

Set of Problems for Exam # 3

Problem 1. Suppose two dice (one red, one green) are rolled. Consider the following events. A: the red die shows 2; B: the numbers add to 3; C: at least one of the numbers is 1; and D: the numbers do not add to 11. Express the given event in symbols:

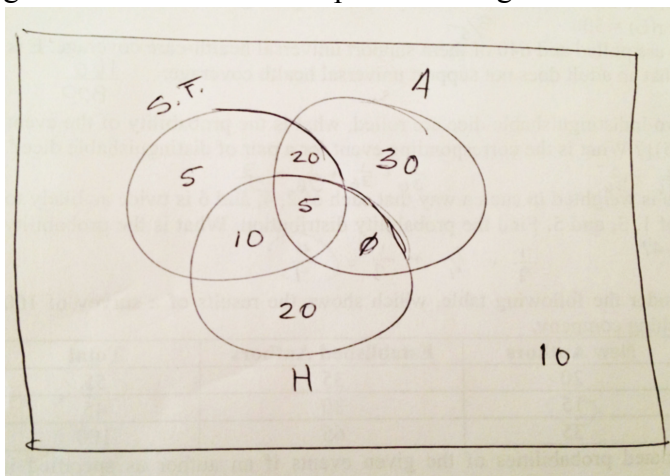
- **Event 1:** The red die shows 2 and the numbers add to 3.
ANSWER: $A \cap B$
- **Event 2:** The numbers do not add to 3.
ANSWER: B'
- **Event 3:** The numbers add to 11 or the red die shows a 2.
ANSWER: $D' \cup A$

Problem 2 . In February, 2005, a search using the web search engine Google™ for “hans” yielded 41 million websites containing that word. A search for “franz” yielded 18 million sites. A search for sites containing both words yielded 3 million sites. How many websites contained either “hans” or “franz” or both?

ANSWER: 56 Million sites

Problem 3. According to a survey of 100 people regarding their movie attendance in the last year, 40 had seen a science fiction movie, 55 had seen an adventure movie, and 35 had seen a horror movie. Moreover, 25 had seen a science fiction movie and an adventure movie, 5 had seen an adventure movie and a horror movie, and 15 had seen a science fiction movie and a horror movie. Only 5 people had seen a movie from all three categories.

- a. Use the given information to set up a Venn diagram and solve it.



- b. Complete the following sentence: The survey suggests that ANSWER: 37.5 % of science fiction movie fans are also horror movie fans.

Problem 4. If $n(A \cup B) = 100$ and $n(A) = n(B) = 60$, find $n(A \cap B)$.
ANSWER: 20

Problem 5. If $n(A) = 100$, $n(A \cup B) = 150$, and $n(A \cap B) = 40$, find $n(B)$.
ANSWER: 90

Problem 6. You are playing Scrabble and have the following letters to work with: A, S, A, L. Because you are losing the game, you would like to use all your letters to make a single word, but you can't think of any four-letter words using all these letters. In desperation, you decide to list *all* the four-letter sequences possible to see if there are any valid words among them. How large is your list?

ANSWER: 12

Problem 7. A test requires that you answer first Part A and then either Part B or Part C. Part A consists of 4 true-false questions, Part B consists of 4 multiple-choice questions with 1 correct answer out of 5, and Part C consists of 3 questions with one correct answer out of 6. How many different completed answer sheets are possible?

ANSWER: $10,000 + 3456 = 13,456$

Problem 8. Many U.S. license plates display a sequence of 3 letters followed by 3 digits.

- a. How many such license plates are possible?

ANSWER: 17,576,000

- b. In order to avoid confusion of letters with digits, some states do not issue standard plates with the last letter an I, O, or Q. How many license plates are still possible?

ANSWER: 15,548,000

- c. Assuming that the letter combinations VET, MDZ and DPZ are reserved for disabled veterans, medical practitioners and disabled persons, respectively, how many license plates are possible, also taking the restriction in part (b) into account?

ANSWER: 15,545,000

Problem 9. A bag contains 3 red marbles, 2 green ones, 1 lavender one, 2 yellows, and 2 orange marbles.

- a. How many sets of five marbles include one of each color?

ANSWER: 24

- b. How many sets of five marbles include at least two red ones?

ANSWER: $105 + 21 = 126$

- c. How many sets of five marbles include at most one of the yellow ones?

ANSWER: $56 + 140 = 196$

Problem 10. If 12 businesspeople have a meeting and each pair exchanges business cards, how many business cards, total, get exchanged?

ANSWER: 132

Problem 11. Suzy grabs 5 marbles from a bag containing 4 red marbles, 3 green ones, 2 white ones, and 2 purple one. She grabs 5 of them. Find the probabilities of the following events, expressing each as a fraction in lowest terms.

Event A: She has all the green ones.

ANSWER: $\frac{28}{462} = \frac{2}{33} \approx 0.0606 \approx 6.06\%$

Event B: She has at least 2 green ones.

$$\text{ANSWER: } \frac{196}{462} = \frac{14}{33} \approx 0.4242 \approx 42.42\%$$

Event C: She has 2 red ones and 1 of each of the other colors.

$$\text{ANSWER: } \frac{72}{462} = \frac{12}{77} \approx 0.1558 \approx 15.58\%$$

Problem 12. Calculate the estimated probability $P(E)$ using the given information.

a. $N = 500$, $f_r(E) = 300$

$$\text{ANSWER: } \frac{3}{5} = 0.6 = 60\%$$

b. 800 adults are polled and 640 of them support universal health-care coverage. E is the event that an adult does not support universal health coverage.

$$\text{ANSWER: } \frac{1}{5} = 0.2 = 20\%$$

Problem 13. If two indistinguishable dice are rolled, what is the probability of the event $\{(5, 5), (2, 5), (3, 5)\}$? $\text{ANSWER: } \frac{5}{36} \approx 0.1389 \approx 13.89\%$

What is the corresponding event for a pair of distinguishable dice?

$$\text{ANSWER: } \frac{1}{12} \approx 0.0833 \approx 8.33\%$$

Problem 14. A die is weighted in such a way that each of 2, 4, and 6 is twice as likely to come up as each of 1, 3, and 5. Find the probability distribution.

ANSWER:

$$P(\{1\}) = \frac{1}{9}; \quad P(\{2\}) = \frac{2}{9}; \quad P(\{3\}) = \frac{1}{9}; \quad P(\{4\}) = \frac{2}{9}; \quad P(\{5\}) = \frac{1}{9}; \quad P(\{6\}) = \frac{2}{9}$$

What is the probability of rolling less than 4?

$$\text{ANSWER: } \frac{4}{9} \approx 0.4444 \approx 44.44\%$$

Problem 15. Consider the following table, which shows the results of a survey of 100 authors by a publishing company.

	New Authors	Established Authors	Total
Successful	20	35	55
Unsuccessful	15	30	45
Total	35	65	100

Compute the estimated probabilities of the given events if an author as specified is chosen at random.

a. An author is established and successful.

$$\text{ANSWER: } \frac{7}{20} \approx 0.35 \approx 35\%$$

- b. An author is unsuccessful and new.
ANSWER: $\frac{3}{20} \approx 0.15 \approx 15\%$
- c. An author is a new author.
ANSWER: $\frac{7}{20} \approx 0.35 \approx 35\%$
- d. An author is successful.
ANSWER: $\frac{11}{20} \approx 0.55 \approx 55\%$
- e. An author is unsuccessful.
ANSWER: $\frac{9}{20} \approx 0.45 \approx 45\%$
- f. An author is established.
ANSWER: $\frac{13}{20} \approx 0.65 \approx 65\%$
- g. A successful author is established.
ANSWER: $\frac{7}{11} \approx 0.6364 \approx 63.64\%$
- h. An unsuccessful author is established.
ANSWER: $\frac{2}{3} \approx 0.6667 \approx 66.67\%$

Problem 17. Use the given information to find the indicated probability.

- a. $P(A) = .1$, $P(B) = .6$, $P(A \cap B) = .05$. Find $P(A \cup B)$.
ANSWER: 0.65
- b. A and B are mutually exclusive. $P(A) = .4$, $P(B) = .4$. Find $P((A \cup B)')$.
ANSWER: $1 - 0.8 = 0.2$
- c. $A \cup B = S$ and $A \cap B = \emptyset$. Find $P(A) + P(B)$.
ANSWER: 1
- d. $P(A \cup B) = .3$ and $P(A \cap B) = .1$. Find $P(A) + P(B)$.
ANSWER: 0.4