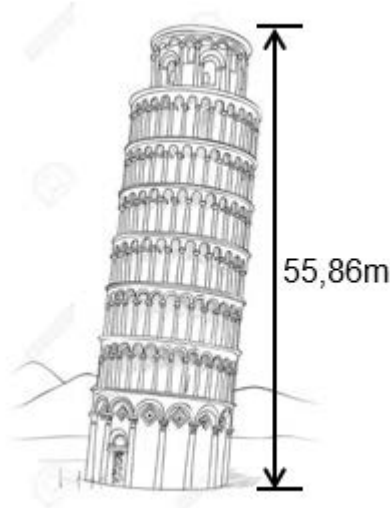
b. The leaning tower of Pisa currently "leans" at an angle of $4°$ and has a perpendicular
    height of $55{,}8$ m. Determine how tall the tower was when it was originally built. (4)

55,86m

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Question 10

A cylindrical peg with radius 4 units fits snugly into a box, the base of which is a right-angled triangle. If $KP = x; K\hat{J}L = 90°$ and the hypotenuse is 24 units, determine with reasons, the perimeter of ΔJKL. (5)

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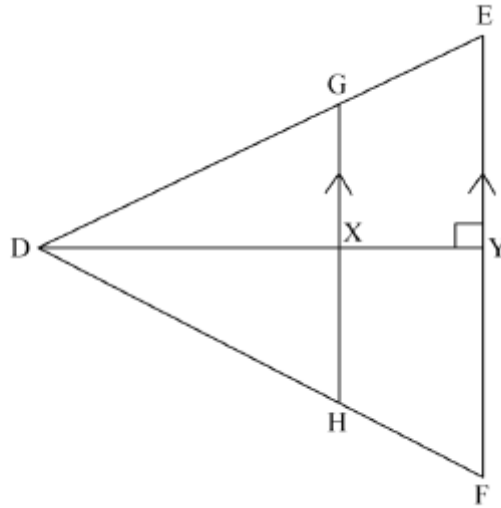_____
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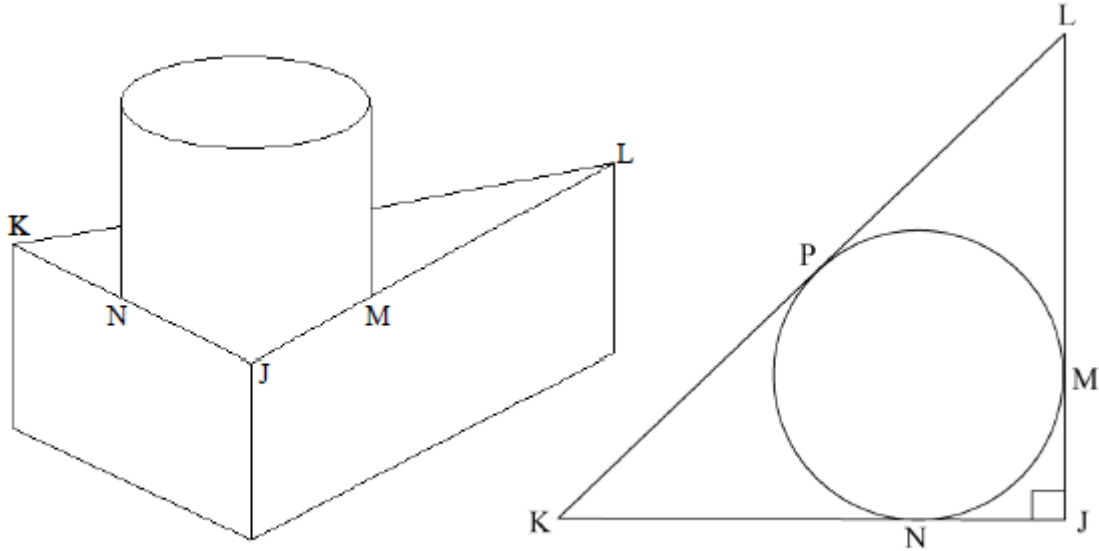
[5]

Reasons to be given unless otherwise stated in Question 11 – 12

Question 11

In the figure, GH is drawn parallel to EF. DY is perpendicular to EF and cuts GH at X.

  a. Prove:

     1. $\Delta DGH \;///\; \Delta DEF$ (3)

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2. $\dfrac{DX}{DY} = \dfrac{GH}{EF}$ (4)

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b. If the area of $\Delta\,GHD$ is equal to the area of quadrilateral $GHFE$:
   1. Express the area of $\Delta\,DEF$ in terms of $\Delta\,GHD$ and $GHFE$. (1)

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   2. Hence, or otherwise, prove that $\dfrac{1}{2}EF.DY = GH.DX$ (4)

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   3. Prove that $\dfrac{DG}{DE} = \dfrac{1}{\sqrt{2}}.$ (4)
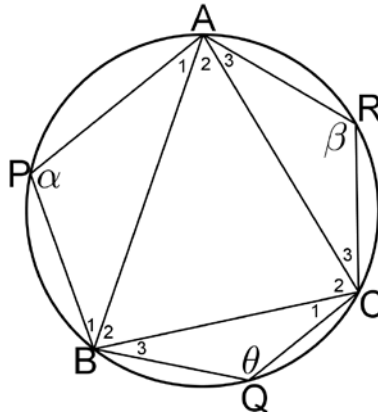
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[16]

Question 12

a. In the diagram below A, R, C, Q, B and P are points on the circumference of the circle. Angles α, β and θ are shown.

Prove that: $\alpha + \beta + \theta = 360°$ (5)

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Additional Working Pages