Physics I

March 30-April 3

Time Allotment: 40 minutes per day

Student Name: ___________________________________

Teacher Name: ___________________________________

Physics I
March 30-April 3

Packet Overview

Date	Objective(s)	Page Number
Monday, March 23	1. Define constant angular acceleration. 2. Memorize the kinematic equations of rotational motion.	2-4
Tuesday, March 24	1. Apply concepts of angular velocity and acceleration to a rotating bicycle wheel – a case where there is both linear and rotational motion. 2. Find angular quantities of an accelerating bicycle wheel.	5-7
Wednesday, March 25	1. Define torque and lever arm. 2. Explain why a force applied to longer lever arm gives a greater effect.	8-10
Thursday, March 26	1. Demonstrate mastery of angular kinematics on your quiz. 2. Draw diagrams showing applied force, a line of action, and a lever arm.	10-13
Friday, March 27	1. Derive the equation for torque. 2. Explain why we use the sine function in the torque equation.	14-16

Additional Notes: The guided worksheets in this packet will follow the textbook readings from Giancoli found at the end of the packet. The final page of this packet will contain an answer key for all Problems and answers to quiz questions.

Khan Academy is a great online resource for physics, though this packet does not require access to the Internet. The Physics videos can help with rotational motion concepts, while the algebra and geometry videos can help with the concept of radians.

Another great resource is a YouTube channel called "Doc Schuster". Dr. Schuster is a high school physics teacher in St. Louis who makes great video lectures with magic markers and paper. His playlist "AP Ch 10 – Rotational Motion and Energy" will cover most of we will in these packets.

Academic Honesty

I certify that I completed this assignment independently in accordance with the GHNO Academy Honor Code.

Student signature:

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I certify that my student completed this assignment independently in accordance with the GHNO Academy Honor Code.

Parent signature:

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Physics I
March 30-April 3

Monday, March 30

Physics Unit: Rotational Motion
Lesson 1: Constant Angular Acceleration
Requirements: Read p. 201 in the textbook provided in the back of the packet and complete the worksheet below.

Unit Overview: Rotational Motion
In this new unit, we will be taking what we have already learned about linear velocity, acceleration, and momentum, and apply them to rotational cases. This will be different from our Chapter 5 unit on circular motion, because as you remember, objects in that chapter orbited in circles (think about the tennis ball on the string and the Moon orbiting around the Earth). In this chapter, we will be concerned with the rotation of the bodies themselves. These rotating bodies can be anything from a penny spinning on its side, you and your friends riding a Merry-Go-Round, a planet making its daily rotation, or an electron spinning. You should be getting excited! Towards the end of this chapter, we will get to see how the fundamentals of rotational motion we will learn leads to one of the most stunning demonstrations in all of mechanics. Stay tuned.

Lesson 1 Objective: Be able to do this by the end of this lesson.
1. Define constant angular acceleration.
2. Memorize the kinematic equations of rotational motion.

Introduction to Lesson 1
The reading for Lesson 1 will be **p.201** in the Giancoli text provided in this packet. Read these pages carefully, and then fill out the worksheet below.

Questions to ponder:
What is motion? Do we need to broaden our definition of motion to include rotating bodies? Think about this: Newton's First Law tells us a body in motion stays in motion unless acted upon by an outside force. What if I'm spinning a tennis ball on a frictionless table but not rolling the ball across the table? Is it moving? Will it take an outside force to stop the ball rotating? Do we then need to include rotation into Newton's Laws of Motion?

1. Review from last week. Remember these quantities? Section 8-1 from last week's packet will be reprinted for you so you can look back and write down what each of them means.

θ -

ω –

α -

2. Make a table of equations like the one on the top of p. 201. Write down the linear equations of motion first in the right-hand column. Remember them from Quarter 1? Now write down the angular equations found in the left-hand column.

2. Look at the two columns you made. What do the two sets of equations have in common? Which variables are substituted when we move from the linear equations to the angular ones?

3. Copy the two columns 3 more times in the space below. You will have to memorize these six equations and write them down on a quiz later this week.

4. Copy question for Example 8-6. Then work all the steps below and box your answers.

Finally, do Problems 15-17 on p. 219. Remember as always: make a list of your knowns and unknowns, and draw and label a diagram before you do anything else. Then write down the equations you need to solve each part, solve for the unknown variable algebraically, and finally plug in numbers and box your final answer. Have a great Monday!

15.

16.

17.

Tuesday, March 31

Physics Unit: Rotational Motion
Lesson 2: Rolling Motion (without slipping)
Requirements: Read p. 202 carefully.

Objective: Be able to do this by the end of this lesson.
1. Apply concepts of angular velocity and acceleration to a rotating bicycle wheel – a case where there is both linear and rotational motion.
2. Find angular quantities of an accelerating bicycle wheel.

Introduction to Lesson 2
Yesterday, we compared angular and linear kinematic equations of motion. Then we solved some problems requiring us to use the angular equations of motion. Today, we're going to look at a case where we have both rotational and linear motion. Can you think of such a case? The one that comes to mind most apparently is a wheel on a car or bicycle wheel. Think about how your bicycle wheel rotates and moves linearly down your driveway as you ride it. If you have a bike in your garage, go take a look at it! Push it across the floor and watch the tire rotate and the axis of rotation translate across the surface you're pushing it on. We'll go through Section 8-3 on p. 202 carefully and then you'll be able to work an example problem at the end of the section.

Dr. Schuster has a great video explaining rolling without slipping. While it's not required to watch the video, it could be helpful for your understanding: https://www.youtube.com/watch?v=ZOynwQ75pms

Before we begin, in the space below, write the 4 linear and 4 rotational kinematic equations of motion three times.

1. Rolling without slipping involves both ____________________________ and

________________________.

2. Draw and label Figures 8-8(a) and 8-8(b) below. Describe what is different about the two cases. Draw a velocity vector at the top of the wheel on Figure 8-8(b).

3. What equation can we use to relate linear velocity to angular velocity if we wanted to calculate the angular velocity of the rotating tire in Figure 8-8(b)?

In the space below, write the question, draw and label a diagram, and work all the steps for Example 8-7 on p. 202-203.

Do Problem 18 on p. 219.
18)

Wednesday, April 1

Physics Unit: Rotational Motion
Lesson 3: Torque
Requirements: Read p. 203, fill in the worksheet below, and do Questions 4-5 on p. 217-218.

Objective: Be able to do this by the end of this lesson.
1. Define torque and lever arm.
2. Explain why a force applied to longer lever arm gives a greater effect.

Introduction to Lesson 3
Today we are going to begin our studies of torque. As you read p. 203, be asking yourself how torque relates to the rotational kinematic equations of motion.

Speaking of those equations, in the space below, write the 8 rotational and linear kinematic equations at least one time. Try to do it from memory. You'll have a quiz on them tomorrow.

8-4 Torque (p. 203)

1. If rotational kinematics describe rotational motion in terms of angular displacement, angular velocity, and angular acceleration, what do rotational dynamics describe?

2. What is required to make an object start rotating about an axis?

3. Draw and label Figure 8-10 in the space below. What is that figure an illustration of?

4. Explain the difference in effects when you apply F_A to the door and F_B to the door. Why is there a difference?

5. The angular acceleration of the door is proportional not only to ______________________________,

but also directly proportional to ____.

This distance is called ____________________________________.

6. Define lever arm in your own words -

7. Redraw Figure 8-10 and circle the two lever arm distances and state which force each lever arm corresponds to.

8. If a lever arm is 5 times as long as another lever arm, the corresponding angular acceleration will be how many times greater?

Turn to p. 204 in your packet.
9. The angular acceleration is proportional to the product of ____________________ times

_________________________________.

10. Define torque -

Lastly, do Questions 4-5 on p. 217-218. **Write in complete sentences for full credit**.

4)

5)

Thursday, April 2

Physics Unit: Rotational Motion
Lesson 4: Torque
Requirements: Take Quiz, read p. 204, and fill in worksheet below.

Objectives: Be able to do this by the end of this lesson.
1. Demonstrate mastery of angular kinematics on your quiz.
2. Draw diagrams showing applied force, a line of action, and a lever arm.

Introduction to Lesson 4: Today we're going to learn more about torque. But first, let's take a quiz on everything we've learned so far this week. Turn the page when you're ready to start the quiz.

Physics I – Quiz on 8-1, 8-2, and Torque Name: ___________________________

1) Write the linear kinematic equations in one column and the rotational kinematic equations in the table provided below.

Linear Kinematic Equations	Rotational Kinematic Equations
1.	1.
2.	2.
3.	3.
4.	4.

2) Define the following quantities

θ -

ω –

α -

3) In your own words and in at least one complete sentence, define torque. Draw and label a diagram to illustrate your point.

When you're finished, get out your red pen. Look back in your packet and textbook reading to correct your quiz. Email me with any questions.
Now turn to p. 204 in your textbook packet and let's learn more about torque!

1) Define once more torque –

2) Net applied torque is proportional to ___________________________. In the space below, write the proportionality relationship found at the top of p. 204.

3) Define once more lever arm –

4) Draw and label Figure 8-12(a). Why do you think Fc be less effective than F_A?

5) Draw and label Figure 8-12(b). What is a line of action? What is the distance r_c in Figure 8-12(b)?

6) The ___________________________ is perpendicular both to ________________________________

and to the ________________________________.

7) Label the lever arm in your Figure 8-12(b) and have a great rest of the day!

Friday, April 3

Physics Unit: Rotational Motion
Lesson 5: Torque
Requirements: Read p. 204-205 in Giancoli, and complete the worksheet below.

Objective: Be able to do this by the end of this lesson.
1. Derive the equation for torque.
2.

Introduction: Today we will finish our discussion of torque. This discussion will culminate in our deriving the torque equation: $\tau = r\,F\,\sin\theta$. You instincts are correct: you will have to memorize this equation.

A question to think about as you read is how is torque related to work? Do they have the same units? What is similar and different?

Read p. 204 and work through these questions as you read.

1) Once more, define torque –

2) In the space below, redraw Figure 8-12(b). The magnitude of torque associate with Fc is related to what two quantities multiplied together?

3) The short lever arm r_c is consistent with the observation that Fc is more/less (circle one) effective in accelerating the door than is F_A. What about F_A is different than Fc?

4) Draw F_A, its line of action, and its lever arm on your Figure 8-12(b) above in 2). Again, what do you notice is different about F_A and Fc?

5) Why is the torque associated with F_D zero?

6) Write Equation (8-10a). Explain what each variable means, including the perpendicular symbol.

7) Draw Figure 8-13(a). Next to the figure, write Equation 8-10(a) again. Draw arrows from each variable in the equation to the part of the figure the variable refers to.

8) Draw and label Figure 8-13(b).

9) What is an equivalent way of determining the torque associated with a force?

10) Why does the parallel component of force exert no torque?

11) Write Equation (8-10c). Why do we use the sin function in this equation? Why is it equivalent to Equation (8-10a)?

12) Write down the units for torque below, and have a great weekend!

You too can experience rapid rotation—if your stomach can take the high angular velocity and centripetal acceleration of some of the faster amusement park rides. If not, try the slower merry-go-round or Ferris wheel. Rotating carnival rides have rotational KE as well as angular momentum.

CHAPTER 8

Rotational Motion

Until now, we have been concerned mainly with translational motion. We discussed the kinematics and dynamics of translational motion (the role of force), and the energy and momentum associated with it. In this Chapter we will deal with rotational motion. We will discuss the kinematics of rotational motion and then its dynamics (involving torque), as well as rotational kinetic energy and angular momentum (the rotational analog of linear momentum). We will find many analogies with translational motion, which will make our study easier. Our understanding of the world around us will be increased significantly—from rotating bicycle wheels and compact disks to amusement park rides, a spinning skater, the rotating Earth, and a centrifuge—and there may be a few surprises.

We will consider mainly the rotation of rigid objects. A **rigid object** is an object with a definite shape that doesn't change, so that the particles composing it stay in fixed positions relative to one another. Any real object is capable of vibrating or deforming when a force is exerted on it. But these effects are often very small, so the concept of an ideal rigid object is very useful as a good approximation.

8–1 Angular Quantities

We saw in Chapter 7 (Section 7–8) that the motion of a rigid object can be analyzed as the translational motion of the object's center of mass, plus rotational motion *about* its center of mass. We have already discussed translational motion in detail, so now we focus on purely rotational motion. By *purely rotational motion*, we mean that all points in the object move in circles, such as the point P in the rotating wheel of Fig. 8–1, and that the centers of these circles all lie on a line called the **axis of rotation**. In Fig. 8–1 the axis of rotation is perpendicular to the page and passes through point O.

Every point in an object rotating about a fixed axis moves in a circle (shown dashed in Fig. 8–1 for point P) whose center is on the axis and whose radius is r, the distance of that point from the axis of rotation. A straight line drawn from the axis to any point sweeps out the same angle θ in the same time.

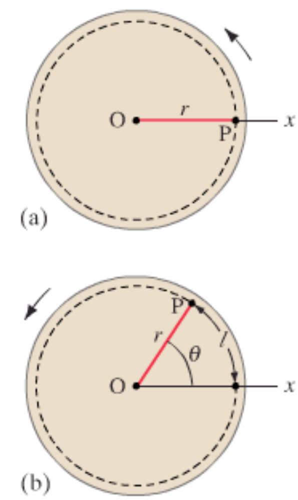


FIGURE 8–1 Looking at a wheel that is rotating counterclockwise about an axis through the wheel's center at O (axis perpendicular to the page). Each point, such as point P, moves in a circular path; l is the distance P travels as the wheel rotates through the angle θ.

To indicate the angular position of a rotating object, or how far it has rotated, we specify the angle θ of some particular line in the object (red in Fig. 8–1) with respect to a reference line, such as the x axis in Fig. 8–1. A point in the object, such as P in Fig. 8–1, moves through an angle θ when it travels the distance l measured along the circumference of its circular path. Angles are commonly measured in degrees, but the mathematics of circular motion is much simpler if we use the *radian* for angular measure. One **radian** (abbreviated rad) is defined as the angle subtended by an arc whose length is equal to the radius. For example, in Fig. 8–1b, point P is a distance r from the axis of rotation, and it has moved a distance l along the arc of a circle. The arc length l is said to "subtend" the angle θ. If $l = r$, then θ is exactly equal to 1 rad. In radians, any angle θ is given by

$$\theta = \frac{l}{r}, \tag{8–1a}$$

θ in radians

where r is the radius of the circle, and l is the arc length subtended by the angle specified in radians. If $l = r$, then $\theta = 1$ rad.

1 rad: arc length = radius

The radian is dimensionless since it is the ratio of two lengths. Nonetheless when giving an angle in radians, we always mention rad to remind us it is not degrees. It is often useful to rewrite Eq. 8–1a in terms of arc length l:

$$l = r\theta. \tag{8–1b}$$

Radians can be related to degrees in the following way. In a complete circle there are 360°, which must correspond to an arc length equal to the circumference of the circle, $l = 2\pi r$. Thus $\theta = l/r = 2\pi r/r = 2\pi$ rad in a complete circle, so

$$360° = 2\pi \text{ rad}.$$

Conversion, degrees to rad

One radian is therefore $360°/2\pi \approx 360°/6.28 \approx 57.3°$. An object that makes one complete revolution (rev) has rotated through 360°, or 2π radians:

1 rad ≈ 57.3°

$$1 \text{ rev} = 360° = 2\pi \text{ rad}.$$

EXAMPLE 8–1 **Bike wheel.** A bike wheel rotates 4.50 revolutions. How many radians has it rotated?

APPROACH All we need is a straightforward conversion of units using

$$1 \text{ revolution} = 360° = 2\pi \text{ rad} = 6.28 \text{ rad}.$$

SOLUTION

$$4.50 \text{ revolutions} = (4.50 \text{ rev})\left(2\pi \frac{\text{rad}}{\text{rev}}\right) = 9.00\pi \text{ rad} = 28.3 \text{ rad}.$$

EXAMPLE 8–2 **Birds of prey—in radians.** A particular bird's eye can just distinguish objects that subtend an angle no smaller than about 3×10^{-4} rad. (*a*) How many degrees is this? (*b*) How small an object can the bird just distinguish when flying at a height of 100 m (Fig. 8–2a)?

APPROACH For (*a*) we use the relation $360° = 2\pi$ rad. For (*b*) we use Eq. 8–1b, $l = r\theta$, to find the arc length.

SOLUTION (*a*) We convert 3×10^{-4} rad to degrees:

$$(3 \times 10^{-4}\,\text{rad})\left(\frac{360°}{2\pi\,\text{rad}}\right) = 0.017°.$$

(*b*) We use Eq. 8–1b, $l = r\theta$. For small angles, the arc length l and the chord length are approximately[†] the same (Fig. 8–2b). Since $r = 100$ m and $\theta = 3 \times 10^{-4}$ rad, we find

$$l = (100\,\text{m})(3 \times 10^{-4}\,\text{rad}) = 3 \times 10^{-2}\,\text{m} = 3\,\text{cm}.$$

A bird can distinguish a small mouse (about 3 cm long) from a height of 100 m. That is good eyesight.

NOTE Had the angle been given in degrees, we would first have had to convert it to radians to make this calculation. Equation 8–1 is valid *only* if the angle is specified in radians. Degrees (or revolutions) won't work.

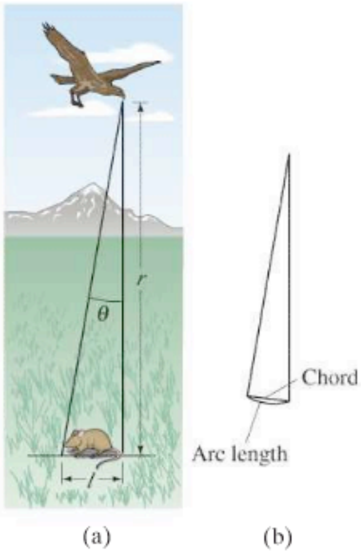


FIGURE 8–2 (a) Example 8–2. (b) For small angles, arc length and the chord length (straight line) are nearly equal.

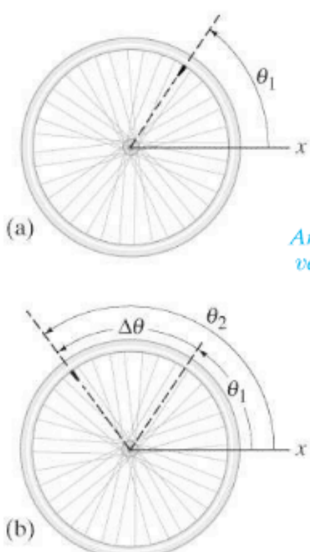
To describe rotational motion, we make use of angular quantities, such as angular velocity and angular acceleration. These are defined in analogy to the corresponding quantities in linear motion, and are chosen to describe the rotating object as a whole, so they are the same for each point in the rotating object. Each point in a rotating object may also have translational velocity and acceleration, but they have different values for different points in the object.

Angular displacement (rad)

When an object, such as the bicycle wheel in Fig. 8–3, rotates from some initial position, specified by θ_1, to some final position, θ_2, its *angular displacement* is

$$\Delta\theta = \theta_2 - \theta_1.$$

FIGURE 8–3 A wheel rotates from (a) initial position θ_1 to (b) final position θ_2. The angular displacement is $\Delta\theta = \theta_2 - \theta_1$.



The *angular velocity* (denoted by ω, the Greek lowercase letter omega) is defined in analogy with linear (translational) velocity that was discussed in Chapter 2. Instead of linear displacement, we use the angular displacement. Thus the **average angular velocity** is defined as

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}, \tag{8–2a}$$

where $\Delta\theta$ is the angle through which the object has rotated in the time interval Δt. We define the **instantaneous angular velocity** as the very small angle $\Delta\theta$, through which the object turns in the very short time interval Δt:

Angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}. \tag{8–2b}$$

Angular velocity is generally specified in radians per second (rad/s). Note that *all points in a rigid object rotate with the same angular velocity*, since every position in the object moves through the same angle in the same time interval.

An object such as the wheel in Fig. 8–3 can rotate about a fixed axis either clockwise or counterclockwise. The direction can be specified with a + or − sign, just as we did in Chapter 2 for linear motion along the $+x$ or $-x$ axis. The usual convention is to choose the angular displacement $\Delta\theta$ and angular velocity ω as positive when the wheel rotates counterclockwise. If the rotation is clockwise, then θ would decrease, so $\Delta\theta$ and ω would be negative.[‡]

[†] Even for an angle as large as 15°, the error in making this estimate is only 1%, but for larger angles the error increases rapidly.

[‡] The vector nature of angular velocity and other angular quantities is discussed in Section 8–9 (optional).

Angular acceleration (denoted by α, the Greek lowercase letter alpha), in analogy to linear acceleration, is defined as the change in angular velocity divided by the time required to make this change. The **average angular acceleration** is defined as

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t}, \qquad \textbf{(8–3a)}$$

where ω_1 is the angular velocity initially, and ω_2 is the angular velocity after a time interval Δt. **Instantaneous angular acceleration** is defined in the usual way as the limit of this ratio as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}. \qquad \textbf{(8–3b)}$$

Angular acceleration

Since ω is the same for all points of a rotating object, Eq. 8–3 tells us that α also will be the same for all points. Thus, ω and α are properties of the rotating object as a whole. With ω measured in radians per second and t in seconds, α will be expressed as radians per second squared $(\mathrm{rad/s^2})$.

Each point or particle of a rotating rigid object has, at any moment, a linear velocity v and a linear acceleration a. We can relate the linear quantities at each point, v and a, to the angular quantities of the rotating object, ω and α. Consider a point P located a distance r from the axis of rotation, as in Fig. 8–4. If the object rotates with angular velocity ω, any point will have a linear velocity whose direction is tangent to its circular path. The magnitude of that point's linear velocity is $v = \Delta l/\Delta t$. From Eq. 8–1b, a change in rotation angle $\Delta\theta$ (in radians) is related to the linear distance traveled by $\Delta l = r\,\Delta\theta$. Hence

$$v = \frac{\Delta l}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

or

$$v = r\omega. \qquad \textbf{(8–4)}$$

Linear and angular velocity related

Thus, although ω is the same for every point in the rotating object at any instant, the linear velocity v is greater for points farther from the axis (Fig. 8–5). Note that Eq. 8–4 is valid both instantaneously and on the average.

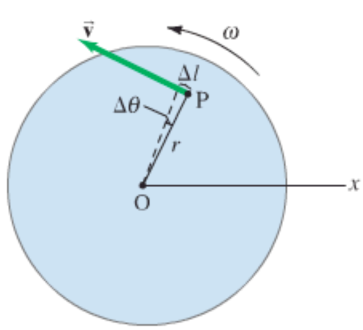


FIGURE 8–4 A point P on a rotating wheel has a linear velocity $\vec{\mathbf{v}}$ at any moment.

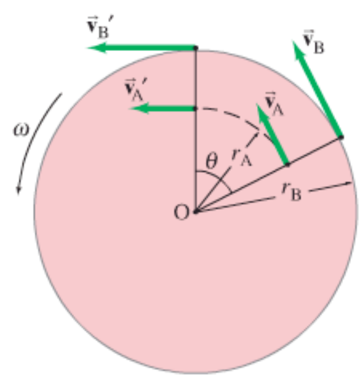


FIGURE 8–5 A wheel rotating uniformly counterclockwise. Two points on the wheel, at distances r_A and r_B from the center, have the same angular velocity ω because they travel through the same angle θ in the same time interval. But the two points have different linear velocities because they travel different distances in the same time interval. Since $r_B > r_A$, then $v_B > v_A$ $(v = r\omega)$.

CONCEPTUAL EXAMPLE 8–3 **Is the lion faster than the horse?** On a rotating carousel or merry-go-round, one child sits on a horse near the outer edge and another child sits on a lion halfway out from the center. (*a*) Which child has the greater linear velocity? (*b*) Which child has the greater angular velocity?

RESPONSE (*a*) The *linear* velocity is the distance traveled divided by the time interval. In one rotation the child on the outer edge travels a longer distance than the child near the center, but the time interval is the same for both. Thus the child at the outer edge, on the horse, has the greater linear velocity.
(*b*) The *angular* velocity is the angle of rotation divided by the time interval. In one rotation both children rotate through the same angle ($360° = 2\pi$ radians). The two children have the same angular velocity.

If the angular velocity of a rotating object changes, the object as a whole—and each point in it—has an angular acceleration. Each point also has a linear acceleration whose direction is tangent to that point's circular path. We use Eq. 8–4 ($v = r\omega$) to show that the angular acceleration α is related to the tangential linear acceleration a_{tan} of a point in the rotating object by

$$a_{\text{tan}} = \frac{\Delta v}{\Delta t} = r\frac{\Delta \omega}{\Delta t}$$

or

Tangential acceleration

$$a_{\text{tan}} = r\alpha. \tag{8–5}$$

In this equation, r is the radius of the circle in which the particle is moving, and the subscript "tan" in a_{tan} stands for "tangential."

The total linear acceleration of a point is the vector sum of two components:

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\text{tan}} + \vec{\mathbf{a}}_{\text{R}},$$

where the radial† component, $\vec{\mathbf{a}}_{\text{R}}$, is the radial or "centripetal" acceleration and its direction is toward the center of the point's circular path; see Fig. 8–6. We saw in Chapter 5 (Eq. 5–1) that $a_{\text{R}} = v^2/r$, and we can rewrite this in terms of ω using Eq. 8–4:

Centripetal (or radial) acceleration

$$a_{\text{R}} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r. \tag{8–6}$$

Thus the centripetal acceleration is greater the farther you are from the axis of rotation: the children farthest out on a carousel feel the greatest acceleration. Equations 8–4, 8–5, and 8–6 relate the angular quantities describing the rotation of an object to the linear quantities for each point of the object. Table 8–1 summarizes these relationships.

FIGURE 8–6 On a rotating wheel whose angular speed is increasing, a point P has both tangential and radial (centripetal) components of linear acceleration. (See also Chapter 5.)

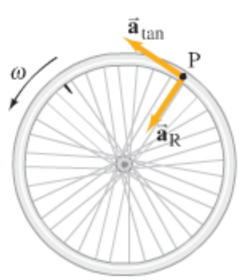


TABLE 8–1 Linear and Rotational Quantities

Linear	Type	Rotational	Relation
x	displacement	θ	$x = r\theta$
v	velocity	ω	$v = r\omega$
a_{tan}	acceleration	α	$a_{\text{tan}} = r\alpha$

† "Radial" means along the radius—that is, toward or away from the center or axis.

EXAMPLE 8–4 **Angular and linear velocities and accelerations.** A carousel is initially at rest. At $t = 0$ it is given a constant angular acceleration $\alpha = 0.060\ \text{rad/s}^2$, which increases its angular velocity for 8.0 s. At $t = 8.0\ \text{s}$, determine the following quantities: (*a*) the angular velocity of the carousel; (*b*) the linear velocity of a child (Fig. 8–7a) located 2.5 m from the center, point P in Fig. 8–7b; (*c*) the tangential (linear) acceleration of that child; (*d*) the centripetal acceleration of the child; and (*e*) the total linear acceleration of the child.

APPROACH The angular acceleration α is constant, so we can use Eq. 8–3a to solve for ω after a time $t = 8.0\ \text{s}$. With this ω and the given α, we determine the other quantities using the relations we just developed, Eqs. 8–4, 8–5, and 8–6.

SOLUTION (*a*) Equation 8–3a tells us

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t}.$$

We are given $\Delta t = 8.0\ \text{s}$, $\bar{\alpha} = 0.060\ \text{rad/s}^2$, and $\omega_1 = 0$. Solving for ω_2, we get

$$\begin{aligned} \omega_2 &= \omega_1 + \bar{\alpha}\,\Delta t \\ &= 0 + (0.060\ \text{rad/s}^2)(8.0\ \text{s}) = 0.48\ \text{rad/s}. \end{aligned}$$

During the 8.0-s interval, the carousel has accelerated from $\omega_1 = 0$ (rest) to $\omega_2 = 0.48\ \text{rad/s}$.

(*b*) The linear velocity of the child with $r = 2.5\ \text{m}$ at time $t = 8.0\ \text{s}$ is found using Eq. 8–4:

$$v = r\omega = (2.5\ \text{m})(0.48\ \text{rad/s}) = 1.2\ \text{m/s}.$$

Note that the "rad" has been dropped here because it is dimensionless (and only a reminder)—it is a ratio of two distances, Eq. 8–1b.

(*c*) The child's tangential acceleration is given by Eq. 8–5:

$$a_{\tan} = r\alpha = (2.5\ \text{m})(0.060\ \text{rad/s}^2) = 0.15\ \text{m/s}^2,$$

and it is the same throughout the 8.0-s acceleration interval.

(*d*) The child's centripetal acceleration at $t = 8.0\ \text{s}$ is given by Eq. 8–6:

$$a_R = \frac{v^2}{r} = \frac{(1.2\ \text{m/s})^2}{(2.5\ \text{m})} = 0.58\ \text{m/s}^2.$$

(*e*) The two components of linear acceleration calculated in parts (*c*) and (*d*) are perpendicular to each other. Thus the total linear acceleration at $t = 8.0\ \text{s}$ has magnitude

$$\begin{aligned} a &= \sqrt{a_{\tan}^2 + a_R^2} \\ &= \sqrt{(0.15\ \text{m/s}^2)^2 + (0.58\ \text{m/s}^2)^2} = 0.60\ \text{m/s}^2. \end{aligned}$$

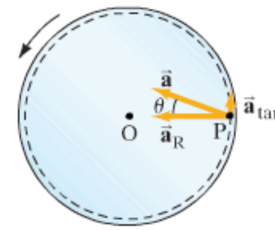
Its direction (Fig. 8–7b) is

$$\theta = \tan^{-1}\left(\frac{a_{\tan}}{a_R}\right) = \tan^{-1}\left(\frac{0.15\ \text{m/s}^2}{0.58\ \text{m/s}^2}\right) = 0.25\ \text{rad},$$

so $\theta \approx 15°$.

NOTE The linear acceleration is mostly centripetal, keeping the child moving in a circle with the carousel. The tangential component that speeds up the motion is smaller.

(a)



(b)

FIGURE 8–7 Example 8–4. The total acceleration vector $\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\tan} + \vec{\mathbf{a}}_R$, at $t = 8.0\ \text{s}$.

We can relate the angular velocity ω to the frequency of rotation, f. The **frequency** is the number of complete revolutions (rev) per second, as we saw in Chapter 5. One revolution (of a wheel, say) corresponds to an angle of 2π radians, and thus $1\,\text{rev/s} = 2\pi\,\text{rad/s}$. Hence, in general, the frequency f is related to the angular velocity ω by

Frequency

$$f = \frac{\omega}{2\pi}$$

or

$$\omega = 2\pi f. \tag{8–7}$$

The unit for frequency, revolutions per second (rev/s), is given the special name the hertz (Hz). That is

$$1\,\text{Hz} = 1\,\text{rev/s}.$$

Note that "revolution" is not really a unit, so we can also write $1\,\text{Hz} = 1\,\text{s}^{-1}$.

The time required for one complete revolution is called the **period** T, and it is related to the frequency by

Period

$$T = \frac{1}{f}. \tag{8–8}$$

If a particle rotates at a frequency of three revolutions per second, then the period of each revolution is $\frac{1}{3}$ s.

EXERCISE A In Example 8–4, we found that the carousel, after 8.0 s, rotates at an angular velocity $\omega = 0.48\,\text{rad/s}$, and continues to do so after $t = 8.0\,\text{s}$ because the acceleration ceased. What are the frequency and period of the carousel?

PHYSICS APPLIED
Hard drive and bit speed

EXAMPLE 8–5 **Hard drive.** The platter of the hard drive of a computer rotates at 7200 rpm (revolutions per minute = rev/min). (*a*) What is the angular velocity of the platter? (*b*) If the reading head of the drive is located 3.00 cm from the rotation axis, what is the linear speed of the point on the platter just below it? (*c*) If a single bit requires 0.50 μm of length along the direction of motion, how many bits per second can the writing head write when it is 3.00 cm from the axis?

APPROACH We use the given frequency f to find the angular velocity ω of the platter and then the linear speed of a point on the platter ($v = r\omega$). The bit rate is found by dividing the linear speed by the length of one bit (v = distance/time).

SOLUTION (*a*) First we find the frequency in rev/s, given $f = 7200\,\text{rev/min}$:

$$f = \frac{(7200\,\text{rev/min})}{(60\,\text{s/min})} = 120\,\text{rev/s} = 120\,\text{Hz}.$$

Then the angular velocity is

$$\omega = 2\pi f = 754\,\text{rad/s}.$$

(*b*) The linear speed of a point 3.00 cm out from the axis is given by Eq. 8–4:

$$v = r\omega = \left(3.00 \times 10^{-2}\,\text{m}\right)(754\,\text{rad/s}) = 22.6\,\text{m/s}.$$

(*c*) Each bit requires 0.50×10^{-6} m, so at a speed of 22.6 m/s, the number of bits passing the head per second is

$$\frac{22.6\,\text{m/s}}{0.50 \times 10^{-6}\,\text{m/bit}} = 45 \times 10^{6}\ \text{bits per second},$$

or 45 megabits/s (Mbps).

8–2 Constant Angular Acceleration

In Chapter 2, we derived the useful kinematic equations (Eqs. 2–11) that relate acceleration, velocity, distance, and time for the special case of uniform linear acceleration. Those equations were derived from the definitions of linear velocity and acceleration, assuming constant acceleration. The definitions of angular velocity and angular acceleration are the same as those for their linear counterparts, except that θ has replaced the linear displacement x, ω has replaced v, and α has replaced a. Therefore, the angular equations for **constant angular acceleration** will be analogous to Eqs. 2–11 with x replaced by θ, v by ω, and a by α, and they can be derived in exactly the same way. We summarize them here, opposite their linear equivalents (we've chosen $x_0 = 0$, and $\theta_0 = 0$ at the initial time $t = 0$):

Angular	Linear			
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	[constant α, a]	**(8–9a)**	*Kinematic equations*
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	[constant α, a]	**(8–9b)**	*for constant*
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$	[constant α, a]	**(8–9c)**	*angular acceleration*
$\overline{\omega} = \dfrac{\omega + \omega_0}{2}$	$\overline{v} = \dfrac{v + v_0}{2}$	[constant α, a]	**(8–9d)**	$(x_0 = 0, \theta_0 = 0)$

Note that ω_0 represents the angular velocity at $t = 0$, whereas θ and ω represent the angular position and velocity, respectively, at time t. Since the angular acceleration is constant, $\alpha = \overline{\alpha}$.

EXAMPLE 8–6 **Centrifuge acceleration.** A centrifuge rotor is accelerated from rest to 20,000 rpm in 30 s. (*a*) What is its average angular acceleration? (*b*) Through how many revolutions has the centrifuge rotor turned during its acceleration period, assuming constant angular acceleration?

APPROACH To determine $\overline{\alpha} = \Delta\omega/\Delta t$, we need the initial and final angular velocities. For (*b*), we use Eqs. 8–9 (recall that one revolution corresponds to $\theta = 2\pi$ rad).

SOLUTION (*a*) The initial angular velocity is $\omega = 0$. The final angular velocity is

$$\omega = 2\pi f = (2\pi\,\text{rad/rev})\frac{(20{,}000\,\text{rev/min})}{(60\,\text{s/min})} = 2100\,\text{rad/s}.$$

Then, since $\overline{\alpha} = \Delta\omega/\Delta t$ and $\Delta t = 30\,\text{s}$, we have

$$\overline{\alpha} = \frac{\omega - \omega_0}{\Delta t} = \frac{2100\,\text{rad/s} - 0}{30\,\text{s}} = 70\,\text{rad/s}^2.$$

That is, every second the rotor's angular velocity increases by 70 rad/s, or by $(70/2\pi) = 11$ revolutions per second.

(*b*) To find θ we could use either Eq. 8–9b or 8–9c, or both to check our answer. The former gives

$$\theta = 0 + \tfrac{1}{2}(70\,\text{rad/s}^2)(30\,\text{s})^2 = 3.15 \times 10^4\,\text{rad},$$

where we have kept an extra digit because this is an intermediate result. To find the total number of revolutions, we divide by 2π rad/rev and obtain

$$\frac{3.15 \times 10^4\,\text{rad}}{2\pi\,\text{rad/rev}} = 5.0 \times 10^3\,\text{rev}.$$

NOTE Let us calculate θ using Eq. 8–9c:

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(2100\,\text{rad/s})^2 - 0}{2(70\,\text{rad/s}^2)} = 3.15 \times 10^4\,\text{rad}$$

which checks our answer perfectly.

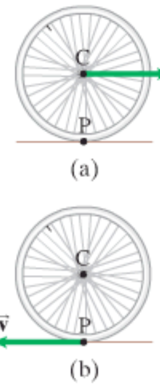


FIGURE 8–8 (a) A wheel rolling to the right. Its center C moves with velocity $\vec{\mathbf{v}}$. Point P is at rest at this instant. (b) The same wheel as seen from a reference frame in which the axle of the wheel C is at rest—that is, we are moving to the right with velocity $\vec{\mathbf{v}}$ relative to the ground. Point P, which was at rest in (a), here in (b) is moving to the left with velocity $-\vec{\mathbf{v}}$ as shown. (See also Section 3–8 on relative velocity.)

8–3 Rolling Motion (Without Slipping)

The rolling motion of a ball or wheel is familiar in everyday life: a ball rolling across the floor, or the wheels and tires of a car or bicycle rolling along the pavement. Rolling *without slipping* is readily analyzed and depends on static friction between the rolling object and the ground. The friction is static because the rolling object's point of contact with the ground is at rest at each moment.

Rolling without slipping involves both rotation and translation. There is then a simple relation between the linear speed v of the axle and the angular velocity ω of the rotating wheel or sphere: namely, $v = r\omega$ (where r is the radius) as we now show. Figure 8–8a shows a wheel rolling to the right without slipping. At the moment shown, point P on the wheel is in contact with the ground and is momentarily at rest. The velocity of the axle at the wheel's center C is $\vec{\mathbf{v}}$. In Fig. 8–8b we have put ourselves in the reference frame of the wheel—that is, we are moving to the right with velocity $\vec{\mathbf{v}}$ relative to the ground. In this reference frame the axle C is at rest, whereas the ground and point P are moving to the left with velocity $-\vec{\mathbf{v}}$ as shown. Here we are seeing pure rotation. So we can use Eq. 8–4 to obtain $v = r\omega$, where r is the radius of the wheel. This is the same v as in Fig. 8–8a, so we see that the linear speed v of the axle relative to the ground is related to the angular velocity ω by

$$v = r\omega. \qquad \text{[rolling without slipping]}$$

This relationship is valid only if there is no slipping.

EXAMPLE 8–7 **Bicycle.** A bicycle slows down uniformly from $v_0 = 8.40\,\text{m/s}$ to rest over a distance of 115 m, Fig. 8–9. Each wheel and tire has an overall diameter of 68.0 cm. Determine (*a*) the angular velocity of the wheels at the initial instant ($t = 0$); (*b*) the total number of revolutions each wheel rotates before coming to rest; (*c*) the angular acceleration of the wheel; and (*d*) the time it took to come to a stop.

APPROACH We assume the bicycle wheels roll without slipping and the tire is in firm contact with the ground. The speed of the bike v and the angular velocity of the wheels ω are related by $v = r\omega$. The bike slows down uniformly, so the angular acceleration is constant and we can use Eqs. 8–9.

SOLUTION (*a*) The initial angular velocity of the wheel, whose radius is 34.0 cm, is

$$\omega_0 = \frac{v_0}{r} = \frac{8.40\,\text{m/s}}{0.340\,\text{m}} = 24.7\,\text{rad/s}.$$

(*b*) In coming to a stop, the bike passes over 115 m of ground. The circumference of the wheel is $2\pi r$, so each revolution of the wheel corresponds to a distance traveled of $2\pi r = (2\pi)(0.340\,\text{m})$. Thus the number of revolutions the wheel makes in coming to a stop is

$$\frac{115\,\text{m}}{2\pi r} = \frac{115\,\text{m}}{(2\pi)(0.340\,\text{m})} = 53.8\,\text{rev}.$$

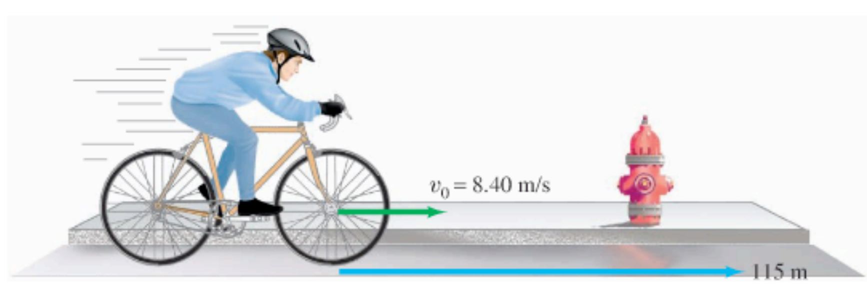


FIGURE 8–9 Example 8–7.

(c) The angular acceleration of the wheel can be obtained from Eq. 8–9c, for which we set $\omega = 0$ and $\omega_0 = 24.7\,\text{rad/s}$. Because each revolution corresponds to 2π radians of angle, then $\theta = 2\pi\,\text{rad/rev} \times 53.8\,\text{rev}$ ($= 338\,\text{rad}$) and

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{0 - (24.7\,\text{rad/s})^2}{2(2\pi\,\text{rad/rev})(53.8\,\text{rev})} = -0.902\,\text{rad/s}^2.$$

(d) Equation 8–9a or b allows us to solve for the time. The first is easier:

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 24.7\,\text{rad/s}}{-0.902\,\text{rad/s}^2} = 27.4\,\text{s}.$$

NOTE When the bike tire completes one revolution, the bike advances linearly a distance equal to the outer circumference ($2\pi r$) of the tire, as long as there is no slipping or sliding.

8–4 Torque

We have so far discussed rotational kinematics—the description of rotational motion in terms of angle, angular velocity, and angular acceleration. Now we discuss the dynamics, or causes, of rotational motion. Just as we found analogies between linear and rotational motion for the description of motion, so rotational equivalents for dynamics exist as well.

To make an object start rotating about an axis clearly requires a force. But the direction of this force, and where it is applied, are also important. Take, for example, an ordinary situation such as the overhead view of the door in Fig. 8–10. If you apply a force $\vec{\mathbf{F}}_A$ to the door as shown, you will find that the greater the magnitude, F_A, the more quickly the door opens. But now if you apply the same magnitude force at a point closer to the hinge—say, $\vec{\mathbf{F}}_B$ in Fig. 8–10—the door will not open so quickly. The effect of the force is less: where the force acts, as well as its magnitude and direction, affects how quickly the door opens. Indeed, if only this one force acts, the angular acceleration of the door is proportional not only to the magnitude of the force, but is also directly proportional to *the perpendicular distance from the axis of rotation to the line along which the force acts*. This distance is called the **lever arm**, or **moment arm**, of the force, and is labeled r_A and r_B for the two forces in Fig. 8–10. Thus, if r_A in Fig. 8–10 is three times larger than r_B, then the angular acceleration of the door will be three times as great, assuming that the magnitudes of the forces are the same. To say it another way, if $r_A = 3r_B$, then F_B must be three times as large as F_A to give the same angular acceleration. (Figure 8–11 shows two examples of tools whose long lever arms are very effective.)

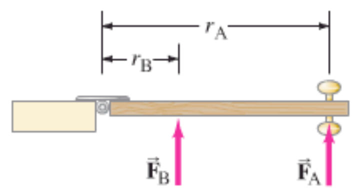


FIGURE 8–10 Applying the same force with different lever arms, r_A and r_B. If $r_A = 3r_B$, then to create the same effect (angular acceleration), F_B needs to be three times F_A, or $F_A = \frac{1}{3}F_B$.

Lever arm

(a) (b)

FIGURE 8–11 (a) A plumber can exert greater torque using a wrench with a long lever arm. (b) A tire iron too can have a long lever arm.

The angular acceleration, then, is proportional to the product of the *force times the lever arm*. This product is called the *moment of the force* about the axis, or, more commonly, it is called the **torque**, and is represented by τ (Greek lowercase letter tau). Thus, the angular acceleration α of an object is directly proportional to the net applied torque τ:

Torque defined

$$\alpha \propto \tau,$$

and we see that it is torque that gives rise to angular acceleration. This is the rotational analog of Newton's second law for linear motion, $a \propto F$.

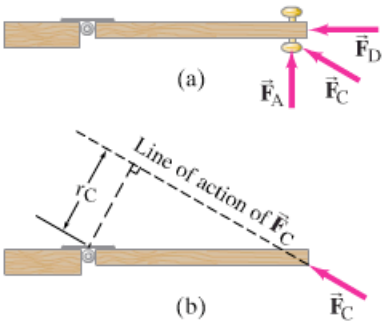


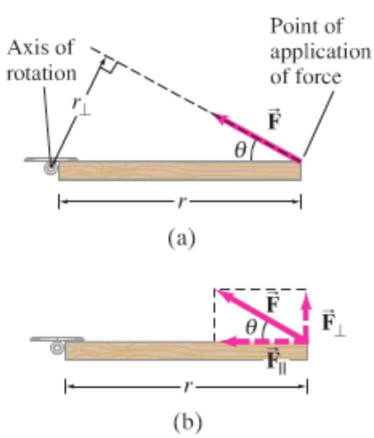
FIGURE 8–12 (a) Forces acting at different angles at the doorknob. (b) The lever arm is defined as the perpendicular distance from the axis of rotation (the hinge) to the line of action of the force.

We defined the lever arm as the *perpendicular* distance from the axis of rotation to the line of action of the force—that is, the distance which is perpendicular both to the axis of rotation and to an imaginary line drawn along the direction of the force. We do this to take into account the effect of forces acting at an angle. It is clear that a force applied at an angle, such as $\vec{\mathbf{F}}_C$ in Fig. 8–12, will be less effective than the same magnitude force applied perpendicular to the door, such as $\vec{\mathbf{F}}_A$ (Fig. 8–12a). And if you push on the end of the door so that the force is directed at the hinge (the axis of rotation), as indicated by $\vec{\mathbf{F}}_D$, the door will not rotate at all.

The lever arm for a force such as $\vec{\mathbf{F}}_C$ is found by drawing a line along the direction of $\vec{\mathbf{F}}_C$ (this is the "line of action" of $\vec{\mathbf{F}}_C$). Then we draw another line, perpendicular to this line of action, that goes to the axis of rotation and is perpendicular also to it. The length of this second line is the lever arm for $\vec{\mathbf{F}}_C$ and is labeled r_C in Fig. 8–12b. The lever arm is perpendicular both to the line of action of the force and, at its other end, perpendicular to the rotation axis.

The magnitude of the torque associated with $\vec{\mathbf{F}}_C$ is then $r_C F_C$. This short lever arm r_C and the corresponding smaller torque associated with $\vec{\mathbf{F}}_C$ is consistent with the observation that $\vec{\mathbf{F}}_C$ is less effective in accelerating the door than is $\vec{\mathbf{F}}_A$. When the lever arm is defined in this way, experiment shows that the relation $\alpha \propto \tau$ is valid in general. Notice in Fig. 8–12 that the line of action of the force $\vec{\mathbf{F}}_D$ passes through the hinge, and hence its lever arm is zero. Consequently, zero torque is associated with $\vec{\mathbf{F}}_D$ and it gives rise to no angular acceleration, in accord with everyday experience.

FIGURE 8–13
Torque $= r_\perp F = rF_\perp$.



In general, then, we can write the magnitude of the torque about a given axis as

$$\tau = r_\perp F, \tag{8–10a}$$

where $r_\perp$ is the lever arm, and the perpendicular symbol ($\perp$) reminds us that we must use the distance from the axis of rotation that is perpendicular to the line of action of the force (Fig. 8–13a).

An equivalent way of determining the torque associated with a force is to resolve the force into components parallel and perpendicular to the line that connects the axis to the point of application of the force, as shown in Fig. 8–13b. The component $F_\parallel$ exerts no torque since it is directed at the rotation axis (its moment arm is zero). Hence the torque will be equal to $F_\perp$ times the distance r from the axis to the point of application of the force:

$$\tau = rF_\perp. \tag{8–10b}$$

That this gives the same result as Eq. 8–10a can be seen from the relations $F_\perp = F \sin\theta$ and $r_\perp = r \sin\theta$. [Note that θ is the angle between the directions of $\vec{\mathbf{F}}$ and r (radial line from the axis to the point where $\vec{\mathbf{F}}$ acts)]. So

Magnitude of a torque

$$\tau = rF \sin\theta \tag{8–10c}$$

in either case. We can use any of Eqs. 8–10 to calculate the torque, whichever is easiest.

Since torque is a distance times a force, it is measured in units of m·N in SI units,[†] cm·dyne in the cgs system, and ft·lb in the English system.

[†]Note that the units for torque are the same as those for energy. We write the unit for torque here as m·N (in SI) to distinguish it from energy (N·m) because the two quantities are very different. An obvious difference is that energy is a scalar, whereas torque has a direction and is a vector. The special name *joule* (1 J = 1 N·m) is used only for energy (and for work), *never* for torque.

Summary

When a rigid object rotates about a fixed axis, each point of the object moves in a circular path. Lines drawn perpendicularly from the rotation axis to various points in the object all sweep out the same angle θ in any given time interval.

Angles are conveniently measured in **radians**, where one radian is the angle subtended by an arc whose length is equal to the radius, or

$$2\pi \text{ rad} = 360^\circ$$
$$1 \text{ rad} \approx 57.3^\circ.$$

Angular velocity, ω, is defined as the rate of change of angular position:

$$\omega = \frac{\Delta\theta}{\Delta t}. \qquad \textbf{(8–2)}$$

All parts of a rigid object rotating about a fixed axis have the same angular velocity at any instant.

Angular acceleration, α, is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}. \qquad \textbf{(8–3)}$$

The linear velocity v and acceleration a of a point fixed at a distance r from the axis of rotation are related to ω and α by

$$v = r\omega, \qquad \textbf{(8–4)}$$
$$a_{\tan} = r\alpha, \qquad \textbf{(8–5)}$$
$$a_{\mathrm{R}} = \omega^2 r, \qquad \textbf{(8–6)}$$

where $a_{\tan}$ and a_{R} are the tangential and radial (centripetal) components of the linear acceleration, respectively.

The frequency f is related to ω by

$$\omega = 2\pi f, \qquad \textbf{(8–7)}$$

and to the period T by

$$T = 1/f. \qquad \textbf{(8–8)}$$

The equations describing uniformly accelerated rotational motion (α = constant) have the same form as for uniformly accelerated linear motion:

$$\omega = \omega_0 + \alpha t, \qquad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta, \qquad \overline{\omega} = \frac{\omega + \omega_0}{2}. \qquad \textbf{(8–9)}$$

The dynamics of rotation is analogous to the dynamics of linear motion. Force is replaced by **torque** τ, which is defined as the product of force times lever arm (perpendicular distance from the line of action of the force to the axis of rotation):

$$\tau = rF\sin\theta = r_\perp F = rF_\perp. \qquad \textbf{(8–10)}$$

Mass is replaced by **moment of inertia** I, which depends not only on the mass of the object, but also on how the mass is distributed about the axis of rotation. Linear acceleration is replaced by angular acceleration. The rotational equivalent of Newton's second law is then

$$\Sigma\tau = I\alpha. \qquad \textbf{(8–14)}$$

The **rotational kinetic energy** of an object rotating about a fixed axis with angular velocity ω is

$$\text{KE} = \tfrac{1}{2}I\omega^2. \qquad \textbf{(8–15)}$$

For an object both translating and rotating, the total kinetic energy is the sum of the translational kinetic energy of the object's center of mass plus the rotational kinetic energy of the object about its center of mass:

$$\text{KE} = \tfrac{1}{2}Mv_{\mathrm{CM}}^2 + \tfrac{1}{2}I_{\mathrm{CM}}\omega^2 \qquad \textbf{(8–16)}$$

as long as the rotation axis is fixed in direction.

The **angular momentum** L of an object about a fixed rotation axis is given by

$$L = I\omega. \qquad \textbf{(8–18)}$$

Newton's second law, in terms of angular momentum, is

$$\Sigma\tau = \frac{\Delta L}{\Delta t}. \qquad \textbf{(8–19)}$$

If the net torque on the object is zero, $\Delta L/\Delta t = 0$, so L = constant. This is the **law of conservation of angular momentum** for a rotating object.

The following Table summarizes angular (or rotational) quantities, comparing them to their translational analogs.

Translation	Rotation	Connection
x	θ	$x = r\theta$
v	ω	$v = r\omega$
a	α	$a = r\alpha$
m	I	$I = \Sigma mr^2$
F	τ	$\tau = rF\sin\theta$
$\text{KE} = \frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	
$p = mv$	$L = I\omega$	
$W = Fd$	$W = \tau\theta$	
$\Sigma F = ma$	$\Sigma\tau = I\alpha$	
$\Sigma F = \frac{\Delta p}{\Delta t}$	$\Sigma\tau = \frac{\Delta L}{\Delta t}$	

Questions

1. A bicycle odometer (which measures distance traveled) is attached near the wheel hub and is designed for 27-inch wheels. What happens if you use it on a bicycle with 24-inch wheels?
2. Suppose a disk rotates at constant angular velocity. Does a point on the rim have radial and/or tangential acceleration? If the disk's angular velocity increases uniformly, does the point have radial and/or tangential acceleration? For which cases would the magnitude of either component of linear acceleration change?
3. Could a nonrigid body be described by a single value of the angular velocity ω? Explain.
4. Can a small force ever exert a greater torque than a larger force? Explain.

5. If a force $\vec{\mathbf{F}}$ acts on an object such that its lever arm is zero, does it have any effect on the object's motion? Explain.

6. Why is it more difficult to do a sit-up with your hands behind your head than when your arms are stretched out in front of you? A diagram may help you to answer this.

7. A 21-speed bicycle has seven sprockets at the rear wheel and three at the pedal cranks. In which gear is it harder to pedal, a small rear sprocket or a large rear sprocket? Why? In which gear is it harder to pedal, a small front sprocket or a large front sprocket? Why?

8. Mammals that depend on being able to run fast have slender lower legs with flesh and muscle concentrated high, close to the body (Fig. 8–34). On the basis of rotational dynamics, explain why this distribution of mass is advantageous.

FIGURE 8–34 Question 8. A gazelle.

FIGURE 8–35 Question 9.

9. Why do tightrope walkers (Fig. 8–35) carry a long, narrow beam?

10. If the net force on a system is zero, is the net torque also zero? If the net torque on a system is zero, is the net force zero?

11. Two inclines have the same height but make different angles with the horizontal. The same steel ball is rolled down each incline. On which incline will the speed of the ball at the bottom be greater? Explain.

12. Two solid spheres simultaneously start rolling (from rest) down an incline. One sphere has twice the radius and twice the mass of the other. Which reaches the bottom of the incline first? Which has the greater speed there? Which has the greater total kinetic energy at the bottom?

13. A sphere and a cylinder have the same radius and the same mass. They start from rest at the top of an incline. Which reaches the bottom first? Which has the greater speed at the bottom? Which has the greater total kinetic energy at the bottom? Which has the greater rotational KE?

14. We claim that momentum and angular momentum are conserved. Yet most moving or rotating objects eventually slow down and stop. Explain.

15. If there were a great migration of people toward the Earth's equator, how would this affect the length of the day?

16. Can the diver of Fig. 8–29 do a somersault without having any initial rotation when she leaves the board?

17. The moment of inertia of a rotating solid disk about an axis through its center of mass is $\frac{1}{2}MR^2$ (Fig. 8–21c). Suppose instead that the axis of rotation passes through a point on the edge of the disk. Will the moment of inertia be the same, larger, or smaller?

18. Suppose you are sitting on a rotating stool holding a 2-kg mass in each outstretched hand. If you suddenly drop the masses, will your angular velocity increase, decrease, or stay the same? Explain.

19. Two spheres look identical and have the same mass. However, one is hollow and the other is solid. Describe an experiment to determine which is which.

* **20.** In what direction is the Earth's angular velocity vector as it rotates daily about its axis?

* **21.** The angular velocity of a wheel rotating on a horizontal axle points west. In what direction is the linear velocity of a point on the top of the wheel? If the angular acceleration points east, describe the tangential linear acceleration of this point at the top of the wheel. Is the angular speed increasing or decreasing?

* **22.** Suppose you are standing on the edge of a large freely rotating turntable. What happens if you walk toward the center?

* **23.** A shortstop may leap into the air to catch a ball and throw it quickly. As he throws the ball, the upper part of his body rotates. If you look quickly you will notice that his hips and legs rotate in the opposite direction (Fig. 8–36). Explain.

FIGURE 8–36 Question 23. A shortstop in the air, throwing the ball.

* **24.** On the basis of the law of conservation of angular momentum, discuss why a helicopter must have more than one rotor (or propeller). Discuss one or more ways the second propeller can operate to keep the helicopter stable.

Problems

8–1 Angular Quantities

1. (I) Express the following angles in radians: (*a*) 30°, (*b*) 57°, (*c*) 90°, (*d*) 360°, and (*e*) 420°. Give as numerical values and as fractions of π.

2. (I) Eclipses happen on Earth because of an amazing coincidence. Calculate, using the information inside the Front Cover, the angular diameters (in radians) of the Sun and the Moon, as seen on Earth.

3. (I) A laser beam is directed at the Moon, 380,000 km from Earth. The beam diverges at an angle θ (Fig. 8–37) of 1.4×10^{-5} rad. What diameter spot will it make on the Moon?

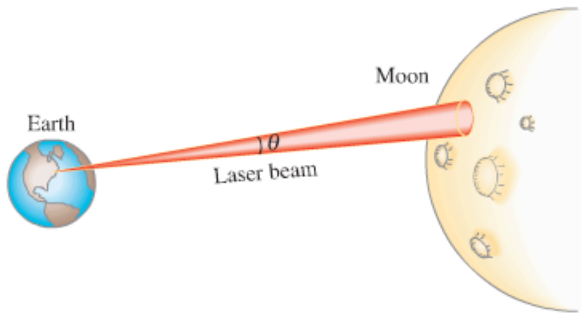


FIGURE 8–37 Problem 3.

4. (I) The blades in a blender rotate at a rate of 6500 rpm. When the motor is turned off during operation, the blades slow to rest in 3.0 s. What is the angular acceleration as the blades slow down?

5. (II) A child rolls a ball on a level floor 3.5 m to another child. If the ball makes 15.0 revolutions, what is its diameter?

6. (II) A bicycle with tires 68 cm in diameter travels 8.0 km. How many revolutions do the wheels make?

7. (II) (*a*) A grinding wheel 0.35 m in diameter rotates at 2500 rpm. Calculate its angular velocity in rad/s. (*b*) What are the linear speed and acceleration of a point on the edge of the grinding wheel?

8. (II) A rotating merry-go-round makes one complete revolution in 4.0 s (Fig. 8–38). (*a*) What is the linear speed of a child seated 1.2 m from the center? (*b*) What is her acceleration (give components)?

FIGURE 8–38 Problem 8.

9. (II) Calculate the angular velocity of the Earth (*a*) in its orbit around the Sun, and (*b*) about its axis.

10. (II) What is the linear speed of a point (*a*) on the equator, (*b*) on the Arctic Circle (latitude 66.5° N), and (*c*) at a latitude of 45.0° N, due to the Earth's rotation?

11. (II) How fast (in rpm) must a centrifuge rotate if a particle 7.0 cm from the axis of rotation is to experience an acceleration of 100,000 *g*'s?

12. (II) A 70-cm-diameter wheel accelerates uniformly about its center from 130 rpm to 280 rpm in 4.0 s. Determine (*a*) its angular acceleration, and (*b*) the radial and tangential components of the linear acceleration of a point on the edge of the wheel 2.0 s after it has started accelerating.

13. (II) A turntable of radius R_1 is turned by a circular rubber roller of radius R_2 in contact with it at their outer edges. What is the ratio of their angular velocities, ω_1/ω_2?

14. (III) In traveling to the Moon, astronauts aboard the *Apollo* spacecraft put themselves into a slow rotation to distribute the Sun's energy evenly. At the start of their trip, they accelerated from no rotation to 1.0 revolution every minute during a 12-min time interval. The spacecraft can be thought of as a cylinder with a diameter of 8.5 m. Determine (*a*) the angular acceleration, and (*b*) the radial and tangential components of the linear acceleration of a point on the skin of the ship 5.0 min after it started this acceleration.

8–2 and 8–3 Constant Angular Acceleration; Rolling

15. (I) A centrifuge accelerates uniformly from rest to 15,000 rpm in 220 s. Through how many revolutions did it turn in this time?

16. (I) An automobile engine slows down from 4500 rpm to 1200 rpm in 2.5 s. Calculate (*a*) its angular acceleration, assumed constant, and (*b*) the total number of revolutions the engine makes in this time.

17. (I) Pilots can be tested for the stresses of flying high-speed jets in a whirling "human centrifuge," which takes 1.0 min to turn through 20 complete revolutions before reaching its final speed. (*a*) What was its angular acceleration (assumed constant), and (*b*) what was its final angular speed in rpm?

18. (II) A wheel 33 cm in diameter accelerates uniformly from 240 rpm to 360 rpm in 6.5 s. How far will a point on the edge of the wheel have traveled in this time?

19. (II) A cooling fan is turned off when it is running at 850 rev/min. It turns 1500 revolutions before it comes to a stop. (*a*) What was the fan's angular acceleration, assumed constant? (*b*) How long did it take the fan to come to a complete stop?

20. (II) A small rubber wheel is used to drive a large pottery wheel, and they are mounted so that their circular edges touch. The small wheel has a radius of 2.0 cm and accelerates at the rate of 7.2 rad/s^2, and it is in contact with the pottery wheel (radius 25.0 cm) without slipping. Calculate (*a*) the angular acceleration of the pottery wheel, and (*b*) the time it takes the pottery wheel to reach its required speed of 65 rpm.

21. (II) The tires of a car make 65 revolutions as the car reduces its speed uniformly from 95 km/h to 45 km/h. The tires have a diameter of 0.80 m. (*a*) What was the angular acceleration of the tires? (*b*) If the car continues to decelerate at this rate, how much more time is required for it to stop?

8–4 Torque

22. (I) A 55-kg person riding a bike puts all her weight on each pedal when climbing a hill. The pedals rotate in a circle of radius 17 cm. (*a*) What is the maximum torque she exerts? (*b*) How could she exert more torque?

23. (I) A person exerts a force of 55 N on the end of a door 74 cm wide. What is the magnitude of the torque if the force is exerted (*a*) perpendicular to the door, and (*b*) at a 45° angle to the face of the door?

24. (II) Calculate the net torque about the axle of the wheel shown in Fig. 8–39. Assume that a friction torque of 0.40 m·N opposes the motion.

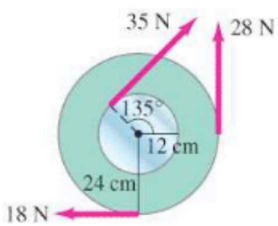


FIGURE 8–39 Problem 24.

25. (II) Two blocks, each of mass m, are attached to the ends of a massless rod which pivots as shown in Fig. 8–40. Initially the rod is held in the horizontal position and then released. Calculate the magnitude and direction of the net torque on this system.

FIGURE 8–40 Problem 25.

26. (II) The bolts on the cylinder head of an engine require tightening to a torque of 88 m·N. If a wrench is 28 cm long, what force perpendicular to the wrench must the mechanic exert at its end? If the six-sided bolt head is 15 mm in diameter, estimate the force applied near each of the six points by a socket wrench (Fig. 8–41).

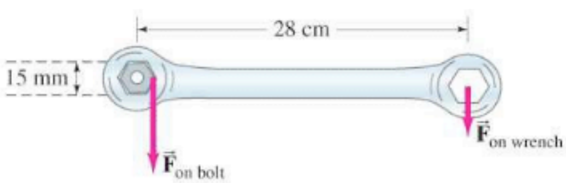


FIGURE 8–41 Problem 26.

8–5 and 8–6 Rotational Dynamics

27. (I) Determine the moment of inertia of a 10.8-kg sphere of radius 0.648 m when the axis of rotation is through its center.

28. (I) Calculate the moment of inertia of a bicycle wheel 66.7 cm in diameter. The rim and tire have a combined mass of 1.25 kg. The mass of the hub can be ignored (why?).

29. (II) A small 650-gram ball on the end of a thin, light rod is rotated in a horizontal circle of radius 1.2 m. Calculate (*a*) the moment of inertia of the ball about the center of the circle, and (*b*) the torque needed to keep the ball rotating at constant angular velocity if air resistance exerts a force of 0.020 N on the ball. Ignore the rod's moment of inertia and air resistance.

30. (II) A potter is shaping a bowl on a potter's wheel rotating at constant angular speed (Fig. 8–42). The friction force between her hands and the clay is 1.5 N total. (*a*) How large is her torque on the wheel, if the diameter of the bowl is 12 cm? (*b*) How long would it take for the potter's wheel to stop if the only torque acting on it is due to the potter's hand? The initial angular velocity of the wheel is 1.6 rev/s, and the moment of inertia of the wheel and the bowl is 0.11 kg·m^2.

FIGURE 8–42 Problem 30.

31. (II) Calculate the moment of inertia of the array of point objects shown in Fig. 8–43 about (*a*) the vertical axis, and (*b*) the horizontal axis. Assume $m = 1.8$ kg, $M = 3.1$ kg, and the objects are wired together by very light, rigid pieces of wire. The array is rectangular and is split through the middle by the horizontal axis. (*c*) About which axis would it be harder to accelerate this array?

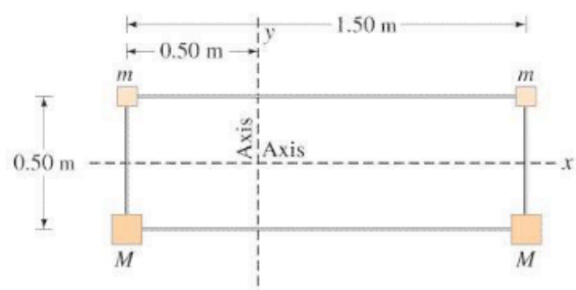


FIGURE 8–43 Problem 31.

Physics I
March 30-April 3

Answer Key

Lesson 1

15. 27,500 rev

16. a) -140 rad/s^2 b) 120 rev

17. 0.07 rad/s^2 b) 40 rpm

Lesson 2

18. 37.3 m

Lesson 3
Questions on p. 217-218

4.Yes, if the lever arm is sufficiently long enough.

5. If the lever arm is zero, there will be no torque. It's similar to the case of work, where if we apply a force to an object but it doesn't move, no work is done.

Lesson 4

Quiz answers are located within the guided worksheets in your packet and in your textbook reading. Email me if you have any questions or can't find an answer.

Lesson 5

All answers are found in your textbook reading for today. Email if you have trouble finding an answer.