# SET 1

## OLYMPIAD 1

**1A**
*3 MINUTES*
*79%*

What number can replace the square to make the statement true?

$$5 \times 11 = \square + 12$$

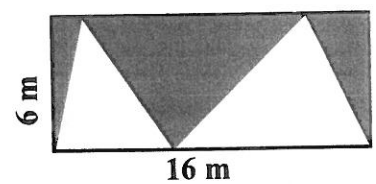
**1B**
*6 MINUTES*
*16%*

The sum of the 3-digit number AAA and the 2-digit number BB is the 4-digit number CD6E. A, B, C, D, and E are different digits. What 4-digit number does CD6E represent?

$$\begin{array}{r} A\ A\ A \\ +\quad B\ B \\ \hline C\ D\ 6\ E \end{array}$$

**1C**
*7 MINUTES*
*47%*

What is the area, in square meters, of the shaded part of the rectangle?




**1D**
*5 MINUTES*
*6%*

In simplest form, the fraction $\frac{60}{N}$ represents a whole number.
N is also a whole number.
What is the total number of different values that $N$ can be?

**1E**
*5 MINUTES*
*24%*

A bowl contains 100 pieces of colored candy: 48 green, 30 red, 12 yellow, and 10 blue. They are all wrapped in foil, so you do not know the color of any piece of candy. What is the least number of pieces you must take to be certain that you have at least 15 pieces of the same color?

***Solutions start on page 132.***

# SET 1
## OLYMPIAD 2

**2A**
*4 MINUTES*
*81%*

If two different counting numbers have the same digits but in reverse order, each number is called the palimage of the other. For example, 738 and 837 are palimages of each other; so are 1234 and 4321. What two different numbers between 40 and 60 are palimages of each other?

**2B**
*4 MINUTES*
*50%*

A cricket chirps 6 times every 8 seconds. At that rate, how many times does the cricket chirp in 2 minutes?

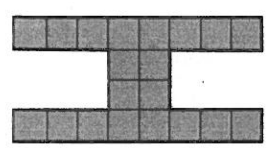
**2C**
*6 MINUTES*
*39%*

The odd numbers from 1 through 17 are placed in the magic square so that the sum of the numbers in each row, column and diagonal are equal. What number goes in the square marked “X”?

| | | |
|---|---|---|
| | 1 | |
| 5 | | 13 |
| X | | 3 |

**2D**
*5 MINUTES*
*15%.*

Each small region in the figure shown is a square. The area of the entire figure is 320 sq cm. What is the number of cm in the perimeter of the entire figure?



**2E**
*6 MINUTES*
*7%*

The pages of a book are numbered consecutively, beginning with 1. The digit 7 is printed 25 times in numbering the pages. What is the largest number of pages the book can have?

*Solutions start on page 133.*

# SET 1
## OLYMPIAD 3

**3A**
*4 MINUTES*
*61%*

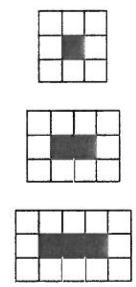
What is the greatest number of Mondays that can occur in 45 consecutive days?

**3B**
*5 MINUTES*
*49%*

The arithmetic mean (average) of five numbers is 8. Two of the numbers are 2 and 5. The other three numbers are equal. What is the value of one of the three equal numbers?

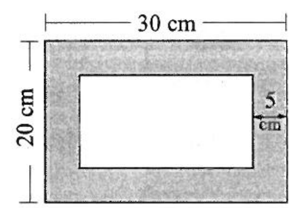
**3C**
*6 MINUTES*
*46%*

Each figure shown is formed by surrounding one row of black squares with white squares. How many white squares will surround one row of 50 black squares?



**3D**
*6 MINUTES*
*6%*

A picture frame is 20 cm by 30 cm. This includes a border (the shaded region) 5 cm wide surrounding the picture itself. What is the area of this shaded border in sq cm?




**3E**
*6 MINUTES*
*11%*

Two standard dice (number cubes) are rolled. One is red and one is green. What is the probability that the product of the two numbers on top is divisible by 3?

***Solutions start on page 135.***

# SET 1

## OLYMPIAD 4

**4A**
*4 MINUTES*
*82%*

Five students (Amy, Beth, Corey, Diego, Emily) sit in that order in a circle, counting down to 1. Amy starts by saying, "34". Then Beth says, "33", and so on. They continue around the circle to count down by ones. Who says, "1"?

**4B**
*5 MINUTES*
*61%*

What whole number may be used in place of to make this statement true?

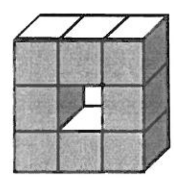
$$\frac{3}{5} < \frac{\square}{7} < \frac{4}{5}$$

**4C**
*6 MINUTES*
*32%*

Bay Street has from 2 through 15 houses, numbered 1, 2, 3, and so on. Mr. Sullivan lives in one of the houses. The sum of all the house numbers less than his equals the sum of all the house numbers greater than his. How many houses are there on Bay Street?

**4D**
*6 MINUTES*
*13%*

Eight cubes are glued together to form the figure shown. The length of an edge of each cube is 3 centimeters. The entire figure is covered in paint. How many square centimeters are covered in paint?



**4E**
*7 MINUTES*
*9%*

The whole number $N$ is divisible by 7. If $N$ is divided by 2 or 3 or 4 or 5, the remainder is 1 in each case. What is the smallest value that $N$ can be?

***Solutions start on page 137.***

# SET 1
## OLYMPIAD 5

**5A**
*6 MINUTES*
*34%*

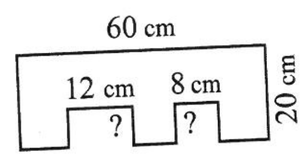
Mr. Red, Mr. White, Mr. Blue, and Mr. Gray each wears a shirt that is the same color as the name of one of the other three men. Each man wears a different color. Mr. Red and Mr. White are older than the man in gray. The man in red is next to Mr. White. Which man wears a white shirt?

**5B**
*4 MINUTES*
*17%*

A, B, and C represent three different numbers.
Each is chosen from the set $\{3, 5, 7, 9\}$.
What is the least possible value of $\frac{A}{B+C}$ ?
Express your answer as a simple fraction.

**5C**
*5 MINUTES*
*48%*

Two rectangles with equal heights are cut from a rectangular piece of paper as shown. The area of the remaining piece of paper is 980 sq cm. What is the height of each cut, in cm?




***(not drawn to scale)***

**5D**
*7 MINUTES*
*10%*

Alexis bakes 90 identical pizzas. Each pizza is cut either into 8 small slices or 6 large slices. There are 5 small slices for every 3 large slices. How many of the 90 pizzas are cut into small slices?

**5E**
*5 MINUTES*
*20%*

Lin has 8 marbles. Each marble weighs either 20 grams or 40 grams or 50 grams. He has a different number of marbles (at least one) of each weight. What is the smallest possible total weight of Lin's marbles?

***Solutions start on page 139.***

# SET 1 SOLUTIONS

## Set 1 Olympiad 1

**1A** <u>Strategy</u>: *First evaluate the left side of the equation.*
$5 \times 11 = 55$, so $55 = \square + 12$.
Then $\square = 55 - 12 = 43$.
**To make the statement true, replace the square by 43.**

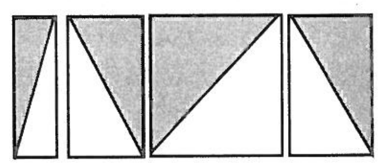
*FOLLOW-UP: Given $15 \times \square = \square + 84$. What one number can replace <u>both</u> squares to make the statement true?* [6]

**1B** <u>Strategy</u>: *Work left to right.*
The sum of a three-digit number and a two-digit number is less than 1100. The sum is 1000 or more only if the three-digit number is greater than 900. So C = 1, A = 9 and D = 0. The problem now is

$$\begin{array}{r} AAA \\ +\ BB \\ \hline CD6E \end{array} \quad \longrightarrow \quad \begin{array}{r} 999 \\ +\ BB \\ \hline 106E \end{array}$$

The sum of 9 and B results in a "carry" into the tens column. Thus, in the tens' column, the sum of 10 and B ends in 6, so that B = 6. This makes the addition 999 + 66 = 1065. **The four-digit number is 1065.**

**1C** <u>Strategy</u>: *Split the region into simpler figures.*
Draw line segments as shown to split the given rectangle into four smaller rectangles. Each small rectangle is cut in half by its diagonal. Half of each small rectangle is shaded. Therefore half of the whole original rectangle is shaded. The area of the original rectangle is $16 \times 6 = 96$ sq m, so **the area of the shaded part of the rectangle is 48 sq m.**



**1D** <u>Strategy</u>: *Make an organized table of factor pairs of 60.*
For $\frac{60}{N}$ to be a whole number, $N$ must be a factor of 60. The factor pairs for 60 are listed in the table at the right. If $N$ is any of these twelve factors, $\frac{60}{N}$ represents a whole number.
***N* can be any of 12 different values.**

| Factor Pairs |
|---|
| 1 × 60 |
| 2 × 30 |
| 3 × 20 |
| 4 × 15 |
| 5 × 12 |
| 6 × 10 |

# SET 1 SOLUTIONS

***Follow-Ups:*** *(1) How many factors (divisors) does each of the following have: 120? 180? 300? (Note that these are 60 × 2, 60 × 3, and 60 × 5.)* [16 factors; 18 factors; 18 factors] *(2) (Exploration) Find the 3 least numbers that have exactly three factors. What property do they share?* [4, 9, 25; They are squares of prime numbers.] *Repeat for numbers that have exactly five factors.* [16, 81, 625; they are fourth powers of primes.]

**1E** *Strategy: Consider the worst case.*
Determine the largest number of pieces you could take and still not have 15 of the same color. You could take all 12 yellow, all 10 blue, 14 of the green, and 14 of the red pieces and still not have 15 of the same color; so far you have a total of 50 pieces. The next piece you take, whether green or red, gives you 15 matching pieces. **The least number of pieces you must take to be sure that you have 15 pieces of the same color is 51.**

***Follow-Up:*** *Ana has 8 pennies, 3 quarters, 6 nickels, and 5 dimes in her piggy bank. She needs a dollar to buy a card. What is the greatest number of coins Ana can take out of the bank and still not have enough for the card?* [19] *What is the fewest number of coins that will get her the card?* [6]

## Olympiad 2

**2A** *Strategy: Consider the tens digits first.*
The numbers are between 40 and 60, so the tens digit is 4 or 5. Because the numbers are palimages, 5 and 4 are the only possible units digits. The numbers are different, so **the numbers are 45 and 54.**

***Follow–Up:*** *A number is called a* PALINDROME *if it reads the same when written left-to-right and right-to-left. For example 13831 is a palindrome. How many palindromes are there between 1000 and 2000?* [10] *Between 30,000 and 50,000?* [200]

**2B METHOD 1:** *Strategy: Find the number of 8-second periods in the interval.*
Two minutes contain 120 seconds. Because 120 ÷ 8 = 15, a two-minute interval has 15 periods of 8 seconds each. Because the cricket chirps 6 times in each period, **the cricket chirps** 15 × 6 = **90 times in two minutes.**

# SET 1 SOLUTIONS

**METHOD 2:** *Strategy: Set up a proportion using chirps per second.*
Let $N$ = the number of chirps in 120 seconds.

Then $\frac{\text{chirps}}{\text{sec.}}$ is equal to $\frac{6}{8} = \frac{N}{120}$.

Since $120 \div 8 = 15$, $\frac{6}{8} = \frac{6 \times 15}{8 \times 15} = \frac{90}{120}$. Then $N = 90$.

The cricket chirps 90 times in two minutes.

> **Follow–Up:** *Jim can cut a log into 5 pieces in 6 minutes. At that rate, how long will it take him to cut a log of the same thickness into 25 pieces?* [36 minutes (Hint: how many cuts are made?)] *Why is this problem different from 2B?*

**2C METHOD 1:** *Strategy: Find the "magic sum."*
To find the sum of the numbers in each row ("the magic sum"), divide the sum of all the numbers by the number of rows. $1 + 3 + 5 + \ldots + 17 = 81$, so the magic sum is $81 \div 3 = 27$.

| | | |
|---|---|---|
| | 1 | |
| 5 | A | 13 |
| X | B | 3 |

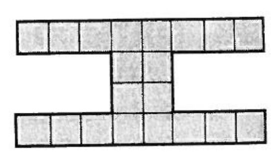
Since 5 + A + 13 = 27, square A contains 9. Then 1 + 9 + B = 27, so square B contains 17. Finally, X + 17 + 3 = 27, so **7 goes in the square marked "X".**

**METHOD 2:** *Strategy: Find the number in the middle square.*
The mean of the odd numbers 1 through 17 is 9. Therefore, 9 is the value in the middle box of this $3 \times 3$ magic square. The sum of the numbers in the middle row then is 27, which is the magic sum. Proceed as in Method 1 to find that the square marked "X" contains 7.

> **Follow–Ups:** *(1) Complete the magic square at the right using the first nine even numbers.* [The rows, from left to right, are 12, 14, 4; 2, 10, 18; 16, 6, 8] *(2)* **Exploration:** *Note that the sum of the first nine odd numbers is 81. What is the sum of the first ten odd numbers?* [100] *The first eleven?* [121] *What is the sum of the odd numbers from 1 through 27?* [196] *From 1 through 99?* [2500]

| | | |
|---|---|---|
| 12 | | |
| | | |
| | 6 | |

**2D** *Strategy: Determine the length of the side of a small square.*
The figure consists of 20 small congruent squares. The area of each small square is $\frac{1}{20}$ of the total area of 320 sq cm, which is 16 sq cm. Then the length of a side of each small square is 4 cm. **The perimeter of the entire figure** consists of 36 sides, so it **is** $36 \times 4 =$ **144 cm.**



# SET 1 SOLUTIONS

**2E** *Strategy: Count the number of 7s in an organized way.*

**METHOD 1:** From pages 1 through 100, the digit seven appears 10 times as a units digit and 10 times as a tens digit (the 70s) for a total of 20 times. The remaining 5 sevens are used to number pages 107, 117, 127, 137, and 147. A 26th seven would be required to number page 157. Therefore, **the largest number of pages the book can have is 156.**

**METHOD 2:** This table separates the numbers in which 7 appears in the tens place from the rest of the numbers. Adjust the cumulative total once it passes 25 sevens.

| pages | Number of Sevens | | | |
|---|---|---|---|---|
| | units place | tens place | subtotals | cum. |
| 1 to 69 | 7 | 0 | 7 | 7 |
| 70 to 79 | 1 | 10 | 11 | 18 |
| 80 to 169 | 9 | 0 | 9 | 27 |

Numbering up to page 169 requires 27 sevens. This is two more than are available. The two pages less than 169 that should not be numbered are pages 167 and 157. The largest number of pages the book can have is 156.

## Olympiad 3

**3A** **METHOD 1:** *Strategy: Use the definition of a week.*
In 45 consecutive days there are 6 weeks and 3 days. Each of the 6 weeks contains one Monday. In order to have the greatest number of Mondays, one of the 3 days left must also be a Monday. **The greatest number of Mondays that can occur in 45 consecutive days is 7.**

**METHOD 2:** *Strategy: Start at 1 and count by sevens.*
Suppose day 1 is a Monday. Mondays will occur on days 1, 8, 15, 22, 29, 36, 43. The next Monday would be after day 45. The greatest number of Mondays in 45 consecutive days is 7.

> **FOLLOW-UP:** *Today is Tuesday. What day is it 100 days from now? 1000?* [Thursday; Monday]

**3B** *Strategy: Find the sum of the five numbers.*
Because the average of the five numbers is 8, the sum of those five numbers is $5 \times 8 = 40$. The sum of the other three numbers is $40 - 2 - 5 = 33$. Then **the value of any one of the three equal numbers is** $33 \div 3 =$ **11.**

> **FOLLOW-UPS:** *The average of five different counting numbers is 8. Consider the greatest of these numbers. (1) What is its greatest possible value?* [30] *(2) What is its least possible value?* [10]

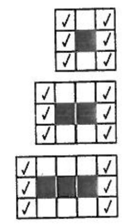
# SET 1 SOLUTIONS

**3C METHOD 1:** *Strategy: Find a pattern from a table of simpler cases.*

| Number of black squares | 1 | 2 | 3 | 4 | 5 | ... | 50 |
|---|---|---|---|---|---|---|---|
| Number of white squares | 8 | 10 | 12 | 14 | 16 | ... | ? |

According to the table, for every black square added, two white squares are added. To jump from 5 to 50 black squares, add 45 black squares. So to jump from 16 white squares, add $2 \times 45 = 90$ white squares to get **106 white squares** that **surround one row of 50 black squares**.

**METHOD 2:** *Strategy: Cut the ends off the figures; look for a pattern.*

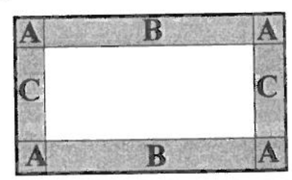


There are 6 white squares at the ends of each figure, 3 at each end. Each end square is checked. The center section contains 2 white squares for every black square. Therefore, the number of white squares is 6 more than twice the number of black squares. (Algebraically, this is written $W = 2 \times B + 6$) Thus, 106 white squares will surround one row of 50 black squares.

**FOLLOW-UP:** *Suppose, instead of a single row of black squares, the white squares surround a square of black squares. How many white squares surround a square block of 2500 black squares?* [204]

**3D METHOD 1:** *Strategy: Find the areas of the two rectangles and subtract.*
The area of the whole 20 × 30 framed rectangle is 600 sq cm. To find the dimensions of the picture itself, subtract 10 cm (5 cm from each end) from the dimensions of the whole rectangle. The picture is 10 cm by 20 cm. Its area is 200 sq cm. **The area of the shaded border is** 600 – 200 = **400 sq cm.**

**METHOD 2:** *Strategy: Split the desired region into more familiar figures.*




The diagram at the right shows one of several ways to partition the shaded region. The area of each region **A** is 5 × 5 = 25 sq cm, of each region **B** is 5 × 20 = 100 sq cm, and of each region **C** is 5 × 10 = 50 sq cm. The area of the shaded border is then (4 × 25) + (2 × 100) + (2 × 50) = 400 sq cm.

**3E METHOD 1:** *Strategy: Count in an organized way.*
The product is divisible by 3 if either number is 3 or 6. The chart shows all possible outcomes when two dice are rolled. An "**x**" marks each outcome in which the product is divisible by 3.

Green Die

| Red Die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | X | | | X |
| 2 | | | X | | | X |
| 3 | X | X | X | X | X | X |
| 4 | | | X | | | X |
| 5 | | | X | | | X |
| 6 | X | X | X | X | X | X |

There are 36 possible outcomes when two dice are rolled. In 20 of them the product of the two top numbers is divisible by 3. **The probability that the product of the two top numbers is divisible by 3 is $\frac{20}{36}$ or $\frac{5}{9}$.**

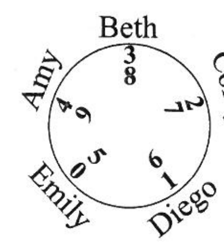
# SET 1 SOLUTIONS

**METHOD 2:** *Strategy: Examine each die separately and subtract duplications.*
As shown above, there are $6 \times 6 = 36$ possible outcomes. The rows (across) show the 12 times when the red die displays either 3 or 6. Similarly, the columns show the 12 times when the green die displays either 3 or 6. However, there are 4 times which appear in *both* the rows *and* the columns. Thus, there are 20, not 24, outcomes in which the product is divisible by 3. The probability is $\frac{20}{36}$ or $\frac{5}{9}$.

**METHOD 3:** *Strategy: Count the number of products which are NOT divisible by 3.*
If the product is not divisible by 3, each die shows one of the four numbers: 1, 2, 4, or 5. With two dice there are $4 \times 4 = 16$ outcomes in which the product is ***not*** a multiple of 3. This is shown by the empty spaces in the chart on page 10. Thus there are $36 - 16 = 20$ outcomes in which the product ***is*** a multiple of 3. As above the probability is then $\frac{20}{36}$ or $\frac{5}{9}$.

## Olympiad 4

**4A METHOD 1:** *Strategy: Count up.*
With five people in the circle, the person who says "1" will also say 6, 11, 16, 21, 26, and 31. If Amy says 34, and Beth says 33, then Corey says 32, and Diego says 31. Therefore, **the person who says "1" is Diego.**




**METHOD 2:** *Strategy: Count down, noticing the units digits.*
There are five students, so after counting twice around the table, they have counted down ten numbers. After that, the students count the same units digits as they did the first two rounds. The student who says "1" is the same student who says "31", Diego.

> **FOLLOW-UP:** *Suppose the counting numbers 1 through 1000 alternate direction in every other row as indicated in the table at the right. In which column does 49 appear? 1000?* [A, D]

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 12 | 11 | 10 | 9 | 8 | 7 |
| 13 | 14 | 15 | … | | |

*… and so on.*

**4B** *Strategy: Find a common denominator.*
The least common denominator is 35, which is the least common multiple of 5 and 7. Raising the terms, the statement becomes $\frac{21}{35} < \frac{5 \times \square}{35} < \frac{28}{35}$. The numerator of the middle fraction is a multiple of 5. The only multiple of 5 between 21 and 28 is 25. If $5 \times \square$ is 25, **the whole number used for $\square$ is 5.**

# SET 1 SOLUTIONS

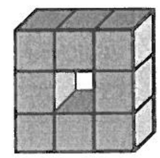
**4C** *Strategy: Make a table.*
Consider the house Mr. Sullivan might live in, the houses "before" his, and then the first few houses that come after that.

| Beginning of Street | | Mr. Sullivan | End of Street | |
|---|---|---|---|---|
| 1 + 2 + 3 | = **6** | 4 | 5 + 6 = **11** | *Too big* |
| 1 + 2 + 3 + 4 | = **10** | 5 | 6 + 7 = **13** | *Too big* |
| 1 + 2 + 3 + 4 + 5 | = **15** | 6 | 7 + 8 = **15** | *Equal sums* |

Therefore, **there are 8 houses on Bay Street.**

*FOLLOW-UP: Suppose the product of the house numbers before Mr. Sullivan's is the same as that of the house numbers after his. How many houses are on Bay Street?* [10]

**4D** **METHOD 1:** *Strategy: Count the number of cubes painted on 4 faces.*



Since every one of the eight cubes is painted on four of its six faces, 32 square faces are painted. The area of each face is $3 \times 3 = 9$ sq cm, so a total of $32 \times 9 =$ **288 sq cm are covered in paint.**

**METHOD 2:** *Strategy: Count the number of cube faces that are exposed.*
The front of the figure has 8 faces of cubes and the back has 8 faces. The top, bottom, and two sides each have 3 faces. The middle "hole" has 4 exposed faces. Therefore, a total of $(2 \times 8) + (4 \times 3) + 4 = 32$ exposed faces are painted. The area of each face is $3 \times 3$ or 9 sq cm, so $32 \times 9 = 288$ sq cm are covered in paint.

**METHOD 3:** *Strategy: Count the number of cube faces that are not exposed.*
Each cube has 6 faces and there are 8 cubes in the figure for a total of 48 faces. There are 8 places where 2 cube faces are glued together; thus $8 \times 2$ faces are not painted. Then $48 - 16 = 32$ faces are painted. As above, 288 sq cm are covered in paint.

**4E** **METHOD 1:** *Strategy: Use the least common multiple.*
$N$ leaves a remainder of 1 when divided by 2, 3, 4, or 5. Suppose we subtract 1 from $N$ to eliminate the remainder. The result is a multiple of 2, 3, 4, and 5, which have a least common multiple of 60. Moreover, *all* common multiples of 2, 3, 4, and 5 are multiples of 60. Then $N$ is 1 more than a multiple of 60. $N$ is in the set {61, 121, 181, 241, 301, 361, ...}. Divide each of these by 7. The first multiple of 7 in this set is 301. **The smallest value that N can be is 301.**

**METHOD 2:** *Strategy: Determine the units digit and then the possible multiples of 7.*
$N$ leaves a remainder of 1 when divided by 5, so $N$ has a units digit of 1 or 6. $N$ leaves a remainder of 1 when divided by 2, so $N$ is odd and its units digit is 1. The multiples of 7 that have a units digit of 1 are the product of 7 and a number with a units digit of 3; i.e. $7 \times 3$, $7 \times 13$, etc. Then $N$ is one of the numbers in the set {21, 91, 161, 231, 301, 371, ...}. The smallest of these that leaves a remainder of 1 when divided by 3 or by 4 is 301.

***Follow-Ups:** (1) A class has more than 10 students. The teacher tries to group them for a game. If she forms groups of 3, 4, 6, or 8, one student is left out. How many students are in the class?* [25] *(2) What is the smallest number that leaves a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, a remainder of 5 when divided by 6, and a remainder of 6 when divided by 7?* [419; Hint: What happens if 1 is added to the number?]

## Olympiad 5

**5A METHOD 1:** *<u>Strategy</u>: Use reasoning.*

1. "*Mr. Red and Mr. White are older than the man in gray.*" Neither Mr. Red nor Mr. White wears gray. Mr. Gray does not wear gray. So Mr. Blue wears the gray shirt.
2. "*The man in red is next to Mr. White.*" Mr. White does not wear red, Mr. Red does not wear red, and Mr. Blue is known to be wearing gray. So Mr. Gray wears the red shirt.
3. Mr. White does not wear white, so he is wearing the blue shirt.
4. Then **Mr. Red wears a white shirt.**

Some students may find it convenient to organize their thoughts by using a table, entering **X** for a combination that is ruled out by the clues and **O** for a match. One way to develop the table is shown below. The darkened boxes show new entries.

| | red | white | blue | gray |
|---|---|---|---|---|
| Mr. R | X | | | X |
| Mr. W | | X | | X |
| Mr. B | | | X | O |
| Mr. G | | | | X |

→

| | red | white | blue | gray |
|---|---|---|---|---|
| Mr. R | X | | | X |
| Mr. W | X | X | O | X |
| Mr. B | | | X | O |
| Mr. G | | | | X |

→

| | red | white | blue | gray |
|---|---|---|---|---|
| Mr. R | X | O | X | X |
| Mr. W | X | X | O | X |
| Mr. B | X | X | X | O |
| Mr. G | O | X | X | X |

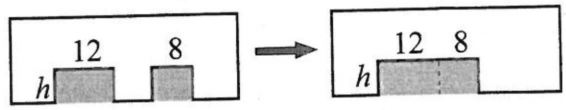
**5B** *<u>Strategy</u>: Minimize the numerator; maximize the denominator.*
A fraction has minimum value if the numerator is as small as possible and the denominator is as large as possible. **The least possible value of the fraction is $\frac{3}{9+7} = \frac{3}{16}$.**

***Follow-Up:** What is the largest possible value of the fraction?* [$\frac{9}{8}$]

**5C** *<u>Strategy</u>: Find the total area of the missing pieces.*
The original area of the piece of paper is $60 \times 20 = 1200$ sq cm. The total area of the regions cut out is then $1200 - 980 = 220$ sq cm. Then:

# SET 1 SOLUTIONS

**METHOD 1:** *Strategy: Change the figure to create a simpler problem.*
"Slide" the two shaded cutout rectangles together, as shown. None of the areas will change.




The total area of the cutouts is the same as the area of a single rectangle with base 12 + 8 = 20 cm. The area of this single rectangle is 220 so **the height of each cut is** 220 ÷ 20 = **11 cm.**

**METHOD 2:** *Strategy: Use algebra.*
Let $h$ = the height of each cut.
The areas of the cutout rectangles are $12h$ and $8h$. $12h + 8h = 220$
Add $12h$ and $8h$: $20h = 220$
Divide each side of the equation by 20: $h = 11$
The height of each cut is 11 cm.

**5D** For ease, call a pizza cut into large slices a *large pie* and call one cut into small slices a *small pie*.

**METHOD 1:** *Find a relationship between the numbers of large and small pies.*
Each small pie is cut into 8 slices, so the number of small slices is a multiple of 8. With 5 small slices for each 3 large slices, the number of small slices is also a multiple of 5. Then the number of small slices is a multiple of 40, the LCM of 8 and 5. Now, group the small slices by 40s. For each (8 × 5) = 40 small slices there are (8 × 3) = 24 large slices. But 40 small slices form 5 small pies and 24 large slices form 4 large pies. That is, of every 9 pies, 5 are small and 4 are large. Alexis has 10 groups of 9 pies, so **there are** 10 × 5 = **50 small pies** (and 40 large pies).

**METHOD 2:** *Strategy: Guess and check.*
With 5 small slices for every 3 large slices, then $\frac{1}{5}$ of the number of small slices equals $\frac{1}{3}$ of the number of large slices. In the table at the right we try first 45 pies of each size and then adjust by fives.

There are 50 small pies.

| | | |
|---|---|---|
| Number of small pies | 45 | **50** |
| Number of large pies | 45 | **40** |
| Number of small slices | 360 | **400** |
| Number of large slices | 270 | **240** |
| $\frac{1}{5}$ of the number of small slices | 72 | **80** |
| $\frac{1}{3}$ of the number of large slices | **No** | **YES** |

**Follow-Ups:** *(1) Suppose there are equal numbers of small and large slices. What is the smallest possible number of whole pizzas?* [7] *(2) Is it possible to have 90 pizzas with the same number of small slices as large slices?* [No; the number of pizzas would have to be a multiple of seven]

# SET 1 SOLUTIONS

**5E** *Strategy: Find three different counting numbers with a sum of 8.*
The numbers of each type of marble can be 1, 3, and 4, or they can be 1, 2 and 5. To get the least possible total weight, assign the greatest weight to the least number of marbles and the second greatest weight to the second least possible number of marbles. Then the least total weight occurs when Lin has one 50 gram marble, two 40 g marbles, and five 20 g marbles. **The smallest possible total weight of Lin's marbles is** $(1 \times 50) + (2 \times 40) + (5 \times 20) =$ **230 g.**

**FOLLOW-UP:** *What is the largest possible weight of Lin's marbles?* [350 g]