

## WORKING WITH INTEGERS:

### 1. Adding Rules:

Positive + Positive = Positive:  $5 + 4 = 9$

Negative + Negative = Negative:  $(-7) + (-2) = -9$

The sum of a negative and a positive number: First subtract: The answer gets the sign of the larger number

$$(-7) + 4 = -3$$

$$(-3) + 7 = 4$$

$$6 + (-9) = -3$$

$$5 + (-3) = 2$$

### 2. Subtracting Rules:

Negative - Positive = Negative:  $(-5) - 3 = -5 + (-3) = -8$

Positive - Negative = Positive + Positive = Positive:  $5 - (-3) = 5 + 3 = 8$

Negative - Negative = Negative + Positive = Use the sign of the larger number and subtract  
(Change double negatives to a positive)

$$(-5) - (-3) = (-5) + 3 = -2$$

$$(-3) - (-5) = (-3) + 5 = 2$$

### 3. Multiplying Rules:

Positive x Positive = Positive:  $3 \times 2 = 6$       Negative x Negative = Positive:  $(-2) \times (-8) = 16$

Negative x Positive = Negative:  $(-3) \times 4 = -12$

Positive x Negative = Negative:  $3 \times (-4) = -12$

### 4. Dividing Rules:

Positive  $\div$  Positive = Positive:  $12 \div 3 = 4$       Negative  $\div$  Negative = Positive:  $(-12) \div (-3) = 4$

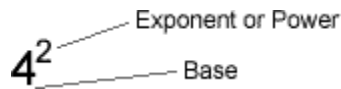
Negative  $\div$  Positive = Negative:  $(-12) \div 3 = -4$

Positive  $\div$  Negative = Negative:  $12 \div (-3) = -4$

### Tips:

1. When working with rules for positive and negative numbers try and think of weight loss or balancing a check book to help solidify why this works.
2. Using a number line showing both sides of "0" is very helpful to help develop the understanding of working with positive and negative numbers/integers.

## WHAT ARE EXPONENTS?



Exponents are sometimes referred to as powers and mean the number of times the 'base' is being multiplied. In algebra, exponents are used frequently. In the example below, you would say: Four to the power of 2 or four raised to the second power or four to the second power. This would mean  $4 \times 4$  or  $(4)(4)$  or  $4 \cdot 4$ . Simplified the example would be 16.

If the power/exponent of a number is 1, the number will always equal itself. In other words, in our example if the exponent 2 was a 1, simplified the example would then be 4.

### Exponent Rules

When working with exponents there are certain rules you'll need to remember.

When you are *multiplying* terms with the same base you can add the exponents.

$$4^2 \times 4^5 = 4^7$$

This means:  $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$  or  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

When you are *dividing* terms with the same base you can subtract the exponents.

$$4^5 \div 4^2 = 4^3$$

This means:  $4 \times 4 \times 4$  or  $4 \cdot 4 \cdot 4$

When *parentheses* are involved - you multiply.  $(8^3)^2 = 8^6$

$$y^a y^b = y^{(a+b)}$$

$$y^a x^a = (yx)^a$$

### Squared ( $x^2$ ), Cubed ( $x^3$ ), and 0's

When you multiply a number by itself it is referred to as being 'squared'.  $4^2$  is the same as saying "4 squared" which is equal to 16. If you multiply  $4 \times 4 \times 4$  which is  $4^3$  it is called 4 cubed.

Squaring is raising to the second power, cubing is raising to the third power. Raising something to a power of "1" means nothing at all; the base term remains the same. Now for the part that doesn't seem logical. When you raise a base to the power of 0, it equals 1. Any number raised to the power of "0" equals 1 and 0 raised to any exponent or power is 0!

## Order of Operations: Rules

1. Calculations must be done from left to right.
2. Calculations in brackets (parenthesis) are done first. When you have more than one set of brackets, do the inner brackets first. *NOTE: Brackets and parenthesis are known also known as grouping symbols.*
3. Exponents (or radicals) must be done next.
4. Multiply and divide in the order the operations occur. So, what you **SEE** first, **DO** first!
5. Add and subtract in the order the operations occur. So, what you **SEE** first, **DO** first!

## Acronyms to Help you Remember

How will you remember this order? Try the following Acronyms:

Please **E**xcuse **M**y **D**ear **A**unt **S**ally (*Parenthesis, Exponents, Multiply, Divide, Add, Subtract*)

**BEDMAS** (*Brackets, Exponents, Divide, Multiply, Add, Subtract*) **OR**

**B**ig **E**lephants **D**estroy **M**ice **A**nd **S**nails (*Brackets, Exponents, Divide, Multiply, Add, Subtract*)

**P**ink **E**lephants **D**estroy **M**ice **A**nd **S**nails (*Parenthesis, Exponents, Divide, Multiply, Add, Subtract*)

**The point is to find SOMETHING that works for you. These are only suggestions.**

## Examples

$12 \div 4 + 3^2$ $12 \div 4 + 9$ $3 + 9$ 12	Rule 3: Exponent first Rule 4: Multiply or Divide as they appear Rule 5: Add or Subtract as they appear
$(4^2 + 5) - 3$ $21 - 3$ 18	Rule 2: Everything in the brackets first Rule 5: Add or Subtract as they appear
$20 \div (12 - 2) \times 3^2 - 2$ $20 \div 10 \times 9 - 2$ $20 \div 10 \times 9 - 2$ $18 - 2$ 16	Rule 2: Everything in the brackets first Rule 3: Exponents Rule 4: Multiply and Divide as they appear Rule 5: Add or Subtract as they appear

### Does It Make a Difference? What If I Don't Use the Order of Operations?

Mathematicians were very careful when they developed the order of operations. Without the correct order, watch what happens:

**15 + 5 X 10 -- Without following the correct order, I know that 15+5=20 multiplied by 10 gives me the answer of 200.**

**15 + 5 X 10 -- Following the order of operations, I know that 5X10 = 50 plus 15 = 65. *This is the correct answer, the above is not!***

You can see that it is **absolutely critical** to follow the order of operations. Some of the most frequent errors students make occur when they do not follow the order of operations when solving mathematical problems. Students can often be fluent in computational work yet do not follow procedures. Use the handy acronyms to ensure that you never make this mistake again.

# Laws of Exponents

Here are the Laws (explanations follow):

<b>Law</b>	<b>Example</b>
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = 1/x$	$4^{-1} = 1/4$
$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
$x^m / x^n = x^{m-n}$	$x^6 / x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2 \times 3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
$(x/y)^n = x^n / y^n$	$(x/y)^2 = x^2 / y^2$
$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$


And the law about Fractional Exponents:

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$
$= (\sqrt[n]{x})^m$	$= (\sqrt[3]{x})^2$

## Laws Explained

The first three laws above ( $x^1 = x$ ,  $x^0 = 1$  and  $x^{-1} = 1/x$ ) are just part of the natural sequence of exponents. Have a look at this:

<b>Example: Powers of 5</b>		
.. etc..		
<b>5<sup>2</sup></b>	<b>1 × 5 × 5</b>	25
<b>5<sup>1</sup></b>	<b>1 × 5</b>	5
<b>5<sup>0</sup></b>	<b>1</b>	1
<b>5<sup>-1</sup></b>	<b>1 ÷ 5</b>	0.2
<b>5<sup>-2</sup></b>	<b>1 ÷ 5 ÷ 5</b>	0.04
.. etc..		



Look at that table for a while ... notice that positive, zero or negative exponents are really part of the same pattern, i.e. 5 times larger (or 5 times smaller) depending on whether the exponent gets larger (or smaller).

## The law that $x^m x^n = x^{m+n}$

**Example:**  $x^2 x^3 = (xx)(xxx) = xxxxx = x^5$

So,  $x^2 x^3 = x^{(2+3)} = x^5$

## The law that $x^m / x^n = x^{m-n}$

**Example:**  $x^4 / x^2 = (xxxx) / (xx) = xx = x^2$

So,  $x^4 / x^2 = x^{(4-2)} = x^2$

(Remember that  $x/x = 1$ , so every time you see an  $x$  "above the line" and one "below the line" you can cancel them out.)

This law can also show you why  $x^0 = 1$  :

**Example:**  $x^2 / x^2 = x^{2-2} = x^0 = 1$

## The law that $(x^m)^n = x^{mn}$

**Example:**  $(x^3)^4 = (xxx)^4 = (xxx)(xxx)(xxx)(xxx) = xxxxxxxxxxxxxx = x^{12}$

So  $(x^3)^4 = x^{3 \times 4} = x^{12}$

## The law that $(xy)^n = x^n y^n$

**Example:**  $(xy)^3 = (xy)(xy)(xy) = xyxyxy = xxxyyy = (xxx)(yyy) = x^3 y^3$

## The law that $(x/y)^n = x^n / y^n$

**Example:**  $(x/y)^3 = (x/y)(x/y)(x/y) = (xxx)/(yyy) = x^3 / y^3$

## The law that $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

$x^{1/n} =$  The  $n$ -th Root of  $x$

And so a fractional exponent like  $4^{3/2}$  is really saying to do a **cube** (3) and a **square root** (1/2), in any order.

Just remember from fractions that  $m/n = m \times (1/n)$ :

**Example:**  $x^{\frac{m}{n}} = x^{(m \times \frac{1}{n})} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$

The order does not matter, so it also works for  $\mathbf{m/n = (1/n) \times m}$ :

**Example:**  $x^{\frac{m}{n}} = x^{(\frac{1}{n} \times m)} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$

Let's look at a few examples of how this is applied.

**Example: What is  $9^{1/2} \times 9^{1/2}$  ?**

$$9^{1/2} \times 9^{1/2} = 9^{(1/2+1/2)} = 9^{(1)} = 9$$

So  $9^{1/2}$  times itself gives 9.

What do we call a number that, when multiplied by itself gives another number? The [square root](#)!

See:

$$\sqrt{9} \times \sqrt{9} = 9$$

So  $9^{1/2}$  is the same as  $\sqrt{9}$

**Example: What is  $4^{3/2}$  ?**

$$4^{3/2} = 4^{3 \times (1/2)} = \sqrt{(4^3)} = \sqrt{(4 \times 4 \times 4)} = \sqrt{(64)} = \mathbf{8}$$

or

$$4^{3/2} = 4^{(1/2) \times 3} = (\sqrt{4})^3 = (2)^3 = \mathbf{8}$$

Either way gets the same result.

**Example: What is  $27^{4/3}$  ?**

$$27^{4/3} = 27^{4 \times (1/3)} = \sqrt[3]{(27^4)} = \sqrt[3]{(531441)} = \mathbf{81}$$

or

$$27^{4/3} = 27^{(1/3) \times 4} = (\sqrt[3]{27})^4 = (3)^4 = \mathbf{81}$$

It was certainly easier the 2nd way!

## Powers of 10

"Powers of 10" is a very useful way of writing down large or small numbers. Instead of having lots of zeros, you show how many **powers of 10** you need to make that many zeros

**Example:**  $5,000 = 5 \times 1,000 = 5 \times 10^3$

- 5 thousand is 5 times a thousand. And a thousand is  $10^3$ . So 5 times  $10^3 = 5,000$
- Can you see that  $10^3$  is a handy way of making 3 zeros?

Scientists and Engineers (who often use very big or very small numbers) like to write numbers this way.

### **Example: The Mass of the Sun**

The Sun has a Mass of  $1.988 \times 10^{30}$  kg.

It would be too hard for scientists to write 1,988,000,000,000,000,000,000,000,000 kg

(And very easy to make a mistake counting the zeros!)

### **Example: A Light Year (the distance light travels in one year)**

It is easier to use  $9.461 \times 10^{15}$  meters, rather than 9,461,000,000,000,000 meters

It is commonly called [Scientific Notation](#), or **Standard Form**.

## “The Trick”

While at first it may look hard, there is an easy "trick":

The index of 10 says ..... *how many places to move the decimal point to the right.*

**Example: What is  $1.35 \times 10^4$  ?**

You can calculate it as:  $1.35 \times (10 \times 10 \times 10 \times 10) = 1.35 \times 10,000 = 13,500$

But it is easier to think "move the decimal point 4 places to the right" like this:

1.35 → 13.5 → 135. → 1350. → 13500.



## Negative Powers of 10

Negative? What could be the opposite of multiplying? **Dividing!**

A negative power means **how many times to divide** by the number.

Negatives just go the other way!

**Example:**  $5 \times 10^{-3} = 5 \div 10 \div 10 \div 10 = 0.005$

Just remember for negative powers of 10:

**For negative powers of 10, move the decimal point to the left.**

**Example: What is  $7.1 \times 10^{-3}$  ?**

Well, it is really  $7.1 \times (\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}) = 7.1 \times 0.001 = 0.0071$

But it is easier to think "move the decimal point 3 places to the left" like this:

$$7.1 \rightarrow 0.71 \rightarrow 0.071 \rightarrow 0.0071$$

## Addition and Subtraction of Polynomials

### **Adding Polynomials**

To add polynomials, you must clear the parenthesis, combine (add or subtract) the like terms. In some cases you will need to remember the order of operations. Remember, when adding and subtracting like parts, **the variable never changes**.

Here are a couple of examples:

$$\begin{aligned}(5x + 7y) + (2x - 1y) \\ &= 5x + 7y + 2x - 1y \quad (\text{Clear the parenthesis}) \\ &= 5x + 2x + 7y - 1y \quad (\text{Combine the like terms}) \\ &= 7x + 6y\end{aligned}$$

Another Example:

$$\begin{aligned}(y^2 - 3y + 6) + (y - 3y^2 + y^3) \\ &y^2 - 3y + 6 + y - 3y^2 + y^3 \quad (\text{Clear the parenthesis}) \\ &y^3 + y^2 - 3y^2 - 3y + y + 6 \quad (\text{Combine the like terms}) \\ &y^3 - 2y^2 - 2y + 6\end{aligned}$$

### **Subtracting Polynomials**

To subtract polynomials, you must change the sign of terms being subtracted, clear the parenthesis, and combine the like terms. Here's an example:

$$\begin{aligned}(4x^2 - 4) - (x^2 + 4x - 4) \\ &(4x^2 - 4) + (-x^2 - 4x + 4) \quad (\text{Change the signs by multiplying by -1; clear the parenthesis}) \\ &4x^2 - 4 + -x^2 - 4x + 4 \quad (\text{Clear the parenthesis}) \\ &4x^2 - x^2 - 4x - 4 + 4 \quad (\text{Combine the like terms})\end{aligned}$$

$$3x^2 - 4x$$

Another Example:

$$(5x^2 + 2x + 1) - (3x^2 - 4x - 2)$$

$$5x^2 + 2x + 1 - 3x^2 + 4x + 2 \quad (\text{Change the signs by multiplying by } -1; \text{ clear the parenthesis})$$

$$5x^2 - 3x^2 + 2x + 4x + 1 + 2 \quad (\text{Combine the like terms})$$

$$2x^2 + 6x + 3$$

### **Polynomial Definitions of Terms:**

A monomial has one term:  $5y$  or  $-8x^2$  or  $3$ .

A binomial has two terms:  $-3x^2 + 2$ , or  $9y - 2y^2$

A trinomial has 3 terms:  $-3x^2 + 2 + 3x$ , or  $9y - 2y^2 + y$

The degree of the term is the exponent of the variable:  $3x^2$  has a degree of 2.

When the variable does not have an exponent - always understand that there's a '1' e.g.,  $y$

## **Multiplying Polynomials**

### **Examples of Polynomials**

- $3x - 2$ ; Monomials:  $3x$ ,  $-2$  (NOTE: *these are also the terms of the polynomial*)
- $5xy^2 + 36xy + 52x + 6$ ; Monomials:  $5xy^2$ ,  $36xy$ ,  $52x$ ,  $6$
- $\Pi r^2 + 2\Pi h$ ; Monomials:  $\Pi r^2$ ,  $2\Pi h$

### **When to Multiply Polynomials**

The instructions will ask you to **multiply** or **simplify** exercises that look like this:

- (Polynomial)  $\times$  (Polynomial)
- (Polynomial)  $\cdot$  (Polynomial)
- (Polynomial)  $*$  (Polynomial)
- (Polynomial)<sup>x</sup>
- (Polynomial)(Polynomial)

**Note:** When there is no multiplication symbol between 2 sets of parentheses, realize that you are being asked to multiply.

## When to *Not* Multiply Polynomials

(Polynomial) + (Polynomial)

(Polynomial) – (Polynomial)

Yes, I understand that parentheses encompass the polynomials, but pay attention to what the exercise is asking you to do.

**NOTE:**  $(3x + 5y) + (2x + -6y)$  does not equal  $(3x + 5y)(2x + -6y)$ .

## Practice with Constants

Multiply:  $(8 + 6)(-2 + 5)$

Use order of operations:

1. Parentheses  
 $(8 + 6) = 14$   
 $(-2 + 5) = 3$
2. Multiply  
 $14 * 3 = 42$

## Using the **F.O.I.L.** Method

Here's another way of looking at it:

FOIL is method to multiply polynomials. It is an acronym for **First Outer Inner Last**.

1. **F**irst:  $(8 + 6)(-2 + 5)$   
Multiply the first terms:  $8 * -2 = -16$
2. **O**uter:  $(8 + 6)(-2 + 5)$   
Multiply the outer terms:  $8 * 5 = 40$
3. **I**nnner:  $(8 + 6)(-2 + 5)$   
Multiply the inner terms:  $6 * -2 = -12$
4. **L**ast:  $(8 + 6)(-2 + 5)$   
Multiply the outer terms:  $6 * 5 = 30$

**Next, add the results:**

$$-16 + 40 + -12 + 30$$

**Simplify:**

$$-16 + 40 + -12 + 30 = 42$$

Now, let's practice multiplying polynomials with variables.

**Practice with Positives**

Simplify the following polynomials.

$$(x + 5)(x + 4)$$

1. *First. Outer. Inner. Last.*  
 $x * x + x * 4 + 5 * x + 5 * 4$
2. *Multiply.*  
 $x^2 + 4x + 5x + 20$
3. *Simplify.*  
 $x^2 + 9x + 20$

**Practice with Negatives**

Simplify the following polynomials.

$$(x - 5)(x - 4)$$

1. Before you start FOILing, change the negative signs (only do this if you need to):  
 $(x + -5)(x + -4)$
2. Now FOIL: *First. Outer. Inner. Last.*  
 $x * x + x * -4 + -5 * x + -5 * -4$
3. *Multiply.*  
 $x^2 + -4x + -5x + 20$
4. *Simplify.*  
 $x^2 + -9x + 20$

**Practice Exercises**

1)  $(x + 3)(x - 3) =$

$$2) (x - 6)(x + 4) =$$

$$3) (x - 8)(x - 9) =$$

$$4) (5j + 11)(j + 1) =$$

$$5) (5p - 7)(4p + 3) =$$

### Answers to Practice Exercises

$$1. (x + 3)(x - 3)$$

$$(x + 3)(x - 3) = x^2 + -9$$

1. Rewrite the minus sign. (Only do this if you have a problem with your signs)

$$(x + 3)(x + -3)$$

2. **First. Outer. Inner. Last.**

$$x * x + x * -3 + 3 * x + 3 * -3$$

3. **Multiply.**

$$x^2 + -3x + 3x + -9$$

4. **Simplify.**

$$x^2 + 0 + -9$$

$$x^2 + -9$$

$$2. (x - 6)(x + 4)$$

$$(x - 6)(x + 4) = x^2 + -2x + -24$$

1. Rewrite the minus sign. (Only do this if you have a problem with your signs)

$$(x + -6)(x + 4)$$

2. **First. Outer. Inner. Last.**

$$x * x + x * 4 + -6 * x + -6 * 4$$

3. **Multiply.**

$$x^2 + 4x + -6x + -24$$

4. **Simplify.**

$$x^2 + -2x + -24$$

**3.  $(x - 8)(x - 9)$**

$$(x - 8)(x - 9) = x^2 + -17x + 72$$

1. Rewrite the minus signs. (*Only do this if you have a problem with your signs*)

$$(x + -8)(x + -9)$$

2. **First. Outer. Inner. Last.**

$$x * x + x * -9 + -8 * x + -8 * -9$$

3. Multiply.

$$x^2 + -9x + -8x + 72$$

4. Simplify.

$$x^2 + -17x + 72$$

**4.  $(5j + 11)(j + 1)$**

$$(5j + 11)(j + 1) = 5j^2 + 16j + 11$$

1. **First. Outer. Inner. Last.**

$$5j * j + 5j * 1 + 11 * j + 11 * 1$$

2. Multiply.

$$5j^2 + 5j + 11j + 11$$

3. Simplify.

$$5j^2 + 16j + 11$$

**5.  $(5p - 7)(4p + 3)$**

$$(5p - 7)(4p + 3) = 20p^2 + -13p + -21$$

1. Rewrite the minus sign. (*Only do this if you have a problem with your signs*)

$$(5p + -7)(4p + 3)$$

2. **First. Outer. Inner. Last.**

$$5p * 4p + 5p * 3 + -7 * 4p + -7 * 3$$

3. Multiply.

$$20p^2 + 15p + -28p + -21$$

4. Simplify.

$$20p^2 + -13p + -21$$

## Dividing Polynomials

1)

$$\begin{aligned} \frac{100}{25} &= \frac{4 \times 5 \times 5}{5 \times 5} \\ &= \frac{4 \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5}} \\ &= 4 \end{aligned}$$

Working with division in Arithmetic is a lot like division of monomials in Algebra. In arithmetic, you use your knowledge of factors to help you. Look at this example of division using factors. When you review the strategy you use in Arithmetic, algebra will make more sense. Simply

show the factors, cancel out the common factors (which is division) and you will be left with your solution.

2)

$$\frac{24ab}{8a} = \frac{\cancel{24}^3 \cancel{a}b}{\cancel{8}^1 \cancel{a}} \text{ (think } 24 \div 8 = 3\text{)}$$
$$= 3b$$

Here's a basic monomial, notice that when you divide the monomial, you're dividing the numerical coefficients (the 24 and the 8) and you're dividing the literal coefficients (a and b).

3)

$$\frac{-20a^5}{5a^2} = 4a^3$$

Once again you divide the numerical and literal coefficients and you'll also divide the *like* variable factors by subtracting their exponents (5-2).

4)

$$\frac{-150ab^4c}{10ab^2c}$$
$$= \frac{\cancel{-150} \cancel{a}b^4 \cancel{c}}{\cancel{10} \cancel{a}b^2 \cancel{c}}$$
$$= 15b^2$$

Divide the numerical and literal coefficients, divide the like variable factors by subtracting the exponents and you're done!

5)

$$\frac{16a^5b^2}{24a^2b^2} = \frac{\cancel{16}a^5 \cancel{b^2}}{\cancel{24}a^2 \cancel{b^2}}$$
$$= \frac{2}{3}a^3$$

Divide the numerical and literal coefficients, divide the like variable factors by subtracting the exponents and you're done!