2-1 Position, Displacement, and Distance

In describing an object's motion, we should first talk about position – where is the object? A position is a vector because it has both a magnitude and a direction: it is some distance from a zero point (the point we call **the origin**) in a particular direction. With one-dimensional motion, we can define a straight line along which the object moves. Let's call this the *x*-axis, and represent different locations on the *x*-axis using variables such as $\vec{x}_0$ and $\vec{x}_1$, as in Figure 2.1.

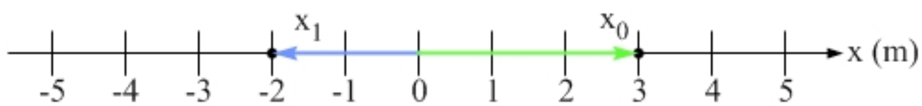


Figure 2.1: Positions $\vec{x}_0 = +3$ m and $\vec{x}_1 = -2$ m, where the + and – signs indicate the direction.

If an object moves from one position to another we say it experiences a **displacement**.

Displacement: a vector representing a change in position. A displacement is measured in length units, so the MKS unit for displacement is the meter (m).

We generally use the Greek letter capital delta (Δ) to represent a change. If the initial position is $\vec{x}_i$ and the final position is $\vec{x}_f$ we can express the displacement as:

$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$. (Equation 2.1: **Displacement in one dimension**)

In Figure 2.1, we defined the positions $\vec{x}_0 = +3$ m and $\vec{x}_1 = -2$ m. What is the displacement in moving from position $\vec{x}_0$ to position $\vec{x}_1$? Applying Equation 2.1 gives $\Delta\vec{x} = \vec{x}_1 - \vec{x}_0 = -2\text{ m} - (+3\text{ m}) = -5\text{ m}$. This method of adding vectors to obtain the displacement is shown in Figure 2.2. Note that the negative sign comes from the fact that the displacement is directed left, and we have defined the positive *x*-direction as pointing to the right.

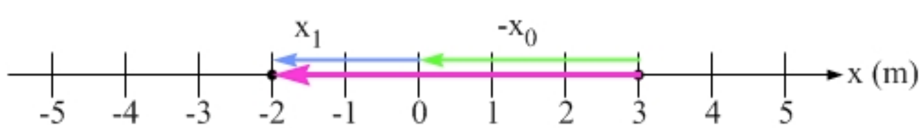


Figure 2.2: The displacement is –5 m when moving from position $\vec{x}_0$ to position $\vec{x}_1$. Equation 2.1, the displacement equation, tells us that the displacement is $\Delta\vec{x} = \vec{x}_1 - \vec{x}_0$, as in the figure. The bold arrow on the axis is the displacement, the vector sum of the vector $\vec{x}_1$ and the vector $-\vec{x}_0$.

To determine the displacement of an object, you only have to consider the change in position between the starting point and the ending point. The path followed from one point to the other does not matter. For instance, let's say you start at $\vec{x}_0$ and you then have a displacement of 8 meters to the left followed by a second displacement of 3 meters right. You again end up at $\vec{x}_1$, as shown in Figure 2.4. The total distance traveled is the sum of the magnitudes of the individual displacements, 8 m + 3 m = 11 m. The net displacement (the vector sum of the individual displacements), however, is still 5 meters to the left: $\Delta\vec{x} = -8\text{ m} + (+3\text{ m}) = -5\text{ m} = \vec{x}_1 - \vec{x}_0$.

Figure 2.3: The net displacement is still –5 m, even though the path taken from $\vec{x}_0$ to $\vec{x}_1$ is different from the direct path taken in Figure 2.2.

EXAMPLE 2.1 – Interpreting graphs

Another way to represent positions and displacements is to graph the position as a function of time, as in Figure 2.4. This graph could represent your motion along a sidewalk.

(a) What happens at a time of $t = 40$ s?

(b) Draw a diagram similar to that in Figure 2.3, to show your motion along the sidewalk. Add circles to your diagram to show your location at 10-second intervals, starting at $t = 0$.

Using the graph in Figure 2.4, find (c) your net displacement and (d) the total distance you covered during the 50-second period.

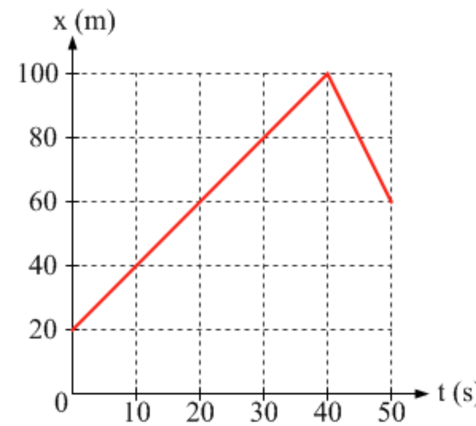


Figure 2.4: A graph of the position of an object versus time over a 50-second period. The graph represents your motion in a straight line as you travel along a sidewalk.

SOLUTION

(a) At a time of $t = 40$ s, the graph shows that your motion changes from travel in the positive x-direction to travel in the negative x-direction. In other words, at $t = 40$ s you reverse direction.

(b) Figure 2.5 shows one way to turn the graph in Figure 2.4 into a vector diagram to show how a series of individual displacements adds together to a net displacement. Figure 2.5 shows five separate displacements, which break your motion down into 10-second intervals.

(c) The displacement can be found by subtracting the initial position, +20 m, from the final position, +60 m. This gives a net displacement of $\Delta\vec{x}_{net} = \vec{x}_f - \vec{x}_i = +60\text{ m} - (+20\text{ m}) = +40\text{ m}$.

A second way to find the net displacement is to recognize that the motion consists of two displacements, one of +80 m (from +20 m to +100 m) and one of –40 m (from +100 m to +60 m). Adding these individual displacements gives $\Delta\vec{x}_{net} = \Delta\vec{x}_1 + \Delta\vec{x}_2 = +80\text{ m} + (-40\text{ m}) = +40\text{ m}$.

(d) The total distance covered is the sum of the magnitudes of the individual displacements. Total distance = 80 m + 40 m = 120 m.

Related End-of-Chapter Exercises: 7 and 9

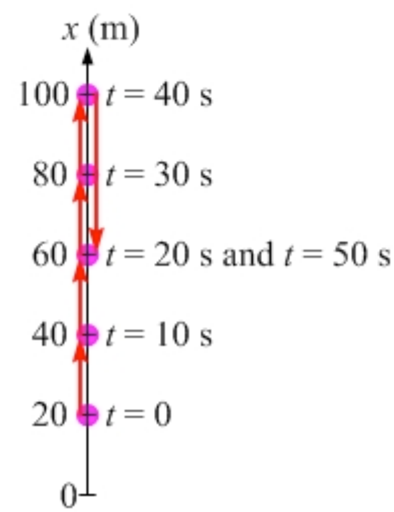


Figure 2.5: A vector diagram to show your displacement, as a sequence of five 10-second displacements over a 50-second period. The circles show your position at 10-second intervals.

Essential Question 2.1: In the previous example, the magnitude of the displacement is less than the total distance covered. Could the magnitude of the displacement ever be larger than the total distance covered? Could they be equal? Explain. *(The answer is at the top of the next page.)*

Answer to Essential Question 2.1: The magnitude of the net displacement is always less than or equal to the total distance. The two quantities are equal when the motion occurs without any change in direction. In that case, the individual displacements point in the same direction, so the magnitude of the net displacement is equal to the sum of the magnitudes of the individual displacements (the total distance). If there is a change of direction, however, the magnitude of the net displacement is less than the total distance, as in Example 2.1.

2-2 Velocity and Speed

In describing motion, we are not only interested in where an object is and where it is going, but we are also generally interested in how fast the object is moving and in what direction it is traveling. This is measured by the object's velocity.

Average velocity: a vector representing the average rate of change of position with respect to time. The SI unit for velocity is m/s (meters per second).

Because the change in position is the displacement, we can express the average velocity as:

$$\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\text{net displacement}}{\text{time interval}} .$$ (Equation 2.2: **Average velocity**)

The bar symbol ($\bar{}$) above a quantity means the average of that quantity. The direction of the average velocity is the direction of the displacement.

"Velocity" and "speed" are often used interchangeably in everyday speech, but in physics we distinguish between the two. Velocity is a vector, so it has both a magnitude and a direction, while speed is a scalar. Speed is the magnitude of the instantaneous velocity (see the next page). Let's define average speed.

Average Speed = $\bar{v} = \frac{\text{total distance covered}}{\text{time interval}}$ (Equation 2.3: **Average speed**)

In Section 2-1, we discussed how the magnitude of the displacement can be different from the total distance traveled. This is why the magnitude of the average velocity can be different from the average speed.

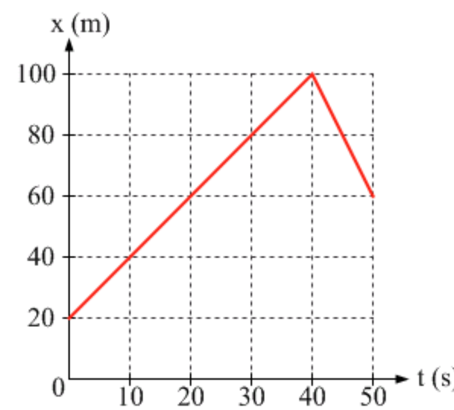


Figure 2.6: A graph of your position versus time over a 50-second period as you move along a sidewalk.

EXAMPLE 2.2A – Average velocity and average speed

Consider Figure 2.6, the graph of position-versus-time we looked at in the previous section. Over the 50-second interval, find:

(a) the average velocity, and (b) the average speed.

SOLUTION

(a) Applying Equation 2.2, we find that the average velocity is:

$$\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t} = \frac{+40\,\text{m}}{50\,\text{s}} = +0.80\ \text{m/s} .$$

The net displacement is shown in Figure 2.7. We can also find the net displacement by adding, as vectors, the displacement of +80 meters, in the first 40 seconds, to the displacement of –40 meters, which occurs in the last 10 seconds.

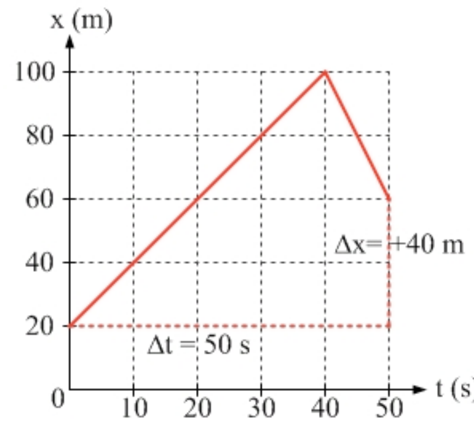


Figure 2.7: The net displacement of +40 m is shown in the graph.

(b) Applying Equation 2.3 to find the average speed,

$$\bar{v} = \frac{\text{total distance covered}}{\text{time interval}} = \frac{80\,\text{m} + 40\,\text{m}}{50\,\text{s}} = \frac{120\,\text{m}}{50\,\text{s}} = 2.4\,\text{m/s}\,.$$

The average speed and average velocity differ because the motion involves a change of direction. Let's now turn to finding instantaneous values of velocity and speed.

Instantaneous velocity: a vector representing the rate of change of position with respect to time at a particular instant in time. A practical definition is that the instantaneous velocity is the slope of the position-versus-time graph at a particular instant. Expressing this as an equation: $\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$. (Equation 2.4: **Instantaneous velocity**) Δt is sufficiently small that the velocity can be considered to be constant over that time interval.
Instantaneous speed: the magnitude of the instantaneous velocity.

EXAMPLE 2.2B – Instantaneous velocity

Once again, consider the motion represented by the graph in Figure 2.6. What is the instantaneous velocity at (a) $t = 25$ s? (b) $t = 45$ s?

SOLUTION

(a) Focus on the slope of the graph, as in Figure 2.8, which represents the velocity. The position-versus-time graph is a straight line for the first 40 seconds, so the slope, and the velocity, is constant over that time interval. Because of this, we can use the entire 40-second interval to find the value of the constant velocity at any instant between $t = 0$ and $t = 40$ s.
Thus, the velocity at t = 25 s is

$$\vec{v}_1 = \frac{\text{rise}}{\text{run}} = \frac{\text{displacement}}{\text{time}} = \frac{+80\text{ m}}{40\text{ s}} = +2.0\text{ m/s}\,.$$

(b) We use a similar method to find the constant velocity between $t = 40$ s and $t = 50$ s:
At $t = 45$ s, the velocity is

$$\vec{v}_2 = \frac{\text{displacement}}{\text{time}} = \frac{-40\text{ m}}{10\text{ s}} = -4.0\text{ m/s}\,.$$

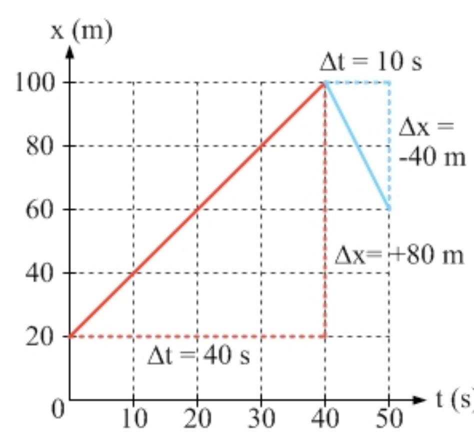


Figure 2.8: The velocity at any instant in time is determined by the slope of the position-versus-time graph at that instant.

Related End-of-Chapter Exercises: 2, 3, 8, 10, and 11

Essential Question 2.2: For the motion represented by the graph in Figure 2.6, is the average velocity over the entire 50-second interval equal to the average of the velocities we found in Example 2.2B for the two different parts of the motion? Explain.

Answer to Essential Question 2.2: If we take the average of the two velocities we found in Example 2.2B, $\vec{v}_1 = +2.0\,\text{m/s}$ and $\vec{v}_2 = -4.0\,\text{m/s}$, we get $-1.0\,\text{m/s}$. This is clearly not the average velocity, because we found the average velocity to be +0.80 m/s in Example 2.2A . The reason the average velocity differs from the average of the velocities of the two parts of the motion is that one part of the motion takes place over a longer time interval than the other (4 times longer, in this case). If we wanted to find the average velocity by averaging the velocity of the different parts, we could do a weighted average, weighting the velocity of the first part of the motion four times more heavily because it takes four times as long, as follows:

$$\bar{\vec{v}} = \frac{4\times(+2.0\,\text{m/s}) + 1\times(-4.0\,\text{m/s})}{4+1} = \frac{+4.0\,\text{m/s}}{5} = +0.8\,\text{m/s}\,.$$

2-3 Different Representations of Motion

There are several ways to describe the motion of an object, such as explaining it in words, or using equations to describe the motion mathematically. Different representations give us different perspectives on how an object moves. In this section, we'll focus on two other ways of representing motion, drawing motion diagrams and drawing graphs. We'll do this for motion with constant velocity - motion in a constant direction at a constant speed.

EXPLORATION 2.3A – Learning about motion diagrams

A motion diagram is a diagram in which the position of an object is shown at regular time intervals as the object moves. It's like taking a video and over-laying the frames of the video.

Step 1 - ***Sketch a motion diagram for an object that is moving at a constant velocity.*** An object with constant velocity travels the same distance in the same direction in each time interval. The motion diagram in Figure 2.9 shows equally spaced images along a straight line. The numbers correspond to times, so this object is moving to the right with a constant velocity.

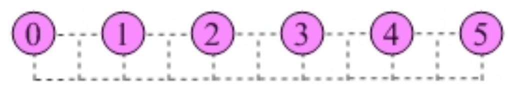


Figure 2.9: Motion diagram for an object that has a constant velocity to the right.

Step 2 - ***Draw a second motion diagram next to the first, this time for an object that is moving parallel to the first object but with a larger velocity.*** To be consistent, we should record the positions of the two objects at the same times. Because the second object is moving at constant velocity, the various images of the second object on the motion diagram will also be equally spaced. Because the second object is moving faster than the first, however, there will be more space between the images of the second object on the motion diagram – the second object covers a greater distance in the same time interval. The two motion diagrams are shown in Figure 2.10.

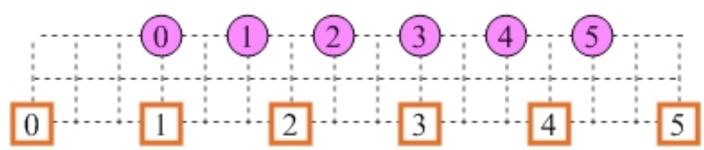


Figure 2.10: Two motion diagrams side by side. These two motion diagrams show objects with a constant velocity to the right but the lower object (marked by the square) has a higher speed, and it passes the one marked by the circles at time-step 3.

Key ideas: A motion diagram can tell us whether or not an object is moving at constant velocity. The farther apart the images, the higher the speed. Comparing two motion diagrams can tell us which object is moving fastest and when one object passes another.

Related End-of-Chapter Exercises: 23 and 24

EXPLORATION 2.3B – Connecting velocity and displacement using graphs

As we have investigated already with position-versus-time graphs, another way to represent motion is to use graphs, which can give us a great deal of information. Let's now explore a velocity-versus-time graph, for the case of a car traveling at a constant velocity of +25 m/s.

Step 1 - ***How far does the car travel in 2.0 seconds***? The car is traveling at a constant speed of 25 m/s, so it travels 25 m every second. In 2.0 seconds the car goes 25 m/s × 2.0 s, which is 50 m.

Step 2 – ***Sketch a velocity-versus-time graph for the motion. What on the velocity-versus-time graph tells us how far the car travels in 2.0 seconds?*** Because the velocity is constant, the velocity-versus-time graph is a horizontal line, as shown in Figure 2.11.

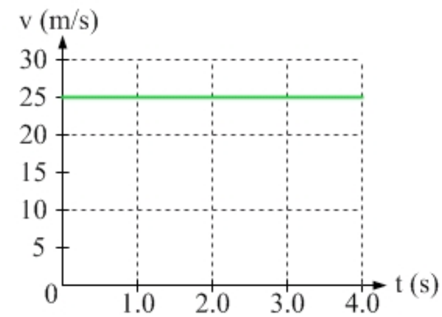


Figure 2.11: The velocity-versus-time graph for a car traveling at a constant velocity of +25 m/s.

To answer the second question, let's re-arrange Equation 2.2, $\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t}$, to solve for the displacement from the average velocity.

$$\Delta\vec{x} = \bar{\vec{v}}\,\Delta t\,.$$ (Equation 2.5: **Finding displacement from average velocity**)

When the velocity is constant, the average velocity is the value of the constant velocity. This method of finding the displacement can be visualized from the velocity-versus-time graph. The displacement in a particular time interval is the area under the velocity-versus-time graph for that time interval. "The area under a graph" means the area of the region between the line or curve on the graph and the *x*-axis. As shown in Figure 2.12, this area is particularly easy to find in a constant-velocity situation because the region we need to find the area of is rectangular, so we can simply multiply the height of the rectangle (the velocity) by the width of the rectangle (the time interval) to find the area (the displacement).

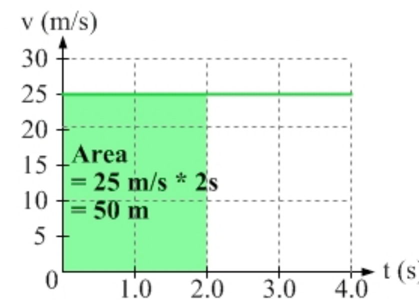


Figure 2.12: The area under the velocity-versus-time graph in a particular time interval equals the displacement in that time interval.

Key idea: The displacement is the area under the velocity-versus-time graph. This is true in general, not just for constant-velocity motion.

Deriving an equation for position when the velocity is constant

Substitute Equation 2.1, $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$, into Equation 2.5, $\Delta\vec{x} = \vec{v}\,\Delta t$.

This gives: $\vec{x}_f - \vec{x}_i = \vec{v}\,\Delta t = \vec{v}\left(t_f - t_i\right)$.

Generally, we define the initial time t_i to be zero: $\vec{x}_f - \vec{x}_i = \vec{v}\,t_f$.

Remove the "f" subscripts to make the equation as general as possible: $\vec{x} - \vec{x}_i = \vec{v}\,t$.

$$\vec{x} = \vec{x}_i + \vec{v}\,t\,.$$ (Equation 2.6: **Position for constant-velocity motion**)

Such a position-as-a-function-of-time equation is known as an **equation of motion**.

Related End-of-Chapter Exercises: 3, 17, and 48.

Essential Question 2.3: What are some examples of real-life objects experiencing constant-velocity motion? *(The answer is at the top of the next page.)*

Answer to Essential Question 2.3: Some examples of constant velocity (or at least almost-constant velocity) motion include (among many others):

- A car traveling at constant speed without changing direction.
- A hockey puck sliding across ice.
- A space probe that is drifting through interstellar space.

2-4 Constant-Velocity Motion

Let's summarize what we know about constant-velocity motion. We will also explore a special case of constant-velocity motion - that of an object at rest.

EXPLORATION 2.4 – Positive, negative, and zero velocities

Three cars are on a straight road. A blue car is traveling west at a constant speed of 20 m/s; a green car remains at rest as its driver waits for a chance to turn; and a red car has a constant velocity of 10 m/s east. At the time $t = 0$, the blue and green cars are side-by-side at a position 20 m east of the red car. Take east to be positive.

Step 1 – ***Picture the scene: sketch a diagram showing this situation.*** In addition to showing the initial position of the cars, the sketch at the middle left of Figure 2.13 shows the origin and positive direction. The origin was chosen to be the initial position of the red car.

Step 2 - ***Sketch a set of motion diagrams for this situation***. The motion diagrams are shown at the top of Figure 2.13, from the perspective of someone in a stationary helicopter looking down on the road from above. Because the blue car's speed is twice as large as the red car's speed, successive images of the blue car are twice as far apart as those of the red car. The cars' positions are shown at 1-second intervals for four seconds.

Step 3 - ***Write an equation of motion (an equation giving position as a function of time) for each car.*** Writing equations of motion means substituting appropriate values for the initial position $\vec{x}_i$ and the constant velocity $\vec{v}$ into Equation 2.5, $\vec{x} = \vec{x}_i + \vec{v}\,t$. The equations are shown above the graphs in Figure 2.13, using the values from Table 2.1.

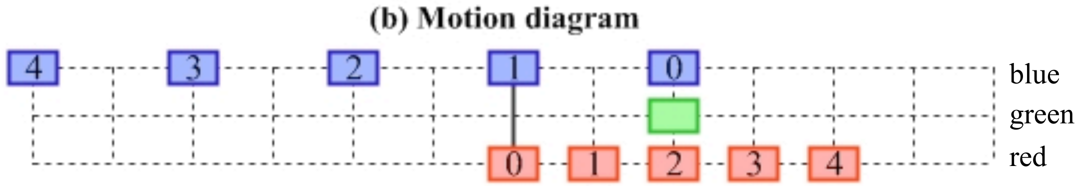
	Blue car	**Green car**	**Red car**
Initial position, $\vec{x}_i$	+20 m	+20 m	0
Velocity, $\vec{v}$	-20 m/s	0	+10 m/s

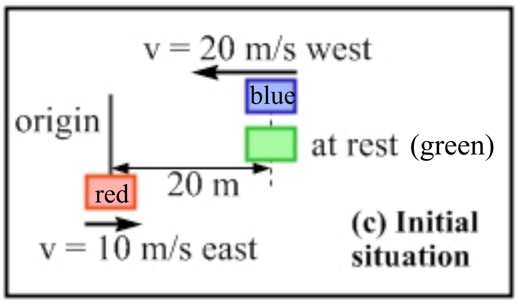
Table 2.1: Organizing the data for the three cars.

Step 4 - ***For each car sketch a graph of its position as a function of time and its velocity as a function of time for 4.0 seconds.*** The graphs are shown at the bottom of Figure 2.13. Note that the position-versus-time graph for the green car, which is at rest, is a horizontal line because the car maintains a constant position. An object at rest is a special case of constant-velocity motion: the velocity is both constant and equal to zero.

Key ideas: The at-rest situation is a special case of constant-velocity motion. In addition, all we have learned about constant-velocity motion applies whether the constant velocity is positive, negative, or zero. This includes the fact that an object's displacement is given by $\vec{x} = \vec{x}_i + \vec{v}\,t$; the displacement is the area under the velocity-versus-time graph; and the velocity is the slope of the position-versus-time graph. **Related End-of-Chapter Exercise: 43**

(a) Description of the motion in words: Three cars are on a straight road. A blue car has a constant velocity of 20 m/s west; a green car remains at rest; and a red car has a constant velocity of 10 m/s east. At t = 0 the blue and green cars are side-by-side, 20 m east of the red car.





(d) Equations of motion

Blue car: $\vec{x}_B = +20\text{ m} - (20\text{ m/s})t$

Green car: $\vec{x}_G = +20\text{ m}$

Red car: $\vec{x}_R = +(10\text{ m/s})t$

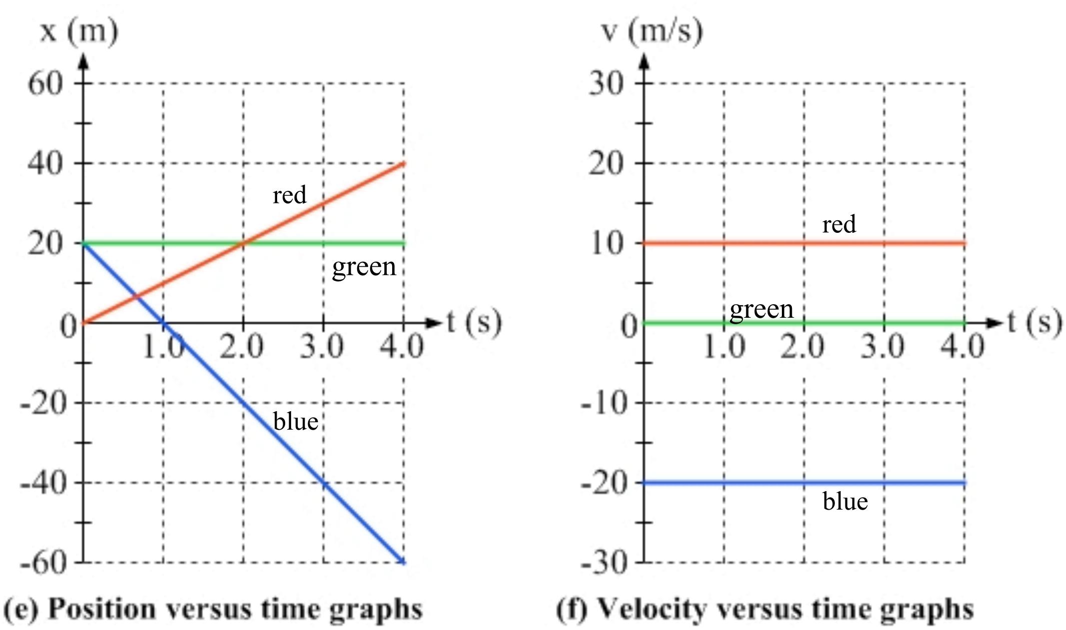


Figure 2.13: Multiple representations of the constant-velocity motions of three cars. These include (a) a description of the motion in words; (b) a motion diagram; (c) a diagram of the initial situation, at t = 0 (this is shown in a box); (d) equations of motion for each car; (e) graphs of the position of each car as a function of time; and (f) graphs of the velocity of each car as a function of time. Each representation gives us a different perspective on the motion.

Essential Question 2.4: Consider the graph of position-versus-time that is part of Figure 2.13. What is the significance of the points where the different lines cross? *(The answer is at the top of the next page.)*