## Valuation of Dividend Stocks (Preliminary Version)

Floyd Vest, March 2014
Valuation of stocks by a formula is a difficult task but it gives the investor a deeper look at the factors that drive stock prices. You can see how individual variables affect stock valuation.

The Constant growth model assumes the company is growing at a constant rate. This model is best used for companies with consistent growth in Earnings per share and in Dividends . Assuming Payout ratio stays unchanged, the growth in Dividends follows growth in Earnings per share.

The Required rate of return is the return required to encourage investors to invest in stocks instead of investing in what is called Risk-free assets (which may not be risk free).

A general Constant growth model is derived with attendant assumptions. We assume the model takes a future expected Dividend stream that continues indefinitely, and discount it back to a Present value. Using a well known present value of a perpetuity discounted at the rate $r$ with dividends growing at the rate $g$, we have
(.5) Acceptable price $=P=\frac{D_{0}(1+g)}{r-g}$. (See Formula $P_{4}$ in "The Mathematics of Financial and Social Responsibility" in this course.) Substituting $D_{1}=D_{0}(1+g)$, the model takes the form
(1) Acceptable price $=P=\frac{D_{1}}{r-g}, r>g, D_{1} \neq 0 . \quad D_{1}=$ dividend for the coming year.
(2) We will let $r=$ Required rate of return, and $g=$ Growth rate of dividends.

We will set
(3) $r=$ Required rate of return $=$ Risk-free rate $+($ Market risk premium $\times$ beta $)$.
(4) Market risk premium $=$ Historical market return - Risk free rate .

From 1926 to 2012, the S\&P 500 index has earned a total return of $9.9 \%$ and a
"Risk free" rate is for long term Treasuries which was $5.7 \%$ in this time period.
For this history, see "Annual Total Return Table for the S\&P 500 Index of Stocks" July 3013 in this course, and measuringworth.com. But, given the history of stock returns since 1999, we will not be optimistic. Some experts predict $7 \%$ total return.
(5) For sake of an example we will use Market return of $8.5 \%$.

The current long term Treasury rate has been $3.7 \%$. For sake of example, we will compromise and use
(6) Risk-free rate $=5 \%$. These assumptions make
(7) Market risk premium $=8.5 \%-5 \%=3.5 \%=.035$.

Next we will simulate a dividend paying stock Microsoft (MSFT) (3-28-14). From finance.yahoo.com and nasdaq.com we use
(8) beta $=.71$, and a dividend for the next year of $D_{1}=4(\$ .24)=\$ .92$. Beta is an indicator of the volatility of the stock price relative to that of the S\&P 500.

Going back to Formula 3 we have
(9) $\quad r=$ Required rate of return $=.05+(.035 \times .71)=.075=7.5 \%$.

A beta of one reflects the volatility of the market represented by the S\&P 500. A beta of .71 indicated less volatility of stock prices for Microsoft and thus would indicate a lower Required rate of return than that of the market. (See the Exercises.)

Next we need $g=$ Growth rate of dividends. From the last five years, for Microsoft stock an average growth rate of dividend has been $15 \%$. We will not be so optimistic and use
(10) $g=$ Growth rate of dividends $=5.5 \%$.

Putting this all together in Formula 1, we have an Acceptable price
(11) $P=\frac{.92}{.075-.055}=\$ 46.00$. Currently the Microsoft price is $\$ 40.29$ and dividend yield is $2.80 \%$ (finace.yahoo.com, 3-29-14). (See the Exercises for choosing your own numbers.)

Multi-stage dividend growth model. This model considers the possibility of two stages with the first stage at the current values given above, and a future change in dividends and dividend growth rate. We will start with
(12) $D_{1}=\$ .92$ annual dividend from above, and for the second stage lower our estimate of

Dividend growth rate to $g=5 \%$.
(13) Future dividends will be discounted at the Required rate of return $=7.5 \%$ from above.

Table 1 below presents for end of Years 1,2,3, 4: Increasing dividend per share at $5.5 \%$, and for Present value, discounting at $7.5 \%$ to the beginning of Year 1.

| End of Year | Appreciated Dividend per share | Present value |
| :---: | :---: | :---: |
| 1 | $\$ .92$ | $\$ .86$ |
| 2 | .97 | .84 |
| 3 | 1.02 | .82 |
| 4 | 1.08 | . .81 |

We have for sum of Present values of dividends at the beginning of Year 1 a $\$ 3.33$.
To calculate the Present value $P V$ of the remaining dividends for an uncertain long period of time we use

$$
\begin{equation*}
P V=\frac{D_{5}}{r-g} . \text { Substitution gives } \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
P V=\frac{1.08(1+.05)}{.075-.05}=\$ 45.36 \text {, which we will discount back to the beginning of Year } 1 \text { to } \tag{15}
\end{equation*}
$$

(16) be $45.36[1+.075]^{-4}=\$ 34.00$. This give the acceptable price of the stock of
(17) $\quad P=3.33+34.00=\$ 37.33$. The Microsoft stock is currently (3-29-14) priced at $\$ 40.29$.

Earnings value model. We assume that a consistent Dividend $D$ is paid by the company. The model for acceptable Price $P$ is
(18) $P=\frac{E_{1} \times\left(\frac{D}{E}\right)}{r-g}$ where
$E_{1}=$ next year's Earnings per share.
(20) $\frac{D}{E}=\frac{\text { Dividend }}{\text { Earnngspershare }}=$ Expected long term payout ratio . (See the Exercises for a discussion of this model.)
(21) We will simulate Microsoft $($ MSFT $)$ with $E_{1}=$ Earnings per share $=\$ 2.70$.
(22) $\frac{D}{E}=$ Payout ratio $=.36$ and $\mathrm{g}=$ Growth rate of earnings per share of $6 \%$. (See yahoo.finance.com and nasdaq.com.) We will let $r=$ Required rate of return $=8.4 \%$. Formula 18 gives
(23) $\quad P=2.70\left[\frac{.36}{.084-.06}\right]=\$ 40.50$. The current price is $\$ 40.29$.

Exercises: Show your work and formulas. Name variables, numbers, and answers. When appropriate, answer in complete sentences.
\#1. Consult the article "Mathematics and the Stock Market" in this course, and along with given assumptions, derive Formula 1. Name the variables and give the assumptions.
\#2. $\quad$ Let $r=$ Required rate of return $=$ Risk free rate $+[($ Market return - Risk free rate $) \times$ beta $]$. If beta $=1$ what does this imply? What is $r=$ Required rate of return equal to?
\#3. If beta $<1$, how does $r$ relate to Market return? Prove this in general.
\#4. If beta $>1$, how does $r$ relate to Market return? Prove this in general.
\#5. For Microsoft (MSFT), look up (a) Earnings per share, (b) Rate of growth of Earnings per share, (c) Rate of growth of dividends, (d) beta, (e) current price, (f) Dividend yield, and (g) Payout ratio. See finance.yahoo.com and nasdaq.com and other sources.
\#6. For MSFT, look up (a) a Risk free rate of return of your choice, a Market return of your choice. (b) Calculate your $r=$ Required rate of return. Cite the references of your choice. See the References in this article.
\#7. Apply your values from \#5 and \#6 to the Constant growth model. What do you do if $r-g$ is negative?
\#8. Make an example for MSFT and apply it to the Multi-stage dividend growth model.
\#9. Make an example for MSFT and apply it to the Earnings value model.
\#10. Give an argument for applying beta in Formulas 3 and 4.
\#11. To study beta, read and report on parts of the article "Measures of Risk and Performance ... in ..., Beta, and ..." in this course.
\#12. For Formula 1, what do you do if $g>r$ ? Give an example where this happens. Explain what happens to the perpetuity. Provide a new method of calculation an acceptable Price.
\#13. Give an argument for the Earnings value model.
\#14. If Payout ratio remains constant and Earnings per share grows at 6\%, at what rate do Dividends grow?
\#15. If Payout ratio $=.36$, and Earnings per share $=\$ 2.70$, what is annual Dividend $D$ ?
\#16. Calculate the sum in Table 1 with one formula for an annuity with rents increasing at a constant rate.
\#17. Derive Formulas 14 and 15 for the Present value of a perpetuity. Compare to $P_{4}$ and its general form in "The Mathematics of Financial Responsibility" in this course. Which is correct? Consider the time lines.
\#18. Choose your own stock and make an example to apply the Constant growth model.
\#19. As in \#18, apply the Multi-stage dividend growth model.
\#20. As in \#18, apply the Earnings value model.
\#21. In the above example in this article, for the Multi-stage dividend growth model, what is the present value of the dividends received after the end of 20 years?
\#22. Discuss how the various variables in Formulas 1, 2, 3, and 4 affect each other.
\#23. Read the articles about Microsoft stocks (MFST) at finance.yahoo.com. Give your date.
What do the analysts say about MFST? On nasdaq.com, look up the history of quarterly dividends for MFST and calculate an average growth rate of annual dividends. Look up the history of the MFST stock prices for the last five years. Give dates, the average annual rate of
change in price, discuss the volatility, and where the price is now. Print out and study several data sheets for MFST from these and other web sites.
\#24. Can you make this work? Explain the numbers. See the article in this course "Comparing Taxed, Tax Deferred, and Tax Sheltered Investments." From Kiplinger’s Personal Finance, 05/2014, p. 68: "Tom and Elaine both 45, contribute $\$ 5000$ to their 401(k). Tom contributes to a traditional 401(k), while Elaine contributes to a Roth 401(k). They don't take withdrawals until they're 75. If their tax rates and investment returns remain equal ( $28 \%$ and $7 \%$ ), Tom will end with $\$ 27,404$ after tax, while Elaine will have $\$ 38,061$ tax-free. Even if Tom invested the $\$ 1400$ in taxed savings, he'd still lag Elaine by $\$ 2,616$. What wrong impression does this example give?
\#25. Worth $\$ 2,560,000$ : If a twenty year old had invested $\$ 10,000$ over the past 15 years in Dodge and Cox Stocks (DODGX), they would have about $\$ 40,000$. By what factor does the money multiply every 15 years? How much would they have by age 80 at this rate? (Kiplinger's Personal Finance, 5/14, p. 18)
\#26. Can you make this work? Kiplinger's Personal Finance, 5/14, p. 69: "Someone who saved $13 \%$ of a $\$ 40,000$ salary starting at age 25 would be able to replace $50 \%$ of his income in retirement starting at age 65 if he withdrew $4 \%$ of his savings the first year of retirement (assuming an annualized $7 \%$ return and $3 \%$ pay increases). But if he stopped making contributions for five years starting at age 45 , he would then have to boot his savings rate to $19 \%$ to replace the $50 \%$ salary at retirement."
\#27. From "Case Study: Fred combines an annuity with portfolio withdrawals"
Vanguard.com. Fred at age 70 pays $\$ 4000,000$ for an immediate annuity which pays $\$ 1400$ per month for the rest of his life, adjusted for inflation. Using $12 \times \$ 1400=\$ 16,800$ per year, build a table for the rates of return $r$ on the annuity investment if Fred lives to age 80, 90, 95, 100 and inflation $I=2 \%, 2.5 \%, 3 \%$. For example if Fred dies at age 80 , and $I=2 \%$, the $r=-13.74 \%$.

His Social Security is $\$ 19,200$ per year, and with the annuity starting at $\$ 16,800$ per year, Fred has $\$ 36,000$ per year in guaranteed inflation adjusted income. Fred has $\$ 800,000$ in additional retirement savings from which he plans to withdraw $5 \%$ making $\$ 40,000$ for the first year giving $36,000+40,000=\$ 76,000$ in income. He wants to increase his withdrawals with inflation. If $r=7 \%$ and $I=3 \%$, how long would Fred's $\$ 800,000$ last? There have been long periods with inflation $I$ near $5 \%$. How long would Fred's money last?

What do you think of Fred's program and the assumptions in the article? Consider not buying the annuity. Read the related articles in this course on the survivability of retirement funds including the sequence effect.

## Side Bar Notes;

John Maynard Keynes (1883-1946), the famous economist and stock investor was a long term investor in his favorite stocks. When the price came down, he bought more to get better bargains. After his death, his net worth stood at $\$ 16.5$ million in 2009. Keynes was a dividend stock investor. See Wikipedia.org.

Stock prices are way ahead of earnings per share (Fortune, April 7, 2014, p.44). "The S\&P 500 currently trades at around 16 times estimated year - ahead earnings, a multiple that has
increased by $50 \%$ since 2011 - the largest P/E expansion since the late 1990's. The question becomes, Is it 1996 or 1999 again?" (See "Annual Total Return Table for the S\&P 500 Index of Stocks," in this course.) Check from 1999 for S\&P prices and total returns and the "great bear market." Did S\&P price recover the former high?
"Another warning sign is the Q ratio, which compares the market's value of corporate assets with their replacement costs ... (page 56). As of March 6, the Q ratio was indicating the market was overvalued by $76 \% \ldots$. The only times this value has been greater were the late 1920's and the late 1990's." (See Wikipedia.org.)

No-load mutual funds charge no commission. But if you buy through a broker of other sources, you may pay a load. If you want to avoid a load, you may have to buy directly from the fund family (Kiplinger's Personal Finance, 5/14, p. 24).

Capital loss. When her taxable stock fund account fell $\$ 177,000$, she sold, triggering a $\$ 80,000$ capital loss. The great thing about a capital loss, it can be carried forward and posted against capital gains. Even if there are no capital gains, $\$ 3000$ can be posted against ordinary income until the loss is used up (Kiplinger's Personal Finance, 5/14, p. 35).

Recent gains in living standards have been documented. Also from 1979 to 2010, the middle class saw after-tax income gains of $40 \%$ (p.60). What was the average annual rate of gain?

Buying high and selling low. Kiplinger's Personal Finance, 5/2014, p. 18: Ordinary investors tend to buy when stocks are high and sell when prices are low. Vanguard.com points out that in 1992, participants in their retirement plans were allocating $58 \%$ to stocks. Buy 2000 as stocks had quadrupled in value, the proportion rose to $74 \%$. Then as stocks fell sharply over the next two years, the allocation fell to $54 \%$ as a partial result of withdrawals. "Their market timing was backwards," Vanguard writes.

As an exercise, check on good companies such as GE stocks (3/2014) which was down to $\$ 26$, about one third below 2007. Cisco is at $\$ 22$ well below a former high. Bank of America is down to $\$ 17$ (Kiplinger's Personal Finance). Go to finance.yahoo.com and nasdaq.com and check on price history, basic statistics, analyst's reports, and stock pricing models.

## References in this course:

"Measuring Risk and Performance ..., Beta, and ..."
"High Dividend Yields on Stocks and Low Interest Rates on CD's and Bonds."
"Annual Total Return Table for the S\&P 500 Index of Stocks." Study and report the total return beginning in 1999. Discuss the volatility. How long did it take for the price to gain its former high? Give the dates. How long did it take the total return to gain its former high? Give the dates. Where is the S\&P now? What has happened since the deep trough? Name several historical developments which probably moved the market.
"'"Dividend Growth."
"Standard Deviation as a Measure of Risk."
"Mathematics and the Stock Market," which derives a stock pricing model.
"Stock Pricing Models."
Have some fun: See observationsandnotes.blogspot.com/2010/11/100-years-ofbond. There are many interesting financial and other topics. Click on 100 Years of Treasury Bond Interest Rate History, and other topics.

For a free course in financial mathematics, with emphasis on personal finance, for upper high school and undergraduate college, see COMAP.com and click on the box and register. They will email you a password. Just click on an article in the annotated bibliography, download it, and teach it. Unit 1: The Basics of Mathematics of Finance, Unit 2: Managing Your Money, Unit 3: Long-term Financial Planning, Unit 4: Investing in Stocks and Bonds, Unit 5: Investing
in Real Estate, Unit 6: Solving Financial Formulas for Interest Rate. Unit 7: More Advanced or Technical (for about ten articles in this area, see the UMAP Journal at COMAP). Additional articles on Financial Mathematics or Related to Personal Finance.

Your school library can probably provide you with a portal to electronic resources including magazines and journal articles. You might arrange a portal for your home computer.

