CHAPTER4

MODEL OF THREE-PHASE INVERTER

4.1 Introduction

In this chapter the three-phase inverter and its functional operation are discussed. In order to realize the three-phase output from a circuit employing dc as the input voltage a three-phase inverter has to be used. The inverter is build of switching devices, thus the way in which the switching takes place in the inverter gives the required output. In this chapter the concept of switching function and the associated switching matrix is explained. Lastly the alternatives as to how the inverter topologies can be formed is presented.

4.2 Switching Functions

It is well-known that some switching devices exist between the source and the load, the number of which depends on the circuit or the type of the load. In any case the number of switching devices are limited by the complexity. Even the densest circuit has one switch between an input line and the output line. If a converter has ‘n’ inputs and ‘m’ outputs the number of switching devices needed for energy conversion is equal to ‘$m \times n$’. These ‘$m \times n$’ switching devices in the circuit can be arranged according to their connections. The pattern suggests a matrix as shown in Figure 4.1.

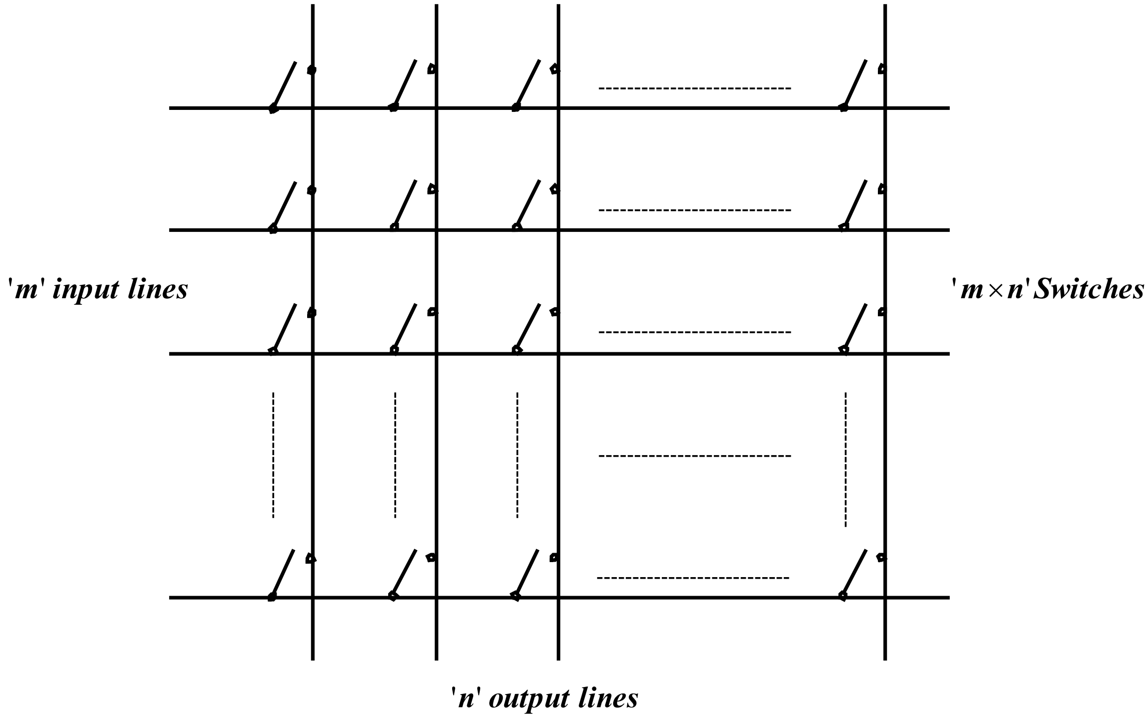


Figure 4.1: The general switch matrix

For example in realizing single phase to dc conversion the single phase has two terminals and the dc has two terminals thus a total of ‘2×2 = 4’ switches are required.

Most power electronic circuits are classified into two types [15]:

1. Direct switch matrix circuits: In these circuits any energy storage elements are connected to the matrix only at the input and output terminals. The storage elements effectively become a part of the source or the load. A full wave rectifier with an output filter is an example of a direct switch matrix circuit.
2. Indirect switch matrix converters also termed as embedded converters: In these circuits, the energy storage elements are connected within the matrix structure. There are usually very few energy storage elements in such case and the indirect switch

matrix circuits are often analyzed as cascade of two direct switch matrix circuits with storage elements in between.

A switch matrix provides a clear way to organize devices for a given application. It also helps to focus the effort in to three major problems areas. Each of these areas must be addressed effectively in order to produce a useful power electronic system.

1.The hardware problem → To build a switch matrix.

2. The software problem → Operate the matrix to achieve the desired conversion.

3.The interface problem → Add energy storage elements to provide the filters or intermediate storage necessary to meet the application requirements.

These problems can more effectively be understood by considering an example of converting ac to dc, in which the hardware problem is as to how many switches have to be used which depends on whether we are performing single phase to dc conversion or three phase to dc conversion.

The software problem lies in the fact that while operating these switches we need to obey the fundamentals laws of energy conversion like the Kirchhoff's voltage and the current laws and take care that these are not violated. Finally as this energy conversion process is not ideal we may not get exact dc voltage at the output terminals, which require the addition of some elements before the final dc voltage is obtained.

4.2.1 Reality Of Kirchhoff's Voltage Law

In determining how the switches operate in the switch matrix, care should be taken to avoid any danger in the operation of the circuit. Consider the circuit in Figure 4.2. This circuit can be operated with the switch 's' closed owing to the fact that when the switch is closed the sum of the voltages around the loop is not equal to zero In reality a very large current will flow and this drop appears across the wires which may result in the burning of the wires that cannot take this huge voltage. Thus this circuit is not correct when the switch is closed as it violates Kirchhoff's Voltage law (KVL). Thus the KVL serves as a warning that never should two unequal voltage sources be connected with out any element in between which can account for the inequality in the two voltages.

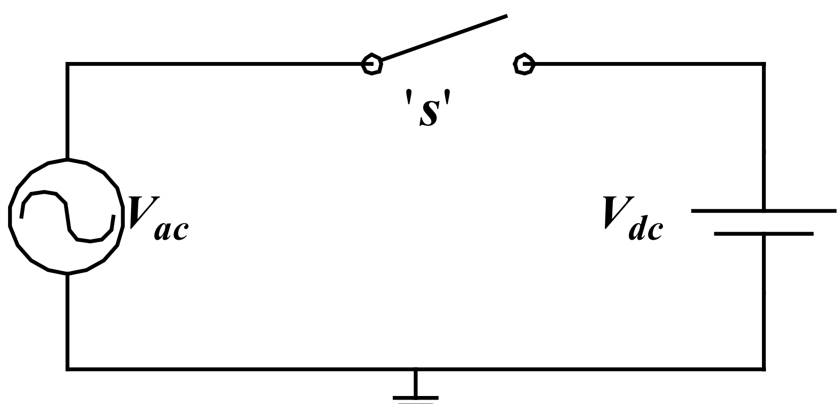


Figure 4.2: Demonstration of KVL.

4.2.2 Reality Of Kirchhoff's Current Law

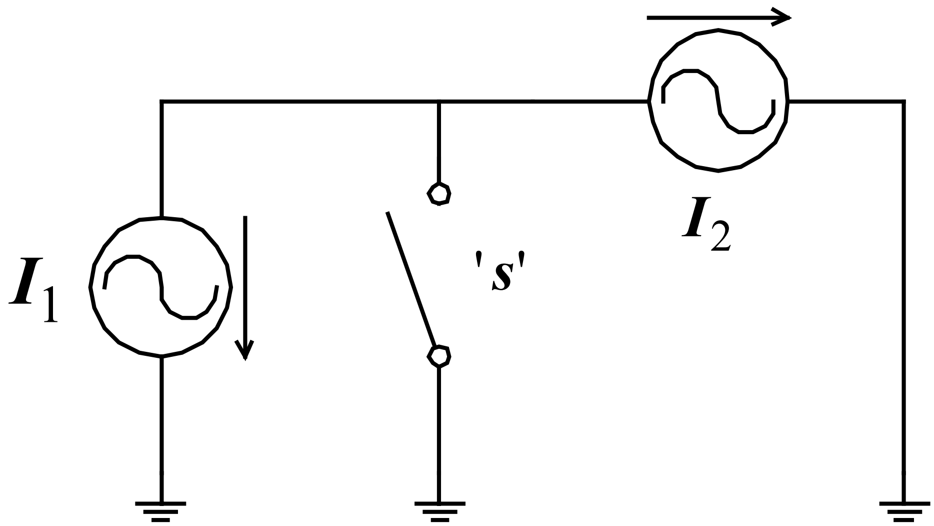


Figure 4.3: Demonstration of Kirchhoff's Current Law.

As is with voltages proper care is to be taken with currents too. According to Kirchhoff's Current Law (KCL) the sum of the currents at a node should be equal to zero at all times. Thus any circuit in which KCL is not proven is dangerous to be operated as short circuit may occur. Consider the circuit in Figure 4.3, which can be operated if the switch 's' is opened. If it is open, that particular path is open so the current is zero and the other two current sources are unequal which violates KCL, the sum of the currents entering the node are not equal to zero.

4.2.3 The Switch State Matrix And Switching Functions

Each switch in a switch matrix is either turned 'on' or turned 'off'. This can be represented with a mathematical matrix called the switch state matrix, which corresponds to the circuit. This is a matrix Q (t), with 'm' rows and 'n' columns, m and n depend on the number of inputs and outputs in the circuit. Each element $q_{ij}(t)$ is '1' when the corresponding switch is turned 'on' and '0' when it is turned ' off'. The elements of $Q(t)$ are referred to as switching functions, and are important in the design of converters.

A general example of a switch state matrix $Q(t)$:

'm' rows (m inputs) 'n' columns (n outputs)

elements $q_{ij}(t) = \begin{cases} 1, & \text{if switch at location } i, j, \text{is on} \\ 0, & \text{if switch is off} \end{cases}$.

At a specific t_o, the matrix will have a particular numerical value, such as

$$Q(t_o) = \begin{bmatrix} 1 & 0 & \ldots & .. & 1 \\ 0 & . & . & . & 1 \\ \vdots & . & . & 0 & 0 \\ . & . & . & . & 0 \\ 1 & . & . & . & 1 \end{bmatrix}.$$

The matrix might often be as small as 2 × 2. The software problem can be stated as choosing the switching functions a desired operation.

4.2.4. Mathematical Representation of Switching Functions

The pattern of switching for the switching devices used in converters/inverters is periodic, therefore the analysis of the switching functions is simple by using the Fourier series[14]. Let the repetition frequency of the pulses be f with a time-period $T = 1/f$; define the angular frequency $\omega = 2\pi f$, then

$\omega T = 2\pi$ radians. If the angular duration of the unit-value period is $2\pi / A$ where A ≥ 1, then the boundaries of the unit- value period with respect to the time zero reference are $-\pi / A$ and π / A radians, the Fourier series for this periodic signal is given in Equation 4.1

$$H(\omega t) = \sum_{n=0}^{n=\infty} [C_n \cos(n\omega t) + S_n \sin(n\omega t)] \qquad (4.1)$$

From the standard determination of co-efficients for a Fourier expansion:

$$\begin{aligned} Sn &= \frac{2}{T} \int_{T/2}^{T/2} H \sin(n\omega t)\, dt \\ &= \frac{1}{\pi} \int_{\pi/A}^{\pi/A} \sin(n\omega t)\, d\omega t \\ &= 0 \end{aligned} \qquad (4.2)$$

$$\begin{aligned} C_0 &= \frac{1}{T} \int_{T/2}^{T/2} H \; dt \\ &= \frac{1}{2\pi} \int_{\pi}^{\pi} H \; d\omega t \\ &= \frac{1}{A} \end{aligned} \qquad (4.3)$$

$$\begin{aligned} C_n \,(n \neq 0) &= \frac{2}{T} \int_{-T/2}^{T/2} H \cos(n\omega t)\, dt \\ &= \frac{1}{\pi} \int_{-\pi/A}^{\pi/A} \cos(n\omega t)\, d\omega t \\ &= \frac{2}{\pi} \sin(n\pi / A) / n \end{aligned} \quad . \qquad (4.4)$$

Thus

$$H(\omega t) = \frac{1}{A} + \frac{2}{\pi} \sum_{n=1}^{n=\infty} [\sin(n\pi / A) / n] \cos(n\omega t) \quad . \qquad (4.5)$$

The average value of the term, 1/A can be brought within the summation to give an alternate form of this expansion,

$$H(\omega t) = \frac{1}{\pi}\sum_{n=-\infty}^{n=\infty}[\sin(n\pi / A)/n]\cos(n\omega t) \tag{4.6}$$

Equation 4.5 can be written equivalently as

$$H(\omega t) = \frac{1}{A} + \frac{2}{\pi}\sum_{n=1}^{n=\infty}[\sin(n\pi / A)/n]\cos(n(\omega t - 2k\pi / A)) \quad . \tag{4.7}$$

The above equation is true owing to the fact that $\cos(\omega t) = \cos(2\pi - \omega t)$. In the above expressions A can be an integer, rational or an irrational number. When A is an integer multiple of A then the term $\sin(n\pi / A)$ vanishes. In Equation 4.7 all those terms in which n is an integer multiple of A vanish, and so it reduces to a fundamental component and a time varying term. Thus the switching pulses can be represented as a dc component and a cos or sine varying term as in Equation 4.8,

$$S = \frac{1}{2}(1 + M) \tag{4.8}$$

where M is called the modulation signal, which can be any sine or cos term (in accordance to Fourier series) depending on the control we want to implement. The more general fundamental component for M is given as

$$M = m_a \cos(\omega t - \alpha) \tag{4.9}$$

in the above expression m_a is called the modulation index which can vary from 0 to 1 and for values of m_a less than ‘1’ a linear modulation range is supposed to exist and for values of m_a greater than ‘1’ it is operated in the over modulation range.

4.3 Three-Phase Inverter

The dc to ac converters more commonly known as inverters, depending on the type of the supply source and the related topology of the power circuit, are classified as voltage source inverters (VSIs) and current source inverters (CSIs). The single-phase inverters and the switching patterns were discussed elaborately in Chapter two and so the three phase inverters are explained in detail here.

Three-phase counterparts of the single-phase half and full bridge voltage source inverters are shown in Figures 4.4 and 4.5. Single-phase VSIs cover low-range power applications and three-phase VSIs cover medium to high power applications. The main purpose of these topologies is to provide a three-phase voltage source, where the amplitude, phase and frequency of the voltages can be controlled. The three-phase dc/ac voltage source inverters are extensively being used in motor drives, active filters and unified power flow controllers in power systems and uninterrupted power supplies to generate controllable frequency and ac voltage magnitudes using various pulse width modulation (PWM) strategies. The standard three-phase inverter shown in Figure 4.5 has six switches the switching of which depends on the modulation scheme. The input dc is usually obtained from a single-phase or three phase utility power supply through a diode-bridge rectifier and LC or C filter.

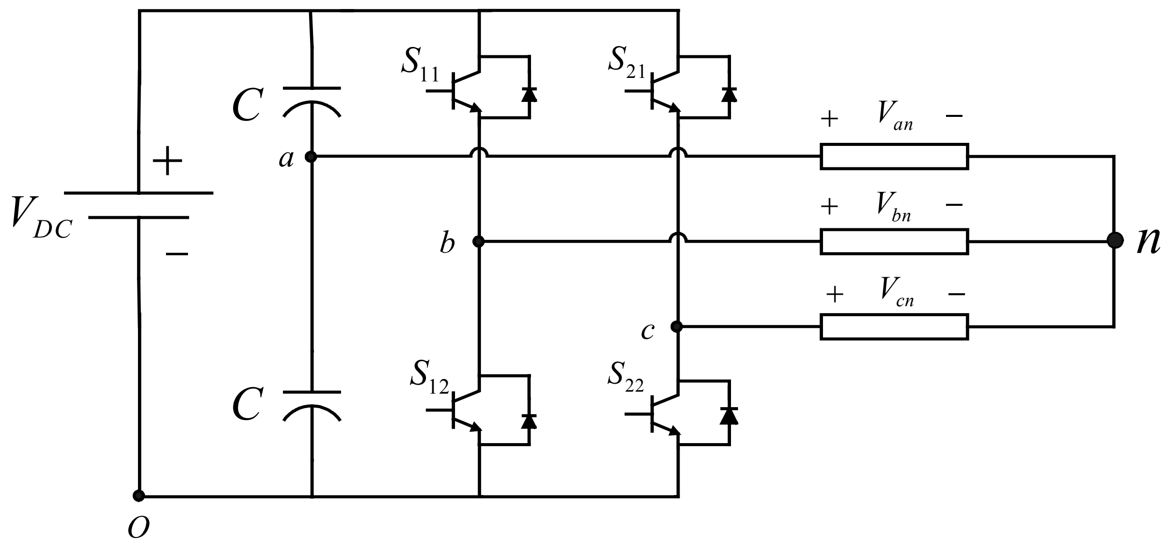


Figure 4.4: Three-Phase Half Bridge Inverter

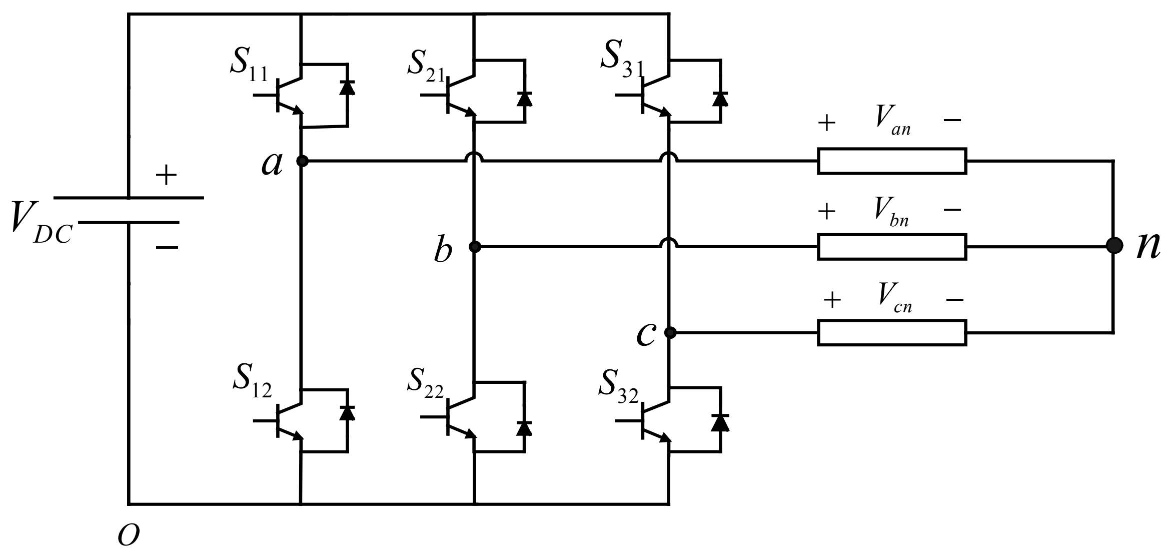


Figure 4.5: Three-phase Full –Bridge Inverter

The inverter has eight switch states given in Table 4.1. As explained earlier in order that the circuit satisfies the KVL and the KCL, both of the switches in the same leg cannot be turned ON at the same time, as it would short the input voltage violating the KVL. Thus the nature of the two switches in the same leg is complementary. In accordance to Figure 4.5,

$$S_{11} + S_{12} = 1 \tag{4.10}$$

$$S_{21} + S_{22} = 1 \tag{4.11}$$

$$S_{31} + S_{32} = 1 \tag{4.12}$$

Table 4.1: The switching states in a three-phase inverter.

S_{11}	S_{12}	S_{31}	V_{ab}	V_{bc}	V_{ca}
0	0	0	0	0	0
0	0	1	0	$-V_{DC}$	V_{DC}
0	1	0	$-V_{DC}$	V_{DC}	0
0	1	1	$-V_{DC}$	0	$-V_{DC}$
1	0	0	V_{DC}	0	$-V_{DC}$
1	0	1	V_{DC}	$- V_{DC}$	0
1	1	0	0	V_{DC}	$-V_{DC}$
1	1	1	0	0	0

Of the eight switching states as shown in Table 4.1 two of them produce zero ac line voltage at the output. In this case, the ac line currents freewheel through either the upper or lower components. The remaining states produce no zero ac output line voltages. In order to generate a given voltage waveform, the inverter switches from one state to another. Thus the resulting ac output line voltages consist of discrete values of voltages, which are $-V_{DC,}$ 0, and $V_{DC.}$

The selection of the states in order to generate the given waveform is done by the modulating technique that ensures the use of only the valid states.

$$\frac{V_{DC}}{2}(S_{11} - S_{12}) = V_{an} + V_{no} \tag{4.13}$$

$$\frac{V_{DC}}{2}(S_{21} - S_{22}) = V_{bn} + V_{no} \tag{4.14}$$

$$\frac{V_{DC}}{2}(S_{31} - S_{32}) = V_{cn} + V_{no}\,. \tag{4.15}$$

Expressing the Equations from 4.13 to 4.15 in terms of modulation signals and making use of conditions from 4.10 to 4.12 gives

$$\frac{V_{DC}}{2}(M_{11}) = V_{an} + V_{no} \tag{4.16}$$

$$\frac{V_{DC}}{2}(M_{21}) = V_{bn} + V_{no} \tag{4.17}$$

$$\frac{V_{DC}}{2}(M_{31}) = V_{cn} + V_{no} \tag{4.18}$$

Adding the Equations from 4.13 to 4.15 together gives Equation 4.19 as

$$\frac{V_{DC}}{2}(S_{11} + S_{21} + S_{31} - S_{12} - S_{22} - S_{32}) = V_{an} + V_{bn} + V_{cn} + 3V_{no} \tag{4.19}$$

As we are dealing with balanced voltages $V_{an} + V_{bn} + V_{cn} = 0$ and making use of the conditions from Equations 4.1 to 4.3, Equation 4.19 becomes

$$\frac{V_{DC}}{6}(2S_{11} + 2S_{21} + 2S_{31} - 3) = V_{no} \tag{4.20}$$

Substituting for V_{no} in Equations 4.13 to 4.15, gives

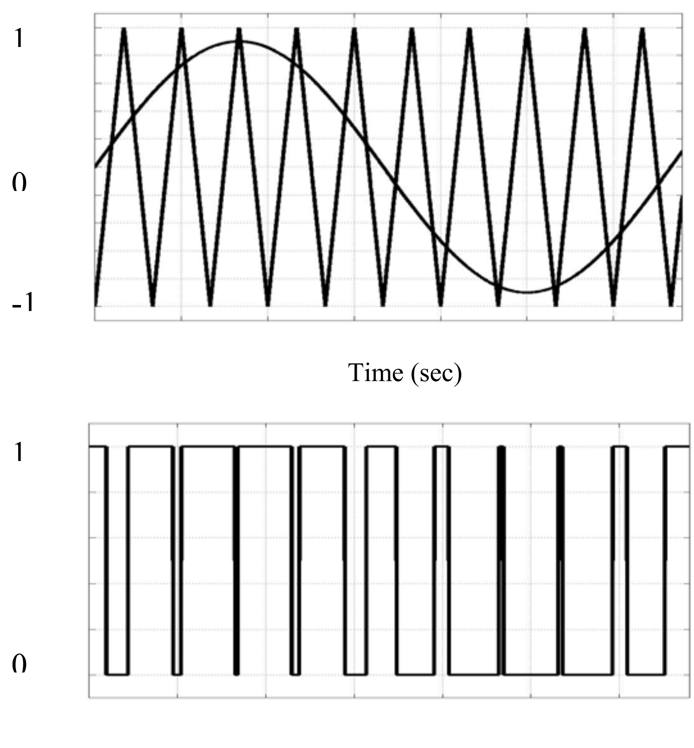
$$\frac{V_{DC}}{3}(2S_{11} - S_{21} - S_{31}) = V_{an} \tag{4.21}$$

$$\frac{V_{DC}}{3}(2S_{21} - S_{21} - S_{31}) = V_{bn} \tag{4.22}$$

$$\frac{V_{DC}}{3}(2S_{31} - S_{21} - S_{11}) = V_{cn}\,. \tag{4.23}$$

4.3.1.Sinusoidal PWM in Three-Phase Voltage Source Inverters

As in the single phase voltage source inverters PWM technique can be used in three-phase inverters, in which three sine waves phase shifted by 120° with the frequency of the desired output voltage is compared with a very high frequency carrier triangle, the two signals are mixed in a comparator whose output is high when the sine wave is greater than the triangle and the comparator output is low when the sine wave or typically called the modulation signal is smaller than the triangle. This phenomenon is shown in Figure 4.6. As is explained the output voltage from the inverter is not smooth but is a discrete waveform and so it is more likely than the output wave consists of harmonics, which are not usually desirable since they deteriorate the performance of the load, to which these voltages are applied.



(a)

(b)

Figure 4.6: PWM illustration by the sine-triangle comparison method (a) sine-triangle comparison (b) switching pulses.

The modulation signals are thus selected so meet some specifications, like harmonic elimination, higher fundamental component and so on. The phase voltages can be obtained from the line voltages as,

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} V_{an} - V_{bn} \\ V_{bn} - V_{cn} \\ V_{cn} - V_{an} \end{bmatrix} \tag{4.24}$$

which can be written as a function of the phase-voltage vector $[V_{an}\ V_{bn}\ V_{cn}]^T$ as

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \tag{4.25}$$

Equation 4.25 represents a linear system where the unknown quantity is the vector $[V_{an}\ V_{bn}\ V_{cn}]^T$, but the matrix is singular and so the phase voltages cannot be found from matrix inversion. However since the phase voltages add to zero, the phase load voltages can be written as

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}. \tag{4.26}$$

which implies

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} V_{ab} \\ V_{bc} \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ 0 \end{bmatrix} \tag{4.27}$$

which when expanded gives the phase voltages in terms as

$$V_{an} = \frac{1}{3}(2V_{ab} + V_{bc}) \tag{4.28}$$

$$V_{bn} = \frac{1}{3}(V_{bc} - V_{ab}) \quad (4.29)$$

$$V_{cn} = \frac{-1}{3}(2V_{bc} + V_{ab})\,. \quad (4.30)$$

4.3.2 Generalized Discontinuous PWM

In the Equations from 4.16 to 4.18 V_{an}, V_{bn} , and V_{cn} are the phase voltages of the load while the voltage of the load neutral to the inverter reference is V_{no}. An alternative carrier based discontinuous modulation scheme is obtained by using the Space Vector methodology to determine the expression for V_{no} [17].

Table 4.2. The eight switching possibilities for the voltage source inverter along with the stationary reference frame voltages

S_{11}	S_{12}	S_{31}	V_{qs}	V_{ds}	V_{os}
0	0	0	0	0	$-V_{DC}/2$
0	0	1	$-V_{DC}/\sqrt{3}$	$V_{DC}/\sqrt{3}$	$-V_{DC}/6$
0	1	0	$-V_{DC}/3$	$-V_{DC}/\sqrt{3}$	$-V_{DC}/6$
0	1	1	$-2V_{DC}/3$	0	$V_{DC}/6$
1	0	0	$2V_{DC}/3$	0	$-V_{DC}/6$
1	0	1	$V_{DC}/3$	$-V_{DC}/\sqrt{3}$	$V_{DC}/6$
1	1	0	$V_{DC}/3$	$V_{DC}/\sqrt{3}$	$V_{DC}/6$
1	1	1	0	0	$V_{DC}/2$

The eight possible switching states for the voltage source inverter along with the qd transformation in stationary reference frame are shown in Table 4.2. The voltages as given in Table 4.2 can be graphically represented as a hexagon whose sides are the switching states.

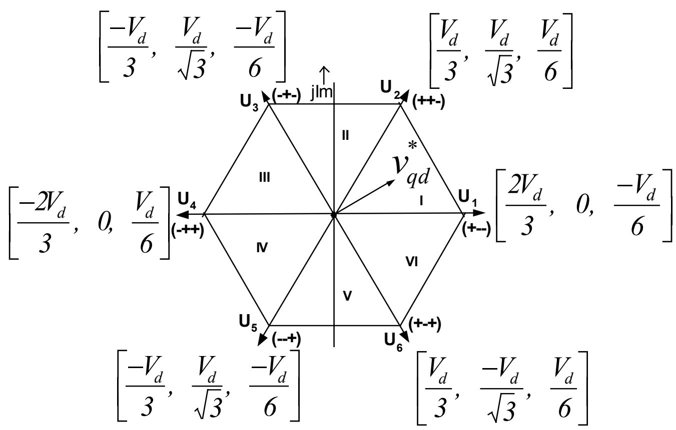


Figure 4.7. Space vector diagram for the eight switching state, including zero sequence voltages

In Figure 4.7 U_1 to U_6 correspond to the six active states, in which the output voltage is not zero and are therefore designated as active states and the two switching states in which the output voltage in the q and d- axis is zero (U_0 and U_7) are designated as null states. The reference zero sequence voltage, V_{no} is approximated by, time-averaging the zero sequence voltages of the two active and two null modes. In general, the three-phase balanced voltages expressed in the stationary reference frame; situated in the appropriate sector in Figure 4.7 are approximated by the time-average over a sampling period (converter switching period, t_s) of the two adjacent active qd voltage inverter vectors and the two zero states U_0 and U_7. The switching turn-on times of the two active and two null states are utilized to determine the duty cycle information to program the active switch gate signals. When the inverter is operating in the linear modulation region, the sum of the times the two active switching modes are utilized is less than the switching period; in which case the remaining time is occupied by using the two null vectors, U_0 and U_7. The reference voltage vector V_{qd}^* is obtained by time averaging two active vectors and the null state vectors. If this vector spends t_a , t_b, t_0, t_7 on V_{qda}, V_{qdb}, V_{qd0}, V_{qd7} then the q and d components of the reference voltage V_{qd}^* are given as

$$V_{qd}^* = V_{qq} + jV_{dd} = V_{qda}t_a + V_{qdb}t_b + V_{qd0}t_0 + Vqd7t7 \quad (4.31)$$

$$t_c = t_0 + t_7 = 1 - t_a - t_b . \quad (4.32)$$

In the above equations t_a , t_b, t_0, t_7 are the normalized times with respect to the converter sampling frequency.

When separated into real and imaginary parts, Equations 4.31 and 4.32 give the expressions for t_a and t_b as

$$t_a = \nabla\ [\ V_{qq} V_{db} - V_{dd} V_{qb}] \quad (4.33)$$

$$t_b = \nabla\ [\ V_{dd} V_{qa} - V_{qq} V_{da}] \quad (4.34)$$

where $\nabla = [\ V_{db} V_{qa} - V_{qb} V_{da}]^{-1}$.

It is observed that both V_{qd0} and V_{qd7} do not influence the values of t_a and t_b. The neutral voltage V_{no} averaged over the switching period t_s is given as :

$$<V_{no}> = V_{oa}t_a + V_{ob}\, t_b + V_{o0}\, t_0 + V_{o7}\, t_7 \quad (4.35)$$

where V_{oa} ,V_{bo} ,V_{o0} , and V_{o7} are the zero-sequence voltages in both the two active and the two null states, respectively. Table 4.3 gives the expression for the averaged neutral voltage $<V_{no}>$ for the six sectors of the space vector. Hence, given the balanced voltage set at any instant, V_{qdo}^* in the stationary reference frame is found and the sector in which V_{qd}^* is located is determined.

Table 4.3: Expressions for the neutral voltage for the six sectors

Sector	**Neutral Voltage** $<V_{no}>$
I	$0.5V_{bn} + 0.5V_{DC}(1-2\alpha) + 0.5(1-2\alpha)[V_{cn} - V_{an}]$
II	$0.5V_{an} + 0.5V_{DC}(1-2\alpha) + 0.5(1-2\alpha)[V_{cn} - V_{bn}]$
III	$0.5V_{cn} + 0.5V_{DC}(1-2\alpha) + 0.5(1-2\alpha)[V_{an} - V_{bn}]$
IV	$0.5V_{bn} + 0.5V_{DC}(1-2\alpha) + 0.5(1-2\alpha)[V_{an} - V_{cn}]$
V	$0.5V_{an} + 0.5V_{DC}(1-2\alpha) + 0.5(1-2\alpha)[V_{bn} - V_{cn}]$
VI	$0.5V_{cn} + 0.5V_{DC}(1-2\alpha) + 0.5(1-2\alpha)[V_{bn} - V_{an}]$

The expression for V_{no} is then selected from Table 4.3, and is subsequently used in Equations 4.16 to 4.18 to determine the modulation signals for the top three devices. In the above table α is the quantity, which can be varied to realize different modulation signals, and typically α ranges from $[0,1]$.

4.3.3 Six Step Voltage Source Inverter

Three-phase bridge inverters are most commonly used in ac motor drives and general-purpose ac supplied. Figure 4.8 (a) and (b) explain the generation of the output voltages in six-step mode of operation. The circuit for the six-step VSI is as shown in Figure 4.5, which consists of three half-bridges, which are mutually phase-shifted by $\frac{2\pi}{3}$ angle to generate the three phase voltages.

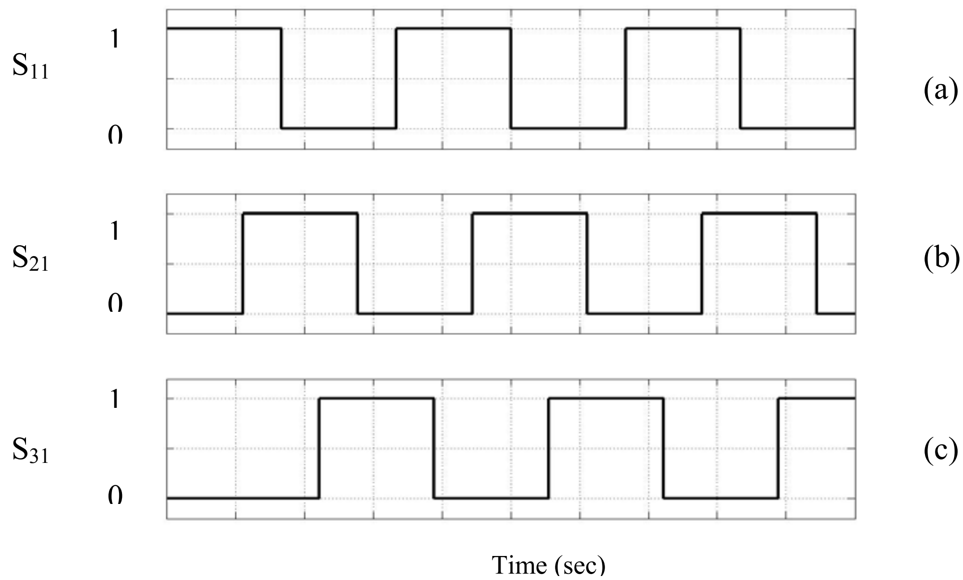


Figure 4.8 (a): Generation of the switching signals for top devices (a) S_{11} (b) S_{21} (c) S_{31}

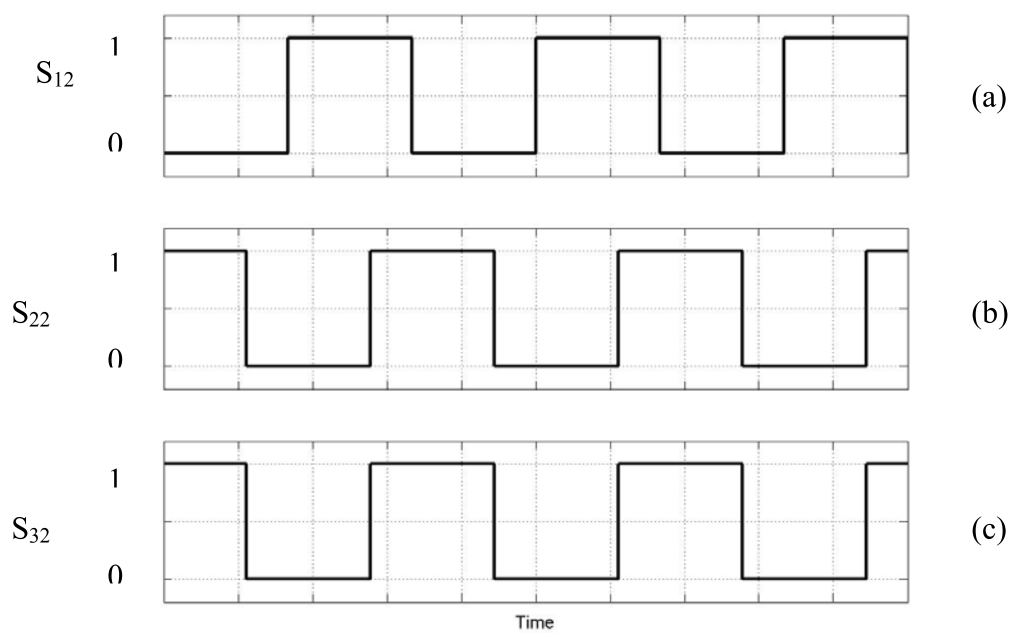


Figure 4.8 (b): Generation of the switching signals for bottom devices (a) S_{12} (b) S_{22}(c) S_{32}

The square wave phase voltages with respect to the fictitious dc center tap can be expressed using Fourier series as,

$$V_{ao} = \frac{2V_{DC}}{\pi}[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - ....] \qquad (4.36)$$

$$V_{bo} = \frac{2V_{DC}}{\pi}[\cos(\omega t - \frac{2\pi}{3}) - \frac{1}{3}\cos 3(\omega t - \frac{2\pi}{3}) + \frac{1}{5}\cos 5(\omega t - \frac{2\pi}{3}) - ....] \qquad (4.37)$$

$$V_{co} = \frac{2V_{DC}}{\pi}[\cos(\omega t + \frac{2\pi}{3}) - \frac{1}{3}\cos 3(\omega t + \frac{2\pi}{3}) + \frac{1}{5}\cos 5(\omega t + \frac{2\pi}{3}) - ....]. \qquad (4.38)$$

The line voltages can thus be obtained from the phase voltages as

$$V_{ab} = V_{an} - V_{bn}$$

$$= \frac{2\sqrt{3}V_{DC}}{\pi}[\cos(\omega t + \frac{\pi}{6}) + 0 - \frac{1}{5}\cos 5(\omega t + \frac{\pi}{6}) - \frac{1}{7}\cos 7(\omega t + \frac{\pi}{6}) + ...] \qquad (4.39)$$

$$V_{bc} = V_{bn} - V_{cn}$$

$$= \frac{2\sqrt{3}V_{DC}}{\pi}[\cos(\omega t - \frac{\pi}{2}) + 0 - \frac{1}{5}\cos 5(\omega t - \frac{\pi}{2}) - \frac{1}{7}\cos 7(\omega t - \frac{\pi}{2}) + ...] \qquad (4.40)$$

$$V_{ca} = V_{cn} - V_{an}$$

$$= \frac{2\sqrt{3}V_{DC}}{\pi}[\cos(\omega t + \frac{5\pi}{6}) + 0 - \frac{1}{5}\cos 5(\omega t + \frac{5\pi}{6}) - \frac{1}{7}\cos 7(\omega t + \frac{5\pi}{6}) + ...] \qquad (4.41)$$

The fundamental of the line voltages is $\sqrt{3}$ times that of the phase voltage, and there is a leading phase-shift of $\frac{\pi}{6}$. The line voltages waves as shown in Figure 4.9 have a characteristic six-step wave shape and thus the name for this inverter. The characteristic harmonics in the waveform are $6n \pm 1$, n being an integer.

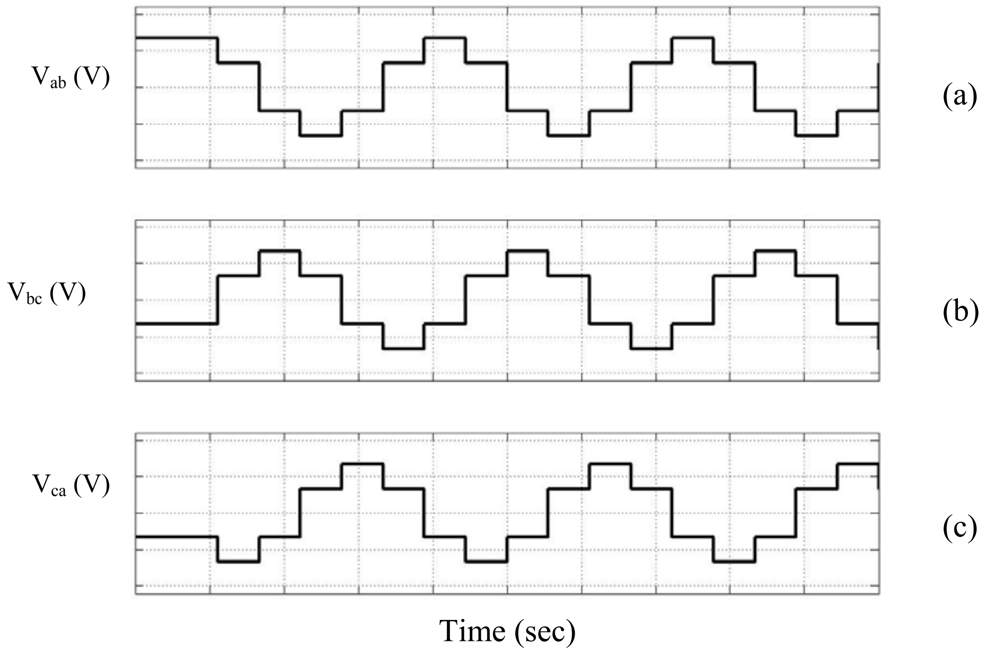


Figure 4.9: Line voltages (a) V_{ab} (b) V_{bc} (c) V_{ca}.

The three-phase fundamental as well as the harmonic components are balanced with a mutual phase-shift angle of $\frac{2\pi}{3}$.

4.3.4. qdo –Transformation For Voltage Source Inverter

The voltage equations in terms of switching signals for the inverter in Figure 4.5 are given in Equations 4.5 to 4.7. As the switching devices in the same leg (a, b, c) are complimentary those Equations can be written as

$$V_{DC}(2S_{11}-1)=V_{an}+V_{no} \tag{4.42}$$

$$V_{DC}(2S_{21}-1)=V_{bn}+V_{no} \tag{4.43}$$

$$V_{DC}(2S_{31}-1)=V_{cn}+V_{no}\,. \tag{4.44}$$

The dc current from the source is as given in Equation 4.45

$$I_{DC} = I_a S_{11} + I_b S_{21} + I_c S_{31} \tag{4.45}$$

The switching functions S_{11}, S_{21} and S_{31} can be expressed in terms of modulation signals as

$$S_{11} = \frac{1}{2}(1 + M_{11}) \tag{4.46}$$

$$S_{21} = \frac{1}{2}(1 + M_{21}) \tag{4.47}$$

$$S_{31} = \frac{1}{2}(1 + M_{31}) \tag{4.48}$$

where M_{11}, M_{21} and M_{31} have the general form of $m \cos \omega t$. As the three phases are phase shifted by $\frac{2\pi}{3}$ radians, the modulation signals are also phase-shifted by $\frac{2\pi}{3}$ from each other. Equations 4.16 to 4.18 are repeated down as

$$M_{11} = \frac{2}{V_{DC}}(V_{an} + V_{no}) \tag{4.49}$$

$$M_{21} = \frac{2}{V_{DC}}(V_{bn} + V_{no}) \tag{4.50}$$

$$M_{31} = \frac{2}{V_{DC}}(V_{cn} + V_{no}) \ . \tag{4.51}$$

Transforming the above equations into synchronous reference frame that is, the reference frame is rotating at the same speed as that of the modulation signals, doing which the transformed m_q and m_d quantities will be constant at steady state.

Thus equations 4.49 to 4.51 after transformation are:

$$\frac{V_{DC}}{2} m_q = V_{qs} \tag{4.52}$$

$$\frac{V_{DC}}{2} m_d = V_{ds} \tag{4.53}$$

$$\frac{V_{DC}}{2} m_{os} = V_{no} \quad . \tag{4.54}$$

V_{no} is the zero sequence voltage which can be selected to be zero or some other value depending on the PWM scheme we which to use, but if there is unbalance in the circuit the unbalance is reflected in V_{no} [13].