

# Mechanical Dynamics, the Swing Equation, Units

## 1.0 Preliminaries

The basic requirement for generator operation is that they must remain “in synchronism.” This means that all generators must have mechanical speeds so as to produce the same “electrical speed.”

Electrical speed and mechanical speed are related as a function of the number of machine poles,  $p$ , or pole-pairs,  $p/2$ .

If  $p=2$ , as in Fig. 1, then there is one magnetic rotation for every one mechanical rotation, i.e., the stator windings see one flux cycle as the rotor turns once.

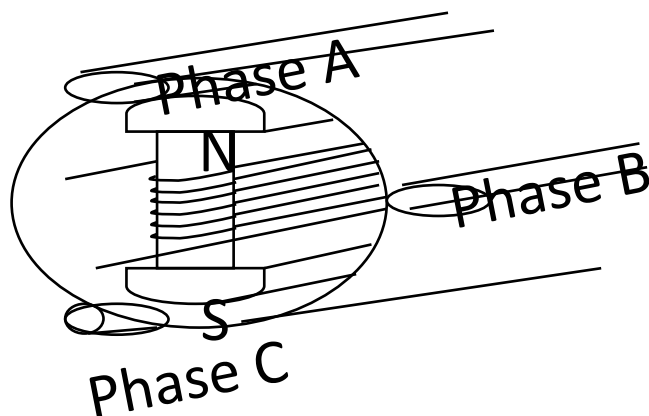


Fig. 1

If  $p=4$ , as in Fig. 2, there are two magnetic rotations for every one mechanical rotations, i.e., the stator windings see two flux cycles as the rotor turns once.

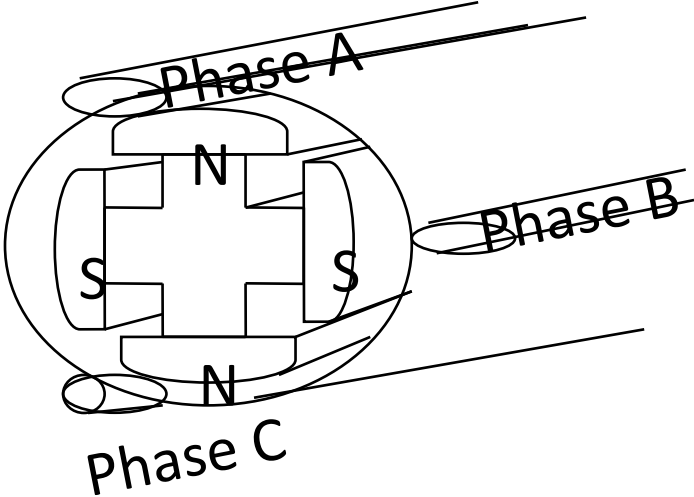


Fig. 2

Therefore, the electrical speed,  $\omega_e$ , will be greater than (if  $p \geq 4$ ) or equal to (if  $p=2$ ) the mechanical speed  $\omega_m$  according to the number of pole-pairs  $p/2$ , i.e.,

$$\omega_e = \frac{p}{2} \omega_m \tag{1}$$

The adjustment for the number of pole-pairs is needed because the electrical quantities (voltage and current) go through one rotation for every one magnetic rotation.

So to maintain synchronized “electrical speed” (frequency) from one generator to another, all synchronous generators must maintain constant mechanical speed. This does not mean all generators have the same mechanical speed, but that their mechanical speed must be constant.

All two-pole machines must maintain

$$\omega_m = (2/2)\omega_e = (2/2)377 = 377 \text{ rad/sec}$$

We can also identify the mechanical speed of rotation in rpm according to

$$N_m = \omega_m \frac{\text{rad}}{\text{sec}} \times \frac{60 \text{ sec/min}}{2\pi \text{ rad/rev}} \quad (2)$$

Substituting for  $\omega_m$  from (1), we get:

$$N_m = \omega_e \frac{2 \text{ rad}}{p \text{ sec}} \times \frac{60 \text{ sec/min}}{2\pi \text{ rad/rev}} \quad (3)$$

Using this expression, we see that a 2-pole machine will have a mechanical synchronous speed of 3600 rpm, and a 4-pole machine will have a mechanical synchronous speed of 1800 rpm.

No. of Poles (p)	Synchronous speed (Ns)
2	3600
4	1800
6	1200
8	900
10	720
12	600
14	514
16	450
18	400
20	360

## 2.0 Causes of rotational velocity change

Because of the synchronism requirement, we are concerned with any conditions that will cause a change in rotational velocity.

But what is “a change in rotational velocity”?

➔ It is acceleration (or deceleration).

What are the conditions that cause acceleration (+ or -)?

To answer this question, we must look at the mechanical system to see what kind of “forces” that are exerted on it.

Recall that with linear motion, acceleration occurs as a result of a body experiencing a “net” force that is non-zero. That is,

$$a = \frac{F}{m} \quad (4)$$

where  $a$  is acceleration (m/sec<sup>2</sup>),  $F$  is force (newtons), and  $m$  is mass (kg). Here, it is important to realize that  $F$  represents the sum of all forces on the body. This is Newton's second law of motion.

The situation is the same with rotational motion, except that here, we speak of torque  $T$  (newton-meters), inertia  $J$  (kg-m<sup>2</sup>), and angular acceleration  $A$  (rad/sec<sup>2</sup>) instead of force, mass, and acceleration. Specifically,

$$A = \frac{T}{J} \quad (5)$$

Here, as with  $F$  in the case of linear motion,  $T$  represents the “net” torque, or the sum of all torques acting on the rotational body.

It is conceptually useful to remember that the torque on a rotating body experiencing a force a distance  $r$  from the center of rotation is given by

$$\vec{T} = \vec{r} \times \vec{F} \quad (6)$$

where  $\vec{r}$  is a vector of length  $r$  and direction from center of rotation to the point on the body where the force is applied,  $\vec{F}$  is the applied force vector, and the “ $\times$ ” operation is the vector cross product. The magnitudes are related through

$$T = rF \sin \gamma \quad (7)$$

where  $\gamma$  is the angle between  $\vec{r}$  and  $\vec{F}$ . If the force is applied tangential to the body, then  $\gamma=90^\circ$  and  $T=rF$ .

Let’s consider that the rotational body is a shaft connecting a turbine to a generator, illustrated in Fig. 3.

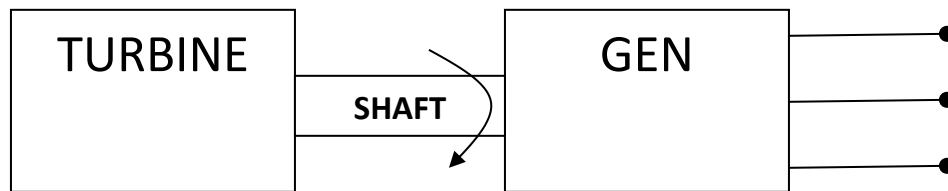


Fig. 3

For purposes of our discussion here, let’s assume that the shaft is rigid (inelastic, i.e., it does not flex), and let’s ignore frictional torques.

What are the torques on the shaft?

- From turbine: The turbine exerts a torque in one direction (assume the direction shown in Fig. 3) which causes the shaft to rotate. This torque is mechanical. Call this torque  $T_m$ .
- From generator: The generator exerts a torque in the direction opposite to the mechanical torque which retards the motion caused by the mechanical torque. This torque is electromagnetic. Call this torque  $T_e$ .

These two torques are in opposite directions. If they are exactly equal, there can be no angular acceleration, and this is the case when the machine is in synchronism, i.e.,

$$T_m = T_e \quad (8)$$

When (8) does not hold, i.e., when there is a difference between mechanical and electromagnetic torques, the machine accelerates (+ or -), i.e., it will change its velocity. The amount of acceleration is proportional to the difference between  $T_m$  and  $T_e$ . We will call this difference the accelerating torque  $T_a$ , i.e.,

$$T_a = T_m - T_e \quad (9)$$

The accelerating torque is defined positive when it produces acceleration in the direction of the applied mechanical torque, i.e., when it increases angular velocity (speeds up).

Now we can ask our original question (page 4) in a somewhat more rigorous fashion:

- Given that the machine is initially operating in synchronism ( $T_m = T_e$ ), what conditions can cause  $T_a \neq 0$ ?

There are two broad types of changes: change in  $T_m$  and change in  $T_e$ . We examine both of these carefully.

### 1. Change in $T_m$ :

- a. Intentionally: through change in steam valve opening, with  $T_m$  either increasing or decreasing.
- b. Disruption in steam flow: typically a decrease in  $T_m$  causing the generator to experience negative acceleration (it would decelerate).

### 2. Change in $T_e$ :

- a. Increase in load: this causes an increase in  $T_e$ , and the generator experiences negative acceleration.



b. Decrease in load: this causes a decrease in  $T_e$ , and the generator experiences positive acceleration.

All of the above changes, 1-a, 1-b, 2-a, and 2-b are typically rather slow, and the generator's turbine-governor will sense the change in speed and compensate by changing the steam flow appropriately.

There is a third way that  $T_e$  can change, that is not slow.

c. Faults: We discuss this in the next section.

### 3.0 Generator under faulted conditions: qualitative

Consider the circuit of Fig. 4. All quantities are in per-unit.

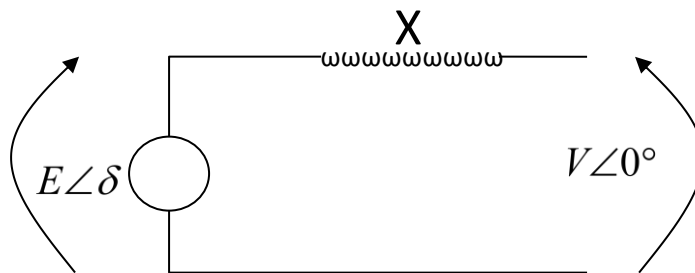


Fig. 4

Here, the voltage  $E \angle \delta$  represents the internal voltage of a synchronous machine and the voltage  $V \angle 0$  represents the terminal voltage of the machine. We are assuming

balanced conditions and therefore we utilize the per-phase equivalent circuit for analysis of the three phase machine.

Assuming a round-rotor machine, we may apply  $S = \bar{V} \bar{I}^*$ , express  $\bar{I}$  in terms of the two voltages using Ohm's law, and then take the real part to show that the steady-state real power supplied at the machine terminals is given by

$$P_e = \frac{EV}{X} \sin \delta \quad (10)$$

Let's assume that a three-phase fault occurs at the machine terminals, so that  $V=0$ .

Then clearly, by (10),  $P_e=0$ .

Recall that torque and power are related by

$$T_e = \frac{P_e}{\omega_m} \quad (11)$$

And so if  $P_e=0$ , it must be the case also that  $T_e=0$ .

By (9), then  $T_a=T_m$ , which means that **all** mechanical torque is being used to accelerate the machine. This is a

very severe situation in that the machine will accelerate at a very high rate.

Of course, faults at the machine terminals are very rare (although they do occur occasionally). Most faults are not so severe in that they occur somewhere in the network rather than at the machine terminals. But even for network faults, the voltage  $V$  at the machine terminals is reduced in magnitude, causing  $P_e$  and therefore  $T_e$  to reduce, causing an imbalance between  $T_m$  and  $T_e$  and therefore a non-zero accelerating torque  $T_a$ .

There are two main influences on the amount of overspeed seen by a synchronous generator under faulted conditions.

- The amount of reduction in  $T_e$ : The greater is the electrical distance between the fault point in the network and the machine terminals, the less will be the reduction on  $V$ , and consequently, the less will be the reduction on  $P_e$  (see (10)) and also  $T_e$  (see (11)). The fault location is something we cannot control of course. But there is another way to prevent reduction in  $V$ , and that is through excitation control. Today's

excitation systems are very fast responding so that terminal voltage reduction is sensed and field current is boosted within just a few cycles following a faulted condition.

- The amount of time that  $T_e$  is reduced: The longer this time, the more the machine accelerates. So we try to minimize this time; this is achieved by removing the faulted condition very quickly. EHV protection systems are typically able to sense and clear a fault within 4 cycles ( $4/60=.0667$  seconds).

This discussion shows that the mechanical dynamics associated with the acceleration of the generator are related intimately to the effect on  $T_e$  of the fault. Such effects can only be properly ascertained by analysis of the network before, during, and after the faulted condition.

In the next section, we therefore derive the relationship between the mechanical dynamics and the electric network.

#### **4.0 Derivation of swing equation**

We begin with (5), repeated here for convenience.

$$A = \frac{T}{J} \quad (5)$$

where we recall that  $T$  is the “net” torque on the rotating body.

We will write the angular acceleration in terms of the angle  $\theta$ , which is here defined as the “absolute angle,” in radians. It gives the position of a tic-mark on the (rotating) shaft relative to a fixed point on the generator. If we make that “tick mark” coincident with the rotor axis, the situation will appear as illustrated in Fig. 5.

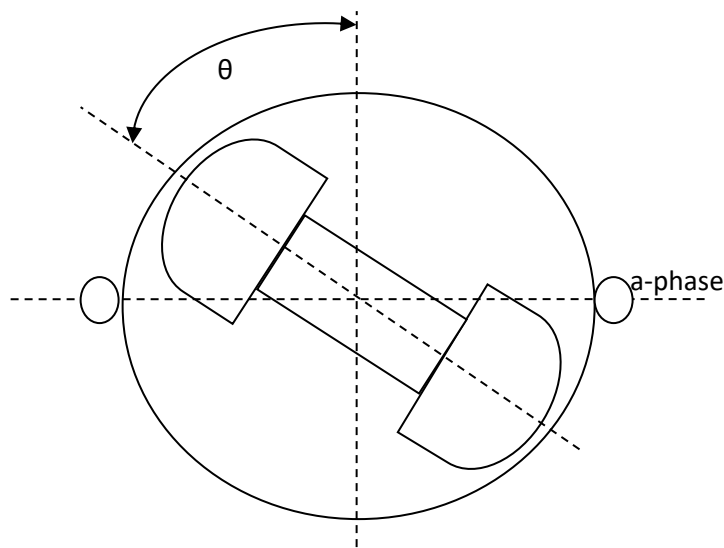


Fig. 5

We can express angular acceleration as  $A = \ddot{\theta} = d^2\theta / dt^2$ , i.e., angular acceleration is the 2<sup>nd</sup> time derivative of  $\theta$ .

Noting that  $T_a$  is the “net” torque on the turbine-generator shaft, we have

$$\ddot{\theta} = \frac{T_a}{J} \quad (12)$$

Here,  $J$  is the moment of inertia of the combined turbine-generator set, in kg-m<sup>2</sup>.

We also define  $\omega_R$  as the rated mechanical angular velocity of the shaft, in rad/sec and note that

$$\omega_R = \frac{\omega_e}{p/2} \quad (13)$$

where  $p$  is the number of poles, as before.

We now define a synchronously rotating reference frame as:

$$\theta_{ref} = \omega_R t + \alpha \quad (14a)$$

where  $\alpha$  is the initial angle at  $t=0$ ; it allows us to position our reference angle  $\theta_{ref}$  wherever we want. We will position it so that it is numerically equal to the angle of the (spatial peak of) resultant flux in the air-gap. We will call this flux  $\boldsymbol{\varphi}_r$ . This flux results from two constituent fluxes:

- The flux from the field circuit; we note this flux as  $\varphi_f$ .
- The composite flux from the currents in the three stator windings, called the flux of armature reaction. We denote this flux as  $\varphi_{ar}$ .

Then we have that

$$\varphi_r = \varphi_f + \varphi_{ar} \quad (14b)$$

Let's define the rotor mechanical *torque angle*,  $\delta_m$ . This is the angle by which the rotor leads the synchronously rotating reference. Since the rotor is in phase with the flux it produces,  $\varphi_f$ , this angle is also the angle of  $\varphi_f$ .

Since

- we have chosen  $\alpha$  so that the synchronously rotating reference is coincident with the resultant flux in the air-gap,  $\varphi_r$ , and
- we know the rotor leads the resultant flux in the air gap (and thus the synchronously rotating reference  $\theta_{ref}$ ) then

we can draw the relationship between these fluxes, as shown in Fig. 6a.

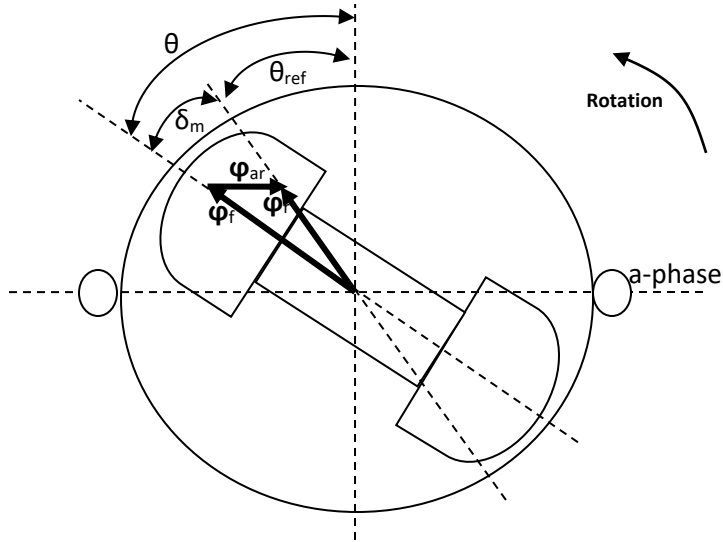


Fig. 6a

Note from (14a) (which is  $\theta_{ref} = \omega_R t + \alpha$ ), that

$$\dot{\theta}_{ref} = \omega_R \quad (15)$$

The implication of (15) is that the reference *speed* is constant, no matter what happens to the rotor.

Recall that each flux  $\varphi_r$ ,  $\varphi_f$ , and  $\varphi_{ar}$  will induce a distinct voltage in the a-phase winding. These voltages are denoted by phasor representations  $\mathbf{V}$ ,  $\mathbf{E}_f$ , and  $\mathbf{V}_{ar}$ , respectively. By Faraday's law, these voltages each lag



their respective fluxes by  $90^\circ$ ; knowing this, we can add the voltages to our vector diagram as shown in Fig. 6b.

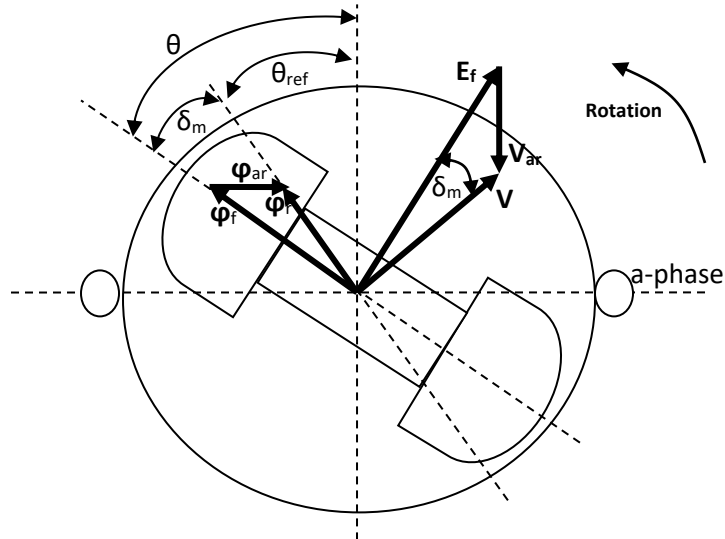


Fig. 6b

From Fig. 6b, one observes that

$$\delta_m = \theta - \theta_{ref} \quad (16)$$

From (16) we can write that

$$\theta = \theta_{ref} + \delta_m \quad (17)$$

Substituting (14a) ( $\theta_{ref} = \omega_R t + \alpha$ ) into (17) yields

$$\theta = \omega_R t + \alpha + \delta_m \quad (18)$$

Under steady-state conditions, the angle between the field flux (at  $\theta$ ) and the resultant flux (at  $\theta_{\text{ref}}$ ) is fixed, and so  $\delta_m$  is constant. Therefore we slightly change (18) to be:

$$\theta(t) = \omega_R t + \alpha + \delta_m \quad (18a)$$

However, under transient conditions, because of rotor acceleration,  $\delta_m = \delta_m(t)$ , and we express (18) as

$$\theta(t) = \omega_R t + \alpha + \delta_m(t) \quad (18b)$$

Considering the transient condition, by taking the first derivative of (18b), we have:

$$\dot{\theta}(t) = \omega_R + \dot{\delta}_m(t) \quad (19)$$

Differentiating again results in

$$\ddot{\theta}(t) = \ddot{\delta}_m(t) \quad (20)$$

Substituting (20) into (12), repeated here for convenience,

$$\ddot{\theta} = \frac{T_a}{J} \quad (12)$$

results in

$$\ddot{\delta}_m(t) = \frac{T_a}{J} \quad \Rightarrow \quad J\ddot{\delta}_m(t) = T_a \quad (21)$$

We observe at this point that all of what we have done is in mechanical radians, and because we have focused on a 2-pole machine, the angles in electrical radians are the same. However, we want to accommodate the general case of a p-pole machine. To do so, recall (1), repeated here for convenience:

$$\omega_e = \frac{p}{2} \omega_m \quad (1)$$

Differentiating, we have

$$\dot{\omega}_e(t) = \frac{p}{2} \dot{\omega}_m(t) \quad (22)$$

which is just

$$\ddot{\delta}_e(t) = \frac{p}{2} \ddot{\delta}_m(t) \quad (23)$$

Substitution of (23) into (21) results in

$$J \frac{2}{p} \ddot{\delta}_e(t) = T_a \quad (24)$$

From now on, we will drop the subscript “e” on  $\delta$  and  $\omega$  with the understanding that both are given in electrical radians. Therefore (24) becomes:

$$\frac{2J}{p} \ddot{\delta}(t) = T_a \quad (25)$$

or since  $\dot{\omega} = \ddot{\delta}$ ,

$$\frac{2J}{p} \dot{\omega}(t) = T_a = T_m - T_e \quad (26)$$

Equation (26) is one form of the swing equation. We shall derive some additional forms in what follows.

## 5.0 A second form of the swing equation

Because power system analysis is more convenient in per-unit, let's normalize (26) by dividing by a base torque chosen to be

$$T_B = \frac{S_{B3}}{\omega_R} \quad (27)$$

where  $S_{B3}$  is a chosen 3-phase MVA rating. Dividing both sides of (26) by  $T_B$  results in

$$\frac{2J\omega_R}{pS_{B3}} \dot{\omega}(t) = \frac{T_a}{T_B} = T_{au} \quad (28)$$

We can express the kinetic energy  $W_k$  of the turbine-generator set, when rotating at  $\omega_R$ , as

$$W_k = \frac{1}{2} J \omega_R^2 \quad (29)$$

where the units are watt-seconds or joules.

Solving (29) for  $J$  results in

$$J = \frac{2W_k}{\omega_R^2} \quad (30)$$

Substituting (30) into (28) yields

$$\frac{2 \frac{2W_k}{\omega_R^2} \omega_R}{pS_{B3}} \dot{\omega}(t) = T_{au} \quad (31)$$

Simplifying:

$$\frac{4W_k}{pS_{B3}\omega_R} \dot{\omega}(t) = T_{au} \quad (32)$$

Let's write one of the 2's in the numerator as a  $\frac{1}{2}$  in the denominator, and group it with  $p$  and  $\omega_R$ , yielding

$$\frac{2W_k}{S_{B3} \left( \frac{p}{2} \omega_R \right)} \dot{\omega}(t) = T_{au} \quad (33)$$

Recalling that  $\omega_R$  is the mechanical reference speed, the reason for the last step is apparent, because we can now identify what is inside the brackets in the denominator as the electrical reference speed, which we can denote as  $\omega_{Re}$ . This would be, in North America, 377 rad/sec. Thus, (33) becomes

$$\frac{2W_k}{S_{B3} \omega_{Re}} \dot{\omega}(t) = T_{au} \quad (34)$$

Now define the inertia constant:

$$H = \frac{W_k}{S_{B3}} \quad (35)$$

Here, when  $S_{B3}$  has units of MVA, and  $W_k$  has units of MW-sec (or Mjoules), then  $H$  has units of MW-sec/MVA or seconds. Note Appendix D gives inertia as  $W_k$ .

When  $S_{B3}$  is chosen as the generator MVA rating,  $H$  falls within a fairly narrow range. I have pulled some numbers from the Appendix D of your text to illustrate.

Unit	$S_{mach}$ (MVA)	$W_k$ (MW-sec)	$H_{mach}$ $=W_k/S_{mach}$ (sec)	$H_{sys}$ $=W_k/S_{sys}$ $S_{sys}=100$
H1	9	23.5	2.61	0.235
H9	86	233	2.71	2.33
H18	615	3166	5.15	31.7
F1	25	125.4	5.02	1.25
F11	270	1115	4.13	11.15
F21	911	2265	2.49	22.65
CF1-HP	128	305	2.38	3.05
CF1-LP	128	787	6.15	7.87
N1	76.8	281.7	3.67	2.82
N8	1340	4698	3.51	47.0
SC1	25	30	1.2	0.3
SC5	75	89.98	1.2	0.9

Notes:

1. On machine base, H generally ranges 1-7 for hydro turbines and most 2-pole steam turbines; 4-pole steam turbines may have slightly higher H (7-9).
2. On system base, H ranges w/ machine size.
3. Cross-compound machines (side-by-side turbines, same steam, different gens) have a high LP H because of large blades required by low pressure steam.
4. Synchronous condensers have no turbine and therefore small H.

## 6.0 A third form of the swing equation

Recall (34):

$$\frac{2W_k}{S_{B3}\omega_{Re}} \dot{\omega}(t) = T_{au} \quad (34)$$

Substitution of  $W_k=HS_{B3}$  (from (35)) results in

$$\frac{2HS_{B3}}{S_{B3}\omega_{Re}} \dot{\omega}(t) = T_{au} \quad (36)$$

or



$$\frac{2H}{\omega_{Re}} \dot{\omega}(t) = T_{au} \quad (37)$$

Equation (37) is equation 2.17 in our text.

Some clarifications:

A. Comments on  $\omega_{Re}$ :

1. It is the rated electrical radian/frequency (377).

2. Clarification in text.

→ *Use of  $\omega_R$  in eq. 2.13, 2.14, 2.15, 2.17: it means  $\omega_{Re}$ . This may be implied by the use of  $\omega_{mR}$  in (2.12); however, the use of  $\omega_R$  in (2.3), because it is an equation in mechanical rad/sec, suggests that the text intended  $\omega_R$  to be in mechanical rad/sec. So it seems that either the use of  $\omega_R$  in (2.13) should be changed to  $\omega_{mR}$ , or the use of  $\omega_R$  in 2.13, 2.14, 2.15, and 2.17 should be changed to  $\omega_{Re}$  (or  $\omega_{eR}$ ). In all of our notes in this document, we have used  $\omega_R$  to mean mechanical reference speed and  $\omega_{Re}$  to mean electrical reference speed.*

B.  $H$  must be given on the same base as  $S_{B3}$  used to normalize the right-hand side torque.

C. You can convert  $H$ 's from one base to another as:

$$W_k = H_{mach} S_{mach} = H_{sys} S_{sys} \quad (38)$$

$$H_{sys} = H_{mach} \frac{S_{mach}}{S_{sys}} \quad (39)$$

## 7.0 Comments on Inertia

We make three additional comments about representing inertia in the swing equation.

### 7.1 Use of $M$ for inertia

Another quantity often used in the literature for inertia is  $M$ , the angular momentum at rated speed, where

$$M = J\omega_R \quad (40)$$

We can see how  $M$  is related to the kinetic energy according to the following. The kinetic energy of the rotor at speed  $\omega_R$  is

$$W_k = \frac{1}{2} J\omega_R^2 \quad (41)$$

Solving for J in (40),

$$J = \frac{M}{\omega_R} \quad (42)$$

Substituting (42) into (41) yields

$$W_k = \frac{1}{2} \frac{M}{\omega_R} \omega_R^2 = \frac{1}{2} M \omega_R \quad (43)$$

Also, from

$$H = \frac{W_k}{S_{B3}} \quad (35)$$

and substituting (43) into (35) results in

$$H = \frac{\frac{1}{2} M \omega_R}{S_{B3}} \quad (44)$$

which, when solved for  $M$ , results in

$$M = \frac{2HS_{B3}}{\omega_R} \quad (45)$$

showing us how to convert from inertia constant  $H$  to angular momentum  $M$ .

Two additional issues to note here:

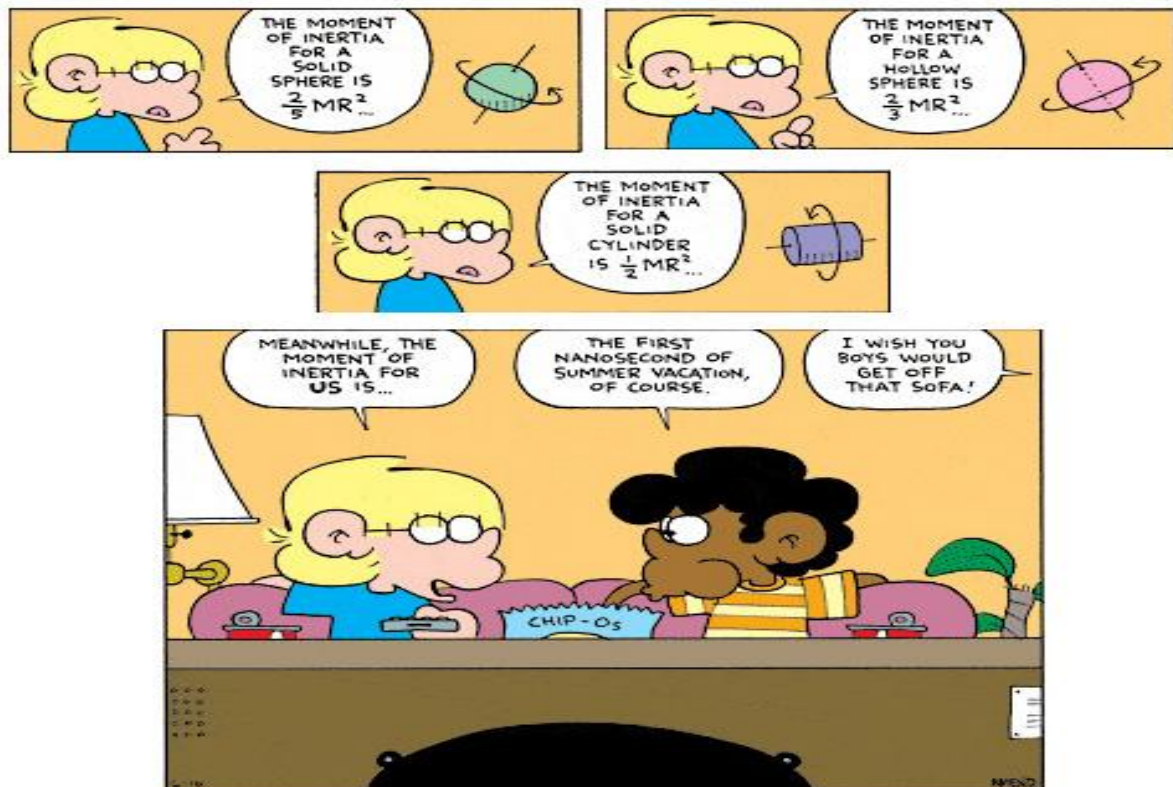
1. A different “ $M$ ” is sometimes used in the literature to denote the “mechanical starting time.” This is the total time required to accelerate the unit from standstill to rated speed  $\omega_R$  if rated torque ( $T_{au}=1.0$ ) is applied as a step function at  $t=0$ . I will denote this as  $T_4$ , nomenclature that is consistent with the VMAF text (see page 596). Kundur, in his book on page 132, shows that this time, in seconds, is given by

$$T_4 = 2H \quad (46)$$

where  $H$  is given on the machine base.

2. The three constants  $M$ ,  $H$ , and  $W_k$ , are defined at the particular angular velocity of  $\omega_R$ . However, the machine speed  $\omega_m$  does deviate from  $\omega_R$  during the transient conditions for which we are interested to study. Therefore, to be rigorous, we should define  $M$ ,  $H$ , and  $W_k$  relative to the machine speed  $\omega_m(t)$  so that  $M$ ,  $H$ , and  $W_k$  vary with time. However, this considerably complicates the swing equation, and does so with negligible improvement in accuracy, since  $\omega_m$ , although time varying during disturbance conditions, does not

deviate much from  $\omega_R$ . On the other hand, the moment of inertia  $J$  is an actual constant, i.e., it is a function of only the machine geometry and mass and does not depend on speed.



## 7.2 W-R Squared

Another constant that is often used by manufacturers (and it will be, usually, what you get from a US manufacturer) is the “W-R-squared,” denoted  $WR^2$ , which is the moment of inertia expressed in English units of  $\text{lb}(m)\text{-ft}^2$ :

$$WR^2 = \underbrace{[\text{mass of rotating parts}]_{\text{lb(m)}}}_{\text{lb(m)}} \underbrace{[\text{radius of gyration}]_{\text{ft}}}_{\text{ft}}^2 \quad (47)$$

The radius of gyration is a root-mean-square average distance of all parts of the rotating object from its axis of rotation.

The conversion of units may be obtained, resulting in the moment of inertia,  $J$ , in  $\text{kg}\cdot\text{m}^2$ , given as

$$J = WR^2 \text{ lb(m)}\text{ft}^2 \frac{0.4536 \text{ kg}}{\text{lb(m)}} \left( \frac{0.3048 \text{ m}}{\text{ft}} \right)^2 = 0.0421 \times WR^2 \quad (48)$$

We may also relate the kinetic energy at rated speed,  $W_k$ , to  $WR^2$ , by substituting (48) into the expression for  $W_k$ :

$$W_k = \frac{1}{2} J \omega_R^2 = \frac{1}{2} (0.0421 WR^2) \omega_R^2 = 0.02105 (WR^2) \omega_R^2 \quad (49)$$

where  $W_k$  is given in joules. If we wanted to write (49) as a function of RPM instead of rad/sec, where  $\omega_R = 2\pi n_R / 60$ ,

$$\begin{aligned} W_k &= 0.02105 (WR^2) (4\pi^2 / 3600) n_R^2 \\ &= 2.31 \times 10^{-4} WR^2 (n_R^2) \end{aligned} \quad (50)$$

where again  $W_k$  is in joules (this is the same as the equation at the top of pg 22 in your text). Expressing  $W_k$  in Mjoules=MW-sec, eqts. (49) and (50) become

$$W_k = 2.105 \times 10^{-8} (WR^2)(\omega_R^2) = 2.31 \times 10^{-10} (WR^2)(n_R^2) \quad (51)$$

A manufacturer's specification sheet for a turbine-generator set will always identify  $WR^2$  and the rated speed  $n_R$  in RPM. With this information, the MW-sec needed in a stability program to characterize the inertia can be obtained from (51).

### 7.3 Summing up

Remember, inertia should account for all masses on the shaft. This will always be the turbine and generator, but it may or may not include an exciter (depends on whether the machine utilizes a rotating exciter or not and whether that rotating exciter is mounted on the same shaft or not).

Also remember that we have five forms in which inertia can be expressed:

$$J, W_k, H, M, \text{ and } WR^2$$

You should be able to convert from any one form to any other form.

## The user manual for the GE PSLF time-domain simulation program includes the below.

### **Does the time constant H have to be in seconds for the H calculation?**

The machine time constant H must be in “seconds” in PSLF dynamic simulation. In many occasions, users may be given data of machine rotating inertia in pound-feet<sup>2</sup> or in kg-meter<sup>2</sup>, instead of H in seconds.

Formula for conversion from rotating inertia X (in pound-ft<sup>2</sup>) to machine time constant H (in seconds):

$$H = (0.231 \times 10^{-6}) * X * (\text{rpm})^2 / \text{KVA rating}$$

Where rpm is machine speed in revolution per minute, e.g., 3600 rpm for a two-pole machine and 1800 rpm for a 4-pole machine.

Example, given machine rating of 234 MVA, rpm of 3600 and machine rotating inertia of 278,620 pound-ft<sup>2</sup>, H (sec.) =  $(0.231 \times 10^{-6}) * 278,620 * 3600^2 / 234000 = 3.56$

Formula for conversion from rotating inertia Y (in kg-m<sup>2</sup>) to machine time constant H (in seconds):

$$H = (0.231 \times 10^{-6}) * Y * 23.73 * (\text{rpm})^2 / \text{KVA rating}$$

Example, given machine rating of 234 MVA, rpm of 3600 and rotating inertia of 15,695 kgm<sup>2</sup>, H(in seconds) =  $(0.231 \times 10^{-6}) * 15,695 * 23.73 * 3600^2 / 234000 = 4.76$