AMORTIZED LOANS

Objectives:

After completing this section, you should be able to do the following:

- Calculate the monthly payment for a simple interest amortized loan.
- Calculate the total interest for a simple interest amortized loans.
- Create an amortization schedule for a simple interest amortized loan.
- Calculate the unpaid balance on an amortized loan.

Vocabulary:

As you read, you should be looking for the following vocabulary words and their definitions:

- amortized loan
- simple interest amortized loan
- amortization schedule
- unpaid balance

Formulas:

You should be looking for the following formulas as you read:

- simple interest amortized loan
- total paid on an amortized loan
- total interest paid on an amortized loan
- interest portion of a payment
- unpaid balance on an amortized loan

An *amortized loan* is a loan whose principal is repaid over the life of the loan usually through equal payments. Meriam-webster.com give the following definition of *amortizing*: "to pay off (as a mortgage) gradually usually by periodic payments of principal and interest".

amortized loan

Some examples of simple interest amortized loans are home loans (mortgages), car loans, and business loans

What we will be calculating with our loans will be the payment amount and total interest paid. In order to do these calculations will need the amount of the loan (principal), the interest rate, and the length of the loan. As with our simple annuities, the payment period and the compounding period will always be the same.

We have seen the formula for amortized loans before. It is the same formula that was used for the present value of an ordinary annuity.

Simple Interest Amortized Loan Formula

$$P\left(1+\frac{r}{n}\right)^{n*t} = pymt\frac{\left(1+\frac{r}{n}\right)^{n*t}-1}{\left(\frac{r}{n}\right)}$$

P = principal or loan amount
FV = future value
$pymt$ = payment amount
r = interest rate in decimal form
n = number of compounding periods in one year
t = time in years

Example 1:

Find the monthly payment and total interest paid for a simple interest amortized loan of \$15000 at an annual interest rate of $6\frac{3}{8}\%$ for 8 years.

Solution:

For this problem we are given the laon amount (\$15000), the interest rate (.06375 in decimal form), the compounding period (monthly or 12 periods per year), and finally the time (8 years). We plug each of these into the appropriate spot in the formula $P\left(1+\frac{r}{n}\right)^{n*t} = pymt\frac{\left(1+\frac{r}{n}\right)^{n*t}-1}{\left(\frac{r}{n}\right)}$. This will give us

$$15000\left(1+\frac{.06375}{12}\right)^{(12*8)} = pymt\frac{\left(1+\frac{.06375}{12}\right)^{(12*8)}-1}{\left(\frac{.06375}{12}\right)}$$

$$24945.67081 = pymt(124.808418)$$

$$\frac{24945.67081}{124.808418} = pymt$$

$$199.8717011 = pymt$$

In this class we will round using standard rounding. This will make the payment amount $199.87.

Total Interest Formula for a Simple Interest Amortized Loan

$$I = pymt * n * t - P$$

I **= interest**
P **= loan amount**
pymt **= payment amount**
n **= number of compounding periods in one year**
t **= time in years**

Now we need to find the total interest paid. This formula (see box at left) is very similar to the ones we have used in the past for interest. In this case the total amount paid will be more than the amount of the loan. From the first part we have the payment amount. When we plug this as well the other values into the formula, we will get

$$I = 199.87 * 12 * 8 - 15000$$
$$I = 4187.52$$

Thus the total interest paid on this loan is $4,187.52.

Example 2:

Gloria bought a house for $267,000. She put 20% down and obtained a simple interest amortized loan for the balance at $4\frac{7}{8}\%$ annually interest for 30 years.

a. Find the amount of Gloria's monthly payment.
b. Find the total interest paid by Gloria.
c. Most lenders will approve a home loan only if the total of all the borrower's monthly payments, including the home loan payment, is no more than 38% of the borrower's monthly income. How much must Gloria make in order to qualify for the loan?

Solution:

a. Our first step here is to find the down payment on the house. To calculate the down payment we need to multiply the price of the house by 20%.

$$\text{down payment} = 267000 * .20 = 53400$$

This means that Gloria will be paying $53,400 in cash for the house and financing the rest with an amortized loan.

Now we need to find the amount of the loan. This will be the difference between the price of the house and the down payment.

$$\text{loan amount} = 267000 - 53400 = 213600$$

Now we are ready to use the amortized loan formula with the loan amount (P), the annual interest rate (r = .04875), and the number of years of the loan (n = 30). This will give us

$$213600\left(1+\frac{.04875}{12}\right)^{(12*30)} = pymt\frac{\left(1+\frac{.04875}{12}\right)^{(12*30)}-1}{\left(\frac{.04875}{12}\right)}$$

$$919327.5006 = pymt(813.2843568)$$

$$\frac{919327.5006}{813.2843568} = pymt$$

$$1130.388766 = pymt$$

In this class we will round using standard rounding. This will make the monthly payment amount $1130.39.

b. To find the total interest paid by Gloria, we will use the formula from example 1 (**Total Interest Formula for a Simple Interest Amortized Loan**) with Gloria's loan amount and the monthly payment that we just calculated. This will give us

$$I = 1130.39 * 12 * 30 - 213600$$

$$I = 193340.40$$

c. To answer this question, we have to make some assumptions. The biggest assumption that we need to make is that Gloria has no other monthly expenses other than the monthly mortgage payment. We need the mortgage payment to be no more than 38% of Gloria's monthly income. We can write this an equation that looks like

$$pymt = (\text{monthly income}) * .38$$

We can solve this equation for monthly income to find out the minimum monthly income allowed for the payment.

$$1130.39 = (\text{monthly income}) * .38$$
$$\frac{1130.39}{.38} = \text{monthly income}$$
$$2974.710526 = \text{monthly income}$$

Thus Gloria would have to have a minimum monthly income of $2974.71 (and no other expenses) in order to qualify for this loan.

Example 3:

Ethan wants to buy a used car that costs $3500. He has two possible loans in mind. One loan is through the car dealer: it is a 3-year add-on interest loan at 5.25% and requires a down payment of $500. The second loan is through his credit union; it is a 3-year simple interest amortized loan at 8.75% and requires a 10% down payment.

a. Find the monthly payment for each loan.
b. Find the total interest paid for each loan.
c. Which loan should Barry choose? Why?

Solution:

> **Simple Interest Future Value Formula**
>
> $$FV = P(1 + rt)$$
>
> ***P*** **= principal or loan amount**
> ***FV*** **= future value**
> ***r*** **= interest rate in decimal form**
> ***t*** **= time in years**

a. We first need to find the monthly payment for each type of loan. We will do the add-on loan first and then the dimple interest amortized loan.

ADD-ON LOAN
Remember that add-on loans are calculated by finding the future value of the loan amount using the simple interest future value formula. Once we have the future value of the loan, we will divide that amount by the number of payments to be made to find out how much each payment will be.

For our add-on loan, the loan amount is cost of the car minus the down payment.

$$P = 3500 - 500 = 3000$$

The interest rate in decimal form is .0525. The time is 3 years. When we plug all this into the simple interest future value formula we get

$$FV = 3000(1 + .0525 * 3)$$
$$FV = 3472.50$$

To find the monthly payment, we divide the future value by the number of payments that will be made. This will give us

$$pymt = \frac{3472.50}{36}$$
$$pymt = 96.4583333$$

Ethan's monthly payments for the add-on loan will be $96.46.

SIMPLE INTEREST AMORTIZED LOAN
Our first step here is to find the down payment on the car. To calculate the down payment we need to multiply the price of the car by 10%.

$$\text{down payment} = 3500 * .10 = 350$$

This means that Ethan will be paying $350 in cash for the car and financing the rest with an amortized loan.

Now we need to find the amount of the loan. This will be the difference between the price of the house and the down payment.

$$\text{loan amount} = 3500 - 350 = 3150$$

Now we are ready to use the amortized loan formula with the loan amount (P), the annual interest rate (r = .0875), and the number of years of the loan (n = 3). This will give us

$$3150\left(1+\frac{.0875}{12}\right)^{(12*3)} = pymt\,\frac{\left(1+\frac{.0875}{12}\right)^{(12*3)}-1}{\left(\frac{.0875}{12}\right)}$$

$$4091.657107 = pymt(40.9973162)$$

$$\frac{4091.657107}{40.9973162} = pymt$$

$$99.80304775 = pymt$$

In this class we will round using standard rounding. This will make the monthly payment amount \$99.80.

b. To find the total interest paid by Ethan, we will use the **Total Interest Formula for a Simple Interest Amortized Loan** with Ethan's loan amount and the monthly payment that we just calculated for each loan.

ADD-ON LOAN

For the add-on loan, the loan amount was \$3000 and the monthly payment was \$96.46.

$$I = 96.46 * 12 * 3 - 3000$$

$$I = 472.56$$

For the add-on loan, Ethan will pay \$472.56 in interest.

SIMPLE INTEREST AMORTIZED LOAN

For the simple interest amortized loan, the loan amount was \$3150 and the monthly payment was \$99.80.

$$I = 99.80 * 12 * 3 - 3150$$

$$I = 442.80$$

For the simple interest amortized loan, Ethan will pay $442.80 in interest.

c. This final question is a little more difficult to answer. Which loan is the better loan depends on your situation. In the simple interest amortized loan, the total amount paid over the three-year loan period is less than with the add-on loan. If total amount paid is all that is important, then Ethan should choose the simple interest amortized loan.

   However, the add-on loan has a lower monthly payment. If this is more important than the total amount paid, then Ethan should choose the add-on loan.

Several things can affect the amount of interest paid on a loan. These things are interest rate, term (length) of the loan, and payment period. In this next example, we will examine one of these – the term of the loan. Some lenders are now offering 15-year home loans. How will this shortened loan term affect the monthly payments and total interest paid?

Example 4:

Find the monthly payment and total interest for a loan of $150,000 at 10.25% for the following terms.

a. 15 years
b. 30 years

Solution:

a. We will need to start by finding the monthly payments on this loan of $150,000 (*P*) at .1025 (*r*) over a term of 15 years (*n*). Plugging all this into the simple interest amortized loan formula, we will get

$$150000\left(1+\frac{.1025}{12}\right)^{(12*15)} = pymt\frac{\left(1+\frac{.1025}{12}\right)^{(12*15)}-1}{\left(\frac{.1025}{12}\right)}$$

$$693399.3126 = pymt(424.1165367)$$

$$\frac{693399.3126}{424.1165367} = pymt$$

$$1634.926377 = pymt$$

In this class we will round using standard rounding. This will make the monthly payment amount $1634.93.

To find the total interest paid, we will use the **Total Interest Formula for a Simple Interest Amortized Loan** with the loan amount and the monthly payment that we just calculated. This will give us

$$I = 1634.93*12*15-150000$$

$$I = 144287.4$$

Thus the total interest paid will be $144,287.40

b. We will need to start by finding the monthly payments on this loan of $150,000 ($P$) at .1025 ($r$) over a term of 30 years (n). Plugging all this into the simple interest amortized loan formula, we will get

$$150000\left(1+\frac{.1025}{12}\right)^{(12*30)} = pymt\frac{\left(1+\frac{.1025}{12}\right)^{(12*30)}-1}{\left(\frac{.1025}{12}\right)}$$

$$6205350.712 = pymt(2384.66397)$$

$$\frac{6205350.712}{2384.66397} = pymt$$

$$1344.151944 = pymt$$

In this class we will round using standard rounding. This will make the monthly payment amount $1344.15.

To find the total interest paid, we will use the **Total Interest Formula for a Simple Interest Amortized Loan** with the loan amount and the monthly payment that we just calculated. This will give us

$$I = 1344.15 * 12 * 30 - 150000$$
$$I = 333894$$

Thus the total interest paid will be \$333,894

Although the 15-year loan has larger monthly payments, the total amount of interest paid on the loan is less than half the total interest paid on a 30-year loan with the same interest rate.