**Rationalizing a denominator:**

- re-writing a fraction so that the denominator contains no radicals (we'll only be working with square roots in this lesson)
  - a fraction such as $\frac{2}{\sqrt{5}}$ can be re-written as $\frac{2\sqrt{5}}{5}$ by simply multiply the original fraction by the denominator over itself $\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$.
    - $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$
  - the reason we multiply by the denominator over itself is because we want to eliminate the square root from the denominator, and also because multiplication by 1 (anything over itself) is always acceptable
  - keep in mind that $\frac{2}{\sqrt{5}}$ and $\frac{2\sqrt{5}}{5}$ are equivalent

**Steps for Rationalizing Denominators:**

1. use the Quotient Rule for Radicals (if possible) to write the numerator and denominator as two separate square roots
2. multiply by 1
   - the square root from the denominator over itself $\left(\frac{\sqrt{\phantom{x}}}{\sqrt{\phantom{x}}}\right)$
     i. in the example shown above, the square root in the denominator was $\sqrt{5}$, so that's why I multiplied by $\frac{\sqrt{5}}{\sqrt{5}}$
3. simplify the square root in the numerator (if possible)
   - Product Rule may be necessary
4. simplify the fraction (if possible)
   - cancel common factors

Keep in mind that when you multiply a square root times itself, you get just the radicand $\left(\sqrt{x} \cdot \sqrt{x} = x\right)$. However when you multiply two squares that are not identical (two different radicands), the square roots do NOT cancel $\left(\sqrt{x} \cdot \sqrt{y} = \sqrt{xy}\right)$.

**Example 1:** Rationalize the denominator of the following expression and simplify your answer completely.

$$\sqrt{\frac{7}{18}}$$

Write the numerator and denominator as two separate square roots using the Quotient Rule for Radicals.

$$\frac{\sqrt{7}}{\sqrt{18}}$$

To rationalize the denominator of a fraction containing a square root, simply multiply both the numerator and denominator by the denominator over itself.

$$\frac{\sqrt{7}}{\sqrt{18}} \cdot \frac{\sqrt{18}}{\sqrt{18}}$$

$$\frac{\sqrt{126}}{18}$$

Be sure to simplify the radical in the numerator completely by removing any factors that are perfect squares.

$$\frac{\sqrt{9} \cdot \sqrt{14}}{18}$$

$$\frac{3 \cdot \sqrt{14}}{18}$$

Be sure to also simplify the fraction by canceling any common factors between the numerator and denominator.

$$\frac{\cancel{3} \cdot \sqrt{14}}{\cancel{18}\ 6}$$

$$\mathbf{\frac{\sqrt{14}}{6}}$$

The final answer should not contain any radicals in the denominator. Also, any radicals in the numerator should be simplified completely. And the fraction should be simplified as well.

Keep in mind that you must always simplify your radicals and your fractions completely. However also keep in mind that in this problem, $\frac{\sqrt{14}}{6}$ cannot be simplified any further because while 14 and 6 both have a common factor of 2, $\sqrt{14}$ and 6 do not.

**<u>Example 2:</u>** Rationalize the denominator of the following expression and simplify your answer completely. (Assume that all variables are positive.)

a. $\frac{1}{\sqrt{3x^5y}}$

b. $\sqrt{\frac{3x}{2y^7}}$

c. $\sqrt{\frac{x^9y}{8x^3y^4}}$

$$\frac{\sqrt{x^9y}}{\sqrt{8x^3y^4}}$$

$$\frac{\sqrt{x^9y}}{\sqrt{8x^3y^4}} \cdot \frac{\sqrt{8x^3y^4}}{\sqrt{8x^3y^4}}$$

$$\frac{\sqrt{8x^{12}y^5}}{8x^3y^4}$$

$$\frac{\sqrt{8}\sqrt{x^{12}}\sqrt{y^5}}{8x^3y^4}$$

$$\frac{\sqrt{4}\sqrt{2}\,x^6\sqrt{y^4}\sqrt{y^1}}{8x^3y^4}$$

$$\frac{2\sqrt{2}\,x^6y^2\sqrt{y}}{8x^3y^4}$$

$$\frac{2x^6y^2\sqrt{2y}}{8x^3y^4}$$

$$\boldsymbol{\frac{x^3\sqrt{2y}}{4y^2}}$$

d. $\frac{x^7\sqrt{y^7}}{\sqrt{3x^2y}}$

$$\frac{x^7\sqrt{y^7}}{\sqrt{3x^2y}} \cdot \frac{\sqrt{3x^2y}}{\sqrt{3x^2y}}$$

$$\frac{x^7\sqrt{3x^2y^8}}{3x^2y}$$

$$\frac{x^7\sqrt{3}\sqrt{x^2}\sqrt{y^8}}{3x^2y}$$

$$\frac{x^7\sqrt{3}xy^4}{3x^2y}$$

$$\frac{x^8y^4\sqrt{3}}{3x^2y}$$

$$\boldsymbol{\frac{x^6y^3\sqrt{3}}{3}}$$

*Answers to Examples:*

*1.* $\frac{\sqrt{14}}{6}$ *; 2a.* $\frac{\sqrt{3xy}}{3x^3y}$ *; 2b.* $\frac{\sqrt{6xy}}{2y^4}$ *; 2c.* $\frac{x^3\sqrt{2y}}{4y^2}$ *; 2d.* $\frac{x^6y^3\sqrt{3}}{3}$