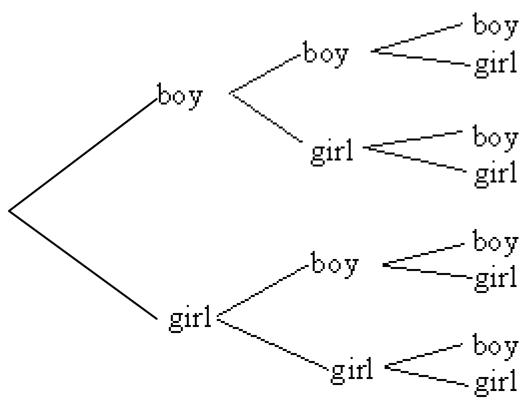
Probability Review Solutions

1. A family has three children. Using b to stand for boy and g to stand for girl, and using ordered triples such as bbg, find the following.

   a. draw a tree diagram to determine the sample space
   b. write the event *E* that the family has exactly two boys
   c. write the event *F* that the family has at least two boys
   d. write the event *G* that the family has three boys
   e. $p(E)$
   f. $p(F)$
   g. $p(G)$
   h the probability that there are exactly two boys given that the first child is a girl.

   Solution:

   a.



   Reading from the beginning to the end of each limb, we have the sample space of
   {bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}.

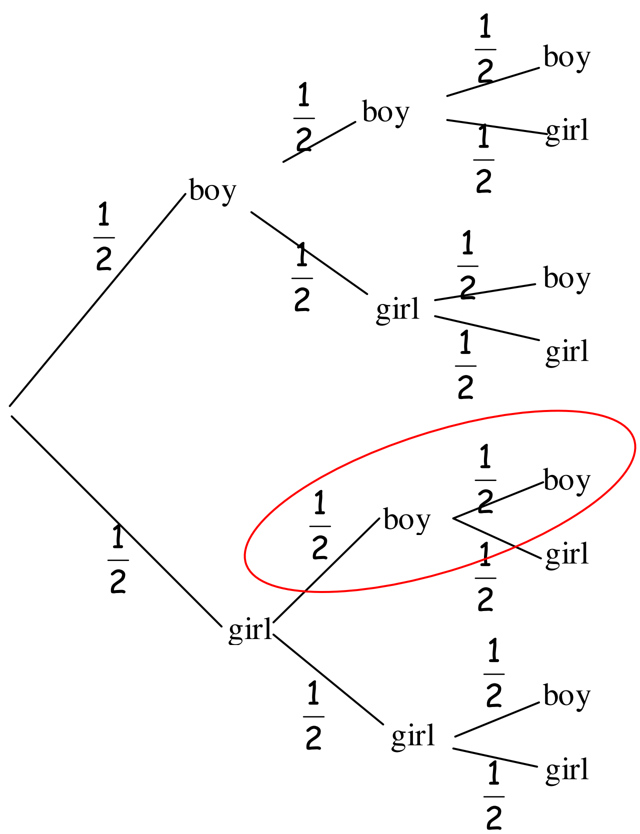
   b. This is asking for the elements in the sample space that have exactly two bs listed. These are {bbg, bgb, gbb}.
   c. This is asking for the elements of the sample space that have twp or more boys. In other words, this is asking for those elements that have exactly two bs and those that have three bs. These are {gbb, bgb, bbg, bbb}.
   d. This is asking for the elements of the sample space that have three boys. In other words, this is asking for those elements that have three bs. This is {bbb}.

e. For this problem, we need to divide the number of things in the event E (exactly two boys) by the number of things in the sample space to find the probability the family will have exactly two boys. This will give us $p(E) = \frac{3}{8} = \frac{\textbf{total number of things in E}}{\textbf{total number of things in the sample space}}$

f. For this problem, we need to divide the number of things in the event F (at least two boys) by the number of things in the sample space to find the probability the family will have at least two boys. This will give us $p(F) = \frac{4}{8} = \frac{\textbf{total number of things in F}}{\textbf{total number of things in the sample space}}$

g. For this problem, we need to divide the number of things in the event G (three boys) by the number of things in the sample space to find the probability the family will have three boys. This will give us $p(G) = \frac{1}{8} = \frac{\textbf{total number of things in G}}{\textbf{total number of things in the sample space}}$

h. For this problem, we once again want to look at a tree diagram.



Since we are asked to determine the probability that there are exactly two boys given that the first child is a girl, we need to only look at the part of the tree diagram where the first child is a give.

We then find all the branches off a first girl that have exactly two boys (this is circled in red). To find the probability of this, we multiply the probabilities of each piece as we work our way out starting with the probability after the first girl. This gives us $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$

2. Mendel found that snapdragons have no color dominance; a snapdragon with one red gene and one white gene will have pink flowers. If a pure red snapdragon is crossed with a pink snapdragon, find the following probabilities.

   a. a red offspring
   b. a white offspring
   c. a pink offspring

   Solution:

   The easiest way to do this problem is to use a Punnet square. The pure red snapdragon will have two red genes (represented by R in the Punnet square). The pink snapdragon will have one red (R) and one white (represented by W in the Punnet square).

		Red Snapdragon	
		R	R
Pink Snapdragon	R	RR	RR
	W	RW	RW

   a. A red offspring would need to have two red (R) genes. As we can see there two offspring with two red genes. Thus $p(red\ offspring)=\frac{2}{4}$.
   b. A white offspring would need to have two white (W) genes. As we can see there no offspring with two white genes. Thus $p(white\ offspring)=\frac{0}{4}$.

c. A red offspring would need to have one red (R) and one white (W) gene. As we can see there two offspring with one red (R) and one white (W) gene. Thus $p(\textit{pink offspring}) = \frac{2}{4}$.

3. If a single card is drawn from a deck of 52 cards, find each of the following probabilities:

   a. a black card
   b. a heart
   c. a queen
   d. a card below a 5 (count an ace as high)
   e. a card above a 9 (count an ace as high)
   f. a card below a 5 **and** above a 9 (count an ace as high)
   g. a card is below a 5 **or** above a 9 (count an ace as high)

   Solution:

   a. To find the probability that the card will be a black card, we need to first determine how many black cards there are and then divide that number by the total number of cards (52). If you look at a deck you will see that there are 26 cards which are black. Thus the probability of a black card is $\frac{26}{52} = \frac{1}{2}$.
   b. To find the probability that the card will be a heart, we need to determine how many hearts are in the deck and then divide that number by 52. If you look at a deck, you will see that there are 13 hearts. Thus the probability of a heart is $\frac{13}{52} = \frac{1}{4}$.
   c. To find the probability that the card will be a queen, we need to determine how many queens are in the deck and then divide that number by 52. If you look at a deck, you will see that there are 4 queens. Thus the probability of a queen is $\frac{4}{52} = \frac{1}{13}$.
   d. To find the probability that the card will be below a five, we need to determine how many cards are below a five in the deck and then divide that number by 52. This means that we are looking for all the 4s, 3s, and 2s. If you look at a deck, you will see that there are 4 fours, 4

threes, and 4 twos. Thus the probability of a card below a 5 is $\frac{12}{52} = \frac{3}{13}$.

e. To find the probability that the card will be above a nine, we need to determine how many cards are above a nine in the deck and then divide that number by 52. This means that we are looking for all the 10s, jacks, queens, kings, and aces. If you look at a deck, you will see that there are 4 tens, 4 jacks, 4 queens, 4 kings, and 4 aces. Thus the probability of a card above a 9 is $\frac{20}{52} = \frac{5}{13}$.

f. To find the probability that the card will be below a five and above a nine, we need to determine how many cards are below a five and above a nine at the same time in the deck and then divide that number by 52. There are no cards that qualify as being both below a five and above a nine at the same time. Thus the probability of a card below a 5 and above a 9 is $\frac{0}{52} = 0$.

g. To find the probability that the card will be below a five or above a nine, we need to determine how many cards are below a five or above a nine at the same time in the deck and then divide that number by 52. Since we see the word "or" in the statement, we can use one of our probability rules $p(E \cup F) = p(E) + p(F) - p(E \cap F)$. If we let E be the event of a card below a five and F be the even that a card is above a 9, then we can use what we got for parts d, e, and f to answer this question. This will give us $\frac{12}{52} + \frac{20}{52} - \frac{0}{52} = \frac{32}{52} = \frac{8}{13}$.

4. Alex is taking two courses, algebra and U.S. history. Student records indicate that the probability of passing algebra is 0.25; that of failing U.S. history is 0.45; and that of passing at least one of the two courses is 0.80. Find the probability of each of the following.

a. Alex will pass history.
b. Alex will pass both courses.
c. Alex will fail both courses.
d. Alex will pass exactly one course.

Solution:

a. Alex passing history is the complement of Alex failing history. Thus we can use what we know about the probability of an event and its complement to find the probability of Alex passing history. It will be 1 - 0.45 = 0.55.

b. The easiest way to figure this out is using the probability formula $p(E \cup F) = p(E) + p(F) - p(E \cap F)$. If we let E be the event that Alex passes history and F be the event that Alex passes algebra, then we have $0.80 = 0.55 + 0.25 - p(E \cap F)$. With a little simplification we get $0.80 = 0.80 - p(E \cap F)$. And we finally determine that $p(E \cap F) = 0$. In other words, the probability that Alex will pass both courses is 0.

c. Alex failing both courses is the complement of Alex passing at least one of the two courses. Thus we can use what we know about complements to find the probability of Alex failing both courses. It will be 1 - 0.80 = 0.20.

d. The probability that Alex will pass exactly one course is the probability that Alex will pass only algebra or Alex will pass only history. Since these two things are mutually exclusive (the probability that Alex will pass both is 0), then we just need to add the probabilities together to get 0.55 + 0.25 = 0.80.

Another way to calculate this is to use a table.

Across the top of this you would write in one column each for pass history, fail history, and total. Along the left you would have one row each for pass algebra, fail algebra, and total. You are told that the probability of failing history is .45. This number would go in the total row at the bottom of the fail history column. You are told that the probability of passing algebra is .25. This number would go in the total column at the end of the passing algebra row. You can now fill in the other piece of the total column at the failing algebra row. Since there are only two things that can happen when it comes to algebra, the two totals must add up to 1. Thus at the end of the failing algebra row in the total column would be 1-.25. Similarly, you can fill in the total row at the bottom of the passing history column with 1-.45. Now the four boxes in the table where the passing and failing history overlap with passing and failing algebra cover all of the possibilities for what can happen. Thus all those 4 probabilities added together add up to 1. Each column must add up to the total at the bottom of the column and each row must add up to the total at the right of each row. Once we fill in one of the 4 pieces, we will be able to fill in the rest. In order to do that we use the information that the probability of passing at least one of the two courses is .80. This number is the sum of the following 3 blocks -- pass history and pass algebra, pass history and fail algebra, pass algebra and fail history. In each of those cases you pass at least one of the courses. This means that the only box not covered is the fail history and fail algebra. This means that we can figure out the probability of failing both courses by calculating 1-.80. Once we have that we can fill in the rest of the boxes and answer the questions.

	pass history	fail history	total
pass algebra	.25-.25	.45-.2	.25
fail algebra	.75-.2	1-.8	1-.25
total	1-.45	.45	1

5. On the basis of his previous experience, the public librarian at Smallville knows that the number of books checked out by a person visiting the library has the following probabilities

Number of Books	0	1	2	3	4	5
Probability	0.05	0.15	0.25	0.35	0.05	0.15

Find the expected number of books checked out by a person visiting this library.

Solution:

To find the expected value of the number of books checked out, we need to find a "weighted average". In other words we need to take into account the probability for each number of books to calculate the average. We do this by multiplying the number of books by the probability that that number will be checked out. We add each of these products together to find the expected value. This gives us

$(0.05 \cdot 0)+(0.15 \cdot 1)+(0.25 \cdot 2)+(0.35 \cdot 3)+(0.05 \cdot 4)+(0.15 \cdot 5)=2.65$.

Thus the expected number of books checked out by a person visiting this library is 2.65.

6. A group of 600 people were surveyed about violence on television. Of those women surveyed, 256 said there was too much violence, 45 said that there was not too much violence, and 19 said they don't know. Of those men surveyed, 162 said there was too much violence, 95 said that there was not too much violence, and 23 said they don't know.

   a. What is the probability that a person surveyed was a woman and thought there was not too much violence on television?
   b. What is the probability that a person thought there was too much violence on television given that the person was a woman?
   c. What is the probability that a person who was a man thought that there was not too much violence on television?

   Solution:
   We want to organize the data into a table. This will make the information easier to use.

	Too Much Violence			
	Yes	No	Don't Know	Total
Men	162	95	23	280
Women	256	45	19	320
Total	418	140	42	600

   a. For this problem we are looking for just how many people are in the women row and also in the no columns. This number is 45. We then take this number and divide it by the total number of people surveyed to get the probability of $\frac{45}{600} = .075$
   b. The part written as "given that the person was a woman" tells us to look only in the women row and use the total number of women as 320 as out total to divide by. In the women row there were 256 who thought that there was too much violence. When we take 256 and divide it by 320 we get the probability that a person though there was too much violence given that the person was a women. This probability is $\frac{256}{320} = .8$.
   c. The part written as "who was a man" tells us to look only in the men row and use the total number of men as 280 as our total to divide by. In the men row there were 95 who thought that there was not too much violence. When we take 95 and divide it by 280 we get the

probability that a person thought there was too much violence given that the person was a man. This probability is $\frac{95}{280} = .3392857143$.