120 Years of Mathematics Through the Eyes of the *American Mathematical Monthly*

Scott Chapman
Editor Emeritus, *American Mathematical Monthly*

Sam Houston State University

June 19, 2017

Prologue

Some resources if you like this talk:

[1] David H. Bailey and Jonathan Borwein. "Pi Day is upon us again and we still do not know if Pi is normal." American Mathematical Monthly 121.3 (2014): 191-206.

[2] Jonathan Borwein and Scott Chapman. "I Prefer Pi: A Brief History and Anthology of Articles in the American Mathematical Monthly." The American Mathematical Monthly 122.03 (2015): 195-216.

[3] John Ewing (Ed.), A Century of Mathematics Through the Eyes of the Monthly, MAA, Washington, D.C., 1994.

[4] P. Baginski and S. T. Chapman, Factorizations of Algebraic Integers, Block Monoids and Additive Number Theory, *Amer. Math. Monthly* **118**(2011), 901–920.

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One of my favorite Monthly papers

A CONTRIBUTION TO THE MATHEMATICAL THEORY OF BIG GAME HUNTING

H. PÉTARD, Princeton, New Jersey

This little known mathematical discipline has not, of recent years, received in the literature the attention which, in our opinion, it deserves. In the present paper we present some algorithms which, it is hoped, may be of interest to other workers in the field. Neglecting the more obviously trivial methods, we shall confine our attention to those which involve significant applications of ideas familiar to mathematicians and physicists.

The present time is particularly fitting for the preparation of an account of the subject, since recent advances both in pure mathematics and in theoretical physics have made available powerful tools whose very existence was unsuspected by earlier investigators. At the same time, some of the more elegant classical methods acquire new significance in the light of modern discoveries. Like many other branches of knowledge to which mathematical techniques have been applied in recent years, the Mathematical Theory of Big Game Hunting has a singularly happy unifying effect on the most diverse branches of the exact sciences.

For the sake of simplicity of statement, we shall confine our attention to Lions (*Felis leo*) whose habitat is the Sahara Desert. The methods which we shall enumerate will easily be seen to be applicable, with obvious formal modifications, to other carnivores and to other portions of the globe. The paper is divided into three parts, which draw their material respectively from mathematics, theoretical physics, and experimental physics.

The author desires to acknowledge his indebtedness to the Trivial Club of St. John's College, Cambridge, England; to the M.I.T. chapter of the Society for Useless Research; to the F. o. P., of Princeton University; and to numerous individual contributors, known and unknown, conscious and unconscious.

One of my favorite Monthly papers

THEOREM I. *There is a lion in the cage.*

Some Basic Facts


THE AMERICAN MATHEMATICAL
MONTHLY MAA
VOLUME 123, NO. 1 JANUARY 2016

A Fair-Bold Gambling Function Is Simply Singular 3
Richard D. Neidinger
The Looping Rate and Sandpile Density of Planar Graphs 19
Adrien Kassel and David B. Wilson
Integration by Parts and by Substitution Unified with Applications to Green's Theorem and Uniqueness for ODEs 40
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A Generalization of Euler's Theorem for $\zeta(2k)$ 53
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Beyond Schur's Conjecture 66
Shivarajkumar
The Diameter of Chess Grows Exponentially 71
Yaroslav Shitov

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77, Monthly Gems; 87, A Partial Solution of Representing 1 as a Sum of n Distinct Unit Fractions; 96, A Synthetic Approach to a USAMO Problem; 112, A Note on the Logarithmic Mean

An Official Publication of the Mathematical Association of America

Founded: 1896

Published Since 1915 by The Mathematical Association of America

The Monthly is the most widely Read Mathematics Journal in the World
Source: JSTOR and Ingenta data


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More Specifically

Of the 16,200 journal titles that Ingenta ranks monthly (by number of downloads), The American Mathematical Monthly constantly ranks in the top 50 (top 1%) and often ranks in the top 20.

The most downloaded article (almost every month): The Problem Section.

A Monthly Problem led to Bill Gate's only mathematical publication.

Dweighter, H. (pseudonym for Jacob E. Goodman). 1975. Problem E2569. American Mathematical Monthly 82:1010.

Gates, W.H., and C.H. Papadimitriou. 1979. Bounds for sorting by prefix reversal. Discrete Mathematics 27:47-57

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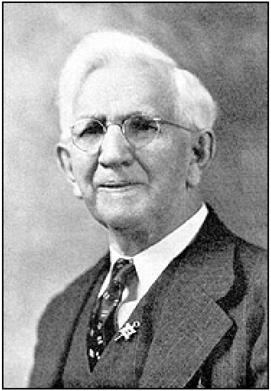

Benjamin Finkel (1865–1947)
Founder of the American Mathematical Monthly
and Editor 1896–1912

Quote From First Issue

"While realizing that the solution of problems is one of the lowest forms of Mathematical research, and that, in general, it has no scientific value, yet its educational value cannot be over estimated. . . . . . . The American Mathematical Monthly will, therefore, devote a due portion of its space to the solution of problems, whether they be the easy problems in Arithmetic, or the difficult problems in the Calculus, Mechanics, Probability, or Modern Higher Mathematics. "

L. E. Dickson

L. E. Dickson (1874–1954)
Editor of the Monthly 1903–1906

Turning Point 1915

In 1915 the American Mathematical Society, by a 3-2 Committee vote, decided not to take control of *The Monthly*.

It did decide to lend support to any other organization which took over the journal.

This led to the formation of the Mathematical Association of America, which still publishes *The Monthly*.

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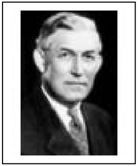
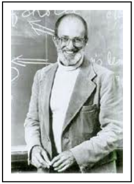
Some Notable Past Monthly Editors

Robert Carmichael (1918)



Herbert Wilf (1987-1991)

Paul Halmos (1982–1986)



John Ewing (1992–1996)

The Current Editor: Susan Colley

Susan Colley
Current Editor of the Monthly

Some Notable Monthly Authors

E. R. Hedrick	D. H. Lehmer	R. W. Hamming
L. R. Ford	Peter Sarnak	Andre Weil
I. Kaplansky	N. Bourbaki	Martin D. Kruskal
G. Pólya	William Feller	E. T. Bell
C. L. Siegel	C. Fefferman	Walter Rudin
Walter Feit	Michael Atiyah	Steve Smale
L. E. Dickson	George D. Birkhoff	Saunders Mac Lane
Andrew Gleason	Felix Browder	George Andrews
Phillip Griffiths	Barry Mazur	S. S. Chern
Stephen Wolfram	Herbert Wilf	David Eisenbud
John Conway	Joseph Silverman	Branko Grünbaum

Most Cited Articles

Most Cited Articles according to Google Scholar (as of June 15, 2017)

1. Cited 4893 times: *College Admissions and the Stability of Marriage*, David Gale and Lloyd Shapley, **69**(1962), 9-15.
2. Cited 4208 times: *Period Three Implies Chaos*, Tien-Yien Li and James A. Yorke, **82**(1975), 985-992.
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In honor of Lloyd Shapley's Nobel Prize, we reprinted in May 2013 his 1962 paper with David Gale, which was cited by the Nobel Awards Committee, in its entirety. The paper is preceded by a Foreword by Ehud Kalai, Distinguished Professor of Game Theory at Northwestern University.

Lloyd Shapley delivers his Nobel Prize Lecture

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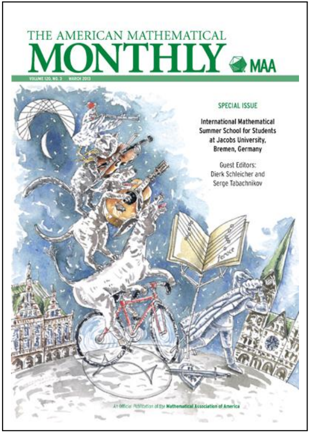
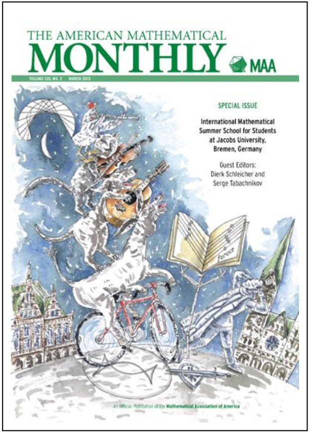
Monthly Past: The Special Issue



The March 2013 issue was the first Special Issue of the Monthly in over 20 years

The issue contains papers written by the contributors to the International Summer School for Students held at Jacobs University in Bremen, Germany in the summer of 2011.

Monthly Past: The Special Issue



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How Tokieda Sees It

The Special Issue includes the Ford-Halmos Award winning paper *Roll Models* by Tadashi Tokieda.

Tokieda's paper contains a wonderful explanation of the mathematics behind simple rolling motion. In fact, he goes to great lengths to explain how a golf ball can completely disappear in the cup and then reemerge without hitting the bottom of the cup.

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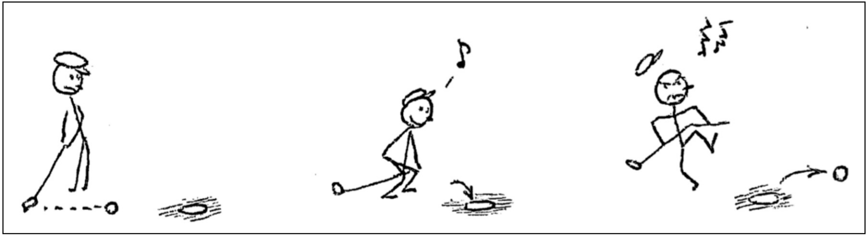


The Inaugural Pi Day Paper

Mathematical Assoc. of America American Mathematical Monthly 121:3 February 4, 2014 6:35 a.m. MONTHLY-D-13-00319RevisedPages.tex page

Pi Day Is Upon Us Again and We Still Do Not Know if Pi Is Normal

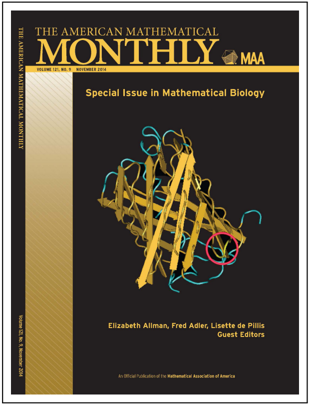
David H. Bailey and Jonathan Borwein

Abstract. The digits of π have intrigued both the public and research mathematicians from the beginning of time. This article briefly reviews the history of this venerable constant, and then describes some recent research on the question of whether π is normal, or, in other words, whether its digits are statistically random in a specific sense.

1. PI AND ITS DAY IN MODERN POPULAR CULTURE. The number π, unique among the pantheon of mathematical constants, captures the fascination both of the public and of professional mathematicians. Algebraic constants such as $\sqrt{2}$ are easier to explain and to calculate to high accuracy (e.g., using a simple Newton iteration scheme). The constant e is pervasive in physics and chemistry, and even appears in financial mathematics. Logarithms are ubiquitous in the social sciences. But none of these other constants has ever gained much traction in the popular culture.

In contrast, we see π at every turn. In an early scene of Ang Lee's 2012 movie adaptation of Yann Martel's award-winning book *The Life of Pi*, the title character

The Math Biology Special Issue



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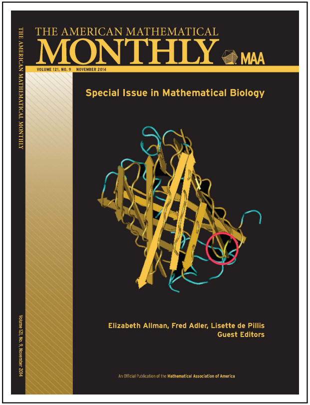

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Some Other Papers of Past Interest

The First 100 Years of the MAA

David E. Zitarelli

Abstract. Why was the MAA founded? What role has the Association played in American mathematics? What were its primary activities? We answer these questions in this overview of the MAA over its 100-year history from its founding in 1915. Along the way, we describe MAA sections, governance, meetings, prizes/awards, and headquarters. The account of MAA activities is divided into two periods, 1916–1955 and 1955–2014 and contains a discussion for the critical role played by the Committee on the Undergraduate Program in Mathematics in this division.

1. INTRODUCTION. This article presents an overview of the history of the Mathematical Association of America as part of the celebration of its centennial in 2015. It describes events this author regards as the most important over the century, but the account is certainly not exhaustive; for example, it makes little mention of competitions conducted under the aegis of the Association or of the expanded book publication program. Our account begins with the founding of the MAA and then describes its sections, governance, and meetings. Overarching activities are outlined in two distinct periods, 1916–1955 and 1955–2014, and I supply an explanation for the partition into disjoint stages. The article then discusses prizes and awards before ending with a brief mention of MAA headquarters.

2. FOUNDING. One of the most historic moments for mathematics in America occurred with the establishment of a national organization on the last two days of 1915.

The Second Pi Paper

I Prefer Pi: A Brief History and Anthology of Articles in the American Mathematical Monthly

Jonathan M. Borwein and Scott T. Chapman

Abstract. In celebration of both a special "big" π Day (3/14/15) and the 2015 centennial of the Mathematical Association of America, we review the illustrious history of the constant π in the pages of the *American Mathematical Monthly*.

1. INTRODUCTION. Once in a century, Pi Day is accurate not just to three digits but to five. The year the MAA was founded (1915) was such a year and so is the MAA's centennial year (2015). To arrive at this auspicious conclusion, we consider the date to be given as month–day–two-digit year.[1] This year, Pi Day turns 26. For a more detailed discussion of Pi and its history, we refer to last year's article [**46**]. We do note that "I prefer pi" is a succinct palindrome.[2]

In honor of this happy coincidence, we have gone back and *selected* roughly 76 representative papers relating to Pi (the constant not the symbol) published in this journal since its inception in 1894 (which predates that of the MAA itself). Those 75 papers listed in three periods (before 1945, 1945–1989, and 1990 on) form the core bibliography of this article. The first author and three undergraduate research students[3] ran a seminar in which they looked at the 75 papers. Here is what they discovered.

Monthly Future!

Here are some titles from 2017.

1. *The Image of a Square*, by Annalisa Crannell et al.
2. *The Euclidean Criterion for Irreducibles*, by Pete Clark.
3. *Friendly Frogs, Stable Marriage, and the Magic of Invariance,* by Maria Deijfen et al.
4. *Mathematics for Human Flourishing* by Francis Su.
5. *On Gauss's First Proof of the Fundamental Theorem of Algebra*, by Soham Basu and Dan Velleman.

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Factorizations of Algebraic Integers, Block Monoids, and Additive Number Theory

Paul Baginski and Scott T. Chapman

Abstract. Let D be the ring of integers in a finite extension of the rationals. The classic examination of the factorization properties of algebraic integers usually begins with the study of norms. In this paper, we show using the ideal class group, $\mathcal{C}(D)$, of D that a deeper examination of such properties is possible. Using the class group, we construct an object known as a block monoid, which allows us to offer proofs of three major results from the theory of nonunique factorizations: Geroldinger's theorem, Carlitz's theorem, and Valenza's theorem. The combinatorial properties of block monoids offer a glimpse into two widely studied constants from additive number theory, the Davenport constant and the cross number. Moreover, block monoids allow us to extend these results to the more general classes of Dedekind domains and Krull domains.

1. INTRODUCTION. In an introductory abstract algebra class, the notion of a unique factorization domain (UFD) is carefully developed and plays an important role. A wide array of UFDs are usually identified in such a course (such as $\mathbb{Z}$, $K[X]$ where K is a field, and $\mathbb{Z}[\imath]$, the Gaussian integers) before deeper algebraic structures, such as Euclidean domains or principal ideal domains, are introduced. To convince a student of the usefulness of the definition of a UFD (also known as a *factorial domain*), it is necessary to provide an example of an integral domain in which the notion of unique factorization fails. While there is an abundance of such examples, the one

The Setting

Let

$$\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

represent the integers

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

represent the natural numbers and

$$\mathbb{N}_0 = \{0, 1, 2, \ldots\}$$

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Notation

This part of the talk will be based on the simple congruence relation on $\mathbb{Z}$ defined by

$$a \equiv b \pmod{n}$$

if and only if

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Why we love $a \equiv b \pmod{n}$

One of our basic arithmetic operations work well here:

If $a \equiv b \pmod{n}$ and $c \in \mathbb{Z}$, then

$$ca \equiv cb \pmod{n}$$

BUT

$ca \equiv cb \pmod{n}$ does not imply that $a \equiv b \pmod{n}$.

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The Sequences

Let's consider a very simple arithmetic sequence:

$$1,\ 5,\ 9,\ 13,\ 17,\ 21,\ 25,\ 29,\ 33,\ 37,\ 41,\ 45,\ 49\ =$$

$$\{1 + 4k \mid k \in \mathbb{N}_0\} =$$

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What is a Monoid?

What is a monoid?

A set S with a binary operation $*$ (like $+$ or $\times$ on the real numbers) which satisfies the following.

1. $*$ is closed on S.
2. $*$ is associative on S ($(a * b) * c = a * (b * c)$)
3. $*$ has an identity element e ($a * e = e * a$)

Examples: $\mathbb{Z}$, $\mathbb{R}$ or $\mathbb{Q}$ under $+$.

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David Hilbert (1912)

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Hilbert's Argument

In $1 + 4\mathbb{N}_0$ we have

$$21 \cdot 33 = 9 \cdot 77$$

$$(3 \cdot 7) \cdot (3 \cdot 11) = (3 \cdot 3) \cdot (7 \cdot 11)$$

and clearly 9, 21, 33 and 77 cannot be factored in $1 + 4\mathbb{N}_0$. But notice that 9, 21, 33 and 77 are not **prime** in the usual sense of the definition in $\mathbb{Z}$.

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An Example in Almost Every Basic Abstract Algebra Textbook

Let $D = \mathbb{Z}[\sqrt{-5}] = \{a + b\sqrt{-5} \mid a, b \in \mathbb{Z}\}$.

In D

$$6 = 2 \cdot 3 = (1 + \sqrt{-5})(1 - \sqrt{-5})$$

represents a nonunique factorization into products of irreducibles in D.

To fully understand this, a student must understand *units* and *norms* in D.

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Basic Notation

Let M be a monoid. Call $x \in M$

(1) *prime* if whenever $x \mid yz$ for x, y, and z in M, then either $x \mid y$ or $x \mid z$.

(2) *irreducible (or an atom)* if whenever $x = yz$ for x, y, and z in M, then either $y \in M^{\times}$ or $z \in M^{\times}$.

As usual,

$$x \text{ prime in } M \implies x \text{ irreducible in } M$$

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How Things Factor in $1 + 4\mathbb{N}_0$

Lemma

The element x is irreducible in $1 + 4\mathbb{N}_0$ if and only if x is either

1. *p where p is a prime and $p \equiv 1 \pmod 4$, or*
2. *$p_1 p_2$ where p_1 and p_2 are primes congruent to 3 modulo 4.*

Moreover, x is prime if and only if it is of type 1.

Corollary

Let $x \in 1 + 4\mathbb{N}_0$. If

$$x = \alpha_1 \cdots \alpha_s = \beta_1 \cdots \beta_t$$

for α_i and β_j in $\mathcal{A}(1 + 4\mathbb{N}_0)$, then $s = t$.

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An Example to Illustrate the Last Two Results

Let's Factor

$$141,851,281 = 4 \times (35,462,820) + 1 \in 1 + 4\mathbb{N}_0$$

Now

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Conclusion of $1 + 4\mathbb{N}_0$

In general, a monoid with this property, i.e.,

$$x = \alpha_1 \cdots \alpha_s = \beta_1 \cdots \beta_t$$

for α_i and β_j in $\mathcal{A}(M)$, then $s = t$, is called *half-factorial.*

Theorem

There is a map

$$\varphi : \mathbb{Z}[\sqrt{-5}] \to 1 + 4\mathbb{N}_0$$

which preserves lengths of factorizations into products of irreducibles. Hence, $\mathbb{Z}[\sqrt{-5}]$ is half-factorial.

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What's in the Monthly About π?

- We see authors of varying notoriety. Many are top-tier research mathematicians whose names are still known.
- Others once famous are no longer known.
- Articles come from small colleges, big ten universities, ivy league schools and everywhere else.
- The process of constructing this selection highlights how much our scholarly life has changed over the past 30 years. Much more can be found and studied easily, but there is even more to find than in previous periods. The ease of finding papers in Google Scholar has the perverse consequence – like Gresham's law in economics – of making less easily accessible material even more likely to be ignored.

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What's in the Monthly About π?

- The Monthly papers on π cover relatively few topics.
- Every few years a 'simple proof' of the irrationality of π is published.
- Many proofs of $\sum_{n\geq 1} 1/n^2 = \pi^2/6$ appear, each trying to be a bit more slick or elementary than the last.
- Articles on mathematics outside the European tradition have appeared since the Monthly's earliest days.
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The Line Up

1. 152 citations: J. M. Borwein, P. B. Borwein, D. H. Bailey, *Ramanujan, modular equations, and approximations to pi or how to compute one billion digits of pi*, **96**(1989) 201–219.

2. 133 citations: G. Almkvist, B. Berndt, *Gauss, Landen, Ramanujan, the arithmetic-geometric mean, ellipses, π, and the ladies diary*, **95**(1988) 585–608.

3. 82 citations: A Kufner, L Maligrand, *The prehistory of the Hardy inequality*, **113**(2006) 715–732.

4. 67 citations: J.M. Borwein, P.B. Borwein, K. Dilcher, *Pi, Euler numbers, and asymptotic expansions*, **96**(1989) 681–687.

5. 61 citations: N.D. Baruah, B.C. Berndt, H.H. Chan, *Ramanujan's series for $1/\pi$: a survey*, **116**(2009) 567–587.

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Pre-Calculus Pi Computations

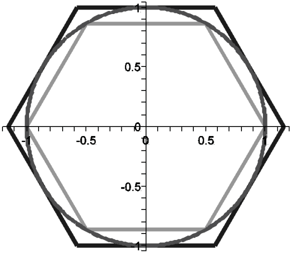
Name	Year	Digits
Babylonians	2000? BC	1
Egyptians	2000? BC	1
Hebrews (1 Kings 7:23)	550? BC	1
Archimedes	250? BC	3
Ptolemy	150	3
Liu Hui	263	5
Tsu Ch'ung Chi	480?	7
Al-Kashi	1429	14
Romanus	1593	15
van Ceulen (**Ludolph's number**)	1615	35

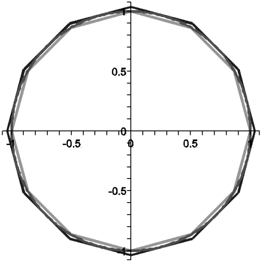
Archimedes' Method: The first rigorous mathematical calculation of π was due to Archimedes, who used a brilliant scheme based on *doubling inscribed and circumscribed polygons*,

$$6 \mapsto 12 \mapsto 24 \mapsto 48 \mapsto 96$$

and computing the perimeters to obtain the bounds

$$3\frac{10}{71} = 3.1408\ldots < \pi < 3\frac{10}{70} = 3.1428\ldots \Rightarrow \ldots.$$





Archimedes' Method

No computational mathematics approached this level of rigour again until the 19th century. Phillips (*AMM* **88**(1981)) calls Archimedes the 'first numerical analyst'.

Archimedes' scheme constitutes the first true algorithm for π, in that it can produce an arbitrarily accurate value for π. It also represents the birth of numerical and error analysis – all without positional notation or modern trigonometry.

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Calculus π-Calculations

Name	Year	Correct Digits
Sharp (and Halley)	1699	71
Machin	1706	100
Strassnitzky and Dase	1844	200
Rutherford	1853	440
Shanks	1874	(707) 527
Ferguson (**Calculator**)	1947	808
Reitwiesner et al. (**ENIAC**)	1949	2,037
Genuys	1958	10,000
Shanks and Wrench	1961	100,265
Guilloud and Bouyer	1973	1,001,250

Why isn't $\pi = 22/7$?

From Calculus we have

$$\int_0^t \frac{x^4(1-x)^4}{1+x^2}\,dx \;=\; \frac{1}{7}t^7 - \frac{2}{3}t^6 + t^5 - \frac{4}{3}t^3 + 4t - 4\arctan(t)\,.$$

From this it easily follows that

$$\int_0^1 \frac{(1-x)^4x^4}{1+x^2}\,dx = \frac{22}{7} - \pi \qquad (1)$$

The integrand is strictly positive on $(0,1)$, so

$$0 < \frac{22}{7} - \pi$$

– despite claims that π is $22/7$ which rage over the millennia (sorry Indiana).

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Post-Calculus π Calculations

Name	Year	Correct Digits
Miyoshi and Kanada	1981	2,000,036
Kanada-Yoshino-Tamura	1982	16,777,206
Gosper	1985	17,526,200
Bailey	Jan. 1986	29,360,111
Kanada et. al	Jan. 1987	134,217,700
Kanada and Tamura	Nov. 1989	1,073,741,799
Chudnovskys	May 1994	4,044,000,000
Kanada and Takahashi	Sep. 1999	206,158,430,000
Takahashi	April. 2009	2,576,980,377,524
Kondo and Yee	Aug. 2010	5,000,000,000,000
Kondo and Yee	Dec. 2013	12,200,000,000,000

What Fueled the Post-Calculus Era?

Much of the progress above was not only due to increased computing capacity, but by improved algorithms.

Many of the algorithms were derived from this fundamental identity of Ramanujan (see the Baruah, Berndt, and Chan highly cited Monthly paper mentioned above):

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}.$$

Each term of this series produces an additional EIGHT correct digits in the result. Gosper used this equation in his 1985 calculation of 17,526,200 digits.

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Two Main Thrusts

Theorem (Lambert 1761)

π is irrational.

Proof: [Breusch, *AMM* **61**(1954)]

Step 1: Assume $\pi = a/b$ with a and b integers. Then, with $N = 2a$, $\sin N = 0$, $\cos N = 1$, and $\cos(N/2) = \pm 1$.

Step 2: Set

$$S(t) = 1 - \frac{(t+1)(t+2)}{(2t+2)(2t+3)}\frac{N^2}{2!} + \frac{(t+1)(t+2)(t+3)(t+4)}{(2t+2)(2t+3)(2t+4)(2t+5)}\frac{N^4}{4!} - \cdots .$$

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Step 3: (HARD) Show that $S(t)$ is an integer for every positive integer t.

Step 4: (Not as Hard) Using the fact that

$$|S(t)| < 1 + N + \frac{N^2}{2!} + \cdots = e^N.$$

show that $| S(t) | < 1$ for all relatively large integers t. Hence $S(t) = 0$ for all such t.

Step 5: But this is impossible, because

$$\lim_{t \to \infty} S(t) = 1 - \frac{1}{2^2} \cdot \frac{N^2}{2!} + \frac{1}{2^4} \cdot \frac{N^4}{4!} - \cdots = \cos(N/2) = \pm 1.$$

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Recall that a complex number is *algebraic* if it is a root of a monic polynomial over $\mathbb{Z}[X]$. Otherwise it is *transcendental*. Hermite proved that the number e was transcendental in 1873.

Theorem (Corollary to the Lindemann–Weierstrass Theorem, 1882)

π is transcendental.

Proof: By Lindemann-Weierstrass, if x is nonzero and algebraic, then e^x is transcendental. Since $e^{\imath\pi} = -1$ is algebraic, $\imath\pi$ must be transcendental. Since $\imath$ is algebraic (a root of $X^2 + 1$) and the product of two algebraic numbers is algebraic, π must be transcendental. □

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There is a wonderful Monthly paper by Ivan Niven (*The transcendence of* π, *AMM* **46**(1939)) which gives a more direct proof of the last theorem in the spirit of Hermite's original proof of the transcendence of e.

The paper is in the appendix of the Borwein-Chapman Monthly paper in its entirety (about 4 pages) but the construction of its proof is sneakingly like the Breusch proof above.

1. Assume that π is algebraic.
2. Using a monic polynomial such that $\theta(\pi) = 0$, construct a complicated but finite summation.
3. Argue that the summation must equal a positive integer.
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Is π Normal?

Given an integer $b \geq 2$, a real number α is said to be *b*-**normal** or **normal base** *b* if every *m*-long string of base-*b*-digits appears in the base-*b* expansion of α with limiting frequency $1/b^{|m|}$.

More precisely, if m is any finite string of base-b digits, then et $N_\alpha(m,n)$ be the number of times the string m appears as a substring in the first n digits of the number α. α is normal if for all finite strings m we have

$$\lim_{n\to\infty} \frac{N_\alpha(m,n)}{n} = \frac{1}{b^{|m|}}.$$

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Is π Normal?

Using measure theory, it is easy to show that almost all real numbers are b-normal for every $b \geq 2$, i.e., the non-normal base-b numbers form a set of measure zero.

That not withstanding, showing that a particular number normal is extremely difficult.

Question

Is π normal to any base b?

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Let m be a finite string of base-10 digits. Does m appear somewhere in the decimal expansion of π?

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Evidence suggests that π might be normal

Decimal Digit	Occurrences in π
0	99999485134
1	99999945664
2	100000480057
3	99999787805
4	100000357857
5	99999671008
6	99999807503
7	99999818723
8	100000791469
9	99999854780
Total	**1000000000000**