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International diversification: A copula approach

Lorán Chollete [a,*], Victor de la Peña [b], Ching-Chih Lu [c]

[a] University of Stavanger, Stavanger N-4036, Norway
[b] Columbia University, New York, NY 10027, USA
[c] National Chengchi University, 64 Sec. 2, Zhi-Nan Road, Taipei 116, Taiwan

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ABSTRACT

The viability of international diversification involves balancing benefits and costs. This balance hinges on the degree of asset dependence. In light of theoretical research linking diversification and dependence, we examine international diversification using two measures of dependence: correlations and copulas. We document several findings. First, dependence has increased over time. Second, we find evidence of asymmetric dependence or downside risk in Latin America, but less in the G5. The results indicate very little downside risk in East Asia. Third, East Asian and Latin American returns exhibit some correlation complexity. Interestingly, the regions with maximal dependence or worst diversification do not command large returns. Our results suggest international limits to diversification. They are also consistent with a possible tradeoff between international diversification and systemic risk.

1. Introduction

The net benefit of international diversification is of great importance in today's economic climate. In general, the tradeoff between diversification's benefits and costs hinges on the degree of dependence across securities, as observed by Samuelson (1967), Ibragimov et al. (2009b), Shin (2009), Veldkamp and Van Nieuwerburgh (2010), and Bai and Green (2010), among others. Economists and investors often assess diversification benefits using a measure of dependence, such as correlation.[1] It is therefore vital to have accurate measures of dependence. There are several measures available in finance, including the traditional correlation and copulas. While each approach has advantages and disadvantages, researchers have rarely compared them in the same empirical study.[2] Such reliance on one dependence measure prevents easy assessment of the degree of international diversification opportunities, and how they differ over time or across regions.

The main goal of this paper is to assess diversification opportunities available in international stock markets, using both correlations and copulas. The recent history of international markets is interesting in itself, due to the large number of financial crises, increasingly globalized markets, and financial contagion.[3] We also examine some basic implications for international asset pricing. In particular, we investigate whether the diversification measures are related to international stock returns. This research is valuable because considerations of diversification and dependence should affect risk premia.

A secondary focus of our paper is the relation between diversification and systemic risk. We motivate this aspect by theoretical research such as Brumelle (1974), Ibragimov et al. (2009b), and Shin (2009), and it concerns two separate, distributional properties: heavy tails and tail dependence. The term 'heavy tail' refers to the tail mass of the marginal, univariate distributions, while 'tail dependence' refers to the connection between marginal distributions at extreme quantiles.[4] While no general theoretical results link

* Corresponding author. Tel.: +47 5183 1500; fax: +47 5183 1550.
E-mail addresses: loran.g.chollete@uis.no (L. Chollete), vp@stat.columbia.edu (V. de la Peña), cclu@nccu.edu.tw (C.-C. Lu).

[1] See Solnik (1974), Ingersoll (1987, Chapter 4); Carrieri et al. (2008); You and Daigler (2010). Moreover, asset prices, which reflect their diversification benefits in equilibrium, are assessed using dependence or covariance. See research on CAPM and stochastic discount methods, such as Sharpe (1964), Lintner (1965), Lucas (1978), and Hansen and Singleton (1982).

[2] Throughout, we use the word dependence as an umbrella to cover any situation where two or more variables move together. We adopt this practice because there are numerous words in use (e.g. correlation, concordance, co-dependency, comovement), and we wish to use a general term. We do not assume that any dependence measure is ideal, and throughout we indicate advantages and disadvantages as the case may be.

[3] See Dungey and Tambakis (2005), Reinhart (2008), Reinhart and Rogoff (2009), Markwat et al. (2009), and Dungey et al. (2010).

[4] We formally define tail dependence and tail indices in Eqs. (5) and (9). Further, we estimate both heavy tails and dependence in Tables 8 and 9, and Table 11, respectively.

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tail dependence and diversification, dependence at extremes is considered important from a risk management, policy and broad economic perspective. This is particularly true in light of the ongoing financial recession.[5] Consequently, tail dependence forms the basis for many measures of systemic risk.[6] Regarding heavy tails, researchers have established results that relate heavy tails, diversification and systemic risk. These results show that when portfolio distributions are heavy-tailed, not only do they represent limited diversification, they may also suggest existence of a wedge between individual risk and systemic risk.[7] Thus, there are aggregate economic ramifications for heavy-tailed assets. Specifically, in a heavy-tailed portfolio environment, diversification may yield both individual benefits and aggregate systemic costs. If systemic costs are too severe, the economy may require a coordinating agency to improve resource allocation.[8] Such policy considerations are absent from previous empirical research on heavy tails in international markets, and provide a further motivation for our paper.

The remaining structure of the paper is as follows. In Section 2, we review theoretical and empirical literature on diversification and dependence. In Section 3, we compare and contrast diversification measures used in empirical finance. Section 4 discusses our data and main results. Section 5 illustrates some financial implications, and Section 6 concludes.

2. Diversification, dependence, and systemic risk

The notion that diversification improves portfolio performance pervades economics, and appears in asset pricing, insurance, and international finance. A central precept is that, based on the law of large numbers, the return variance on a portfolio of a group of securities is lower than that of any single security.[9] An important caveat, noted as early as Samuelson (1967), concerns the dependence structure of security returns, as we discuss below. This theoretical importance of dependence structure motivates our use of copulas in the empirical analysis.

Diversification depends on both the dependence and marginal properties. Since our initial focus is on dependence, we will discuss its implications for diversification *for a given set of marginals*, unless otherwise noted.[10] Therefore when we describe a set of assets with lower dependence as having higher diversification benefits, it is always with the caveat that we are considering the marginals to be fixed.

2.1. Theoretical background

When portfolios are heavy-tailed, diversification may not be optimal.[11] In an early important paper, Samuelson (1967) examines the restrictive conditions needed to ensure that diversification is optimal.[12] He underscores the need for a general definition of negative dependence, framed in terms of the distribution function of security returns. In a significant development, Brumelle (1974) proves that negative correlation is neither necessary nor sufficient for diversification, except in special cases such as normal distributions or quadratic preferences. Brumelle uses a form of dependence as a sufficient condition for diversification, that involves the shape of the entire distribution. Thus, shortly after the inception of modern portfolio theory, both Brumelle (1974) and Samuelson (1967) realize and discuss the need for restrictions on the joint distribution, in order to obtain diversification. However, that discussion has a gap: it stops short of examining multivariate ($n > 2$) asset returns, and the practical difficulty of imposing dependence restrictions on empirical data. The use of copulas may be one way to fill this gap.[13] The research of Embrechts et al. (2002) introduces copulas into risk management. The authors first show that standard Pearson correlations can go dangerously wrong as a risk measure. They then suggest the copula function as a flexible alternative to correlation, which can capture dependence throughout the entire distribution of asset returns. A copula C is by definition a joint distribution with uniform marginals. In the bivariate case, that means

$$C(u, v) = \Pr[U \leqslant u, \quad V \leqslant v], \tag{1}$$

where U and V are uniformly distributed on $[0, 1]$.[14]

The intuition behind copulas is that they "couple" or join marginals into a joint distribution. Copulas often have convenient parametric forms, and summarize the dependence structure between variables.[15] Specifically, for any joint distribution $F_{X,Y}(x, y)$ with marginals $F_X(x)$ and $F_Y(y)$, we can write the distribution as

$$F_{X,Y}(x, y) = C(F_X(x), \ F_Y(y)), \tag{2}$$

for some copula C. The usefulness of (2) is that we can simplify analysis of dependence in a return distribution $F_{X,Y}(x, y)$ by studying instead a copula C. Since copulas represent dependence of arbitrary distributions, in principle they allow us to examine diversification effects for heavy-tailed joint distributions, following the logic of Brumelle (1974) and Samuelson (1967).

The above approaches analyze investor decisions, and say little about systemic risk. Evidently investors' decisions, in aggregate, may have an externality effect on financial and economic markets. The existence of externalities related to "excessive" diversification has been emphasized by several recent papers. We discuss the following three articles, since their results focus on distributional properties.[16] Ibragimov et al. (2009b) develop a

[5] Economic examples of dependence at extreme periods may include the liquidity trap of Keynes (1936) and the nonlinear Phillips curve of Phelps (1968).

[6] For measures of systemic risk derived from tail dependence, see Hartmann et al. (2003), Cherubini et al. (2004, p. 43), and Adrian and Brunnermeier (2008). For a discussion related to tail dependence and portfolio riskiness, see Kortschak and Albrecher (2009).

[7] For evidence on limited diversification, see Embrechts et al. (2002) and Ibragimov and Walden (2007). For evidence on a wedge between individual risk and systemic risk, see Ibragimov et al. (2009a) and Ibragimov et al. (2009b).

[8] For related work, see Ibragimov et al. (2009a), Chollete (2008), and Shin (2009).

[9] The following authors formalize aspects of this precept: Markowitz (1952), Sharpe (1964), Lintner (1965), Mossin (1966), and Samuelson (1967).

[10] In this paper, we transform all marginals to uniform by first ranking the data. Evidently if we were performing a dynamic study the marginals would vary and we would have a further reason to consider both marginals and dependence when discussing diversification.

[11] See Embrechts et al. (2005) and Ibragimov (2009).

[12] Samuelson (1967) discusses several approaches to obtain equal investment in all assets, as well as positive diversification in at least one asset. The distributional assumptions on security returns involve i.i.d. and strict independence of at least one security. Although both utility functions and distributional assumptions are relevant, Samuelson focuses on distributional concerns. A special case of dependence when diversification may be optimal is that of perfect negative correlation. However, if a portfolio consists of more than two assets, some of which are negatively correlated, then at least two must be positively correlated. This could still result in suboptimality of diversification for at least one asset, when there are short sale constraints. See Ibragimov (2009) and Samuelson (1967, p. 7).

[13] Another approach involves extreme value theory, which we explore elsewhere.

[14] See de la Peña et al. (2006, Definition 3.1). It is typical to express the copula in terms of the marginal distributions $F_X(x)$ and $F_Y(y)$. In general, the transformations from X and Y to their distributions F_X and F_Y are known as probability integral transforms, and F_X and F_Y can be shown to be uniformly distributed. See Cherubini et al. (2004, p. 52) and Embrechts (2009).

[15] This result holds for multivariate ($n > 2$) quantities. It is due to Sklar (1959), who proves that copulas uniquely characterize continuous distributions. For non-continuous distributions, the copula will not necessarily be unique. In such situations, the empirical copula approach of Deheuvels (1979) helps narrow down admissible copulas.

[16] Other papers include Krishnamurthy (2009), Shin (2009), Danielsson et al. (2009), and Beine et al. (2010).

model of catastrophic risks. They characterize the existence of *non-diversification traps*: situations where insurance providers may not insure catastrophic risks nor participate in reinsurance even though there is a large enough market for complete risk-sharing. Conditions for this market failure to occur comprise limited liability and heavy left-tailedness of risk distributions. Economically speaking, if assets have infinite second moments, this represents potentially unbounded downside risk and upside gain. In the face of this, insurers prefer to ration insurance rather than decide coverage unilaterally.[17] The authors go onto say that, if the number of insurance providers is large but finite, then non-diversification traps can arise only with distributions that have heavy left tails. In a related paper, Ibragimov and Walden (2007) examine distributional considerations that limit the optimality of diversification. The authors show that non-diversification may be optimal when the number of assets is small relative to their distributional support. They suggest that such considerations can explain market failures in markets for assets with possibly large negative outcomes. They also identify theoretical non-diversification regions, where risk-sharing will be difficult to create, and risk premia may appear anomalously large. The authors show that this result holds for many dependent risks as well, in particular convolutions of dependent risks with joint truncated α-symmetric distributions.[18] Since these convolutions exhibit heavy-tailedness and dependence, copula models are potentially useful in empirical applications of this result, by extracting the dependence structure of portfolio risks. In a recent working paper, Ibragimov et al. (2009a) discuss the importance of characterizing the potential for externalities transmitted from individual bank risks to the distribution of systemic risk. Their model highlights the phenomenon of *diversification disasters*: for some distributions, there is a wedge between the optimal level of diversification for individual agents and for society. This wedge depends crucially on the degree of heavy-tailedness: for very small or very large heavy-tailedness, individual rationality and social optimality agree, and the wedge is small. The wedge is potentially largest for moderately heavy-tailed risks.[19] This result continues to hold for risky returns with uncertain dependence or correlation complexity.[20] Economically speaking, when risk distributions are moderately heavy tailed, this represents potentially unbounded downside risk and upside gain. In such a situation, some investors might wish to invest in several asset classes, even though this contributes to an increased fragility of the entire financial system. Thus, individual and social incentives are not aligned. A similar situation exists when the structure of asset correlations is complex and uncertain.[21] The authors provide a calibration illustrating a diversification disaster where society prefers concentration, while individuals prefer diversification. As in Ibragimov et al. (2009b), they explain that their results hold for general distributions, including the Student-*t*, logistic, and symmetric stable distributions, all of which generally exhibit dependence.

2.2. Relation of theoretical results to heavy tails and copulas

The research above emphasizes on theoretical grounds the importance of isolating both heavy tails in the marginals and dependence in the joint distribution of asset returns. Regarding dependence, most economic measures of systemic risk involve tail dependence.[22] At the same time, the theoretical link between tail dependence and diversification is still developing. In light of the research of Samuelson (1967), Brumelle (1974), and Shin (2009) it appears that some type of negative dependence across assets in general enhances diversification, while asymmetric dependence limits diversification. These conditions can be examined empirically using copulas since, as shown in (2), copulas characterize dependence.[23] This motivates our estimation of dependence in Section 4. It should be noted, however, that these results are phrased in terms of the distributions, not copulas directly. Therefore, copulas can at best help an empirical study by showing that the dependence in the data satisfies a necessary condition. For example, if the estimated copulas exhibit tail dependence, then it is possible for limited diversification, diversification traps and diversification disasters to occur. Regarding heavy tails, their link to diversification is well established, although of secondary importance in this paper. The research of Ibragimov and Walden (2007) and Ibragimov et al. (2009b) establishes that heavy tails are a precondition for the results on limited diversification and systemic risk. Thus in Section 4.4 we will also estimate whether there are heavy tails in international security returns. Finally, in light of results by Ibragimov et al. (2009a), we also examine correlation complexity, as evidenced by disagreement in our estimated dependence measures.[24] Such disagreement may be consistent with a wedge between diversification and systemic risk.

2.3. Related empirical research

Previous research on dependence generally falls into either correlation or copula frameworks.[25] The literature in each area applied to international finance is vast and growing, so we summarize only some key contributions.[26] With regard to correlation, a major finding of Longin and Solnik (1995) and Ang and Bekaert (2002) is that international stock correlations tend to increase over time. Moreover, Cappiello et al. (2006) document that international stock and bond correlations increase in response to negative returns, although part of this apparent increase may be due to an inherent volatility-induced bias.[27] You and Daigler (2010) examine international diversi-

[17] This parallels the credit rationing literature of Jaffee and Russell (1976) and Stiglitz and Weiss (1981).

[18] This class contains spherical distributions, including multinormal, multivariate *t*, and multivariate spherically symmetric α-stable distributions.

[19] The authors define a distribution $F(x)$ to be moderately heavy-tailed if it satisfies the following relation, for $1 < \alpha < \infty : \lim_{x \to +\infty} F(-x) = \frac{c+o(1)}{x^{\alpha}} l(x)$. Here c and α are positive constants and $l(x)$ is a slowly varying function at infinity. The parameter α is the tail index, and characterizes the heavy-tailedness of F. We calibrate α in Tables 8 and 9. For more details, see de Haan and Ferreira (2006) and Embrechts et al. (1997).

[20] Other research documents the biases arising from complex assets, see Skreta and Veldkamp (2009) and Tarashev (2010).

[21] Individuals have an incentive to diversify because they do not bear all the costs in the event of systemic crises. That is, the aggregate risk is an externality, as examined by Shin (2009).

[22] See Hartmann et al. (2003), Cherubini et al. (2004), and Adrian and Brunnermeier (2008).

[23] It is possible to estimate the full joint distributions directly, but this leads to a problem of misspecification in both the marginals and dependence. Using copulas with standardized empirical marginals removes the problem of misspecification in the marginals. Therefore the only misspecification relates to dependence, which can be ameliorated with goodness of fit tests for copulas of different shapes. For further background on issues related to choosing copulas, see Chen and Fan (2006), Cherubini et al. (2004), Embrechts (2009), Joe (1997), Mikosch (2006), and Nelsen (1998).

[24] See Skreta and Veldkamp (2009) for related research on the impact of complexity in financial decision making.

[25] There is also a related literature that examines dependence using extreme value theory, as well as threshold correlations or dynamic skewness. These papers all find evidence that dependence is nonlinear, increasing more during market downturns for many countries, and for bank assets as well as stock returns. For approaches that build on extreme value techniques, see Longin and Solnik (2001), Hartmann et al. (2003), Poon et al. (2004), and Beine et al. (2010). For threshold correlations, see Ang and Chen (2002). For dynamic skewness, see Harvey and Siddique (1999).

[26] For summaries of copula literature, see Cherubini et al. (2004), Embrechts et al. (2005), Jondeau et al. (2007), and Patton (2009). For more general information on dependence in finance, see Embrechts et al. (1997) and Cherubini et al. (2004).

[27] See Forbes and Rigobon (2002).

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fication and document that the assumption of constant correlation overstates the true amount of diversification. The main sources of bias are existence of time-varying correlations and data non-normalities. Regarding copula-based studies of dependence, an early paper by Mashal and Zeevi (2002) shows that equity returns, currencies and commodities exhibit tail dependence. Patton (2004) uses a conditional form of the copula relation (2) to examine dependence between small and large-cap US stocks. He finds evidence of asymmetric dependence in the stock returns. Patton (2004) also documents that knowledge of this asymmetry leads to significant gains for investors who do not face short sales constraints. Patton (2006) uses a conditional copula to assess the structure of dependence in foreign exchange. Using a sample of Deutschemark and Yen series, Patton (2006) finds strong evidence of asymmetric dependence in exchange rates. Jondeau and Rockinger (2006) successfully utilize a model of returns that incorporates skewed-t GARCH for the marginals, along with a dynamic Gaussian and Student-t copula for the dependence structure. Rosenberg and Schuermann (2006) analyze the distribution of bank losses using copulas to represent, very effectively, the aggregate expected loss from combining market risk, credit risk, and operational risk. Rodriguez (2007) constructs a copula-based model for Latin American and East Asian countries. His model allows for regime switches, and yields enhanced predictive power for international financial contagion. Okimoto (2008) also uses a copula model with regime switching, focusing on the US and UK. Okimoto (2008) finds evidence of asymmetric dependence between stock indices from these countries. Harvey and de Rossi (2009) construct a model of time-varying quantiles, which allow them to focus on the expectation of different parts of the distribution. This model is also general enough to accommodate irregularly spaced data. Harvey and Busetti (2009) devise tests for constancy of copulas. They apply these tests to Korean and Thai stock returns and document that the dependence structure may vary over time. Ning (2008) examines the dependence of stock returns from North America and East Asia. She finds asymmetric, dynamic tail dependence in many countries. Ning (2008) also documents that dependence is higher intra-continent relative to across continents. Ning (2010) analyzes the dependence between stock markets and foreign exchange, and discovers significant upper and lower tail dependence between these two asset classes. Chollete et al. (2009) use general canonical vines in order to model relatively large portfolios of international stock returns from the G5 and Latin America. They find that the model outperforms dynamic Gaussian and Student-t copulas, and also does well at modifying the Value-at-Risk (VaR) for these international stock returns.[28] These papers all contribute to the mounting evidence on significant asymmetric dependence in joint asset returns.

2.4. Contribution of our paper

Our paper has similarities and differences with the previous literature. The main similarity is that, with the aim of gleaning insight on market returns and diversification, we estimate dependence of international financial markets. There are several main differences. First, we assess diversification using both correlation and copula techniques, and we are agnostic ex ante about which technique is appropriate. To the best of our knowledge, ours is the first paper to analyze international dependence using both methods.[29] Second, with the exception of Hartmann et al. (2003), who analyze foreign exchange, our work uses a broader range of countries than most previous studies, comprising both developed and emerging markets. Third, we undertake a preliminary analysis to explore the link between diversification and regional returns.

Finally, our paper builds on specific economic theories of diversification and dependence. Previous empirical research focuses very justifiably on establishing the existence of asymmetric dependence, dynamic dependence and heavy tails. Understandably, these empirical studies are generally motivated by implications for individual market participants and risk management benchmarks such as VaR. By contrast, our work relies on theoretical diversification research, and discusses both individual and systemic implications of asset distributions. Most empirical research assessing market dependence assumes that larger dependence leads to poorer diversification in practice. However, as discussed above, the link between dependence and diversification is still unclear. What is arguably more important from an economic point of view is that there are aggregate ramifications for elevated asset dependence and uncertain asset dependence. Therefore, we present the average dependence across regions and over time, and also evidence on disagreement of dependence measures. In this way we intend to obtain empirical insight on the possibility of a wedge between individual and social desiderata. Such considerations are absent from most previous empirical copula research.

3. Measuring diversification

Diversification is assessed with various dependence measures. If two assets have relatively lower dependence for a given set of marginals, they offer better diversification opportunities than otherwise. In light of the above discussion, we estimate dependence in two ways, using correlations and copulas.[30] The extent of discrepancy between the two can suggest correlation complexity. It can also be informative if we wish to obtain a sense of possible mistakes from using correlations alone. We now define the dependence measures. Throughout, we consider X and Y to be two random variables, with a joint distribution $F_{X,Y}(x, y)$, and marginals $F_X(x)$ and $F_Y(y)$, respectively.

3.1. Correlations

Correlations are the most familiar measures of dependence in finance. If properly specified, correlations tell us about average diversification opportunities over the entire distribution. The Pearson correlation coefficient ρ is the covariance divided by the product of the standard deviations:

$$\rho = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X) \cdot \mathrm{Var}(Y)}}. \tag{3}$$

The main advantage of correlation is its tractability. There are, however, a number of theoretical shortcomings, especially in finance settings.[31] First, a major shortcoming is that correlation is not invariant to monotonic transformations. Thus, the correlation of two return series may differ from the correlation of the squared returns or log returns. Second, there is mixed evidence of infinite

[28] Value-at-Risk or VaR is a measure of diversification related to portfolio losses L. Let the distribution of losses be $F_L(\cdot)$. Then VaR represents the smallest number $\bar{L}$ such that the probability of losses larger than L is less than $(1-\alpha)$. That is, for a given confidence level $\alpha \in (0, 1)$,

$$VaR(\alpha) \equiv \inf\{\bar{L} \in R : \Pr(L > \bar{L} < 1 - \alpha)\} = \inf\{\bar{L} \in R : F_L(\bar{L}) \geqslant \alpha\}.$$

For more details, see Embrechts et al. (2005, Chapter 2).

[29] We assume time-invariant dependence in this study. While a natural next step is time-varying conditional dependence, we start at the unconditional case, since there has been little or no comparative research even at this level. Furthermore, we do analyze whether dependence changes in different parts of the sample.

[30] Readers already familiar with dependence and copula concepts may proceed to Section 4.

[31] Disadvantages of correlation are discussed by Embrechts et al. (2002).

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variance in financial data.[32] From Eq. (3), if either X or Y has infinite variance, the estimated correlation may give little information on dependence, since it will be undefined or close to zero. A related issue is that many financial series have infinite fourth moments, as documented by Gabaix et al. (2003). For such data, it has been shown that auto-correlation estimates are not well-behaved.[33] A third drawback concerns estimation bias: by definition the conditional correlation is biased and spuriously increases during volatile periods.[34] Fourth, correlation is a symmetric measure and therefore may overlook important asymmetric dependence. It does not distinguish, for example, between dependence during up and down markets.[35] Finally, correlation is a linear measure of dependence, and may not capture important nonlinearities. Whether these shortcomings matter in practice is an empirical question that we approach in this paper.

A related, nonlinear measure is the **rank** (or Spearman) **correlation**, ρ_S. This is more robust than the traditional correlation. ρ_S measures dependence of the ranks, and can be expressed as $\rho_S = \frac{\mathrm{Cov}(F_X(X),F_Y(Y))}{\sqrt{\mathrm{Var}(F_X(X))\mathrm{Var}(F_Y(Y))}}$.[36] The rank correlation is especially useful when analyzing data with a number of extreme observations, since it is independent of the levels of the variables, and therefore robust to outliers. Another nonlinear correlation measure is one we term **downside risk**,[37] $d(u)$. This function measures the conditional probability of an extreme event beyond some threshold u. For simplicity, normalize variables to the unit interval [0, 1]. Hence

$$d(u) \equiv \Pr(F_X(X) \leqslant u | F_Y(Y) \leqslant u). \tag{4}$$

A final nonlinear correlation measure is left **tail dependence**, $\lambda(u)$, which is the limit of downside risk as losses become extreme,

$$\lambda(u) \equiv \lim_{u \to 0} \Pr(F_X(X) \leqslant u | F_Y(Y) \leqslant u). \tag{5}$$

Tail dependence is important because it measures the asymptotic likelihood that two variables go down or up at the same time. Such a quantity is evidently important in economics and risk management.[38] Consequently, tail dependence is the basis for many measures of systemic risk.[39]

3.2. Copulas

If we knew the entire joint distribution of international returns, we could summarize all relevant dependence and therefore all diversification opportunities. In a portfolio of two assets with returns X and Y, all dependence is contained in the joint density $f_{X,Y}(x, y)$. This information is often not available, especially for large portfolios, because there might be no simple parametric joint density that characterizes the relationship across all variables. Moreover, there is a great deal of estimation and misspecification error in attempting to find the density parametrically.

We may measure diversification in this setting with the **copula function** $C(u, v)$. From expression (1), a copula is a joint distribution with uniform marginals U and V, $C(u, v) = \Pr[U \leqslant u, V \leqslant v]$. As shown in (2), any joint distribution $F_{X,Y}(x, y)$ with continuous marginals is characterized by a unique copula function C such that $F_{X,Y}(x, y) = C(F_X(x), F_Y(y))$. In the absolutely continuous case, it is often convenient to differentiate Eq. (2) and use a corresponding "canonical" density version

$$f(x,y) = c(F_X(x), F_Y(y)) \cdot f_X(x) \cdot f_Y(y), \tag{6}$$

where $f(x, y)$ and $c(F_X, F_Y)$ are the joint and copula densities, respectively.[40] Eq. (6) is interesting because it empowers us to separate out the joint distribution from the marginals. For example, if we are interested in why extreme events increase risk in a US–UK portfolio, this could come from either the fact that the marginals are heavy-tailed, or they exhibit tail dependence, or both. This distinction is relevant whenever we are interested in the downside risk of the entire portfolio, more than the heavy-tailedness of each security in the portfolio. We estimate (6) in Section 5, for different copula specifications.

Researchers use a number of parametric copula specifications. We focus on three types, the normal, the Student-t, and the Gumbel copulas, for several reasons.[41] The normal specification is a natural benchmark, as the most common distributional assumption in finance, with zero tail dependence.[42] The Student-t is useful since it has symmetric but nonzero tail dependence and nests the normal copula. The Gumbel copula is useful because it has nonlinear dependence and asymmetric tail dependence – the mass in its right tail greatly exceeds the mass in its left tail. Moreover, the Gumbel is a member of two important families, Archimedean copulas and extreme value copulas.[43] Practically, these copulas represent the most important shapes for finance, and are a subset of those frequently used in recent empirical papers.[44] Table 1 provides functional forms of the copulas. We estimate the copulas by maximum likelihood.

We note several main advantages of using copulas in finance. First, they are a convenient choice for modeling potentially nonlinear portfolio dependence, such as correlated defaults. This aspect of copulas is especially attractive since they nest some important forms of dependence, as described in Section 3.3. A second advantage is that copulas can aggregate portfolio risk from disparate sources, such as credit and operational risk. This is possible even for risk distributions that are subjective and objective, as in Rosenberg and Schuermann (2006). In a related sense, copulas permit one to model *joint* dependence in a portfolio without specifying the distribution of individual assets in the portfolio.[45] A third advantage is invariance. Since the copula is based on ranks, it is invariant under strictly increasing transforms. That is, the copula

[32] See Mandelbrot (1963), Fama (1965), and Rachev (2003) for evidence on infinite variance. See Gabaix et al. (2003) for evidence on finite variance in financial series.

[33] See Davis and Mikosch (1998) and Mikosch and Stărică (2000).

[34] See Forbes and Rigobon (2002). After adjusting for such bias, Forbes and Rigobon (2002) document that prior findings of international dependence (contagion) are reversed.

[35] Such asymmetry may be substantial, as illustrated by Ang and Chen (2002) in the domestic context. These researchers document significant asymmetry in downside and upside correlations of US stock returns.

[36] See Cherubini et al. (2004, p. 100).

[37] The concept of downside risk appears in a number of settings without being explicitly named. It is the basis for many measures of systemic risk, see Cherubini et al. (2004, p. 43), Hartmann et al. (2003), and Adrian and Brunnermeier (2008).

[38] Economic examples of extreme dependence may include the liquidity trap of Keynes (1936) and the nonlinear Phillips curve of Phelps (1968).

[39] See Cherubini et al. (2004, p. 43), Hartmann et al. (2003), and Adrian and Brunnermeier (2008).

[40] Specifically, $f(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$, and similarly $c(F_X(x), F_Y(y)) = \frac{\partial^2 C(F_X(x),F_Y(y))}{\partial x \partial y}$. The terms $f_X(x)$ and $f_Y(y)$ are the marginal densities.

[41] Since we wish to investigate left dependence or downside risk, we also utilize the survivor function of the Gumbel copula, denoted Rotated Gumbel.

[42] Tail dependence refers to dependence at the extreme quantiles as in expression (5). See de Haan and Ferreira (2006).

[43] Archimedean copulas represent a convenient bridge to Gaussian copulas since the former have dependence parameters that can be defined through a correlation measure, Kendall's tau. Extreme value copulas are important since they can be used to model joint behavior of the distribution's extremes.

[44] See for example, Embrechts et al. (2002), Patton (2004), and Rosenberg and Schuermann (2006).

[45] This is usually expressed by saying that copulas do not constrain the choice of individual or marginal asset distributions. For example, if we model asset returns of the US and UK as bivariate normal, this automatically restricts both the individual (marginal) US and UK returns to be univariate normal. Our semi-parametric approach avoids restricting the marginals by using empirical marginal distributions, based on ranks of the data. Specifically, first the data for each marginal are ranked to form empirical distributions. These distributions are then used in estimating the parametric copula.

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Table 1
Distribution of various copulas.

Copula	Distribution	Parameter range	Complete dependence	Independence
Normal	$C_N(u, v; \rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v))$	$\rho \in (-1, 1)$	$\rho = 1$, or -1	$\rho = 0$
Student-t	$C_t(u, v; \rho, d) = t_{d,\rho}(t_d^{-1}(u), t_d^{-1}(v))$	$\rho \in (-1, 1)$	$\rho = 1$, or -1	$\rho = 0$
Gumbel	$C_G(u, v; \beta) = \exp\{-[(-\ln(u))^{1/\beta} + (-\ln(v))^{1/\beta}]^\beta\}$	$\beta \in (0, 1)$	$\beta = 0$	$\beta = 1$
RG	$C_{RG}(u, v; \alpha) = u + v - 1 + C_G(1 - u, 1 - v; \alpha)$	$\alpha \in (0, 1)$	$\alpha = 0$	$\alpha = 1$
Clayton	$C_C(u, v; \theta) = \max((u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}, 0)$	$\theta \in [-1, \infty) \setminus \{0\}$	$\theta \to \infty$	$\theta \to 0$
RC	$C_{RC}(u, v; \theta) = u + v - 1 + C_C(1 - u, 1 - v, \theta)$	$\theta \in [-1, \infty) \setminus \{0\}$	$\theta \to \infty$	$\theta \to 0$

RG and RC denote the Rotated Gumbel and Rotated Clayton copulas, respectively. The symbols $\Phi_\rho(x, y)$ and $t_{\nu,\rho}(x, y)$ denote the standard bivariate normal and Student-t cumulative distributions, respectively: $\Phi_\rho(x, y) = \int_{-\infty}^{x}\int_{-\infty}^{y} \frac{1}{2\pi|\Sigma|} \exp\left\{-\frac{1}{2}(xy)\Sigma^{-1}(xy)'\right\} dxdy$, and $t_{\nu,\rho}(x, y) = \int_{-\infty}^{x}\int_{-\infty}^{y} \frac{\Gamma(\frac{\nu+2}{2})}{\Gamma(\nu/2)(\nu\pi)|\Sigma|^{1/2}} \left\{1 + (st)\Sigma^{-1}(st)'/\nu\right\}^{\frac{-(\nu+2)}{2}} dsdt$. The correlation matrix is given by $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$.

extracts the way in which x and y comove, regardless of the scale used to measure them.[46] Fourth, since copulas are rank-based and can incorporate asymmetry, they are also natural dependence measures from a theoretical perspective. The reason is that a growing body of research recognizes that investors care a great deal about the ranks and downside performance of their investment returns.[47] There are two drawbacks to using copulas. First, existing financial models of asset prices are typically expressed in terms of Pearson correlation. Therefore, if a study uses copulas that do not have correlation as a parameter, it is potentially difficult to relate the results to those in existing financial models. This is not an issue for our study, since our model selection chooses a t copula, which contains a correlation parameter. Second, from a statistical perspective, it is not easy to say which parametric copula best fits the data, since some copulas may fit better near the center and others near the tails. This issue is not strongly relevant to our paper, since the theoretical background research from Section 2 focuses on asymmetry and tail dependence. Thus the emphasis is on the shape of copulas, rather than on a specific copula. Further, we use several specification checks, namely AIC, BIC, a mixture model, and the econometric test of Chen and Fan (2006).

3.3. Relationship of diversification measures

We briefly outline the relationship of the diversification measures.[48] If the true joint distribution is bivariate normal, then the copula and traditional correlation give the same information. Once we move far away from normality, there is no clear relation between correlation and the other measures. However, as we show below, the other dependence properties that we discussed can all be related to copulas. We describe relationships for rank correlation ρ_S, downside risk $d(u)$, and tail dependence $\lambda(u)$ in turn. The relation between copulas and rank correlation is given by

$$\rho_S = 12 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 3. \tag{7}$$

This means that if we know the correct copula, we can recover the rank correlation. Therefore, rank correlation is a pure copula property. Regarding downside risk, it can be shown that $d(u)$ satisfies

$$d(u) \equiv \Pr(F_X(X) \leqslant u | F_Y(Y) \leqslant u) = \frac{\Pr(F_X(X) \leqslant u, F_Y(Y) \leqslant u)}{\Pr(F_Y(Y) \leqslant u)} = \frac{C(u, u)}{u}. \tag{8}$$

where the third line uses definition (1) and the fact since $F_Y(Y)$ is uniform, $\Pr[F_Y(Y) \leqslant u] = u$. Thus downside risk is also a pure copula property and does not depend on the marginals at all. Since tail dependence is the limit of downside risk, it follows from (5) and (8) that $\lambda(u) = \lim_{u\downarrow 0} \frac{C(u,u)}{u}$. To summarize, the nonlinear measures are directly related to the copula, and ρ and the normal copula give the same information when the data are jointly normal. While the above discussion describes how to link the various concepts in theory, there is little empirical work comparing the different diversification measures. This provides a rationale for our empirical study.

4. Data and results

We use security market data from fourteen national stock market indices, for a sample period of January 11, 1990 to May 31, 2006. The countries are from the G5, East Asia and Latin America. The G5 countries are France (FR), Germany (DE), Japan (JP), the UK and the US. The East Asian countries are Hong Kong (HK), South Korea (KR), Singapore (SI), Taiwan (TW) and Thailand (TH). The Latin American countries include Argentina (AR), Brazil (BR), Chile (CH) and Mexico (ME). The data comprise return indices from MSCI. These indices are tradeable, therefore the returns represent what an investor could realize by holding these in a portfolio. All non-US returns are denominated in US dollars, and therefore are comparable to the realized returns of a US investor who invested abroad. These countries are chosen because they all have daily data available for a relatively long sample period.[49] We aggregate the data to a weekly frequency (Wednesday–Wednesday returns) in order to avoid time zone differences. Therefore the total number of observations is 831 for the full sample.[50] We briefly overview summary statistics, then discuss the correlation and copula estimates.

Table 2 summarizes our data. From an investment perspective, the most striking point is US dominance, since it has the lowest volatility in each sample. The US also has one of the largest mean returns in the full sample and during the 1990s, dominating all other G5 and East Asian countries. This suggests that recent stock market history is markedly different from previous times such as those examined by Lewis (1999), when US investment overseas had clearer diversification benefits. For the full sample, across all countries mean returns are between 3% and 16%. The smallest and largest returns are for Thailand (−3.7) and Brazil (15.24), respectively. Generally standard deviations are high, at least twice

[46] See Schweizer and Wolff (1981). For more details on copula properties, see Nelsen (1998, Chapter 2).

[47] See Polkovnichenko (2005) and Barberis et al. (2001).

[48] For background and proofs on the relations between dependence measures, see Cherubini et al. (2004, Chapter 3), Embrechts et al. (2005), and Jondeau et al. (2007).

[49] Moreover, many of them are considered integrated with the world market by Bekaert and Harvey (1995).

[50] We also split the sample in two, from 1991 to 2001 and from 2001 to 2006. This division of the sample was chosen so that at least one part of the sample, the first part, covers a complete business cycle in the US, as described by the National Bureau of Economic Research.

Table 2
Average returns for international indices.

	1990–2006	1990–2001	2001–2006
FR	7.10	8.31	4.64
	(20.38)	(18.99)	(22.99)
DE	5.49	6.85	2.69
	(21.97)	(19.92)	(25.69)
JP	0.09	−2.52	5.43
	(22.58)	(23.30)	(21.04)
UK	5.96	6.90	4.05
	(16.38)	(15.81)	(17.52)
US	8.10	12.03	0.09
	(15.49)	(14.69)	(17.00)
HK	7.76	10.61	1.93
	(24.64)	(27.03)	(18.85)
KR	4.68	−4.49	23.41
	(36.60)	(39.38)	(30.03)
SI	3.48	2.78	4.91
	(25.19)	(27.75)	(18.95)
TW	1.16	0.98	1.53
	(32.62)	(34.90)	(27.45)
TH	−3.70	−14.88	19.16
	(37.85)	(42.24)	(26.51)
AR	12.95	14.70	9.35
	(40.53)	(41.38)	(38.81)
BR	15.24	15.37	14.98
	(44.32)	(48.59)	(34.07)
CH	11.16	10.33	12.86
	(22.61)	(24.28)	(18.79)
ME	13.61	12.18	16.54
	(31.80)	(35.14)	(23.58)

The average country portfolio returns are annualized and in percentage points. Standard deviations are in parentheses.
Source: MSCI.

the magnitude of the mean, and often much larger. In the first part of the sample, 1990–2001, average returns are roughly the same as for the entire sample. As in the full sample, the smallest and largest returns are for Thailand (−14.88) and Brazil (15.37), respectively. In the latter sample, 2001–2006, average returns are similar in magnitude to the first sample. However, there is some evidence of a shift upwards: the smallest return is now positive, for the US (0.09), and the maximal return, for Thailand (19.16) is larger than the preceding period. Notably, the US shifted dramatically from having the largest G5 returns in the 1990s to having the lowest of all countries after 2001. Another indication of a dramatic shift in international returns is that Thailand went from having the lowest returns in the 1990s to having the second-largest returns after the turn of the century.

4.1. Correlation estimates of dependence

Table 3 presents correlation and rank correlation estimates. As explained in Section 2, when we discuss implications for diversification in this context, it is with the assumption of fixed marginals. We first consider G5 countries. Panel A shows results for the entire sample, where the average correlation is 0.545. Panel B shows results for the first part of the sample, which features a slightly lower correlation of 0.487. Panel C displays results from the latter part of the sample, where average correlations are much larger, at 0.637. In all sample periods, the maximum and minimum correlations are for the same countries, France–Germany, and Japan–US, respectively. Similar patterns are detected by the rank correlation. Thus, for the G5 average dependence has increased (diversification has fallen) for every country pair over time, the countries affording maximal and minimal diversification benefits are stable over time, and the dependence measures agree on which countries offer the best and worst diversification.

Now we consider the East Asian economies. For the entire sample, in Panel A, the average Pearson correlation of 0.406 is considerably lower than for the G5 economies. Panel B shows results for the first sample. Here, average correlation is slightly lower than for the full sample, at 0.379. The maximum and minimum are also smaller than for the full sample. Panel C shows the latter sample, where correlation has increased substantially to 0.511. Throughout, the country pair with maximal correlation is that of Hong Kong–Singapore. However, the minimal correlation pair switches from Korea–Taiwan in the first half to Hong Kong–Thailand in the latter half, and is Taiwan–Thailand for the entire sample. Therefore the best countries for diversification differ depending on investors' holding periods. Moreover, the dependence measures disagree in the latter sample with regard to the lowest dependence: ρ picks Hong Kong-Thailand, whereas ρ_S chooses Taiwan–Thailand. Thus, for East Asian economies, average dependence has increased over time, the two-country portfolios affording best diversification are not stable, and the dependence measures disagree for the more recent periods.

Finally, we consider the Latin American economies. Panel A shows the full sample estimates, which feature an average correlation of 0.414. Panel B presents the first sample, with an average correlation of 0.416. Panel C shows the latter sample, with a similar correlation of 0.423. The two dependence measures do not agree with regard to which countries have maximal and minimal dependence in the early sample. They also do not agree on maximal dependence in the full sample. Further, there is a switch in the countries offering best dependence: for the early sample it is Argentina–Brazil according to ρ, which switches to Argentina–Chile for the later sample. Thus, for Latin American countries, dependence increases only slightly, the countries with best diversification are not stable over time, and dependence measures disagree in the early and full sample.

In terms of general comparison, the lowest average dependence for the full sample and early period are for East Asia, and for Latin America in the latter period. The specific countries with the very minimum dependence are ambiguous for the full sample: using ρ it is in the G5, while ρ_S selects East Asia. In the early and late periods, the countries with minimal dependence are in East Asia and Latin America, respectively. In purely economic terms, an investor who invests solely in East Asia or Latin America has enhanced diversification benefits, relative to an investor who invests solely in the G5. However, given that the dependence measures sometimes disagree in Latin America and East Asia, this suggests correlation complexity, which may mitigate the apparent benefits.[51]

4.2. Copula results

We now present results from our copula estimation. We consider six copulas, the normal, Student-*t*, Gumbel, Rotated Gumbel, Clayton and Rotated Clayton.[52] We first discuss evidence on heavy-tailedness, based on the shape of the best fitting copulas, and then

[51] We assume an investor holds stock market indices. A separate approach involves holding industry portfolios to diversify sectorally, see Berben and Jansen (2005) and Flavin (2004).

[52] As we mentioned above, there are many other copulas available. We choose these copulas because they have all been used in a number of recent finance studies, and because they represent four important portfolio shapes for finance: symmetric thin tails, symmetric heavy tails, heavy upper tails, and heavy lower tails. The Student-*t* and mixture model have heavy tails on both the upside and downside. The Gumbel and Rotated Gumbel feature only heavy right tail and only heavy left tail, respectively. The Clayton and Rotated Clayton copula have heavy left tail and heavy right tail, respectively. The normal copula is the only one with thin tails.

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Table 3
Correlation estimates of international dependence.

	G5			East Asia			Latin America		
	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min
Panel A: 1990–2006									
ρ	0.545	0.822 (FR–DE)	0.303 (JP–US)	0.406	0.588 (HK–SI)	0.315 (TW–TH)	0.414	0.506 (BR–ME)	0.355 (AR–CH)
ρ_S	0.523	0.772 (FR–DE)	0.304 (JP–US)	0.373	0.539 (HK–SI)	0.271 (TW–TH)	0.376	0.447 (AR–ME)	0.299 (AR–CH)
Panel B: 1990–2001									
ρ	0.487	0.762 (FR–DE)	0.281 (JP–US)	0.379	0.577 (HK–SI)	0.237 (KR–TW)	0.416	0.493 (BR–ME)	0.359 (AR–BR)
ρ_S	0.471	0.709 (FR–DE)	0.267 (JP–US)	0.322	0.511 (HK–SI)	0.176 (KR–TW)	0.366	0.480 (AR–ME)	0.307 (BR–CH)
Panel C: 2001–2006									
ρ	0.637	0.901 (FR–DE)	0.355 (JP–US)	0.511	0.639 (HK–SI)	0.353 (HK–TH)	0.423	0.561 (BR–ME)	0.310 (AR–CH)
ρ_S	0.624	0.887 (FR–DE)	0.389 (JP–US)	0.512	0.641 (HK–SI)	0.376 (TW–TH)	0.405	0.520 (BR–ME)	0.266 (AR–CH)

ρ and ρ_S denote the Pearson and rank correlations, defined in Section 3 of the text. Avg, Max, and Min denote the average, maximum, and minimum dependence for each region. Further details on individual countries are available from the authors upon request.

Table 4
Comparing dependence structures using information criteria.

Models	AIC	BIC
Panel A: G5		
Gumbel	−269.17	−264.44
Rotated Gumbel	−312.37	−307.64
Clayton	−275.46	−270.73
Rotated Clayton	−206.26	−201.53
Normal	−302.82	−298.10
Student-*t*	−316.20	−306.75
Mixed Copula	−318.18	−294.57
Panel B: East Asia		
Gumbel	−111.25	−106.53
Rotated Gumbel	−139.43	−134.71
Clayton	−122.70	−117.98
Rotated Clayton	−87.31	−82.59
Normal	−132.38	−127.66
Student-*t*	−138.47	−129.02
Mixed Copula	−138.98	−115.36
Panel C: Latin America		
Gumbel	−121.23	−116.51
Rotated Gumbel	−183.97	−179.25
Clayton	−171.26	−166.54
Rotated Clayton	−86.50	−81.78
Normal	−153.02	−148.30
Student-*t*	−167.56	−158.12
Mixed Copula	−179.22	−155.61

AIC and BIC are the average Akaike and Bayes Information Criteria for countries in each region.

Table 5
Comparing dependence structures using mixture weights.

Weights	G5	East Asia	Latin America
W_{Gumbel}	0.097 (0.085)	0.145 (0.102)	0.099 (0.084)
$W_{\text{R. Gumbel}}$	0.517 (0.170)	0.384 (0.147)	0.787 (0.160)
W_{Normal}	0.386 (0.177)	0.471 (0.196)	0.114 (0.161)

W_i denotes the average weight on copula i in each region, where i = Gumbel, Rotated Gumbel (R. Gumbel), and normal. The average standard deviation of weights for each region is in parentheses.

estimate dependence parameters. The diagnostic methods we consider for copula shape are AIC, BIC, a mixture model, and the econometric test of Chen and Fan (2006).[53]

4.2.1. Evidence on asymmetric dependence

Table 4 presents evidence on asymmetric dependence using results from AIC and BIC. We first discuss the AIC results. For G5 countries the best model (lowest AIC) is the mixed copula, with an average AIC of −318.18 across countries, closely followed by the Student-*t*. For the East Asian economies, the lowest AIC of −139.43 corresponds to the Rotated Gumbel, followed closely by the mixed copula and Student-*t*. Finally, for Latin American countries, the lowest AIC of −183.97 is for the Rotated Gumbel model, followed by the mixed copula. We now discuss the BIC results. For the G5 countries, the best model on average is the Rotated Gumbel, with an average BIC of −307.64, closely followed by the Student-*t* copula. Similarly, for both the East Asian and Latin American countries, the best model on average is again the Rotated Gumbel, closely followed by the Student-*t*. Thus, according to AIC and BIC, the best fitting copulas all exhibit tail dependence.

The copulas above mainly assume a single dependence structure. In order to address this assumption, we examine more closely the mixed copula, which has normal, Gumbel and Rotated Gumbel components. The results are presented in Table 5.[54] Since the weights on each copula in the mixture reflect the proportion of the data consistent with that copula shape, a large weight on the Gumbel indicates large upside dependence (systemic booms) while a large weight on the Rotated Gumbel copula suggests large

[53] AIC and BIC denote the Akaike and Bayes Information Criteria, respectively. AIC and BIC are not formal statistical tests, although it is customary to use them to give a rough sense of goodness of fit. We therefore include these two information criteria, since they are employed in this literature by many researchers, such as Dias and Embrechts (2004) and Frees and Valdez (1997).

[54] The mixed copula is also useful since the weights can inform us on another aspect of diversification, namely downside risk, as mentioned in the previous section. The mixed copula is estimated by iterative maximum likelihood, as is standard in mixture model research. Another paper that uses mixed copulas is that of Hu (2006), although she uses this framework descriptively, not for model selection or regional comparisons of downside risk. For details on mixture model estimation, see McLachlan and Peel (2000).

Table 6
Comparing dependence structures using likelihood methods.

	FR–DE	FR–JP	FR–UK	FR–US	DE–JP	DE–UK	DE–US	JP–UK	JP–US	UK–US
Panel A: G5 Countries										
Normal vs. Clayton	−1.05	0.14	−2.72	−2.79	1.16	−0.71	−2.33	0.34	−0.57	−2.43
Normal vs. R. Clayton	−6.49	−4.36	−6.25	−4.30	−4.90	−7.01	−5.21	−4.21	−2.25	−4.91
Normal vs. Gumbel	−1.75	−3.19	−3.00	−2.50	−3.10	−3.37	−3.49	−2.39	−0.61	−3.00
Normal vs. R. Gumbel	1.89**	0.73	−0.02	−0.66	1.28*	1.18	−0.52	0.71	0.27	−0.65
Normal vs. *t*	0.00	0.44	0.19	0.88	0.13	0.05	0.25	0.11	0.19	0.62
Normal vs. Mixed	3.34**	1.16	1.85**	1.32*	1.52*	2.44**	1.19	1.46*	1.16	0.82
t vs. Clayton	−0.01	−0.24	−0.98	−2.94	0.07	−0.09	−2.44	−0.07	−0.29	−2.68
t vs. R. Clayton	−0.01	−3.68	−1.96	−4.50	−0.90	−0.37	−5.12	−0.57	−0.57	−5.01
t vs. Gumbel	−1.75	−3.19	−3.00	−2.50	−3.10	−3.37	−3.49	−2.39	−0.61	−3.00
t vs. R. Gumbel	0.00	0.22	−0.18	−0.86	0.14	0.00	−0.57	0.00	−0.12	−0.82
t vs. Mixed	0.00	0.43	0.07	1.06	0.14	0.03	1.17	0.04	0.02	0.50
	HK–KR	HK–SI	HK–TW	HK–TH	KR–SI	KR–TW	KR–TH	SI–TW	SI–TH	TW–TH
Panel B: Asian Countries										
Normal vs. Clayton	−1.39	0.12	−0.46	−0.25	−1.32	−0.93	−1.99	−0.28	−1.36	−0.02
Normal vs. R. Clayton	−3.55	−5.07	−3.50	−3.42	−3.31	−2.84	−2.88	−3.30	−3.46	−2.83
Normal vs. Gumbel	−2.71	−2.50	−2.62	−2.09	−2.46	−2.00	−2.11	−2.00	−1.77	−2.63
Normal vs. R. Gumbel	−0.25	1.75**	0.53	0.98	−0.28	0.17	−0.61	0.43	0.49	0.51
Normal vs. *t*	0.55	0.02	0.69	0.09	0.63	0.89	0.65	0.77	0.07	0.66
Normal vs. Mixed	0.94	2.61**	1.58*	1.95**	0.65	1.14	0.45	1.10	1.90**	0.98
t vs. Clayton	−1.60	−0.02	−0.81	−0.11	−1.53	−1.29	−2.18	−0.55	−0.16	−0.31
t vs. R. Clayton	−3.76	−0.08	−3.68	−0.39	−3.42	−3.09	−3.07	−3.78	−0.30	−3.04
t vs. Gumbel	−2.71	−2.50	−2.62	−2.09	−2.46	−2.00	−2.11	−2.00	−1.77	−2.63
t vs. R. Gumbel	−0.38	0.00	0.25	0.01	−0.39	−0.05	−0.73	0.18	−0.03	0.36
t vs. Mixed	0.78	0.01	1.11	0.06	0.36	0.79	0.13	0.73	0.02	0.81
	AR–BR	AR–CH	AR–ME	BR–CH	BR–ME	CH–ME				
Panel C: Latin American Countries										
Normal vs. Clayton	1.53*	1.44*	−0.45	0.68	2.10**	1.78**				
Normal vs. R. Clayton	−4.35	−4.53	−4.21	−3.84	−6.03	−4.61				
Normal vs. Gumbel	−2.41	−3.34	−2.67	−2.58	−4.89	−3.54				
Normal vs. R. Gumbel	1.91**	1.28*	1.04	1.49*	2.38**	1.85**				
Normal vs. *t*	0.01	0.13	0.08	0.03	0.08	0.04				
Normal vs. Mixed	2.27**	1.32*	1.97**	1.97**	2.53**	2.00**				
t vs. Clayton	0.00	0.15	−0.11	−0.02	0.14	0.02				
t vs. R. Clayton	−0.04	−1.05	−0.46	−0.13	−0.66	−0.17				
t vs. Gumbel	−2.41	−3.34	−2.67	−2.58	−4.89	−3.54				
t vs. R. Gumbel	0.01	0.20	0.03	0.01	0.18	0.04				
t vs. Mixed	0.01	0.20	0.07	0.02	0.19	0.04				

Test statistics are generated using the pseudo-likelihood ratio test of Chen and Fan (2006).
R. Gumbel and R. Clayton represent the Rotated Gumbel and Rotated Clayton copulas, respectively.
* Significance at the 10% level.
** Significance at the 5% level.

downside dependence (systemic downturns). First, consider the G5 estimates. The largest average weight of 0.517 is on the Rotated Gumbel copula, with relatively little weight on the Gumbel copula. This suggests that there is generally asymmetric dependence in the G5, with substantial downside risk. Now consider the East Asian models. Here the weights are closer than for the G5. The largest average weight of 0.471 is on the normal copula, closely followed by the Rotated Gumbel. Finally, for Latin American countries the Rotated Gumbel copula is again dominant, with an average weight of 0.787. Thus, according to the mixed copula results, there is evidence of asymmetric dependence, particularly in the G5 and Latin America. The greatest downside risk is in Latin America, which has nearly eighty percent of the average weight on the Rotated Gumbel.

Table 6 presents formal statistical tests of copula fit, using the approach of Chen and Fan (2006). Goodness of fit is assessed by a pseudo-likelihood ratio test, where each model is compared to two benchmarks, namely the normal and Student-*t* copulas.[55] Panel A presents results for G5 countries. We first discuss the normal benchmark results. For the comparison of normal versus Gumbel, Clayton, Rotated Clayton and Student-*t*, the alternative models are insignificant for all countries, and the normal benchmark is preferred. However, in comparison to the Rotated Gumbel, there is slightly weaker performance of the normal, with significance of the Rotated Gumbel in 2 of the 10 cases. Finally, the mixed model is significant in six of the country pairs. Therefore, there is some evidence against the normal, in favor of a potentially asymmetric model. We now consider the set of comparisons with the Student-*t* as benchmark. In all cases the alternative models are insignificant. Thus, the evidence is in favor of a copula with tail dependence for the G5 economies. Panel B displays the results for East Asian economies. For the normal benchmark, the Clayton, Rotated Clayton, Gumbel and Student-*t* copulas are always insignificant, and the Rotated Gumbel is only significant for one country pair. The mixed model, however, is significant in four cases. When we turn to the *t* benchmark, the other copulas are always insignificant. Therefore, for East Asia there is evidence of symmetric tail dependence. This evidence is not overwhelming, however, because the normal model generally fares very well. Panel C contains the Latin American results. For the normal benchmark, the Rotated Clayton, Gumbel and Student-*t* copulas are always insignificant, while the Clayton copula is significant in four of six cases. The Rotated Gumbel is significant in five cases, and the mixed model is always significant. For the Student-*t* benchmark, as in the G5 and East Asia, the alternative copulas are

[55] For conformity with previous literature, we consider a *p*-value of 0.1 or less to be significant, as in Chen and Fan (2006).

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Table 7
Copula estimates of international dependence.

Parameters	Full sample			1990–2001			2001–2006		
	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min
Panel A: G5									
R. Gumbel: α	0.655	0.444 (FR–DE)	0.813 (JP–US)	0.701	0.516 (FR–DE)	0.831 (JP–US)	0.561	0.299 (FR–DE)	0.756 (JP–US)
Student-t: ρ_t	0.525	0.773 (FR–DE)	0.309 (JP–US)	0.469	0.703 (FR–DE)	0.270 (JP–US)	0.641	0.902 (FR–DE)	0.408 (JP–US)
Panel B: East Asia									
R. Gumbel: α	0.760	0.637 (HK–SI)	0.827 (TW–TH)	0.798	0.648 (HK–SI)	0.896 (KR–TW)	0.661	0.583 (KR–TW)	0.746 (HK–TH)
Student-t: ρ_t	0.385	0.546 (HK–SI)	0.284 (TW–TH)	0.324	0.519 (HK–SI)	0.175 (KR–TW)	0.530	0.628 (HK–SI)	0.402 (HK–TH)
Panel C: Latin America									
R. Gumbel: α	0.727	0.686 (BR–ME)	0.774 (AR–CH)	0.736	0.665 (AR–ME)	0.780 (BR–CH)	0.705	0.611 (BR–ME)	0.800 (AR–CH)
Student-t: ρ_t	0.414	0.477 (AR–ME)	0.349 (AR–CH)	0.398	0.514 (AR–ME)	0.336 (BR–CH)	0.447	0.560 (BR–ME)	0.308 (AR–CH)

The table presents statistics on dependence parameters for Rotated Gumbel (R. Gumbel) and t copulas. Avg, Max, and Min denote the average, maximum, and minimum dependence for each region. As in Section 3, minimum dependence corresponds to best diversification, and vice versa. As mentioned in the text and seen in Table 1, dependence for the Rotated increases as the parameter α goes from 1 to 0. Therefore the greatest dependence (Max) for α entails *smaller* numbers than does the lowest dependence (Min). Further details on individual countries are available from the authors upon request.

all dominated by the t. Therefore, the Latin American countries tend to exhibit asymmetric dependence.

To summarize our diagnostic methods, the main similarity is that in all regions, the symmetric Student-t copula dominates other copulas that have tail dependence. While this is suggestive evidence in favor of symmetric dependence (joint booms and crashes), the Student-t does not perform well against the normal in any region. We therefore pay close attention to regional differences, in comparing the normal to other copulas. In this regard, there are interesting regional differences. For G5 countries the normal copula is not the best description of the data, because the mixed copula does well. Moreover, for the G5 we find only small evidence of asymmetric dependence. In Latin American economies, normality is decisively rejected, and we find evidence of asymmetric dependence. For the East Asian economies we document little evidence of asymmetric dependence, and the normal copula does better than in other regions. This latter finding on East Asian limited downside risk is previously undocumented. In economic terms, Latin America is most prone to situations where investors are unable to diversify during bad times.

4.2.2. Copula estimates of dependence

We now estimate dependence using our best-performing single copula models from above, the Rotated Gumbel and Student-t models. Table 7 presents parameter estimates.[56] We focus on the dependence parameter ρ_t for the t copula, as it is related to the familiar correlation ρ. Panel 1 displays the G5 estimates. For the full sample, average dependence is 0.525. For the first sample, dependence is 0.469, increasing dramatically to 0.641 in the second period. In all sample periods, both dependence measures agree on the maximum and minimum dependence countries, France–Germany and Japan–USA. Panel 2 shows the East Asian results. For the full sample average dependence is much smaller than in the G5, at 0.385. For the first sample, the average dependence is 0.324, which rises substantially to 0.530 in the second sample. In East Asia the two dependence measures agree, except in the latter period, on which countries have the highest dependence. Panel 3 reports the Latin American results. For the full sample the average is 0.414. In the first sample, the average is 0.398, increasing to 0.447 in the late sample. The two dependence measures agree on which countries afford lowest and worst dependence, except for the highest dependence in the full sample. Further, in the second sample there is a switch in countries with minimal dependence from Brazil–Chile to Argentina–Chile.

To summarize Table 7, over time average dependence has increased for each region. East Asian economies have the lowest average dependence for the full sample and early periods, while Latin America dominates for the later period. Similarly, East Asia possesses the lowest dependence pair for the full and early samples, while Latin America does so for the later sample. These results hold regardless of whether we measure dependence with symmetric or asymmetric copulas. In both East Asia and Latin America, there is some disagreement on which countries have largest dependence, and in Latin America, there is a switch in the countries with the highest and lowest dependence. Economically speaking, our copula results suggest that in recent history an international investor has had difficulty ascertaining which developing markets are the worst diversifiers, but also had more clarity about the best opportunities in East Asia and Latin America. The switch in Latin America, and disagreement of dependence measures provide some evidence on correlation complexity, which could reduce the aforementioned diversification opportunities.[57]

4.3. Comparing correlation and copula results

We summarize the results from correlations in Section 4.1 and copulas in Section 4.2.2. Although it is tempting to discuss the results in terms of diversification, in general diversification relates to both dependence and marginal properties. Therefore as mentioned

[56] The Rotated Gumbel dependence parameter α ranges from 0 to 1, with 1 reflecting independence and 0 reflecting maximal dependence. Thus for the Rotated Gumbel, dependence increases as α falls. In addition to ρ_t, the t copula also has another parameter, the degree of freedom (DOF), which increases with the thinness of the tails. We do not report this since we are only interested in dependence. Estimates of DOF as well as individual country pairs are available from the authors upon request.

[57] This Latin American shift may reflect changing economic policies in the aftermath of recent political and economic crises.

in the beginning of Section 2, all our statements about diversification are with the caveat that marginals are fixed. Both correlation and copula results agree that dependence has increased over time in each region. They also agree that the lowest average dependence for the full sample and early period are for East Asia, and for Latin America in the latter period. The correlation approach gives ambiguous results for the full sample but copulas definitely select East Asia as the best diversification region. Both approaches agree that in the early and late periods, the countries with minimal dependence are in East Asia and Latin America, respectively. However, both copulas and correlations show dependence uncertainty, given that the dependence measures sometimes disagree in Latin America and East Asia. This suggests as in our Section 2 discussion, that these countries are prone to systemic risk because of correlation complexity. Although both dependence approaches capture the switch in Latin America, correlations are again ambiguous on the specific countries, while copula-based estimates agree.

More broadly, our results show that correlation signals agree for G5, but not for markets in East Asia and Latin America. This empirical evidence bolsters the theoretical reasons of Embrechts et al. (2002) for using more robust dependence measures in risk management. Comparatively speaking, East Asia and G5 each have only one channel for diversification problems, correlation complexity and downside risk, respectively. By contrast, Latin America is susceptible to through two channels, correlation complexity and downside risk.

4.4. Further perspectives on heavy tails and dependence

We have focused thus far on copulas and correlations. Another complementary perspective concerns direct estimates of tail indices and tail dependence. Regarding tail indices, these parameters measure the thickness of individual tails. As discussed in Section 2, heavy tails have been theoretically linked to failure of diversification and systemic risk.[58] As we discussed in Section 3, tail dependence measures the likelihood of joint down moves during extreme periods, which is evidently a useful quantity for risk managers and policymakers to estimate.

Both heavy tails and tail indices relate to an important regularity in economics, the concept of *power laws*. Consider two variables of interest X and C. Then, as presented by Gabaix (2009), a power law is a relation of the form $C = hX^{\alpha}$, for some unimportant constant h. The quantity α is called the power law exponent and controls extreme behavior of the particular distribution. For example, income research has documented that the proportion of individuals with wealth X above a certain threshold x satisfies the following relationship: $\Pr(X > x) \sim \frac{C}{x^{\alpha}}$, where $\alpha \approx 1$. Power laws are ubiquitous in economics and a source of important new theoretical and empirical research.

4.4.1. Heavy tails

Evidence of heavy-tailed marginals lends further support to the importance of using copulas to separate the analysis of marginals from their dependence. Estimation of heavy-tailedness is conducted using the concept of tail index, which is the same as the power law exponent above.[59] Assume that returns r_t are serially independent with a common distribution function $F(x)$. Consider a sample of size $T > 0$ and denote the sample order statistics as

$$r_{(1)} \leqslant r_{(2)} \leqslant \cdots \leqslant r_{(T)}.$$

Then the asymptotic distribution of the smallest returns $r_{(1)}$, written as $F_1(x)$, can be shown to satisfy

$$\begin{aligned} F_1(x) &= \left\{1 - \exp\left[-(1+kx)^{\frac{1}{k}}\right]\right\}, \quad \text{if } k \neq 0, \\ &= \{1 - \exp[x]\}, \quad \text{if } k = 0. \end{aligned} \tag{9}$$

The parameter k governs the tail behavior of the distribution. It is often more useful to examine the **tail index**α,defined as $\alpha = -1/k$. The distribution will have at most i moments, for $i \leqslant \alpha$. For example, if $\alpha$is estimated to be 1.5, the data will only have well-defined means, but not variances. Thus, the smaller the tail index, the heavier the tails of the particular asset returns. We consider two methods to estimate the tail index. The first, due to Hill (1975), is denoted α^H, and estimated as

$$\alpha^H = -\frac{1}{q}\sum_{i=1}^{q}\{\ln[|-r_{(i)}|] - \ln[|-r_{(q+1)}|]\} \tag{10}$$

where q is a positive integer, as in Chapter 7 of Tsay (2002). The Hill estimator is asymptotically normal, and consistent if q is chosen appropriately.[60]

A more recent estimator is due to Gabaix and Ibragimov (2010), and derived in the following manner. First, arrange the variables in decreasing order, as

$$z_{(1)} \geqslant z_{(2)} \geqslant \cdots \geqslant z_{(T)}.$$

Then, Gabaix and Ibragimov (2010) show that an optimal estimate of the tail index α is obtained from the b's in the regression below:

$$\ln\left(\text{Rank} - \frac{1}{2}\right) = a + b\ln(\text{Size}), \tag{11}$$

where Rank denotes the order of observation t and Size denotes $z_{(t)}$. The authors demonstrate the optimality of their estimator using both parametric and simulation methods. They also construct asymptotic standard errors.

We now discuss our estimates of tail indices, using the two methods outlined above. First, Table 8 shows Hill estimates. These results include right and left tail indices corresponding to the 5%, 7.5% and 10% most extreme observations in the distribution. Since the results are broadly similar at all the percentiles, we focus on the 5% estimates. The main finding is that all the estimates are between 2.2 and 4.0. Given the relatively small standard errors, the data all appear to have first moments. However, the evidence on moments of the second order or greater is mixed. For example Japan has a left tail index (4) that is statistically larger than two at conventional significance levels. By contrast, Brazil's tail index of 2.26 is not statistically larger than two. Table 9 displays tail index estimates using the log–log rank approach of Gabaix and Ibragimov (2010). Qualitatively, these results tend to agree with those of Table 8. In particular, at the 5% cutoff, both right and left tail indices are between 2.7 and 5.2 and significant at conventional levels. Consequently, according to this latter methodology, all the data appear to have both first and second moments. The finding that tail indices are generally between 2 and 4 accords with previous empirical research on heavy-tailed security return distributions in other markets, such as in the research of Loretan and Phillips (1994) and Gabaix et al. (2003). To summarize our results, when we estimate the tail index using both methods, the data all appear to have first moments, with mixed evidence on second moments. Therefore, these results indicate that use of copulas to estimate the dependence structure is potentially valuable.

[58] See Embrechts et al. (2005), Ibragimov and Walden (2007), Ibragimov et al. (2009b), and Ibragimov et al. (2009a).

[59] The material on tail indices follows the exposition of Tsay (2002), and Gabaix and Ibragimov (2010).

[60] For more details, see Tsay (2002), Embrechts et al. (2005), and de Haan and Ferreira (2006).

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Table 8
Tail index measured by the Hill estimator.

	Left tail			Right tail		
	5%	7.5%	10%	5%	7.5%	10%
FR	2.78	2.30	2.45	3.09	3.29	3.17
	(0.43)	(0.29)	(0.27)	(0.48)	(0.42)	(0.35)
DE	2.76	2.33	2.15	3.36	3.18	3.12
	(0.43)	(0.30)	(0.24)	(0.52)	(0.40)	(0.34)
JP	4.00	3.14	2.80	3.16	2.72	2.44
	(0.62)	(0.40)	(0.31)	(0.49)	(0.34)	(0.27)
UK	3.09	3.22	2.76	3.64	3.04	3.15
	(0.48)	(0.41)	(0.30)	(0.56)	(0.39)	(0.35)
US	3.31	3.05	2.25	3.48	3.00	2.37
	(0.51)	(0.39)	(0.25)	(0.54)	(0.38)	(0.26)
HK	2.42	2.17	2.07	3.82	3.14	3.39
	(0.37)	(0.28)	(0.23)	(0.59)	(0.40)	(0.37)
KR	2.86	2.60	2.49	2.79	2.53	2.58
	(0.44)	(0.33)	(0.27)	(0.43)	(0.32)	(0.28)
SI	2.79	2.11	2.21	3.71	2.97	2.62
	(0.43)	(0.27)	(0.24)	(0.57)	(0.38)	(0.29)
TW	2.67	2.80	2.59	2.62	2.62	2.43
	(0.41)	(0.36)	(0.28)	(0.40)	(0.33)	(0.27)
TH	3.44	2.69	2.08	3.14	3.16	2.37
	(0.53)	(0.34)	(0.23)	(0.48)	(0.40)	(0.26)
AR	3.51	2.92	2.55	3.18	2.52	2.08
	(0.54)	(0.37)	(0.28)	(0.49)	(0.32)	(0.23)
BR	2.26	2.36	1.95	3.00	2.60	2.79
	(0.35)	(0.30)	(0.21)	(0.46)	(0.33)	(0.31)
CH	2.92	2.74	2.65	3.23	2.99	2.41
	(0.45)	(0.35)	(0.29)	(0.50)	(0.38)	(0.26)
ME	2.62	2.50	2.26	2.94	2.70	2.41
	(0.40)	(0.32)	(0.25)	(0.45)	(0.34)	(0.26)

The table presents estimates of right and left tail indices for each series, corresponding to the 5%, 7.5%, and 10% most extreme observations in the distribution. The tail index is estimated using the non-parametric estimator of Hill (1975). Standard errors, in parentheses, are calculated using the asymptotic variance of the Hill estimator, and obtained by the Delta method.

Table 9
Tail index measured by OLS log-log rank-size regression.

	Left tail			Right tail		
	5%	7.5%	10%	5%	7.5%	10%
FR	3.48	3.00	2.77	3.21	3.21	3.20
	(0.76)	(0.54)	(0.43)	(0.70)	(0.58)	(0.50)
DE	3.61	3.14	2.78	3.63	3.49	3.39
	(0.79)	(0.56)	(0.43)	(0.79)	(0.63)	(0.53)
JP	5.11	4.32	3.71	3.67	3.44	3.10
	(1.12)	(0.78)	(0.58)	(0.80)	(0.62)	(0.48)
UK	3.84	3.51	3.30	3.28	3.33	3.29
	(0.84)	(0.63)	(0.51)	(0.72)	(0.60)	(0.51)
US	3.79	3.51	3.12	3.89	3.57	3.15
	(0.83)	(0.63)	(0.48)	(0.85)	(0.64)	(0.49)
HK	3.26	2.74	2.52	4.44	4.05	3.76
	(0.71)	(0.49)	(0.39)	(0.97)	(0.73)	(0.58)
KR	2.78	2.74	2.71	3.77	3.14	2.94
	(0.61)	(0.49)	(0.42)	(0.82)	(0.56)	(0.46)
SI	3.13	2.78	2.52	3.71	3.64	3.34
	(0.68)	(0.50)	(0.39)	(0.81)	(0.65)	(0.52)
TW	3.17	3.02	2.88	3.33	3.06	2.89
	(0.69)	(0.54)	(0.45)	(0.73)	(0.55)	(0.45)
TH	4.33	3.57	3.01	3.34	3.28	3.05
	(0.94)	(0.64)	(0.47)	(0.73)	(0.59)	(0.47)
AR	3.73	3.40	3.21	3.50	3.18	2.80
	(0.81)	(0.61)	(0.50)	(0.76)	(0.57)	(0.43)
BR	2.88	2.60	2.46	4.01	3.28	3.06
	(0.63)	(0.47)	(0.38)	(0.87)	(0.59)	(0.47)
CH	3.23	3.06	2.98	3.75	3.49	3.19
	(0.71)	(0.55)	(0.46)	(0.82)	(0.63)	(0.49)
ME	2.83	2.70	2.59	3.49	3.21	2.94
	(0.62)	(0.49)	(0.40)	(0.76)	(0.58)	(0.46)

The table presents estimates of right and left tail indices for each series, corresponding to the 5%, 7.5%, and 10% most extreme observations in the distribution. The tail index is estimated using the log–log rank-size estimator of Gabaix and Ibragimov (2010). Standard deviations are in parentheses.

4.4.2. Tail dependence and Kendall's τ

In Section 3, we discussed that tail dependence measures comovement of assets at the extremes. We can estimate tail dependence using analytical relations with copulas. These relations are useful since they present a theoretical benchmark for our previous estimates. Moreover, a useful dependence concept is Kendall's τ, which measures the difference between positive and negative dependence: $\tau(X,Y) = P[(X-\tilde{X})(Y-\tilde{Y}) > 0] - P[(X-\tilde{X})(Y-\tilde{Y}) < 0]$, where the tildes denote independent copies of the relevant random variable. Although Kendall's τ is not widely applied in economics, it is nevertheless useful since, unlike correlations, it is always defined even if variances are infinite.[61] Table 10 shows the analytical relations between copulas, tail dependence, and τ. Further, Table 11 presents the corresponding empirical calibrations of tail dependence and τ. These results are broadly similar to our correlation and copula estimates from above.[62] Regarding tail dependence, there is some evidence of disagreement. For example, in East Asia the minimal left tail dependence occurs for Taiwan–Thailand according to both the Rotated Gumbel and Clayton copulas. However, the Student-t copula chooses Korea–Thailand as the minimum. Regarding Kendall's τ, the G5 and East Asian estimates agree on maximal and minimal dependence, for all calibrations. However, for Latin America, the Rotated Gumbel and Clayton copulas deliver Brazil–Mexico as the maximally dependent, while the other calibrations indicate Argentina–Mexico. This disagreement of dependence measures supports our previous results on correlation complexity.

Table 10
Tail dependence and Kendall's τ for various copulas.

	Left tail dep.	Right tail dep.	Kendall's τ
Gaussian	0	0	$\frac{2}{\pi}\arcsin\rho$
Student-t	$2t_{d+1}\left(-\sqrt{\frac{(d+1)(1-\rho)}{1+\rho}}\right)$	$2t_{d+1}\left(-\sqrt{\frac{(d+1)(1-\rho)}{1+\rho}}\right)$	$\frac{2}{\pi}\arcsin\rho$
Gumbel	0	$2-2^{\alpha}$	$1-\alpha$
R. Gumbel	$2-2^{\alpha}$	0	$1-\alpha$
Clayton	0	$2^{-\frac{1}{\theta}}$	$\frac{\theta}{\theta-2}$
R. Clayton	$2^{-\frac{1}{\theta}}$	0	$\frac{\theta}{\theta-2}$

The table presents analytical formulas for tail dependence and Kendall's τ. Further information may be obtained from Chapter 5 of Embrechts et al. (2005).
R. Clayton and R. Gumbel denote the Rotated Clayton and Rotated Gumbel copulas. α and θ denote dependence parameters of the Gumbel and Clayton copulas, and d denotes the degrees of freedom of the Student-t copula.

5. Implications for international finance

We noted in Section 3 that higher dependence corresponds to reduced ability to avoid downside risk, for a given set of marginals.

[61] For more details on Kendall's τ, see Embrechts et al. (2005, Chapters 3 and 5).
[62] See Tables 3 and 7.

Table 11
Tail dependence Kendall's τ from different copula models.

Model	Left tail dependence			Right tail dependence			Kendall's τ		
	Ave	Max	Min	Ave	Max	Min	Ave	Max	Min
Panel A: G5									
Gumbel	0.0000			0.4016	0.6177 (FR–DE)	0.2344 (JP–US)	0.3279	0.5329 (FR–DE)	0.1798 (JP–US)
R. Gumbel	0.4200	0.6398 (FR–DE)	0.2434 (JP–US)	0.0000			0.3447	0.5562 (FR–DE)	0.1872 (JP–US)
Clayton	0.4238	0.6949 (FR–DE)	0.1680 (JP–US)	0.0000			0.3000	0.4878 (FR–DE)	0.1627 (JP–US)
R. Clayton	0.0000			0.3579	0.6383 (FR–DE)	0.1359 (JP–US)	0.2625	0.4357 (FR–DE)	0.1479 (JP–US)
Normal	0.0000			0.0000			0.3534	0.5491 (FR–DE)	0.1949 (JP–US)
Student-t	0.1276	0.4658 (FR–DE)	0.0036 (DE–US)	0.1276	0.4658 (FR–DE)	0.0036 (DE–US)	0.3591	0.5625 (FR–DE)	0.2000 (JP–US)
Panel B: East Asia									
Gumbel	0.0000			0.2868	0.4147 (HK–SI)	0.2031 (TW–TH)	0.2242	0.3353 (HK–SI)	0.1545 (TW–TH)
R. Gumbel	0.3058	0.4449 (HK–SI)	0.2261 (TW–TH)	0.0000			0.2403	0.3630 (HK–SI)	0.1731 (TW–TH)
Clayton	0.2681	0.4773 (HK–SI)	0.1581 (TW–TH)	0.0000			0.2103	0.3191 (HK–SI)	0.1582 (TW–TH)
R. Clayton	0.0000			0.2681	0.4773 (HK–SI)	0.1581 (TW–TH)	0.1804	0.2603 (HK–SI)	0.1248 (TW–TH)
Normal	0.0000			0.0000			0.2499	0.3577 (HK–SI)	0.1836 (TW–TH)
Student-t	0.0557	0.2246 (HK–SI)	0.0031 (KR–TH)	0.0557	0.2246 (HK–SI)	0.0031 (KR–TH)	0.2523	0.3678 (HK–SI)	0.1835 (TW–TH)
Panel C: Latin America									
Gumbel	0.0000			0.3075	0.3615 (AR–ME)	0.2562 (AR–CH)	0.2411	0.2877 (AR–ME)	0.1978 (AR–CH)
R. Gumbel	0.3447	0.3912 (BR–ME)	0.2897 (AR–CH)	0.0000			0.2733	0.3140 (BR–ME)	0.2258 (AR–CH)
Clayton	0.3526	0.4330 (BR–ME)	0.2769 (AR–CH)	0.0000			0.2503	0.2928 (BR–ME)	0.2125 (AR–CH)
R. Clayton	0.0000			0.2144	0.3080 (AR–ME)	0.1418 (AR–CH)	0.1839	0.2274 (AR–ME)	0.1507 (AR–CH)
Normal	0.0000			0.0000			0.2719	0.3123 (AR–ME)	0.2331 (AR–CH)
Student-t	0.1207	0.1527 (AR–BR)	0.0617 (AR–CH)	0.1207	0.1527 (AR–BR)	0.0617 (AR–CH)	0.2723	0.3167 (AR–ME)	0.2267 (AR–CH)

Other things being equal, investors should therefore demand higher returns to compensate for increased dependence.[63]

5.1. Relationship between returns and dependence

If investors require higher returns for higher dependence, it is natural to explore which of our dependence measures more closely relates to returns over our sample period. Table 12 displays the relation between average returns and average dependence measures in each region. For simplicity each variable is ranked from low (L) to high (H). Panel A shows the results for the full sample. Regarding dependence, even though the G5 always has the highest dependence by both measures, it never has the highest returns. Indeed, the G5 have the very lowest returns in the latter sample. Regarding return patterns, the Latin American region always has the very largest returns, sometimes double the return of other regions. Nevertheless, its dependence is never highest – indeed it is the lowest in the latter period. However, the East Asian link to returns is clearer: it is the lowest dependence region for the early and full sample and earns lowest returns. When it switches to median dependence in the late sample, this is matched by a concommitant switch to median returns.

To summarize, the results indicate no monotonic relationship between any dependence measure and returns. Indeed, from 2001 to 2006, Latin America exhibits both highest returns and the lowest dependence, while the G5 have the lowest returns and highest dependence. This finding is inconsistent with the notion that investors are averse to downside risk exposure. Such an outcome might arise in the framework of Ibragimov and Walden (2007), where anomalously large returns accompany heavy-tailed data. The fact that the region with light tails is the only one with agreement in ranks for dependence and returns is also consistent

[63] A classic example in finance is the CAPM, which under some conditions, says that for any stock i, its return R_i depends on its dependence (covariance) with the market return R_m:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f], \quad (12)$$

where $\beta = \mathrm{Cov}(R_m, R_i)/\mathrm{Var}(R_m)$. Therefore, the greater its dependence with the market, the higher an asset's own return.

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Table 12
Regional returns and international dependence.

	Return	World beta	ρ	ρ_t
Panel A: Full Sample				
East Asia	2.68 (L)	0.416 (L)	0.406 (L)	0.385 (L)
G5	5.35 (M)	0.739 (H)	0.545 (H)	0.525 (H)
Latin	13.24 (H)	0.426 (M)	0.414 (M)	0.414 (M)
Panel B: 1990–2001				
East Asia	−1.00 (L)	0.358 (L)	0.379 (L)	0.324 (L)
G5	6.31 (M)	0.701 (H)	0.487 (H)	0.469 (H)
Latin	13.15 (H)	0.370 (M)	0.416 (M)	0.398 (M)
Panel C: 2001–2006				
East Asia	10.19 (M)	0.537 (L)	0.511 (M)	0.530 (M)
G5	3.38 (L)	0.812 (H)	0.637 (H)	0.641 (H)
Latin	13.43 (H)	0.544 (M)	0.423 (L)	0.447 (L)

The table presents average returns and average dependence for different regions. The world beta is computed on filtered returns, in similar fashion to Eq. (12). L, M, and H denote the lowest, middle, and highest returns or dependence, compared across regions. ρ and ρ_t denote the Pearson correlation and the dependence parameter for the Student-t copula, respectively.

with this view. Our findings, while suggestive and related to theoretical work on investor behavior during exuberant or costly-information times, are evidently preliminary.[64] These considerations may merit further study in a conditional setting with a wider group of countries.

6. Conclusions

Diversification carries benefits and costs, as noted by a growing body of theoretical literature. Although diversification is measured by dependence when marginals are fixed, few studies compare asset dependence using robust dependence measures. Moreover, when assets have heavy tails, diversification may not be optimal, and individually optimal diversification may differ from social optimality since investors undervalue systemic risk. These observations motivate our empirical study. We examine diversification opportunities in international markets, using two different dependence measures, correlations and copulas.

Empirically, we document several findings. First, although correlations and copulas often agree, they deliver different risk management signals for countries with maximal risk of being undiversified. Further, our analysis indicates some evidence of heavy tails in the data and stronger evidence of infinite fourth moments. These results bolster extant theoretical reasons for using robust dependence measures in risk management. Second, both measures agree that dependence has increased over time for all regions. Third, in our distributional tests we document asymmetric dependence for Latin American countries, which has the interpretation of downside risk for investors. There is less downside risk in the G5, and very little evidence of downside risk in East Asia, a finding that to the best of our knowledge is previously undocumented. Fourth, over our sample period, Latin America experiences a switch between the best and worst dependence countries. Finally, the dependence measures disagree on which countries have largest and smallest diversification benefits, which provides evidence of correlation complexity in East Asia and Latin America. In economic terms, an investor enjoys the largest diversification benefits in East Asian and Latin America, but has difficulty identifying the most risky country pairs therein.

More broadly, the fact that return distributions exhibit heavy tails with correlation complexity implies that they not only represent limited diversification, they are also consistent with the possibility of a wedge between investor diversification and international systemic risk. Such aggregate implications are largely absent from previous empirical research on diversification and dependence in international markets. In a simple application, we find no link between largest dependence and regional stock returns, although the low-dependence region of East Asia always shows matching returns. This latter finding relates to theoretical literature on investor behavior during extreme, information-constrained periods, and suggests that international investors are not compensated for exposure to downside risk.

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[64] For related theoretical work, see Abreu and Brunnermeier (2003), Pavlov and Wachter (2006), and Veldkamp (2006).

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