

CHAPTER 28

THE OPTION TO DELAY AND VALUATION IMPLICATIONS

In traditional investment analysis, a project or new investment should be accepted only if the returns on the project exceed the hurdle rate; in the context of cash flows and discount rates, this translates into investing in projects with positive net present values. The limitation of this view of the world, which analyzes projects on the basis of expected cash flows and discount rates, is that it fails to consider fully the options that are usually associated with many investments.

In this chapter, we will consider an option that is embedded in many projects, namely the option to wait and take the project in a later period. Why might a firm want to do this? If the present value of the cash flows on the project are volatile and can change over time, a project with a negative net present value today may have a positive net present value in the future. Furthermore, a firm may gain by waiting on a project even after a project has a positive net present value, because the option has a time premium that exceeds the cash flows that can be generated in the next period by accepting the project. We will argue that this option is most valuable in projects where a firm has the exclusive right to invest in a project and becomes less valuable as the barriers to entry decline.

There are three cases where the option to delay can make a difference when valuing a firm. The first is undeveloped land in the hands of real estate investor or company. The choice of when to develop rests in the hands of the owner and presumably development will occur when real estate values increase. The second is a firm that owns a patent or patents. Since a patent provides a firm with the exclusive rights to produce the patented product or service, it can and should be valued as an option. The third is a natural resource company that has undeveloped reserves that it can choose to develop at a time of its choosing – presumably when the price of the resource is high.

The Option to Delay a Project

Projects are typically analyzed based upon their expected cash flows and discount rates at the time of the analysis; the net present value computed on that basis is a measure of its value and acceptability at that time. Expected cash flows and discount rates change

over time, however, and so does the net present value. Thus, a project that has a negative net present value now may have a positive net present value in the future. In a competitive environment, in which individual firms have no special advantages over their competitors in taking projects, the fact that net present values can be positive in the future may not be significant. In an environment in which a project can be taken by only one firm because of legal restrictions or other barriers to entry to competitors, however, the changes in the project's value over time give it the characteristics of a call option.

The Payoff on the Option to Delay

Assume that a project requires an initial up-front investment of X and that the present value of expected cash inflows from investing in the project, computed today, is V . The net present value of this project is the difference between the two.

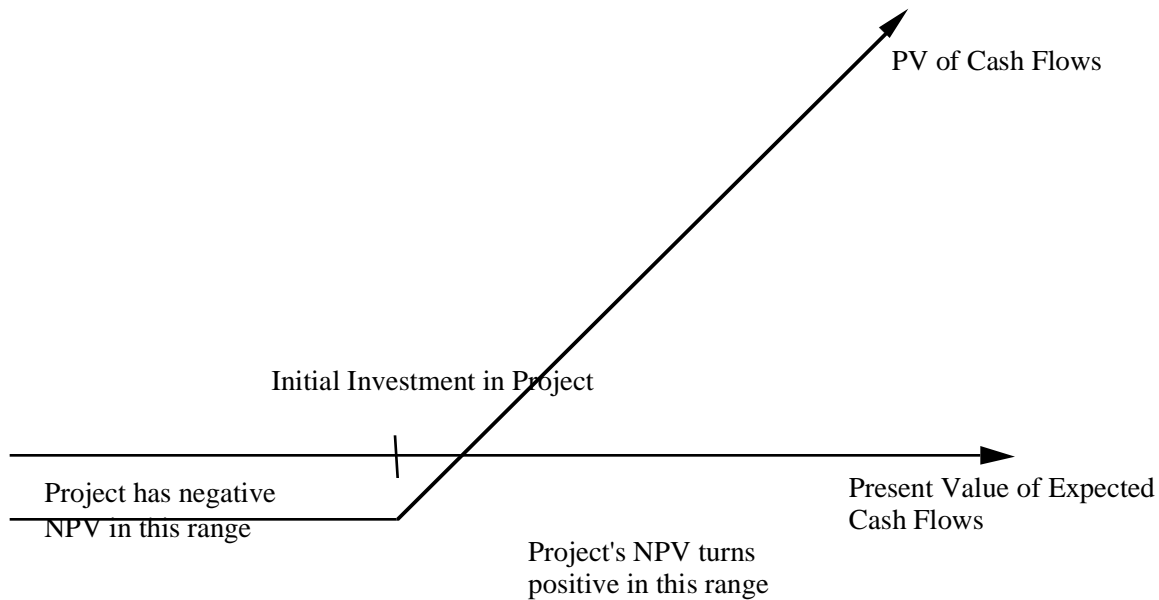
$$NPV = V - X$$

Now assume that the firm has exclusive rights to this project for the next n years and that the present value of the cash inflows may change over that time, because of changes in either the cash flows or the discount rate. Thus, the project may have a negative net present value right now, but it may still be a good project if the firm waits. Defining V again as the present value of the cash flows, the firm's decision rule on this project can be summarized as follows:

If	$V > X$	Invest in the project: Project has positive net present value
	$V < X$	Do not invest in the project: Project has negative net present value

If the firm does not invest in the project over its life, it incurs no additional cash flows, though it will lose what it invested to get exclusive rights to the project. This relationship can be presented in a payoff diagram of cash flows on this project, as shown in Figure 28.1, assuming that the firm holds out until the end of the period for which it has exclusive rights to the project.

Figure 28.1: The Option to Delay a Project



Note that this payoff diagram is that of a call option — the underlying asset is the project, the strike price of the option is the initial investment needed to take the project; and the life of the option is the period for which the firm has rights to the project. The present value of the cash flows on this project and the expected variance in this present value represent the value and variance of the underlying asset.

Inputs for Valuing the Option to Delay

The inputs needed to apply option pricing theory to value the option to delay are the same as those needed for any option using the Black-Scholes model. We need the value of the underlying asset, the variance in that value, the time to expiration on the option, the strike price, the riskless rate and the equivalent of the dividend yield.

Value Of The Underlying Asset

In the case of product options, the underlying asset is the project to which the firm has exclusive rights. The current value of this asset is the present value of expected cash flows from initiating the project now, not including the up-front investment. This present value can be obtained by doing a standard investment analysis. There is likely to be a substantial amount of error in the cash flow estimates and the present value, however. Rather than being viewed as a problem, this uncertainty should be viewed as the reason the project delay option has value. If the expected cash flows on the project were

known with certainty and were not expected to change, there would be no need to adopt an option pricing framework, since there would be no value to the option.

Variance in the value of the asset

As noted in the prior section, there is likely to be considerable uncertainty associated with the cash flow estimates and the present value that measures the value of the project now. This is partly because the potential market size for the product may be unknown and partly because technological shifts can change the cost structure and profitability of the product. The variance in the present value of cash flows from the project can be estimated in one of three ways.

- If we have invested in similar projects in the past, the variance in the cash flows from those projects can be used as an estimate. This may be the way that a consumer product company like Gillette might estimate the variance associated with introducing a new blade for its razors.
- We can assign probabilities to various market scenarios, estimate cash flows and a present value under each scenario and then calculate the variance across present values. Alternatively, the probability distributions can be estimated for each of the inputs into the project analysis - the size of the market, the market share and the profit margin, for instance - and simulations used to estimate the variance in the present values that emerge. This approach tends to work best when there are only one or two sources¹ of significant uncertainty about future cash flows.
- We can use the variance in the value of firms involved in the same business (as the project being considered) as an estimate of the variance. Thus, the average variance in the value of firms involved in the software business can be used as the variance in present value of a software project.

The value of the option is largely derived from the variance in cash flows - the higher the variance, the higher the value of the project delay option. Thus, the value of an option to invest in a project in a stable business will be less than the value of one in an environment where technology, competition and markets are all changing rapidly.

Exercise Price on Option

The option to delay a project is exercised when the firm owning the rights to the project decides to invest in it. The cost of making this initial investment is the exercise price of the option. The underlying assumption is that this cost remains constant (in present value dollars) and that any uncertainty associated with the investment is reflected in the present value of cash flows on the product.

Expiration of the Option and the Riskless Rate

The project delay option expires when the rights to the project lapse. Investments made after the project rights expire are assumed to deliver a net present value of zero as competition drives returns down to the required rate. The riskless rate to use in pricing the option should be the rate that corresponds to the expiration of the option. While expiration dates can be estimated easily when firms have the explicit right to a project (through a license or a patent, for instance), it becomes far more difficult to obtain if the right is less clearly defined. If, for instance, a firm has a competitive advantage on a product or project, the option life can be defined as the expected period over which the advantage can be sustained.

Cost of Delay

In Chapter 5, we noted that an American option generally will not be exercised prior to expiration. When you have the exclusive rights to a project, though, and the net present value turns positive, you would not want the owner of the rights to wait until the rights expire to exercise the option (invest in the project). Note that there is a cost in delaying investing in a project, once the net present value turns positive. If you wait an additional period, you may gain if the variance pushes value higher but you also lose one period of protection against competition. You have to consider this cost when analyzing the option and there are two ways of estimating it.

- Since the project rights expire after a fixed period and excess profits (which are the source of positive present value) are assumed to disappear after that time as new

¹ In practical terms, the probability distributions for inputs like market size and market share can often be obtained from market testing.

competitors emerge, each year of delay translates into one less year of value-creating cash flows.² If the cash flows are evenly distributed over time and the life of the patent is n years, the cost of delay can be calculated.

$$\text{Annual cost of delay} = \frac{1}{n}$$

Thus, if the project rights are for 20 years, the annual cost of delay works out to 5% a year for the very first year. Note, though, that this cost of delay rises each year, to 1/19 in year 2, 1/18 in year 3 and so on, making the cost of delaying exercise larger over time.

- If the cash flows are uneven, the cost of delay can be more generally defined in terms of the change in present value that can be expected to occur over the next period as a percent of the present value today.

$$\text{Cost of delay} = \frac{\text{Present value}_{\text{next period}} - \text{Present value}_{\text{Now}}}{\text{Present value}_{\text{Now}}}$$

In either case, the likelihood that a firm will delay investing in a project is higher early in the exclusive rights period rather than later and the cost will increase as the loss in present value from waiting a period increases.



optvar.xls: There is a dataset on the web that summarizes standard deviations in firm value and equity value by industry group in the United States.

Illustration 28.1: Valuing the Option to Delay a Project

Assume that you are interested in acquiring the exclusive rights to market a new product that will make it easier for people to access their email on the road. If you do acquire the rights to the product, you estimate that it will cost you \$50 million up-front to set up the infrastructure needed to provide the service. Based upon your current projections, you believe that the service will generate only \$10 million in after-tax cash

² A value-creating cashflow is one that adds to the net present value because it is in excess of the required return for investments of equivalent risk.

flows each year. In addition, you expect to operate without serious competition for the next 5 years.

From a static standpoint, the net present value of this project can be computed by taking the present value of the expected cash flows over the next 5 years. Assuming a discount rate of 15% (based on the riskiness of this project), we obtain the following net present value for the project.

$$\begin{aligned} &= -50 \text{ million} + 10 \text{ million}(\text{PV of annuity, 15\%, 5 years}) \\ \text{NPV of project} &= -50 \text{ million} + 33.5 \text{ million} \\ &= -16.5 \text{ million} \end{aligned}$$

This project has a negative net present value.

The biggest source of uncertainty about this project is the number of people who will be interested in the product. While current market tests indicate that you will capture a relatively small number of business travelers as your customers, they also indicate the possibility that the potential market could get much larger over time. In fact, a simulation of the project's cash flows yields a standard deviation of 42% in the present value of the cash flows, with an expected value of \$33.5 million.

To value the exclusive rights to this project, we first define the inputs to the option pricing model.

Value of the Underlying Asset (S) = PV of Cash Flows from Project if introduced now = \$ 33.5 million

Strike Price (K) = Initial Investment needed to introduce the product = \$ 50.0 million

Variance in Underlying Asset's Value = $0.42^2 = 0.1764$

Time to expiration = Period of exclusive rights to product = 5 years

Dividend Yield = $1/\text{Life of the patent} = 1/5 = 0.20$

Assume that the 5-year riskless rate is 5%. The value of the option can be estimated.

$$\text{Call Value} = 33.5e^{-(0.2)(5)}(0.2250) - 50.0e^{-(0.05)(5)}(0.0451) = \$1.019 \text{ million}$$

The rights to this product, which has a negative net present value if introduced today, is \$1.019 million. Note, though, as measured by N(d1) and N(d2), the likelihood is low that this project will become viable before expiration.



delay.xls: This spreadsheet allows you to estimate the value of an option to delay an investment.

Arbitrage Possibilities and Option Pricing Models

In our discussion of option pricing models in chapter 5, we noted that they are based upon two powerful constructs – the idea of replicating portfolios and arbitrage. Models such as the Black Scholes and the Binomial assume that you can create a replicating portfolio, using the underlying asset and riskless borrowing or lending, that has cashflows identical to those on an option. Furthermore, these models assume that since investors can then create riskless positions by buying the underlying option and selling the replicating portfolio, or vice versa, the value of the call should converge on the cost of creating the replicating portfolio. If it does not, investors should be able to create riskless positions and walk away with guaranteed profits – the essence of arbitrage. This is why the interest rate used in option pricing models is the riskless rate.

With listed options on traded stocks or assets, arbitrage is clearly feasible at least for some investors. With options on non-traded assets, it is almost impossible to trade the replicating portfolio though you can create it on paper. In the illustration above, for instance, you would need to buy 0.225 units (the option delta) of the underlying project (a non-traded asset) to create a portfolio that replicates the call option.

There are some who argue that the impossibility of arbitrage makes it inappropriate to use option pricing models to value real options, whereas other try to adjust for this limitation by using an interest rate higher than the riskless rate in the option pricing model. We do not think that either of these responses is appropriate. Note that while you cannot trade on the replicating portfolios in many real options, you still can create them on paper (as we did in illustration 28.1) and value the options. The difficulties in creating arbitrage positions may result in prices that deviate by a large amounts from this value, but that is an argument for using real option pricing models and not for avoiding them. Increasing the

riskless rate to reflect the higher risk associated with real options may seem like an obvious fix, but doing this will only make call options (such as the one valued in illustration 28.1) more valuable, not less.

If you want to be more conservative in your estimate of value for real options to reflect the difficulty of arbitrage, you have two choices. One is to use a higher discount rate in computing the present value of the cash flows that you would expect to make from investing in the project today, thus lowering the value of the underlying asset (S) in the model. In illustration 28.1, using a 20% discount rate rather than a 15% rate would result in a present value of \$29.1 million, which would replace the \$33.5 million as S in the model. The other is to value the option and then apply an illiquidity discount to it (similar to the one we used in valuing private companies) because you cannot trade it easily.

Problems in valuing the Option to Delay

While it is quite clear that the option to delay is embedded in many projects, several problems are associated with the use of option pricing models to value these options. First, the underlying asset in this option, which is the project, is not traded, making it difficult to estimate its value and variance. We have argued that the value can be estimated from the expected cash flows and the discount rate for the project, albeit with error. The variance is more difficult to estimate, however, since we are attempting to estimate a variance in project value over time.

Second, the behavior of prices over time may not conform to the price path assumed by the option pricing models. In particular, the assumption that value follows a diffusion process and that the variance in value remains unchanged over time, may be difficult to justify in the context of a project. For instance, a sudden technological change may dramatically change the value of a project, either positively or negatively.

Third, there may be no specific period for which the firm has rights to the project. Unlike the case of a patent, for instance, in which the firm has exclusive rights to produce the patented product for a specified period, the firm's rights often are less clearly defined, in terms of both exclusivity and time. For instance, a firm may have significant advantages over its competitors, which may, in turn, provide it with the virtually exclusive rights to a

project for a period of time. An example would be a company with strong brand name recognition in retailing or consumer products. The rights are not legal restrictions, however, and will erode over time. In such cases, the expected life of the project itself is uncertain and only an estimate. In the valuation of the rights to the product, in the previous section, we used a life of 5 years for the option, but competitors could in fact enter sooner than we anticipated. Alternatively, the barriers to entry may turn out to be greater than expected and allow the firm to earn excess returns for longer than 5 years. Ironically, uncertainty about the expected life of the option can increase the variance in present value and through it the expected value of the rights to the project.

Implications and Extensions of Delay Options

Several interesting implications emerge from the analysis of the option to delay a project as an option. First, a project may have a negative net present value currently based upon expected cash flows, but the rights to it may still be valuable because of the option characteristics.

Second, a project may have a positive net present value but still not be accepted right away. This can happen because the firm may gain by waiting and accepting the project in a future period, for the same reasons that investors do not always exercise an option that is in the money. A firm is more likely to wait if it has the rights to the project for a long time and the variance in project inflows is high. To illustrate, assume a firm has the patent rights to produce a new type of disk drive for computer systems and building a new plant will yield a positive net present value today. If the technology for manufacturing the disk drive is in flux, however, the firm may delay investing in the project in the hopes that the improved technology will increase the expected cash flows and consequently the value of the project. It has to weigh this benefit against the cost of delaying the project, which will be the cash flows that will be forsaken by not investing in it.

Third, factors that can make a project less attractive in a static analysis can actually make the rights to the project more valuable. As an example, consider the effect of uncertainty about the size of the potential market and the magnitude of excess returns. In a static analysis, increasing this uncertainty increases the riskiness of the project and

may make it less attractive. When the project is viewed as an option, an increase in the uncertainty may actually make the option more valuable, not less. We will consider two cases, product patents and natural resource reserves, where we believe that the project delay option allows us to estimate value more precisely.

Option Pricing Models

Once you have identified the option to delay a project as a call option and identified the inputs needed to value the option, it may seem like a trivial task to actually value the option. There are, however, some serious estimation issues that we have to deal with in valuing these options. In Chapter 5, we noted that while the more general model for valuing options is the Binomial model, many practitioners use the Black-Scholes model, which makes far more restrictive assumptions about price processes and early exercise to value options. With listed options on traded assets, options can be exercised early at fairly low cost. With real options, there can be a substantial cost for the following reasons.

- Unlike listed options, real options tend to be exercised early, if they are in the money. While there are ways in which the Black-Scholes can be adjusted to allow for this early exercise, the Binomial allows for much more flexibility.
- The binomial option pricing model allows for a much wider range of price processes for the underlying asset than the Black-Scholes, which assumes that prices are not only continuous but log-normally distributed. With real options, where the present value of the cashflows is often equivalent to the price, the assumptions of non-normality and continuous distribution may be difficult to sustain.

The biggest problem with the binomial model is that the prices at each node of the binomial tree have to be estimated. As the number of periods expands, this will become more and more difficult to do. You can, however, use the variance estimate to come up with rough measures of the magnitude of the up and down movements, which can be used to obtain the binomial tree.

Having made a case for the binomial model, you may find it surprising that we use the Black-Scholes model to value any real options. We do so not only because the model

is more compact and elegant to present, but because we believe that it will provide a lower bound on the value in most cases. To provide a frame of reference, we will present the values that we would have obtained using a binomial model in each case.

From Black-Scholes to Binomial

It is a fairly simple exercise to convert the inputs to the Black-Scholes model into a binomial model. To make the adjustment, you have to assume a multiplicative binomial process, where the magnitude of the jumps, in percent terms, remains unchanged from period to period. If you assume symmetric probabilities, the up (u) and down (d) movements can be estimated as a function of the annualized variance in the price process and how many periods you decide to break each year into (t).

$$u = e^{\sigma \sqrt{dt} + r - y - \frac{\sigma^2}{2} dt}$$

$$d = e^{-\sigma \sqrt{dt} + r - y - \frac{\sigma^2}{2} dt}$$

$$\text{where } dt = \frac{1}{\text{number of periods each year}}$$

To illustrate, consider the project delay option that we valued in Illustration 28.1. The standard deviation in the value was assumed to be 42%, the riskfree rate was 5% and the dividend yield was 20%. To convert the inputs into a binomial model, we will assume that each year is a time period and estimate the up and down movements.

$$u = e^{0.42\sqrt{1} + 0.05 - 0.20 - \frac{0.42^2}{2} \cdot 1} = 1.1994$$

$$d = e^{-0.42\sqrt{1} + 0.05 - 0.20 - \frac{0.42^2}{2} \cdot 1} = 0.5178$$

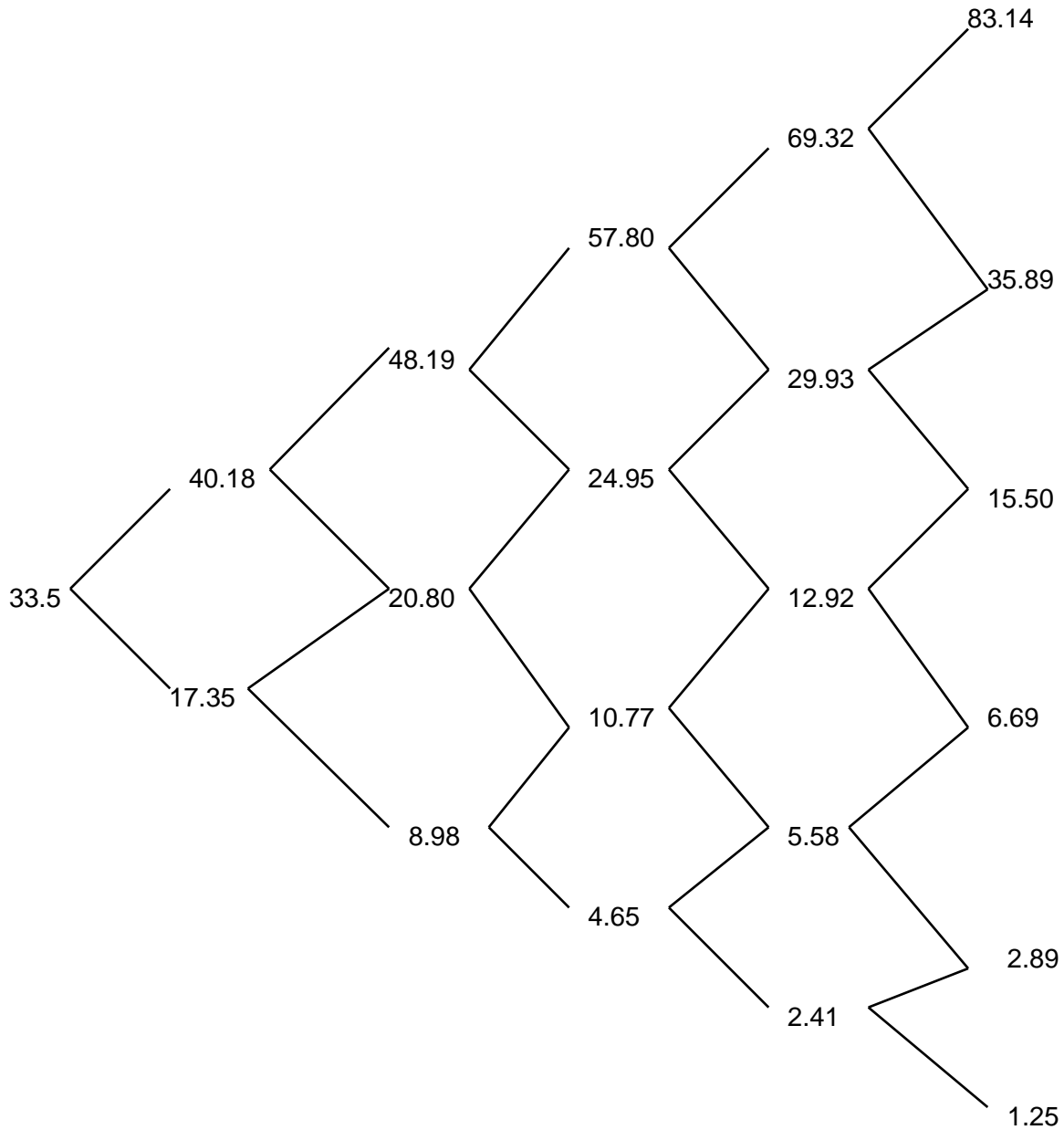
The value today is \$33.5 million. To estimate the end values for the first branch:

$$\text{Value with up movement} = \$33.5 (1.1994) = \$40.20 \text{ million}$$

$$\text{Value with down movement} = \$33.5 (0.5178) = \$17.35 \text{ million}$$

You could use these values then to get the three potential values at the second branch. Note that the value of \$17.35 million growing at 19.94% is exactly equal to the value of \$40.20 million dropping by 48.22%. The binomial tree for the five periods is shown below in Figure 28.2.

Figure 28.2: Binomial tree for delay option



You could estimate the value of the option from this binomial tree to be \$1.58 million, slightly higher than the estimate obtained from the Black Scholes model of \$1.019 million. The differences will narrow as the option becomes more in the money and the time period used in the binomial model shortens.

Valuing a Patent

A number of firms, especially in the technology and pharmaceutical sectors, can patent products or services. A product patent provides a firm with the right to develop and market a product and thus can be viewed as an option.

Patents as call options

The firm will develop a patent only if the present value of the expected cash flows from the product sales exceeds the cost of development, as shown in Figure 28.3. If this does not occur, the firm can shelve the patent and not incur any further costs. If I is the present value of the costs of commercially developing the patent and V is the present value of the expected cash flows from development, then:

$$\begin{aligned} \text{Payoff from owning a product patent} &= V - I && \text{if } V > I \\ &= 0 && \text{if } V \leq I \end{aligned}$$

Thus, a product patent can be viewed as a call option, where the product is the underlying asset.

Figure 28.3: Payoff to Introducing Product

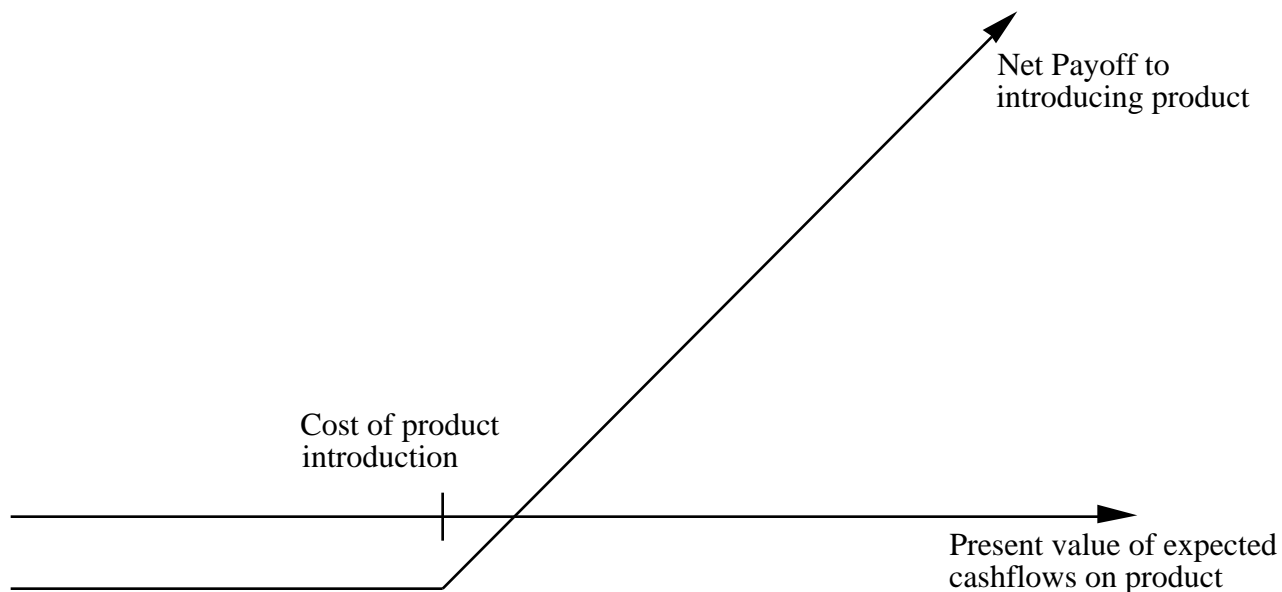


Illustration 28.2: Valuing a Patent: Avonex in 1997

Biogen is a bio-technology firm with a patent on a drug called Avonex, which has received FDA approval for use in treating multiple sclerosis. Assume you are trying to

value the patent and that you have the following estimates for use in the option pricing model.

- An internal analysis of the financial viability of the drug today, based upon the potential market and the price that the firm can expect to charge for the drug, yields a present value of cash flows of \$3.422 billion, prior to considering the initial development cost.
- The initial cost of developing the drug for commercial use is estimated to be \$2.875 billion, if the drug is introduced today.
- The firm has the patent on the drug for the next 17 years and the current long-term treasury bond rate is 6.7%.
- The average variance in firm value for publicly traded bio-technology firms is 0.224.

We assume that the potential for excess returns exists only during the patent life and that competition will eliminate excess returns beyond that period. Thus, any delay in introducing the drug, once it becomes viable, will cost the firm one year of patent-protected returns. (For the initial analysis, the cost of delay will be 1/17, next year it will be 1/16, the year after 1/15 and so on.)

Based on these assumptions, we obtain the following inputs to the option pricing model.

Present Value of Cash Flows from Introducing the Drug Now = $S = \$ 3.422$ billion

Initial Cost of Developing Drug for Commercial Use (today) = $K = \$ 2.875$ billion

Patent Life = $t = 17$ years

Riskless Rate = $r = 6.7\%$ (17-year Treasury Bond rate)

Variance in Expected Present Values = $\sigma^2 = 0.224$

Expected Cost of Delay = $y = 1/17 = 5.89\%$

These yield the following estimates for d and $N(d)$:

$$d1 = 1.1362 \quad N(d1) = 0.8720$$

$$d2 = -0.8152 \quad N(d2) = 0.2076$$

Plugging back into the dividend-adjusted Black-Scholes option pricing model³, we get:

$$\text{Value of the patent} = 3,422e^{(-0.0589)(17)}(0.8720) - 2,875e^{(-0.067)(17)}(0.2079) = \$907 \text{ million}$$

To provide a contrast, the net present value of this project is only \$547 million.

$$\text{NPV} = \$3,422 \text{ million} - \$ 2,875 \text{ million} = \$ 547 \text{ million}$$

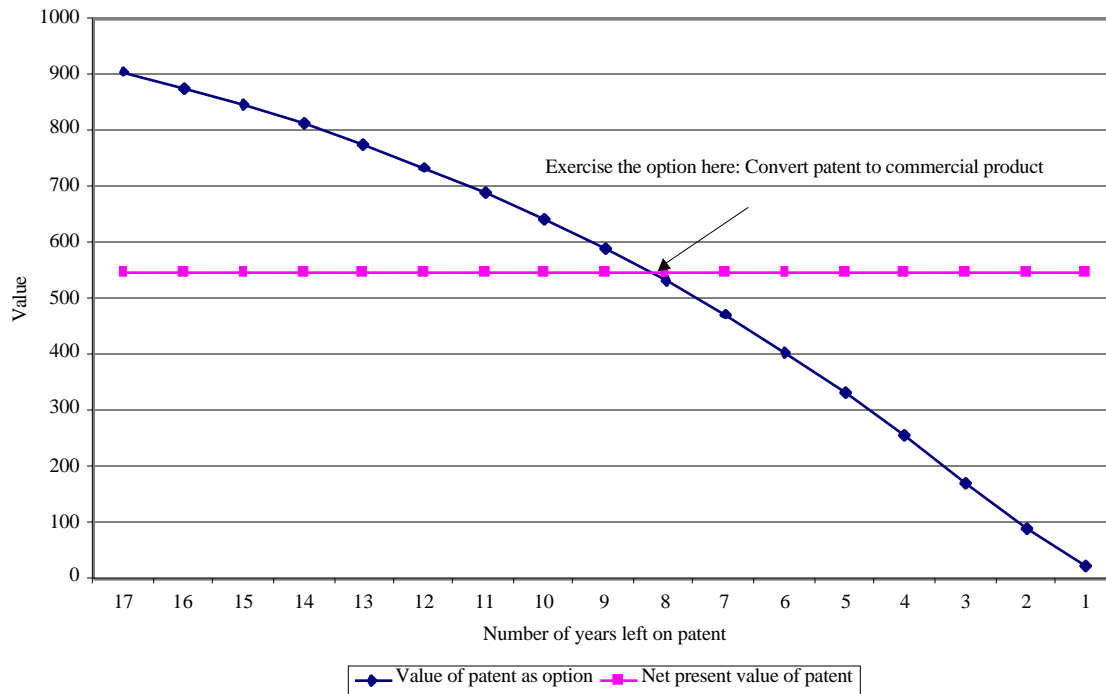
The time premium of \$360 million on this option (\$907-\$547) suggests that the firm will be better off waiting rather than developing the drug immediately, the cost of delay notwithstanding. However, the cost of delay will increase over time and make exercise (development) more likely in future years.

To illustrate, we will value the call option, assuming that all of the inputs, other than the patent life, remain unchanged. For instance, assume that there are 16 years left on the patent. Holding all else constant, the cost of delay increases as a result of the shorter patent life.

$$\text{Cost of delay} = 1/16$$

The decline in the present value of cash flows (which is S) and the increase in the cost of delay (y) reduce the expected value of the patent. Figure 28.4 graphs the option value and the net present value of the project each year.

Figure 28.4: Patent value versus Net Present value



³ With a binomial model, we estimate a value of \$915 million for the same option.

Based upon this analysis, if nothing changes, you would expect Avonex to be worth more as a commercial product than as a patent four years from now, which would also then be the optimal time to commercially develop the product.



product.xls: This spreadsheet allows you to estimate the value of a patent.

Competitive Pressures and Option Values

In the section above, we have taken the view that a firm is protected from competition for the life of the patent. This is generally true only for the patented product or process, but the firm may still face competition from other firms that come up with their own products to serve the same market. More specifically, Biogen can patent Avonex, but Merck or Pfizer can come up with their own drugs to treat multiple sclerosis and compete with Biogen.

What are the implications for the value of the patent as an option? First, the life of the option will no longer be the life of the patent but the lead time that the firm has until a competing product is developed. For instance, if Biogen knows that another pharmaceutical firm is working on a drug to treat MS and where this drug is in the research pipeline (early research or the stage in the FDA approval process), it can use its estimate of how long it will take before the drug is approved for use as the life of the option. This will reduce the value of the option and make it more likely that the drug will be commercially developed earlier rather than later.

The presence of these competitive pressures may explain why commercial development is much quicker with some drugs than with others and why the value of patents is not always going to be greater than a discounted cash flow valuation. Generally speaking, the greater the number of competing products in the research pipeline, the less likely it is that the option pricing model will generate a value that is greater than the traditional discounted cash flow model.

Valuing a firm with patents

If the patents owned by a firm can be valued as options, how can this estimate be incorporated into firm value? The value of a firm that derives its value primarily from

commercial products that emerge from its patents can be written as a function of three variables.

- the cash flows it derives from patents that it has already converted into commercial products,
- the value of the patents that it already possesses that have not been commercially developed and
- the expected value of any patents that the firm can be expected to generate in future periods from new patents that it might obtain as a result of its research.

Value of Firm = Value of commercial products + Value of existing patents + (Value of New patents that will be obtained in the future – Cost of obtaining these patents)

The value of the first component can be estimated using traditional cash flow models. The expected cash flows from existing products can be estimated for their commercial lives and discounted back to the present at the appropriate cost of capital to arrive at the value of these products. The value of the second component can be obtained using the option pricing model described earlier to value each patent. The value of the third component will be based upon perceptions of a firm's research capabilities. In the special case, where the expected cost of research and development in future periods is equal to the value of the patents that will be generated by this research, the third component will become zero. In the more general case, firms such as Cisco and Pfizer that have a history of generating value from research will derive positive value from this component as well.

How would the estimate of value obtained using this approach contrast with the estimate obtained in a traditional discounted cash flow model? In traditional discounted cash flow valuation, the second and the third components of value are captured in the expected growth rate and the expected time in such an expected growth rate in cash flows. Firms such as Cisco are allowed to grow at much higher rates for longer periods because of the technological edge they possess and their research prowess. In contrast, the approach described in this section looks at each patent separately and allows for the option component of value explicitly.

The biggest limitation of the option-based approach is the information that is needed to put it in practice. To value each patent separately, you need access to

proprietary information that is usually available only to managers of the firm. In fact, some of the information, such as the expected variance to use in option pricing may not even be available to insiders and will have to be estimated for each patent separately.

Given these limitations, the real option approach should be used to value small firms with one or two patents and little in terms of established assets. A good example would be Biogen in 1997, which was valued in the last section. For firms such as Cisco and Lucent that have significant assets in place and hundreds of patents, discounted cash flow valuation is a more pragmatic choice. Viewing new technology as options provides insight into Cisco's successful growth strategy over the last decade. Cisco has been successful at buying firms with nascent and promising technologies (options) and converting them into commercial success (exercising these options).

Illustration 28.3: Valuing Biogen as a firm

In Illustration 28.2, the patent that Biogen owns on Avonex was valued as a call option and the estimated value was \$907 million. To value Biogen as a firm, two other components of value would have to be considered.

- Biogen had two commercial products (a drug to treat Hepatitis B and Intron) at the time of this valuation that it had licensed to other pharmaceutical firms. The license fees on these products were expected to generate \$50 million in after-tax cash flows each year for the next 12 years. To value these cash flows, which were guaranteed contractually, the pre-tax cost of debt of 7% of the licensing firms was used.

$$\text{Present Value of License Fees} = \$50 \text{ million} \frac{1 - \frac{1}{1.07^{12}}}{0.07} = \$397.13 \text{ million}$$

- Biogen continued to fund research into new products, spending about \$100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years and 5% thereafter. While it was difficult to forecast the specific patents that would emerge from this research, it was assumed

that every dollar invested in research would create \$1.25 in value in patents⁴ (valued using the option pricing model described above) for the next 10 years and break even after that (i.e., generate \$1 in patent value for every \$1 invested in R&D). There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.⁵ The value of this component was then estimated as follows:

$$\text{Value of Future Research} = \sum_{t=1}^{t=\infty} \frac{\text{Value of Patents}_t - R \& D_t}{(1+r)^t}$$

The table below summarizes the value of patents generated each period and the R&D costs in that period. Note that there is no surplus value created after the tenth year.

Table 28.1: Value of New Research

<i>Year</i>	<i>Value of Patents generated</i>	<i>R&D Cost</i>	<i>Excess Value</i>	<i>Present Value at 15%</i>
1	\$ 150.00	\$ 120.00	\$ 30.00	\$ 26.09
2	\$ 180.00	\$ 144.00	\$ 36.00	\$ 27.22
3	\$ 216.00	\$ 172.80	\$ 43.20	\$ 28.40
4	\$ 259.20	\$ 207.36	\$ 51.84	\$ 29.64
5	\$ 311.04	\$ 248.83	\$ 62.21	\$ 30.93
6	\$ 373.25	\$ 298.60	\$ 74.65	\$ 32.27
7	\$ 447.90	\$ 358.32	\$ 89.58	\$ 33.68
8	\$ 537.48	\$ 429.98	\$ 107.50	\$ 35.14
9	\$ 644.97	\$ 515.98	\$ 128.99	\$ 36.67
10	\$ 773.97	\$ 619.17	\$ 154.79	\$ 38.26
				\$ 318.30

⁴ To be honest, this is not an estimate based upon any significant facts other than Biogen's history of success in coming up with new products. You can estimate this value created from the return and cost of capital for a firm. For instance, if you assume that you can earn a return on capital of 15% in perpetuity with a cost of capital of 10%, you would obtain the following value created for every dollar of investment: Value Created = $\$ 1 + (\text{ROC} - \text{Cost of Capital}) / \text{Cost of Capital} = 1 + (.15 - .10) / .10 = \$ 1.50$

⁵ This discount rate was estimated by looking at the costs of equity of young publicly traded biotechnology firms with little or no revenue from commercial products.

The total value created by new research is \$318.3 million.

The value of Biogen as a firm is the sum of all three components – the present value of cash flows from existing products, the value of Avonex (as an option) and the value created by new research.

$$\begin{aligned} \text{Value} &= \text{CF: Commercial products} + \text{Value: Undeveloped patents} + \text{Value: Future R\&D} \\ &= \$ 397.13 \text{ million} + \$ 907 \text{ million} + \$ 318.30 \text{ million} = \$1622.43 \text{ million} \end{aligned}$$

Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.50 million) to arrive at a value per share.

$$\text{Value per share} = \frac{\$1,622.43 \text{ million}}{35.5} = \$45.70$$

Is there life after the patent expires?

In our valuations, we assumed that the excess returns are restricted to the patent life and that they disappear the instant the patent expires. In the pharmaceutical sector, the expiration of a patent does not necessarily mean the loss of excess returns. In fact, many firms continue to be able to charge a premium price for their products and earn excess returns even after the patent expires, largely as a consequence of the brand name image that they built up over the project life. A simple way of adjusting for this reality is to increase the present value of the cash flows on the project (S) and decrease the cost of delay (y) to reflect this reality. The net effect is a greater likelihood that firms will delay commercial development, while they wait to collect more information and assess market demand.

The other thing that might increase the value of the patent is the capacity that drug companies have shown to lobby legislators to extend the patent life of profitable drugs. If we consider this as a possibility when we value a patent, it will increase the expected life of the patent and its value as an option.

Natural Resource Options

Natural resource companies, such as oil and mining companies, generate cash flows from their existing reserves but also have undeveloped reserves that they can develop if they choose to do so. They will be much more likely to develop these reserves if the price of the resource (oil, gold, copper) increases and these undeveloped reserves

can be viewed as call options. In this section, we will begin by looking at the value of an undeveloped reserve and then consider how we can extend this to look at natural resource companies that have both developed and undeveloped reserves.

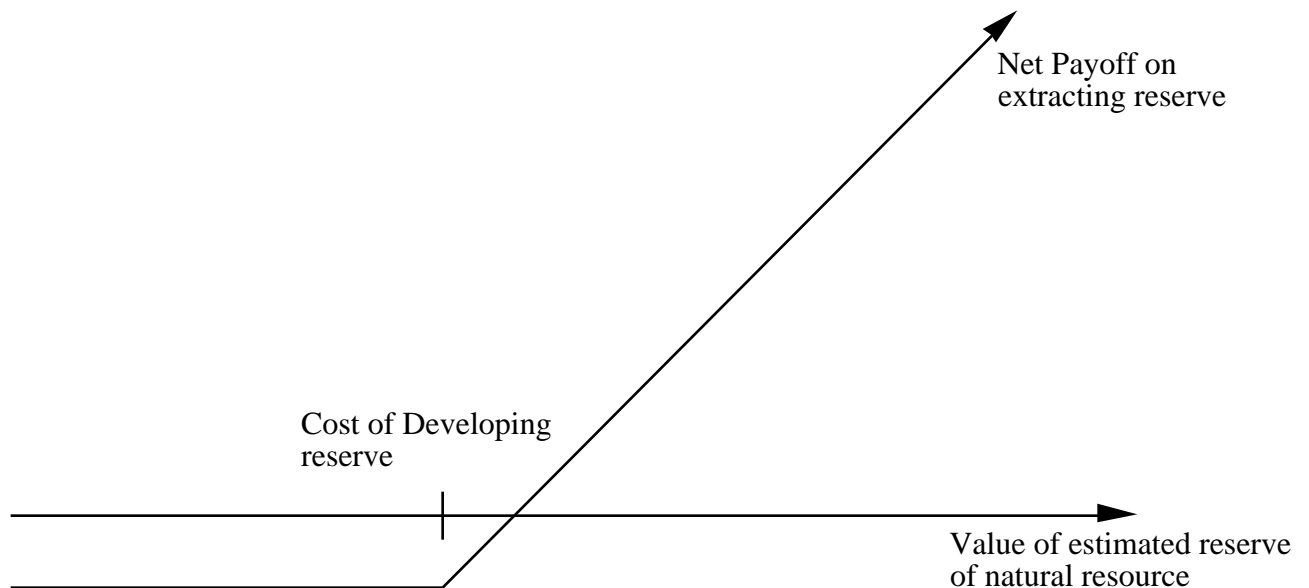
Undeveloped Reserves as Options

In a natural resource investment, the underlying asset is the natural resource and the value of the asset is based upon the estimated quantity and the price of the resource. Thus, in a gold mine, the underlying asset is the value of the estimated gold reserves in the mine, based upon the price of gold. In most such investments, there is an initial cost associated with developing the resource; the difference between the value of the estimated reserves and the cost of the development is the profit to the owner of the resource (see Figure 28.5). Defining the cost of development as X and the estimated value of the resource as V makes the potential payoffs on a natural resource option the following:

$$\begin{aligned} \text{Payoff on natural resource investment} &= V - X && \text{if } V > X \\ &= 0 && \text{if } V \leq X \end{aligned}$$

Thus, the investment in a natural resource option has a payoff function similar to a call option.

Figure 28.5: Payoff from Developing Natural Resource Reserves



Inputs for valuing a natural resource option

To value a natural resource investment as an option, we need to make assumptions about a number of variables.

1. Available reserves of the resource and estimated value if extracted today: Since this is not known with certainty at the outset, it has to be estimated. In an oil tract, for instance, geologists can provide reasonably accurate estimates of the quantity of oil available in the tract. The value of the reserves is then the product of the estimated reserves and the contribution (market price of the resource – variable cost of extraction) per unit of reserve.

2. Estimated cost of developing the resource: The estimated cost of developing the resource reserve is the exercise price of the option. In an oil reserve, this would be the fixed cost of installing the rigs to extract oil from the reserve. With a mine, it would be the cost associated with making the mine operational. Since oil and mining companies have done this before in a variety of settings, they can use their experience to come up with a reasonable measure of development cost.

3. Time to expiration of the option: The life of a natural resource option can be defined in one of two ways. First, if the ownership of the investment has to be relinquished at the end of a fixed period of time, that period will be the life of the option. In many off-shore oil leases, for instance, the oil tracts are leased to the oil company for a fixed period. The second approach is based upon the inventory of the resource and the capacity output rate, as well as estimates of the number of years it would take to exhaust the inventory. Thus, a gold mine with a reserve of 3 million ounces and a capacity output rate of 150,000 ounces a year will be exhausted in 20 years, which is defined as the life of the natural resource option.

4. Variance in value of the underlying asset: The variance in the value of the underlying asset is determined by the variability in the price of the resource and the variability in the estimate of available reserves. In the special case where the quantity of the reserve is known with certainty, the variance in the underlying asset's value will depend entirely upon the variance in the price of the natural resource.

5. Cost of Delay: The net production revenue is the annual cash flow that will be generated, once a resource reserve has been developed, as a percentage of the market value

of the reserve. This is the equivalent of the dividend yield and is treated the same way in calculating option values. An alternative way of thinking about this cost is in terms of a cost of delay. Once a natural resource option is in the money (Value of the reserves > Cost of developing these reserves), by not developing the reserve the firm is costing itself the production revenue it could have generated by doing so.

An important issue in using option pricing models to value natural resource options is the effect of development lags on the value of these options. Since oil or gold or any other natural resource reserve cannot be developed instantaneously, a time lag has to be allowed between the decision to extract the resources and the actual extraction. A simple adjustment for this lag is to reduce the value of the developed reserve for the loss of cash flows during the development period. Thus, if there is a one-year lag in development and you can estimate the cash flow you would make over that year, you can estimate the cash flow as a percent of your reserve value and discount the current value of the developed reserve at that rate. This is the equivalent of removing the first year's cash flow from your investment analysis and lowering the present value of your cash flows.

*In Practice 28.4: Valuing an Oil Reserve*⁶

Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil; the cost of developing the reserve is expected to be \$600 million, and the development lag is two years. Exxon has the rights to exploit this reserve for the next 20 years, and the marginal value (price per barrel - marginal cost per barrel) per barrel of oil is currently \$12⁷. Once developed, the net production revenue each year will be 5% of the value of the reserves. The riskless rate is 8%, and the variance in oil prices is 0.03.

Given this information, the inputs to the Black-Scholes can be estimated

Current Value of the asset = S = Value of the developed reserve discounted back the length of the development lag at the dividend yield = $\frac{(12)(50)}{1.05^2} = \544.22

⁶ The following is a simplified version of the illustration provided by Siegel, Smith and Paddock to value an offshore oil property.

⁷ For simplicity, we will assume that while this marginal value per barrel of oil will grow over time, the present value of the marginal value will remain unchanged at \$12 per barrel. If we do not make this

Exercise Price = Cost of developing reserve = \$ 600 million

Time to expiration on the option = 20 years

Variance in the value of the underlying asset⁸ = 0.03

Riskless rate = 8%

Dividend Yield = Net production revenue / Value of reserve = 5%


Based upon these inputs, the Black-Scholes model provides the following values.

$d_1 = 1.0359$ $N(d_1) = 0.8498$

$d_2 = 0.2613$ $N(d_2) = 0.6030$

Call Value = $544.22e^{-(0.05)(20)}(0.8498) - 600e^{-(0.08)(20)}(0.6030) = \97.10 million

This oil reserve, though not viable at current prices, is still valuable because of its potential to create value if oil prices go up.⁹

 *natres.xls*: This spreadsheet allows you to estimate the value of an undeveloped natural resource reserve.

Multiple Sources of Uncertainty

In the example above, we assumed that there was no uncertainty about the quantity of the reserve. Realistically, the oil company has an estimate of the reserve of 50 million barrels but does not know it with certainty. If we introduce uncertainty about the quantity of the reserve into the analysis, there will be two sources of variance and both can affect value. There are two ways we can address this problem.

1. Combine the uncertainties into one value: If we consider the value of the reserves to be the product of the price of oil and the oil reserves, the variance in the value should reflect the combined effect of the variances in each input. In fact, if the variance in the $\ln(\text{estimated reserve quantity})$ is q^2 and the variance in $\ln(\text{oil prices})$ is p^2 , the variance¹⁰ in the value of the reserve can be calculated.

assumption, we will have to estimate the present value of the oil that will be extracted over the extraction period.

⁸ In this example, we assume that the only uncertainty is in the price of oil and the variance therefore becomes the variance in $\ln(\text{oil prices})$.

⁹ With a binomial model, we arrive at an estimate of value of \$99.15 million.

¹⁰ This is the variance of a product of two variables. If $c = ab$, then $c^2 = a^2 + b^2 +$

$$\sigma_{res}^2 = \sigma_q^2 \frac{\# \text{ barrels}^2}{\# \text{ barrels} + \text{price}/\text{barrel}} + \sigma_p^2 \frac{\text{price}/\text{barrel}^2}{\# \text{ barrels} + \text{price}/\text{barrel}}$$

We are assuming that there is no correlation between the estimated quantity of oil in a reserve and the price per barrel of oil. Thus, in the above example, if we introduced a variance of 0.05 in the estimated reserve of 50 million barrels, we obtain the following estimate for the variance in the value of the reserve.

$$\sigma_{res}^2 = 0.05 \frac{50^2}{50 + 12} + 0.03 \frac{12^2}{50 + 12} = 0.0336$$

This would be the variance we would use in the option pricing model to estimate a new value for the reserve.

2. Keep the variances separate and value the option as a rainbow option: A rainbow option allows explicitly for more than one source of variance and allows us to keep the variances separate and still value the option. While option pricing becomes more complicated, you may need to do this if you expect the two sources of uncertainty to evolve differently over time – the variance from one source (say, market conditions) may increase over time whereas the variance from the other source (say, technology) may decrease over time.

Valuing a firm with undeveloped reserves

The examples provided above illustrate the use of option pricing theory in valuing individual mines and oil tracts. Since the assets owned by a natural resource firm can be viewed primarily as options, the firm itself can be valued using option pricing models.

Individual Reserves versus Aggregate Reserves

The preferred approach would be to consider each option separately, value it and cumulate the values of the options to get the value of the firm. Since this information is likely to be difficult to obtain for large natural resource firms, such as oil companies, which own hundreds of such assets, a variant of this approach is to value the entire firm as one option. A purist would probably disagree, arguing that valuing an option on a portfolio of assets (as in this approach) will provide a lower value than valuing a portfolio

of options (which is what the natural resource firm really own) because aggregating the assets that are correlated yields a lower variance which will lower the value of the portfolio of the aggregated assets. Nevertheless, the value obtained from the model still provides an interesting perspective on the determinants of the value of natural resource firms.

Inputs to option valuation

If you decide to apply the option pricing approach to estimate the value of undeveloped reserves, you have to estimate the inputs to the model. In general terms, while the process resembles the process used to value an individual reserve, there are a few differences.

Value of underlying asset: You should cumulate all of the undeveloped reserves owned by a company and estimate the value of these reserves, based upon the price of the resource today and the average variable cost of extracting these reserves today. The variable costs are likely to be higher for some reserves and lower for others, and weighting the variable costs at each reserve by the quantity of the resource of that reserve should give you a reasonable approximation of this value. At least hypothetically, we are assuming that the company can decide to extract all of its undeveloped reserves at one time and not affect the price of the resource.

Exercise Price: For this input, you should consider what it would cost the company today to develop all of its undeveloped reserves. Again, the costs might be higher for some reserves than for others, and you can use a weighted average cost.

Life of the option: A firm will probably have different lives for each of its reserves. As a consequence, you will have to use a weighted average of the lives of the different reserves.¹¹

Variance in the value of the asset: Here, there is a strong argument for looking at only the oil price as the source of variance, since a firm should have a much more precise estimate of its total reserves than it does of any one of its reserves.

¹¹ If you own some reserves in perpetuity, you should cap the life of the reserve at a large value – say, 30 years – in making this estimate.

Dividend Yield (cost of delay): As with an individual reserve, a firm with viable reserves will be giving up the cash flows it could receive in the next period from developing these reserves if it delays exercise. This cash flow, stated as a percent of the value of the reserves, becomes the equivalent of the dividend yield.

The development lag reduces the value of this option just as it reduces the value of an individual reserve. The logical implication is that undeveloped reserves will be worth more at oil companies that can develop their reserves quicker than at less efficient companies.

Illustration 28.5: Valuing an oil company - Gulf Oil in 1984

Gulf Oil was the target of a takeover in early 1984 at \$70 per share (It had 165.30 million shares outstanding and total debt of \$9.9 billion). It had estimated reserves of 3038 million barrels of oil and the average cost of developing these reserves at that time was estimated to be \$30.38 billion dollars (The development lag is approximately two years). The average relinquishment life of the reserves is 12 years. The price of oil was \$22.38 per barrel, and the production cost, taxes and royalties were estimated at \$7 per barrel. The bond rate at the time of the analysis was 9.00%. If Gulf chooses to develop these reserves, it was expected to have cash flows next year of approximately 5% of the value of the developed reserves. The variance in oil prices is 0.03.

Value of underlying asset = Value of estimated reserves discounted back for period of

$$\text{development lag} = \frac{(3038)(22.38 - 7)}{1.05^2} = \$42,380 \text{ million}$$

Note that you could have used forecasted oil prices and estimated cash flows over the production period to estimate the value of the underlying asset, which is the present value of all of these cash flows. We have used as short cut of assuming that the current contribution margin of \$15.38 a barrel will remain unchanged in present value terms over the production period.

Exercise price = Estimated cost of developing reserves today = \$30,380 million

Time to expiration = Average length of relinquishment option = 12 years

Variance in value of asset = Variance in oil prices = 0.03

Riskless interest rate = 9%

Dividend yield = Net production revenue/ Value of developed reserves = 5%

Based upon these inputs, the Black-Scholes model provides the following value for the call.¹²

$$d_1 = 1.6548$$

$$N(d_1) = 0.9510$$

$$d_2 = 1.0548$$

$$N(d_2) = 0.8542$$

$$\text{Call Value} = 42,380e^{(-0.05)(12)}(0.9510) - 30,380e^{(-0.09)(12)}(0.8542) = \$13,306 \text{ million}$$

This stands in contrast to the discounted cash flow value of \$12 billion that you obtain by taking the difference between the present value of the cash flows of developing the reserve today (\$42.38 billion) and the cost of development (\$30.38 billion). The difference can be attributed to the option possessed by Gulf to choose when to develop its reserves.

This represents the value of the undeveloped reserves of oil owned by Gulf Oil. In addition, Gulf Oil had free cashflows to the firm from its oil and gas production from already developed reserves of \$915 million and assume that these cashflows are likely to be constant and continue for ten years (the remaining lifetime of developed reserves). The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:

$$\text{Value of already developed reserves} = \frac{915 \left(1 - \frac{1}{1.125^{10}}\right)}{0.125} = \$5,066$$

Adding the value of the developed and undeveloped reserves of Gulf Oil provides the value of the firm.

Value of undeveloped reserves	= \$ 13,306 million
Value of production in place	= \$ 5,066 million
Total value of firm	= \$ 18,372 million
Less Outstanding Debt	= \$ 9,900 million
Value of Equity	= \$ 8,472 million
Value per share	= $\frac{\$8,472}{165.3} = \51.25

This analysis would suggest that Gulf Oil was overvalued at \$70 per share.

¹² With a binomial model, we estimate the value of the reserves to be \$13.73 billion.

Price Volatility and Natural Resource Company Valuation

An interesting implication of this analysis is that the value of a natural resource company depends not just on the price of the natural resource but also on the expected volatility in that price. Thus, if the price of oil goes from \$25 a barrel to \$40 a barrel, you would expect all oil companies to become more valuable. If the price drops back to \$25, the values of oil companies may not decline to their old levels, since the perceived volatility in oil prices may have changed. If investors believe that the volatility in oil prices has increased, you would expect an increase in values but the increase will be greatest for companies that derive a higher proportion of their value from undeveloped reserves.

If you regard undeveloped reserves as options, discounted cash flow valuation will generally under estimate the value of natural resource companies, because the expected price of the commodity is used to estimate revenues and operating profits. As a consequence, you miss the option component of value. Again, the difference will be greatest for firms with significant undeveloped reserves and with commodities where price volatility is highest.

Other Applications

While patents and undeveloped reserves of natural resource companies lend themselves best to applying option pricing, there are other assets that we have referenced in earlier chapters that can also be valued as options.

- In Chapter 26, in the context of real estate valuation, we noted that vacant land could be viewed as an option on commercial development.
- In Chapter 27, we presented an argument that copyrights and licenses could be viewed as options, even if they are not commercially viable today.

The following table presents the inputs you would use to value each of these options in an option pricing model.

Table 28.2: Inputs to value other options to delay

	<i>Undeveloped Land</i>	<i>License/ copyright</i>
Value of the	Present value of the cash flows	Present value of the

underlying asset	that would be obtained from commercial development of land today.	cashflows that would be obtained from commercially utilizing the license or copyright today.
Variance in value of underlying asset	Variance in the values of commercial property in the area where the real estate is located.	Variance in the present values from commercial utilization of copyright or license.
Exercise Price	Cost of commercially developing land today.	Up-front cost of commercially utilizing copyright or license today.
Life of the option	If land is under long-term lease, you could use the lease period. If not, you should set the option life equal to the period when the loan that you used to buy the land comes due.	Period for which you have rights to copyright or license.
Cost of delay	Interest payments you have to make on the loan each year.	Cashflow you could generate in next year as a percent of present value of the cash flows today.

Much of what we have said about the other option applications apply here as well. The value is derived from the exclusivity that you have to commercially develop the asset. That exclusivity is obtained by legal sanction in the case of licenses and copyrights and from the scarcity of land in the case of undeveloped land.

Summary

In traditional investment analysis, we compute the net present value of a project's cash flows and conclude that firms should not invest in a project with a negative net present value. This is generally good advice, but it does not imply that the rights to this

project are not valuable. Projects that have negative net present values today may have positive net present values in the future, and the likelihood of this occurring is directly a function of the volatility in the present value of the cash flows from the project.

In this chapter, we valued the option to delay an investment and considered the implications of this option for three valuation scenarios – the value of a firm that derives all or a significant portion of value from patents that have not been commercially exploited yet, the value of a natural resource company with undeveloped reserves of the resource and the value of a real estate firm with undeveloped land. In each case, we showed that using discounted cash flow valuation would result in an understatement of the values for these firms.

Problems

1. A company is considering delaying a project which with after-tax cash flows of \$25 million but costs \$300 million to take (the life of the project is 20 years and the cost of capital is 16%). A simulation of the cash flows leads you to conclude that the standard deviation in the present value of cash inflows is 20%. If you can acquire the rights to the project for the next ten years, what are the inputs for the option pricing model? (The six-month T.Bill rate is 8%, the ten year bond-rate is 12% and the 20-year bond rate is 14%.)
2. You are examining the financial viability of investing in some abandoned copper mines in Chile, which still have significant copper deposits in them. A geologist survey suggests that there might be 10 million pounds of copper in the mines still and that the cost of opening up the mines will be \$3 million (in present value dollars). The capacity output rate is 400,000 pounds a year and the price of copper is expected to increase 4% a year. The Chilean Government is willing to grant a twenty-five year lease on the mine. The average production cost is expected to be 40 cents a pound and the current price per pound of copper is 85 cents. (The production cost is expected to grow 3% a year, once initiated.) The annualized standard deviation in copper prices is 25% and the twenty-five year bond rate is 7%.
 - a. Estimate the value of the mine using traditional capital budgeting techniques.
 - b. Estimate the value of the mine based upon an option pricing model.
 - c. How would you explain the difference between the two values?
3. You have been asked to analyze the value of an oil company with substantial oil reserves. The estimated reserves amount to 10,000,000 barrels and the estimated present value of the development cost for each barrel is \$12. The current price of oil is \$20 per barrel and the average production cost is estimated to be \$6 per barrel. The company has the rights to these reserves for the next twenty years and the twenty year bond rate is 7%. The company also proposes to extract 4% of its reserves each year to meet cashflow needs. The annualized standard deviation in the price of the oil is 20%. What is the value of this oil company?
4. You are analyzing a capital budgeting project. The project is expected to have a PV of cash inflows of \$250 million and will cost \$200 million (in present value dollars) to take on. You have done a simulation of the project cashflows and the

simulation yields a variance in present value of cash inflows of 0.04. You have the rights to this project for the next five years, during which period you have to pay \$12.5 million a year to retain the project rights. The five-year treasury bond rate is 8%.

- a. What is the value of project, based upon traditional NPV?
 - b. What is the value of the project as an option?
 - c. Why are the two values different? What factor or factors determine the magnitude of this difference?
5. Cyclops Inc, a high technology company specializing in state-of-the-art visual technology, is considering going public. While the company has no revenues or profits yet on its products, it has a ten-year patent to a product that will enable contact lens users to get no-maintenance lens that will last for years. While the product is technically viable, it is exorbitantly expensive to manufacture and the potential market for it will be relatively small initially. (A cash flow analysis of the project suggests that the present value of the cash inflows on the project, if adopted now, would be \$250 million, while the cost of the project will be \$500 million.) The technology is rapidly evolving and a simulation of alternative scenarios yields a wide range of present values, with an annualized standard deviation of 60%. To move towards this adoption, the company will have to continue to invest \$10 million a year in research. The ten-year bond rate is 6%.
- a. Estimate the value of this company.
 - b. How sensitive is this value estimate to the variance in project cash flows? What broader lessons would you draw from this analysis?