AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Measurement

Length, Area and Volume

Measurement

"Data from international studies consistently indicate that students are weaker in the area of measurement than any other topic in the mathematics curriculum"

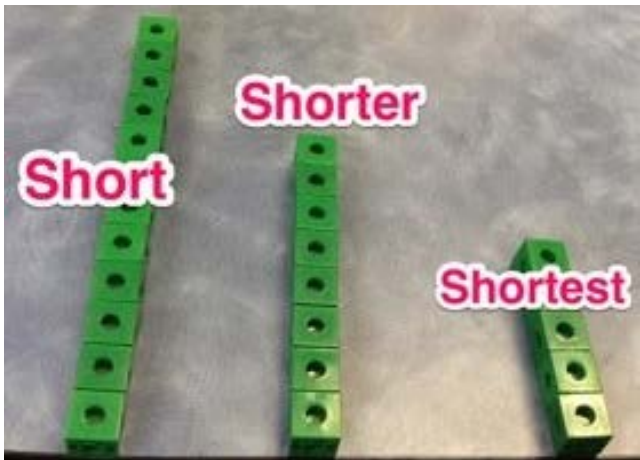
Thompson & Preston, 2004

Measurement

When to use

Foundation

Compares objects directly by placing one object against another to determine which is longer, hefting to determine which is heavier or pours to determine which holds more, and uses terms such as tall, taller, holds more, holds less



Hefting -lift or hold (something) in order to test its weight.

Measurement

When to use

Level 6

Connect decimal representations to the metric system (ACMMG135)

Convert between common metric units of length, mass and capacity (ACMMG136)

Solve problems involving the comparison of lengths and areas using appropriate units(ACMMG137)

Connect volume and capacity and their units of measurement (ACMMG138)

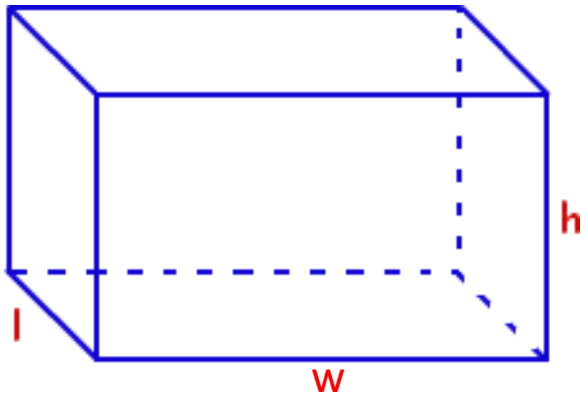
Measurement

When to use

Level 7

Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)

Calculate volumes of rectangular prisms (ACMMG160)



Measurement

Where it fits

Measurement integrates in all subject areas

Number and Place Value – measuring objects connects idea of number to the real world, enhancing number sense. The metric system of measurement is built on the base ten system

Measurement

History

- The decimal metric system was created by the French in 1799
- The British introduced a system based on the centimetre, gram and second in 1874, which was used for scientific experimentation but for everyday use they retained the Imperial System with its feet, inches, miles, furlongs etc. Australia inherited this system at the time of European settlement
- In 1939 an international system was adopted based on the metre, kilogram and second
- In 1970 the Australian parliament passed the Metric Conversion Act and the Australian building trades made it the standard in 1974

Measurement

Where does it fit?

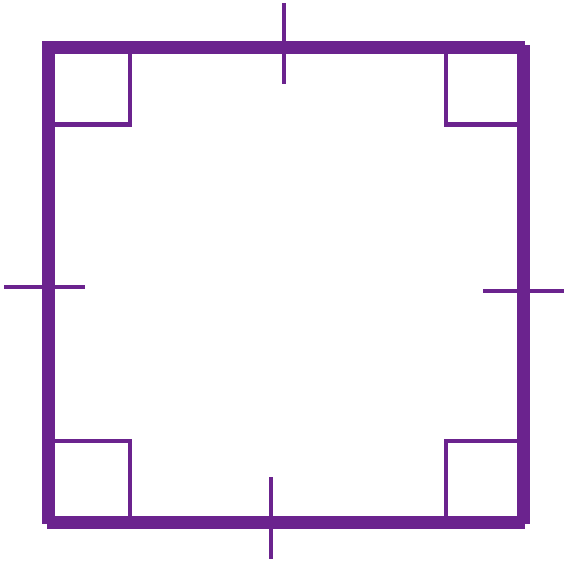
Geometry – measurements play a significant role in the describing and understanding of the properties of shapes. In later levels this is needed for knowledge in trigonometry.

Can a square be a rectangle?

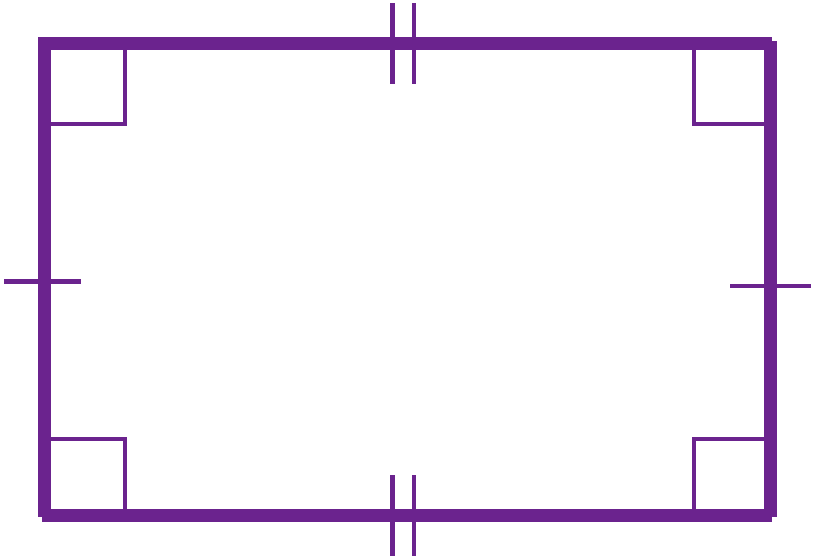
Can a rectangle be a square?

Measurement

Square v Rectangle



Four sided shape
Every angle is a right angle
Opposite sides are parallel
All four sides are equal length



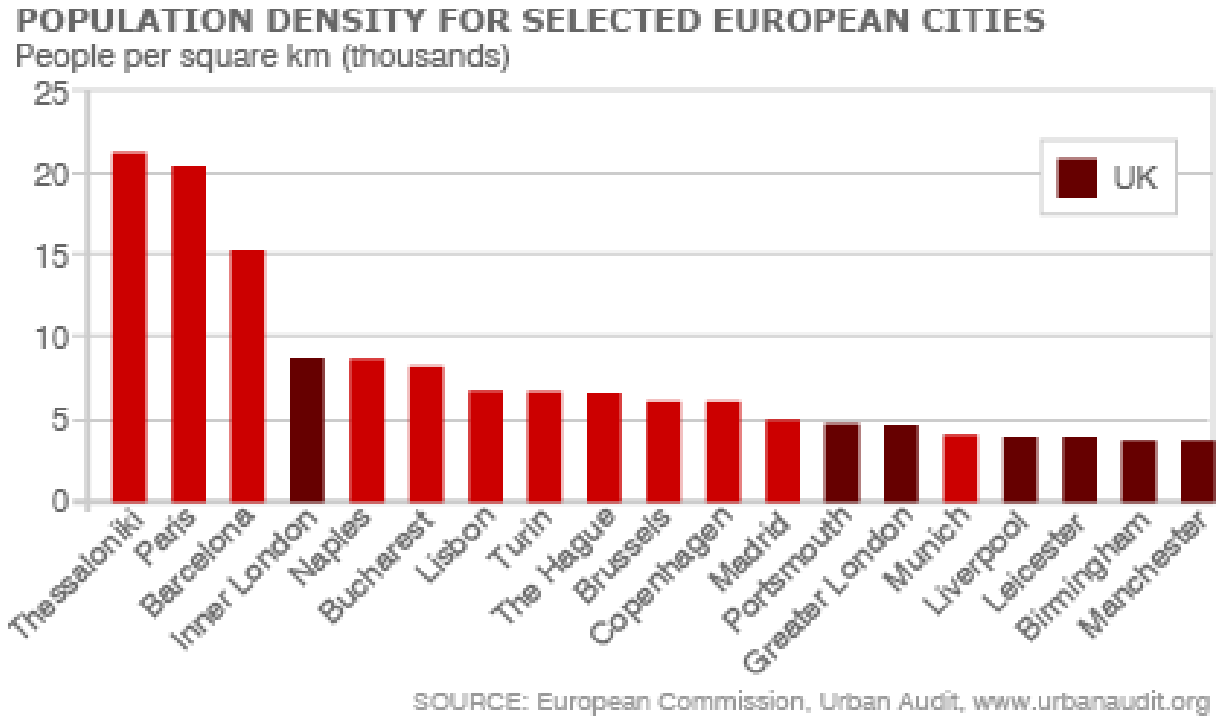
Four sided shape
Every angle is a right angle
Opposite sides are parallel
Opposite sides are equal length

A square is a rectangle as it satisfies all of its properties. However, not every rectangle is a square, to be a square its sides must have the same length.

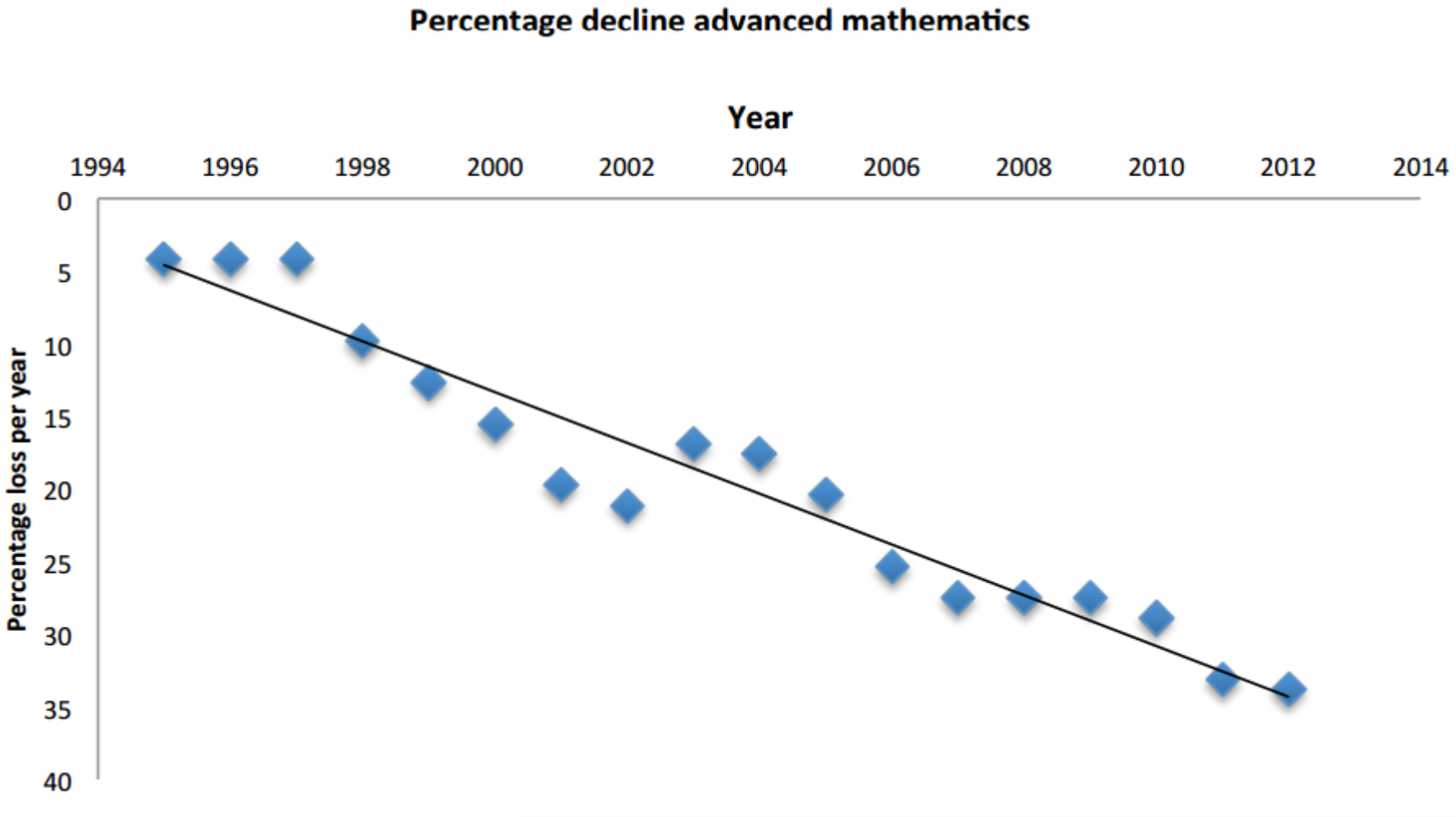
Measurement

Where does it fit?

Data and Statistics - stats and graphs help answer questions and describe our world. Often these descriptions are related to measurement such as time or temperature

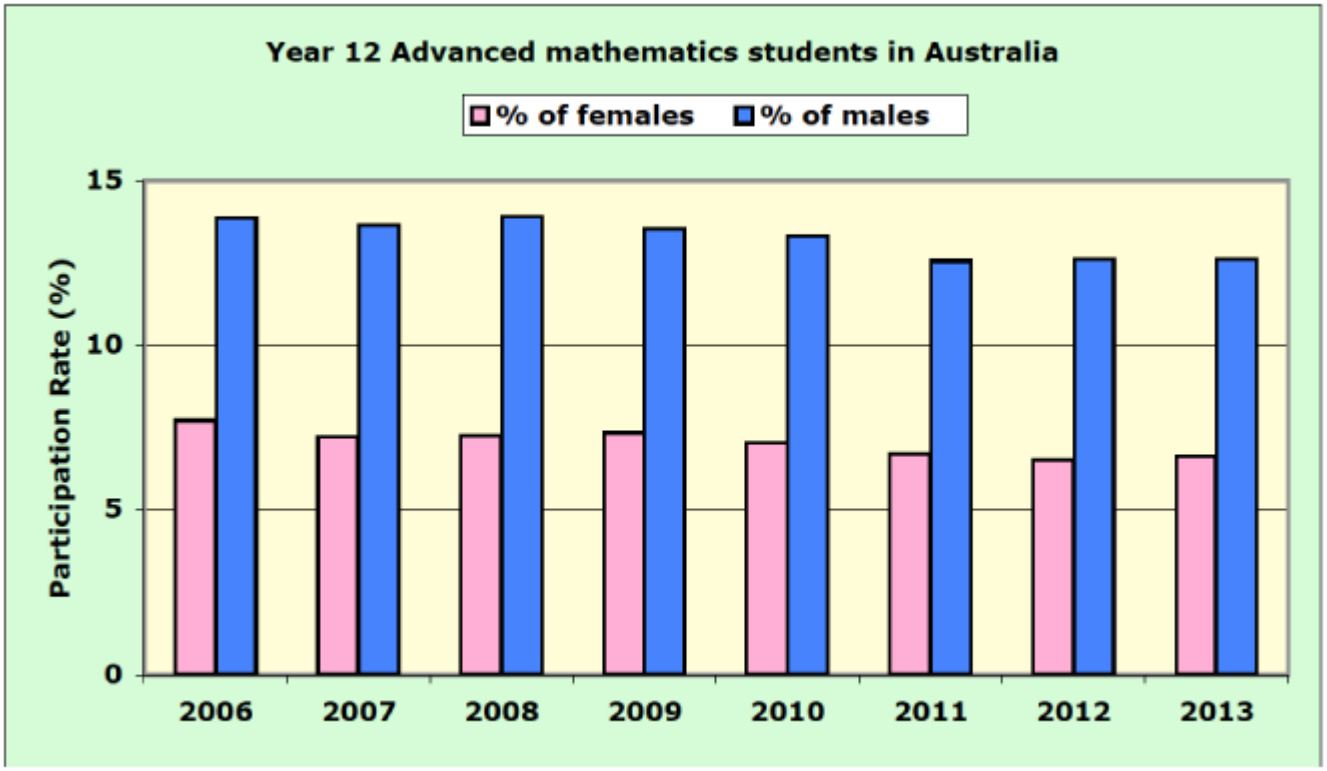


Measurement



Source: Frank Barrington, Year 12 Mathematics Participation Rates in Australia, data collection provided to AMSI

Measurement



Source: Frank Barrington, Year 12 Mathematics Participation Rates in Australia, data collection provided to AMSI

Measurement

How to introduce units

- Familiarity with the unit
- Ability to select an appropriate unit
- Knowledge of relationships between units

(Elementary and Middle School Mathematics)

Measurement

Familiarity

"40% Yr. 4 students were able to identify how many kg a bicycle weighed given the choices were 1.5kg, 15kg, 150kg or 1500kg"

Measurement

Familiarity

Ability to visualise

- How much milk does a carton of milk contain?
- How long is a basketball court?
- How far is the petrol station from school?
- What does a block of chocolate weigh?

Level 5
Chooses appropriate units of measurement for length, area, volume, capacity and mass, recognising that some units of measurement are better suited for some tasks than others, for example, km rather than m to measure the distance between two towns

Measurement

Ability to select appropriate units

What unit would you use to find the weight of the iPad?

A kilograms

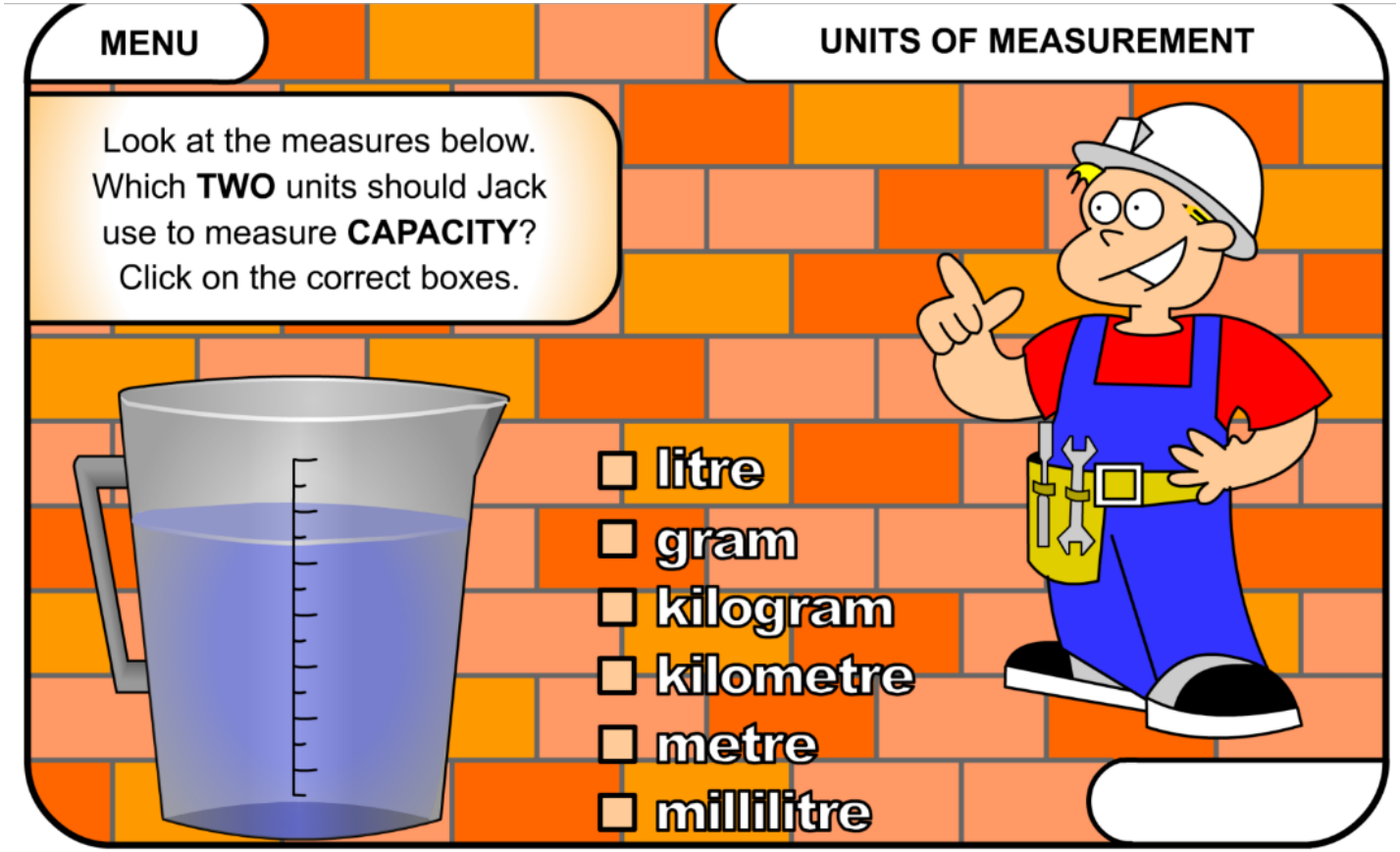
B centimetres

C grams

D cm^2

Measurement

Ability to select appropriate units

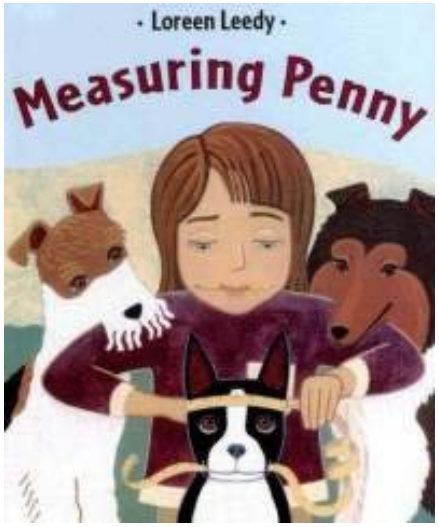


http://www.bgfl.org

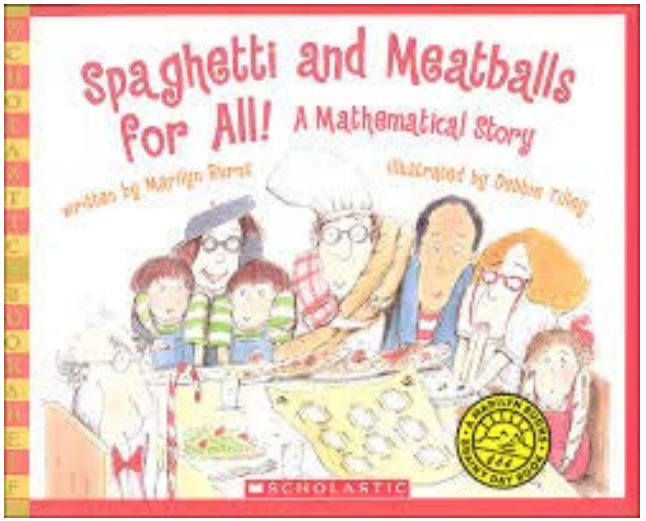
Measurement

Knowledge of relationships

Picture Books



Measuring Penny -*Loreen Leedy*

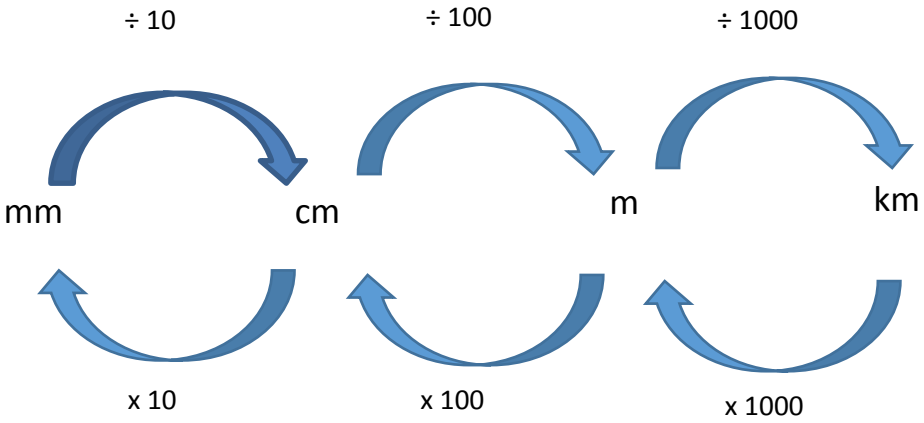


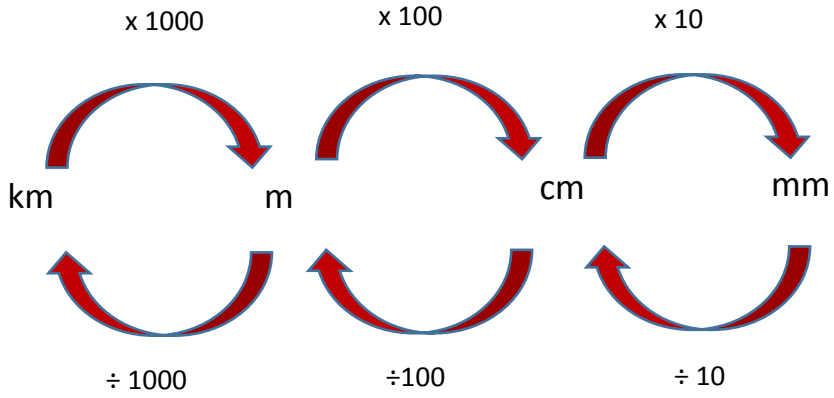
Spaghetti and Meatballs for All!

- *Marilyn Burns*

Measurement

Conversion



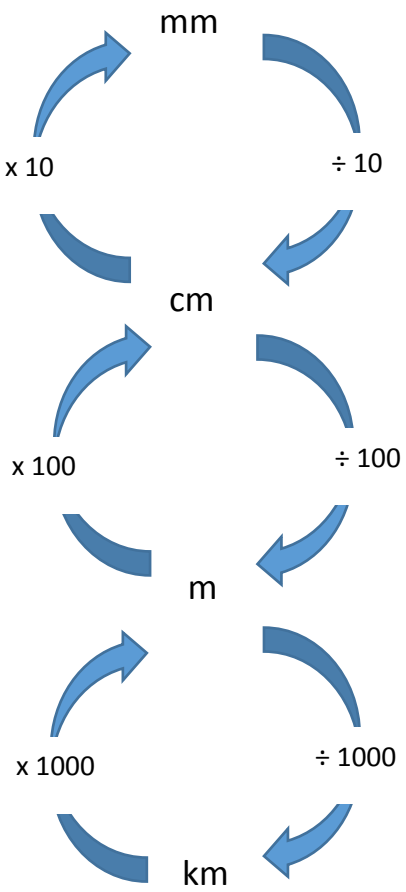


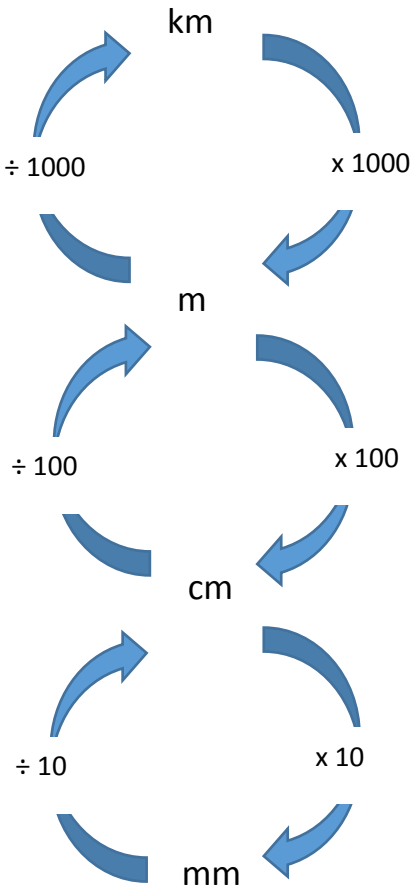
Level 6

Converts between common metric units of length, mass and capacity (identifying and using the correct operations when converting units including millimetres, centimetres, metres, kilometres, milligrams, grams, kilograms, tonnes, millilitres, litres, kilolitres and mega-litres

Measurement

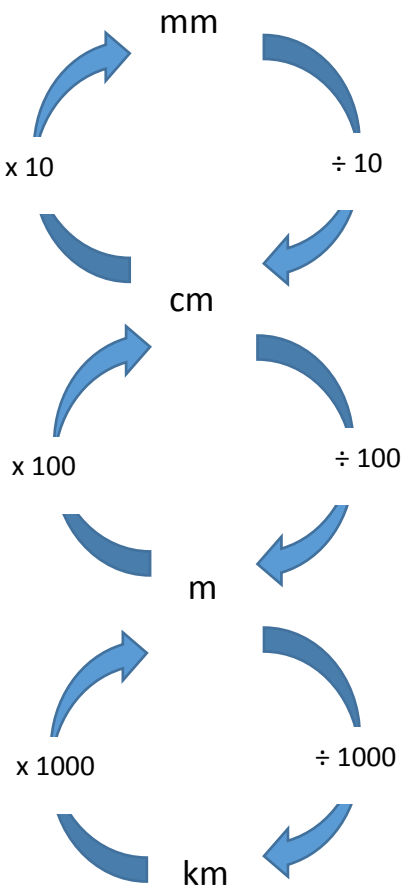
Conversion

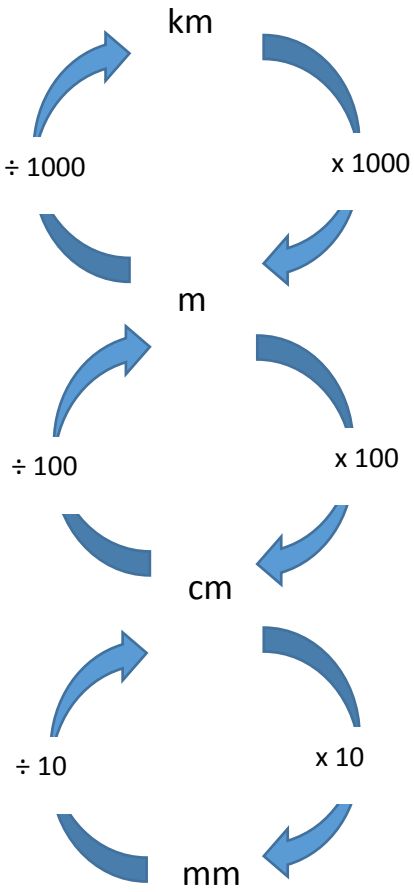




Measurement

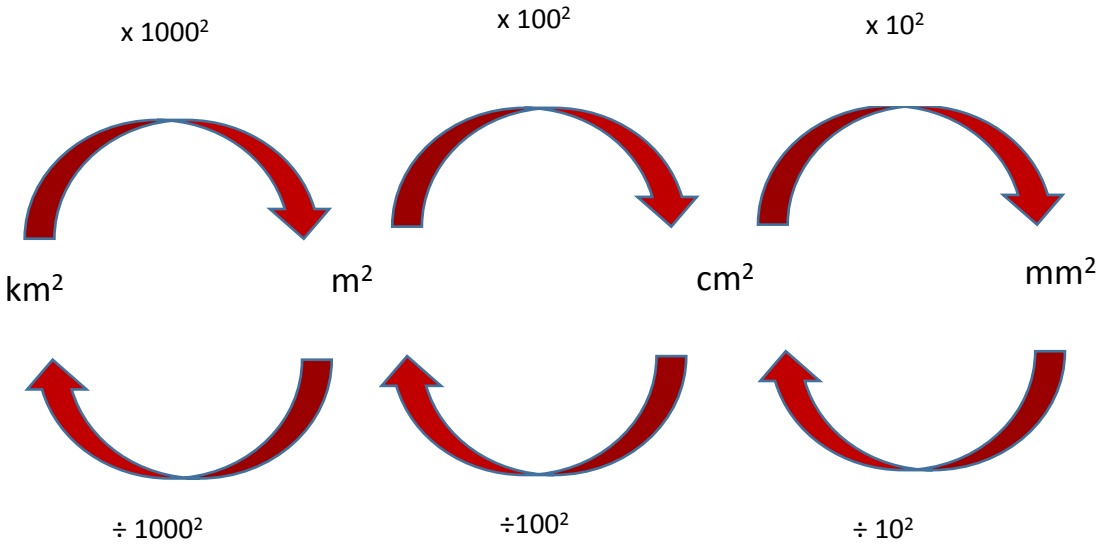
Conversion





Measurement

Conversion



Level 8

Chooses appropriate units of measurement for area and volume and converts from one unit to another. Recognises that the conversion factors for area of units are the squares of those for the corresponding linear units and for volume, units are the cubes of those for the corresponding linear units

Measurement

Prefixes

Prefix	Label	Decimal Value	Scientific	Colloquial
yocto	y	0.000 000 000 000 000 000 000 001	10^{-24}	septillionth
zepto	z	0.000 000 000 000 000 000 001	10^{-21}	sextillionth
atto	a	0.000 000 000 000 000 001	10^{-18}	quintillionth
femto	f	0.000 000 000 000 001	10^{-15}	quadrillionth
pico	p	0.000 000 000 001	10^{-12}	trillionth
nano	n	0.000 000 001	10^{-9}	billionth
micro	μ	0.000 001	10^{-6}	millionth
milli	m	0.001	10^{-3}	thousandth
centi	c	0.01	10^{-2}	hundredth
deci	d	0.1	10^{-1}	tenth
---	---	1	10^{0}	one
deka	da	10	10^{1}	ten
hecto	h	100	10^{2}	hundred
kilo	k	1 000	10^{3}	thousand
mega	M	1 000 000	10^{6}	million
giga	G	1 000 000 000	10^{9}	billion
tera	T	1 000 000 000 000	10^{12}	trillion
peta	P	1 000 000 000 000 000	10^{15}	quadrillion
exa	E	1 000 000 000 000 000 000	10^{18}	quintillion
zetta	Z	1 000 000 000 000 000 000 000	10^{21}	sextillion

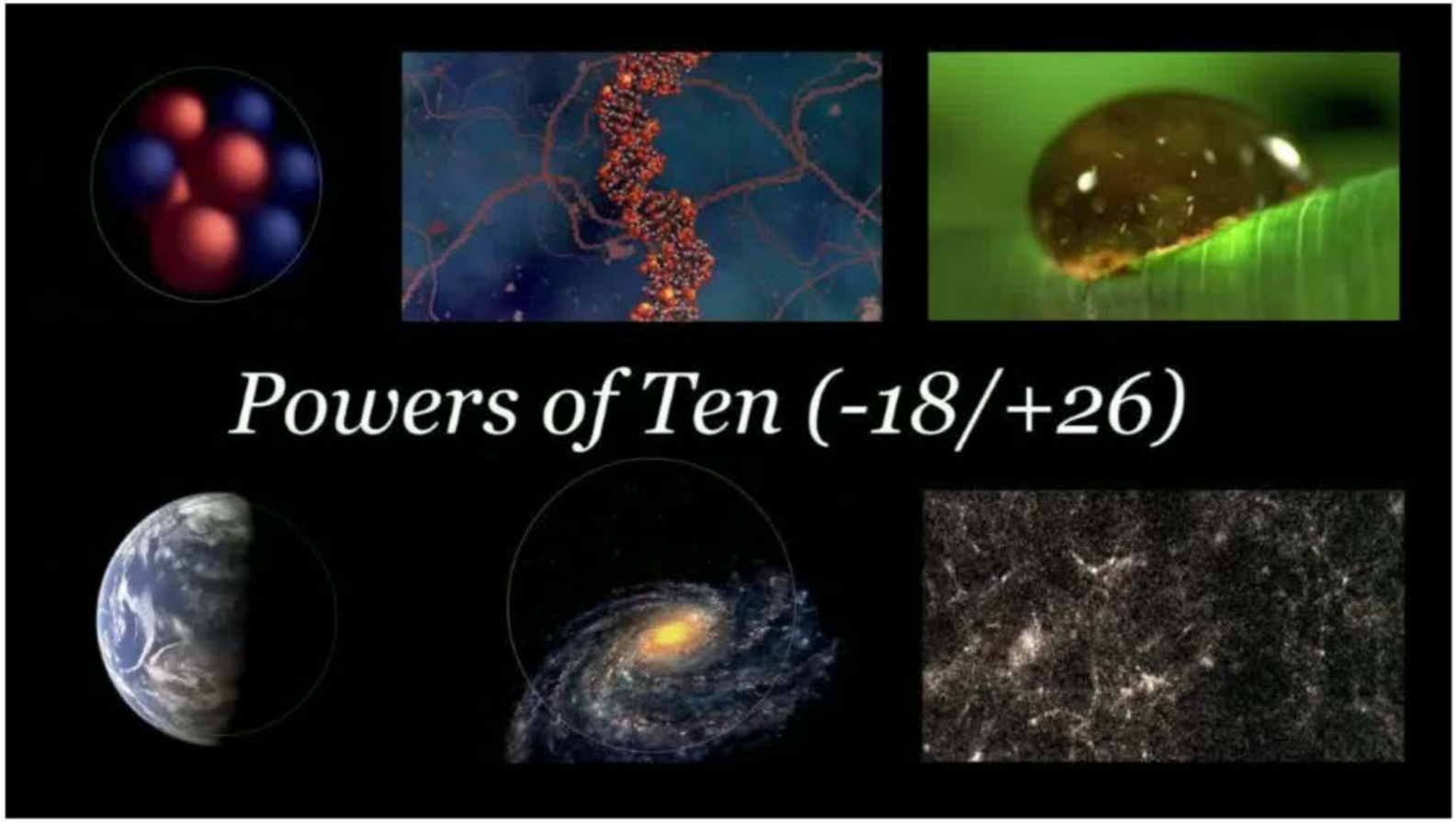
Level 6

Recognises the significance of the prefixes in the units of measurements

e.g.: milli = 1000^{th} ,mega = one million, kilo= 1000, centi = 100^{th}

Measurement

Prefixes



1 light year = 9.4605284×10^{15} metres

https://www.youtube.com/watch?v=bhofN1xX6u0

Measurement

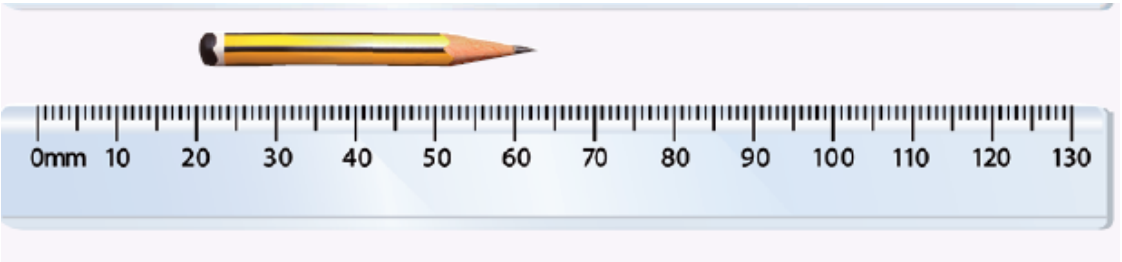
Estimation

http://www.aamt.edu.au/digital-resources/R10267/index.html

Measurement

Using instruments

Measuring when the object is not aligned with the end of the ruler

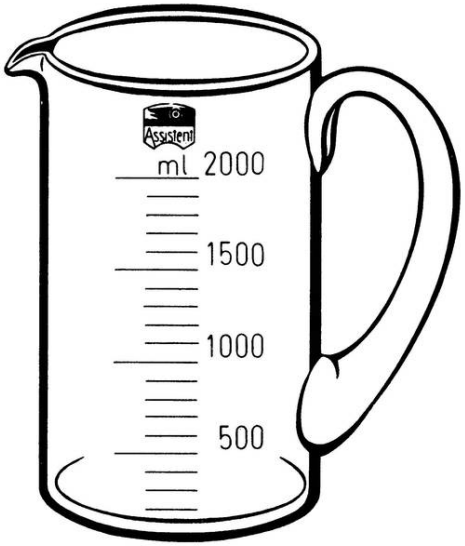


Reading from a tape measure

Measurement

Using instruments

Increments on the measuring device when not one unit.



Level 4

Uses graduated scaled instruments to measure and compare lengths, masses, capacities and temperatures

Measurement

Perimeter

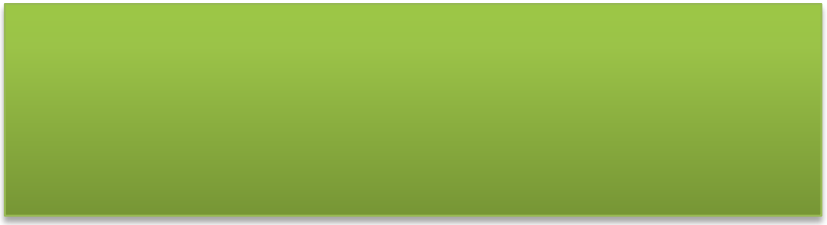
The word perimeter means 'a path that surrounds an area'. It comes from the Greek words *peri* meaning around and *metre* which means measure. Its first recorded usage was during the 15th century.

Perimeter is defined as the distance around a closed two-dimensional shape.

Level 5
Calculates the perimeter and area of rectangles using familiar metric units.
Explores efficient ways of calculating perimeters by adding the length and width together and doubling the result

Measurement

Perimeter



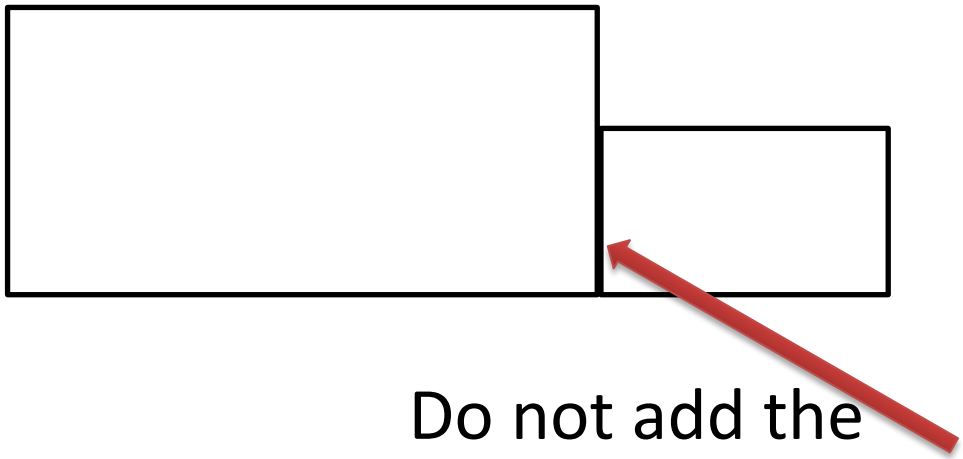
w

l

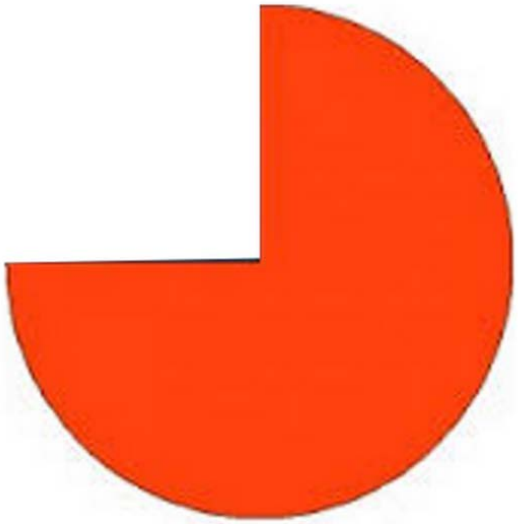
Perimeter = l + l + w + w

= 2l + 2w

= 2 (l + w)



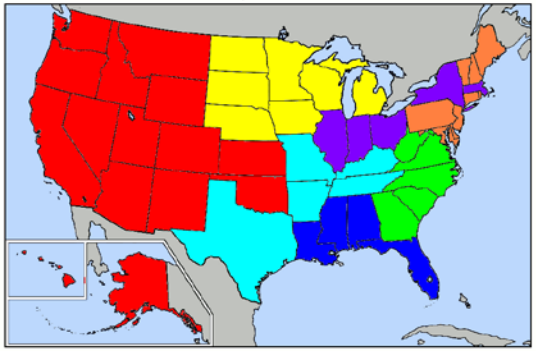
Do not add the internal line

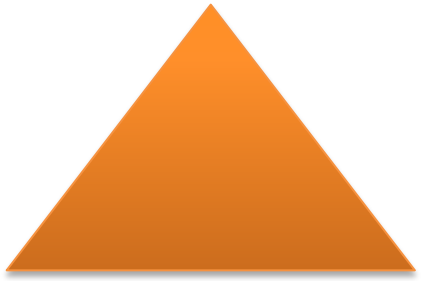


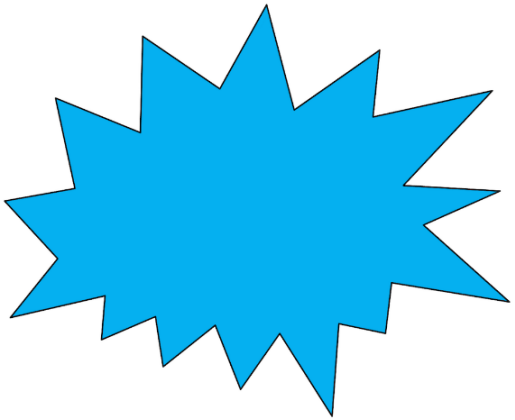
Measurement

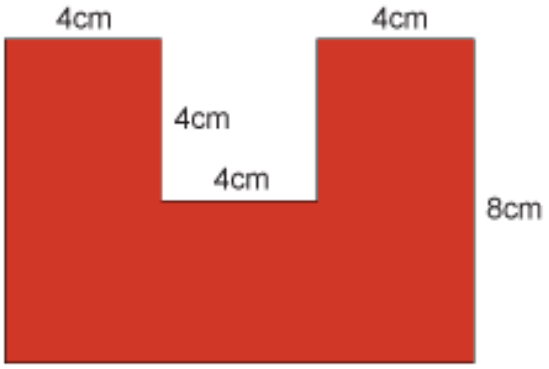
Area

Area is defined as a 2D space inside a region







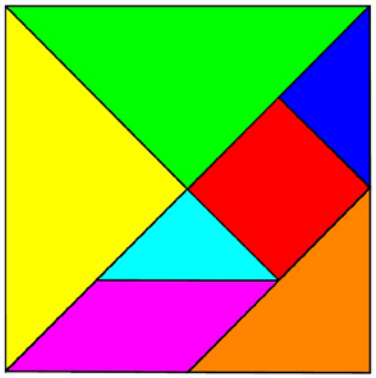


- Measured in units squared

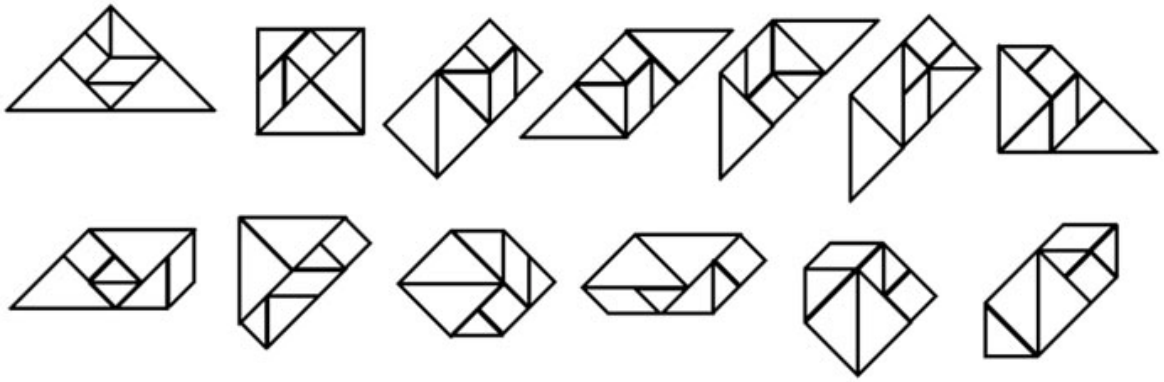
Measurement

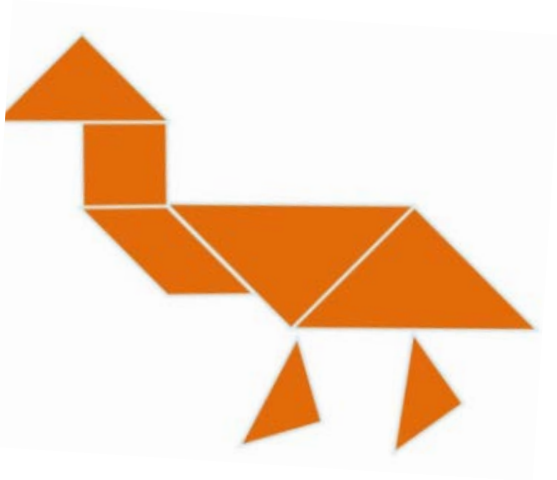
Area

Cutting a shape into different parts and reassembling it shows that different shapes can have the same area



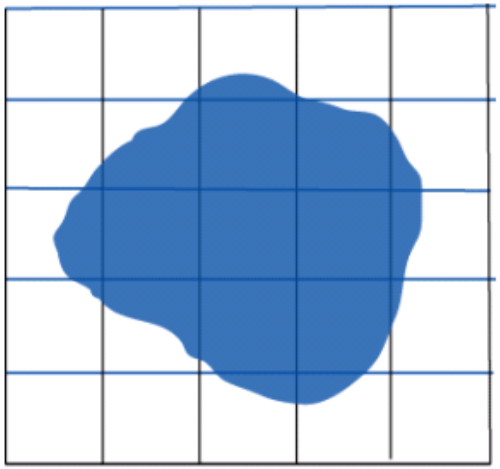
Use of tangrams



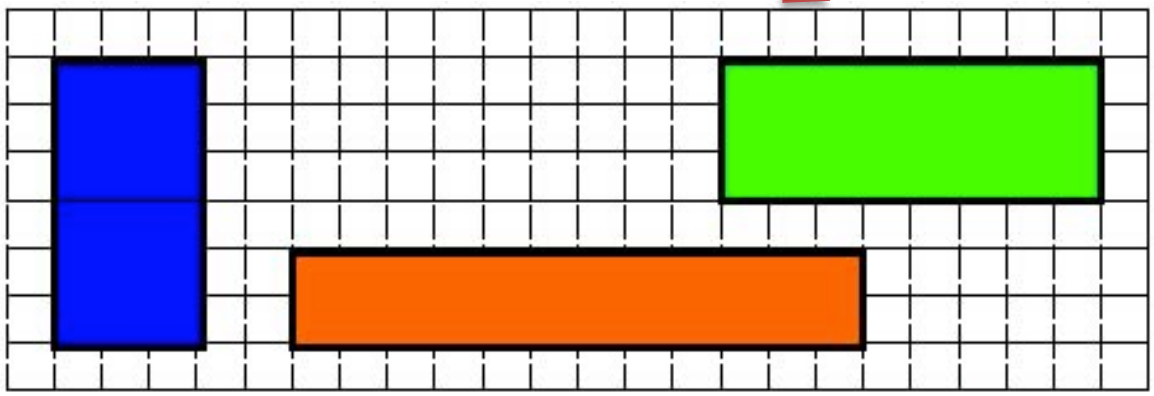


Measurement

Area



Level 2
Compares and orders several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units

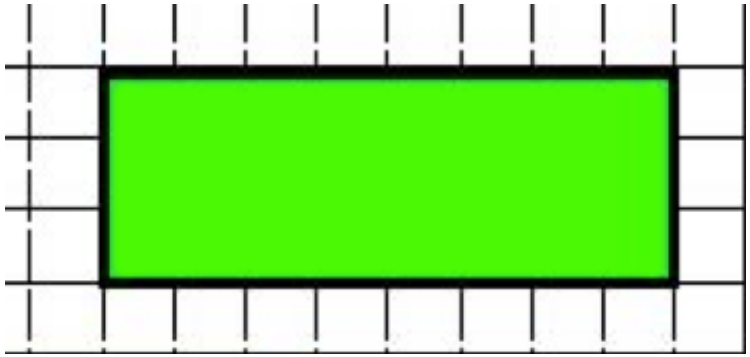


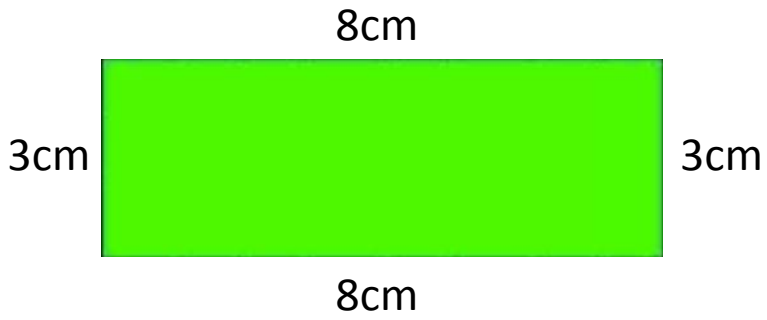
Level 4
Compares objects using familiar metric units of area (grid paper)

Measurement

Area

A 8 cm by 3 cm rectangle contains 8 × 3 = 24 squares, each with an area of 1 square centimetre. So the area of the rectangle is 24 square centimetres, or 24 cm^2





Area = l x w

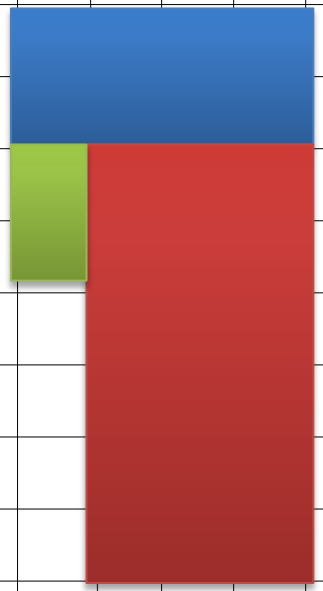
= 8cm x 3cm

= 24 cm^2

Measurement

Area

Multiplication Table grid game



Measurement

Area

GOMGOM

Measurement

Area

R

2 x 5 = 10

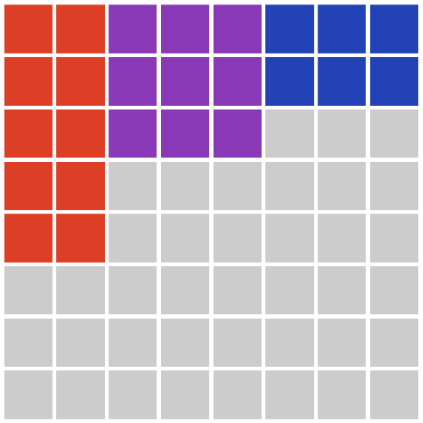
15

3 x 3 = 9

3 x 2 = 6

6

18



20

2 x 2 = 4

2 x 2 = 4

4 x 4 = 16

2 x 2 = 4

4 x 1 = 4

4 x 1 = 4

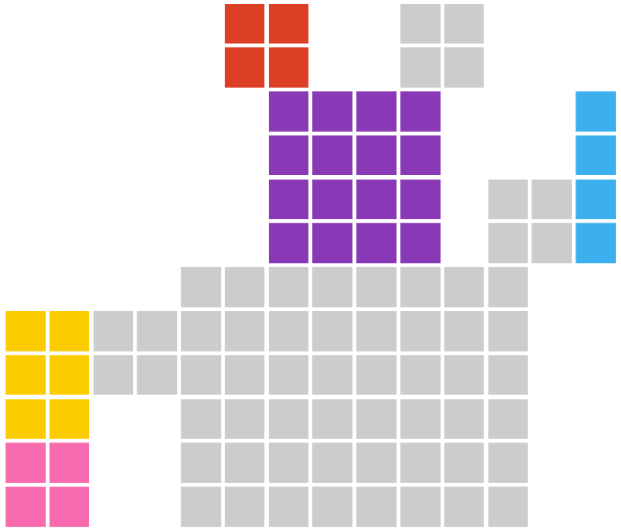
8 x 2 = 16

4 x 2 = 8

3 x 2 = 6

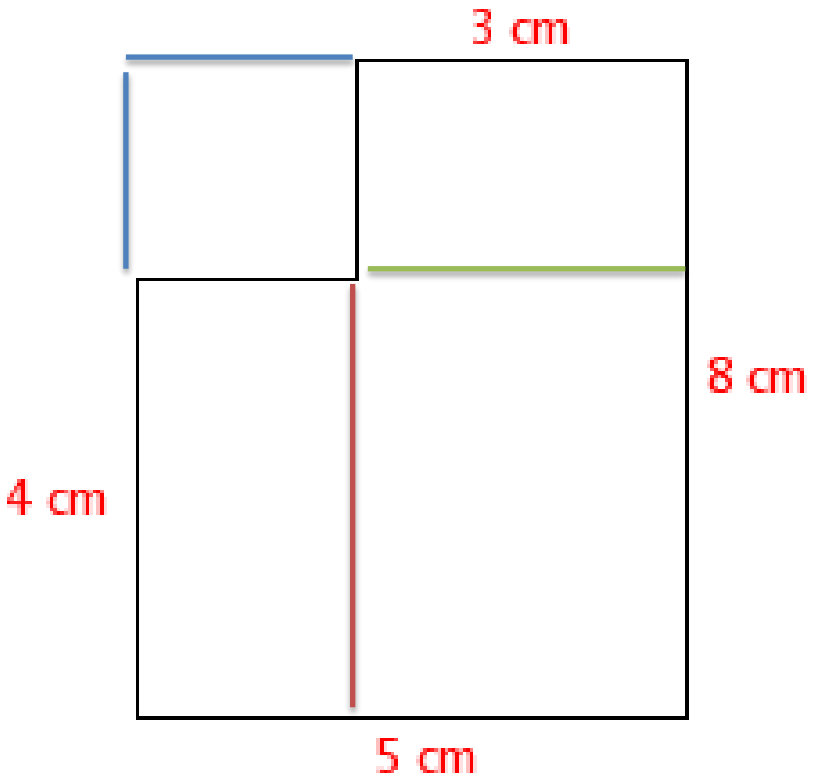
7 x 4 = 28

Stage: 8x



Measurement

Area



Method 1

Area = 4x2 + 3x8

= 8 + 24

= 32 cm^2

Method 2

Area = 3x4 + 4x5

= 12 + 20

= 32 cm^2

Method 3

Area = 5x8 – 4x2

= 40 – 8

= 32cm^2

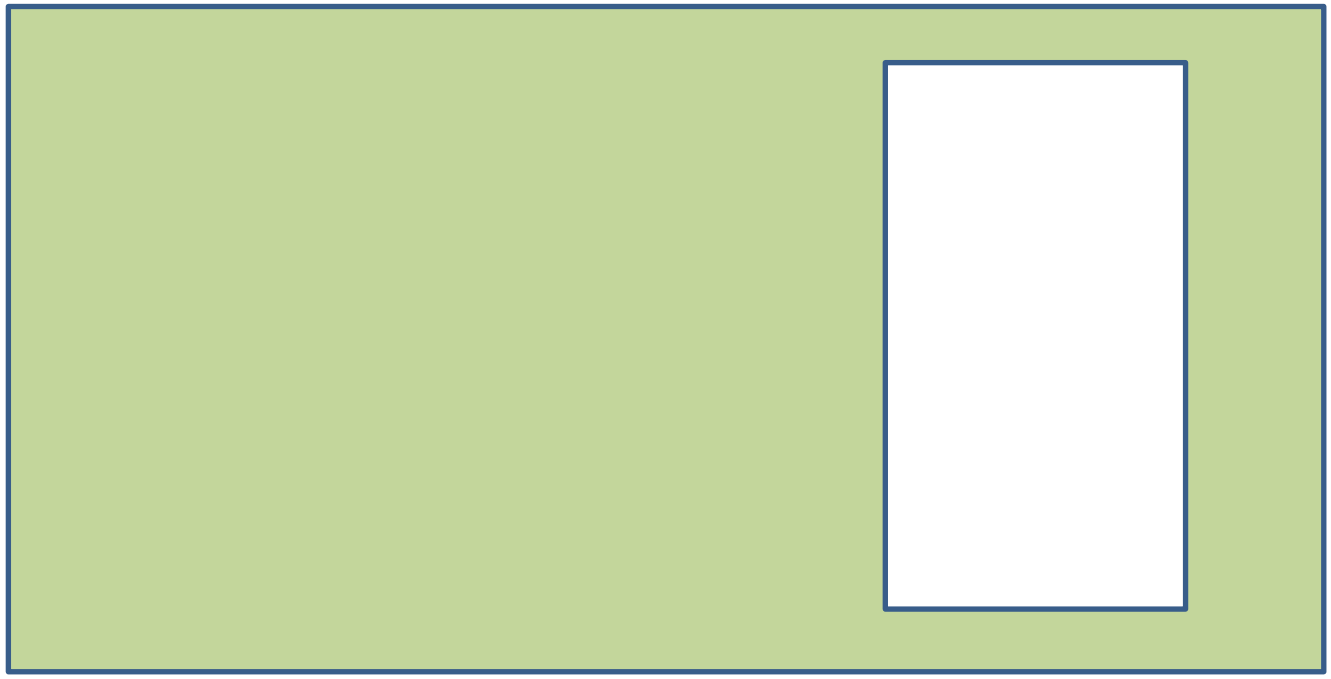
Level 9

Calculates the areas of composite shapes

Measurement

Area

Use of subtraction method



Level 9
Calculates the areas of composite shapes

Measurement

Perimeter and Area Relationship

On the grid paper sketch as many different rectangles you can using 12 squares only

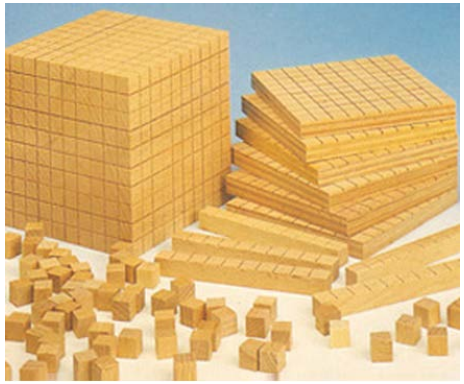
Inside each rectangle write its area and perimeter

What do you notice?

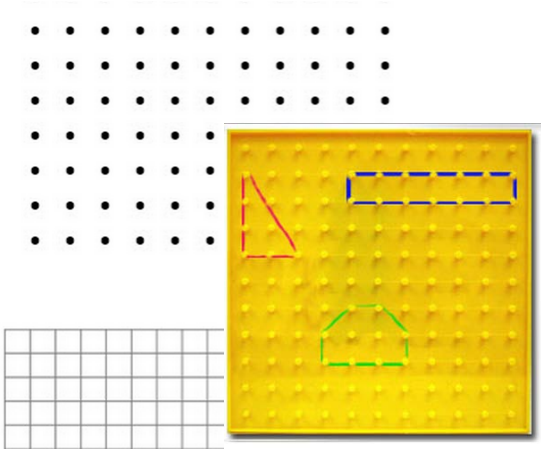
Measurement

Perimeter and Area relationship

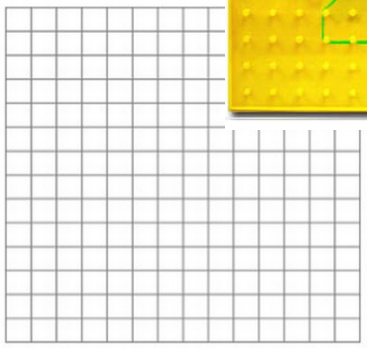
Two shapes with the same perimeter but different areas



Two shapes with the same area but different perimeters



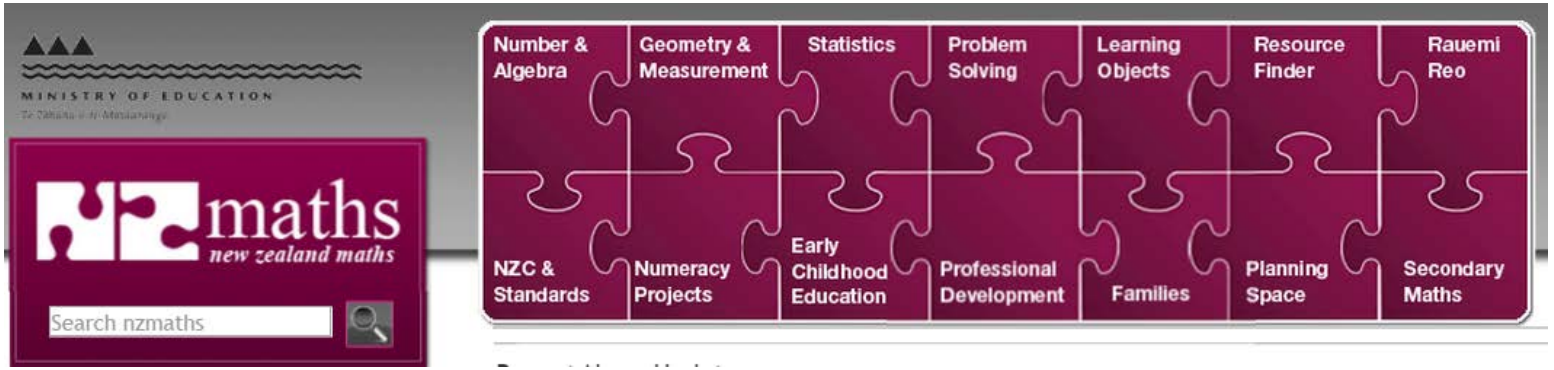
Make a shape – try to change it to a shape that the area decreases but the perimeter increases



Measurement

Problem Solving

The landscape gardeners have thirty-six square paving tiles to make a rest area in the middle of a lawn. To make it easy to mow they want the rest area to be rectangular in shape and have the least perimeter as possible.
How can they arrange the tiles?



http://nzmaths.co.nz/

Measurement

Problem Solving

Level 3 Problems

The problems have been grouped below by Strand.

Geometry

Kitchen Floor
Treasure Map
CopyCats

Measurement

Tims Trip
Adams Watch
ThousandSec
Parking meters

Statistics

Dressing
Grabbing Cds
Coin Shake
Training

Number

Basketball Caps
Pocket Money
Legs in Barn
500 Problem
At the movies
Fathers Day Card
My Dogs
Count to 1000
Super Darts
Darts
Even more pizza
Gulls
The Rock Pool
Lollies, Lollies, Lollies
3c and 5c Stamps
Make 4.253
Invent A Problem
Big Magic Squares
Decimal Magic Squares
Triangle sums

Algebra

Shaking Hands
Toothpick squares
Race to 100
Take Two
Farm Sheep I
Sara's Table
Bill's Number Plate
Multiples of a
Triangular Numbers

Logic and Reasoning

Brian's Pegboard I
No Three in a Line

Measurement

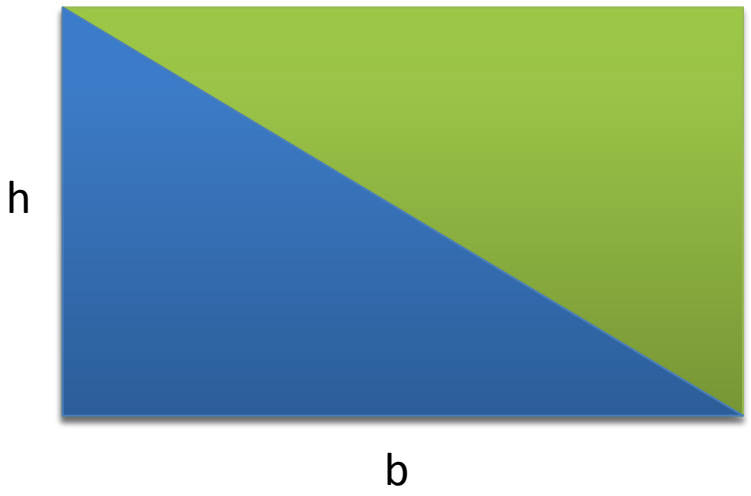
Problem Solving

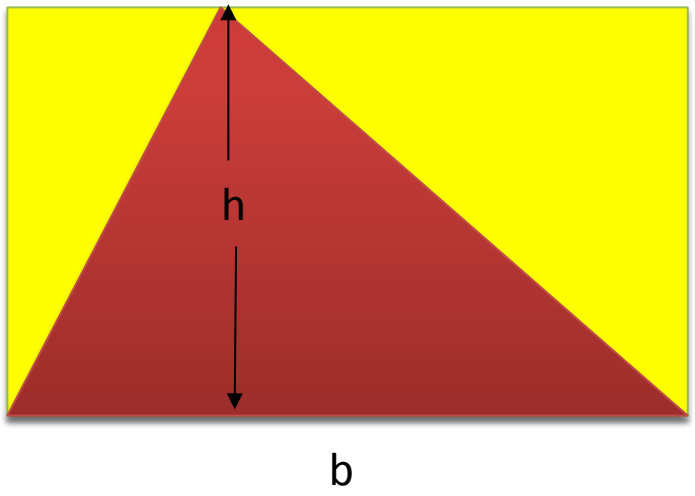
Achievement Objectives	Learning Outcomes	Unit title
GM2-1	• demonstrate a personal benchmark for 1 metre, 1/2 metre • identify and use external benchmarks to carry out practical measuring tasks • discuss the need for having and using standard measures of length • make sensible estimates about the lengths of given objects	Making Benchmarks
	• recognise the need for a standard unit of length • recognise a metre length • estimate and measure to the nearest metre	Pirate Plays
	• carry out practical measuring tasks using appropriate metric units. • make measurement estimates using appropriate metric units • pose measurement questions	Make a measurement trail
	• recognise the need for a standard unit of length • recognise a centimetre length • estimate and measure to the nearest centimetre	All About Me
GM2-1 GM2-2	• estimate using metres and centimetres • measure to the nearest metre and centimetre	Paper Planes L2
	• find objects that they estimate to be a 1cm, 10cm, 50cm and one metre long • measure lengths of approximately one metre to the nearest cm	Scavenger Hunt

Measurement

Triangles

Area of a triangle is $A = \frac{1}{2}bh$





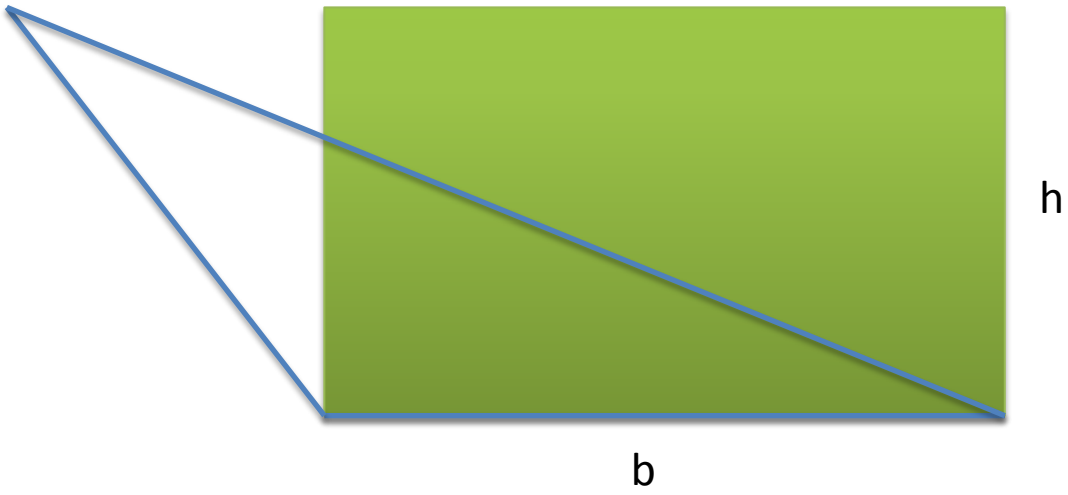
Level 7

Establishes the formulas for areas of rectangles, triangles and parallelograms and uses these in problem solving

Measurement

Triangles

Is the area of this triangle half of the area of this rectangle?



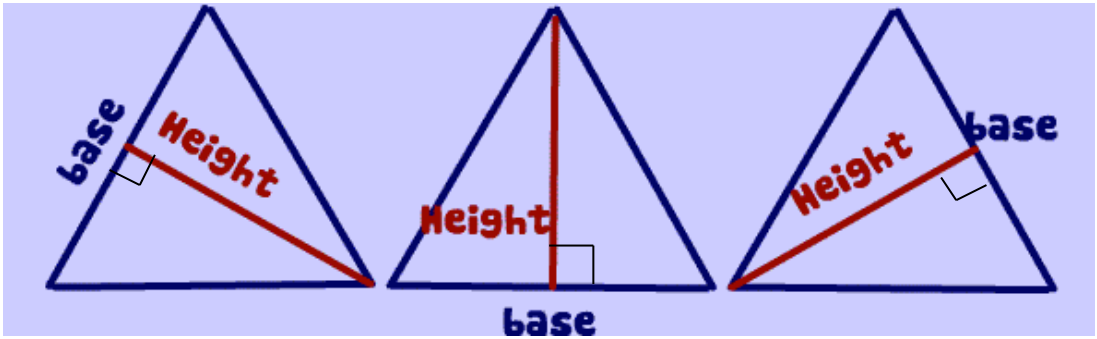
Measurement

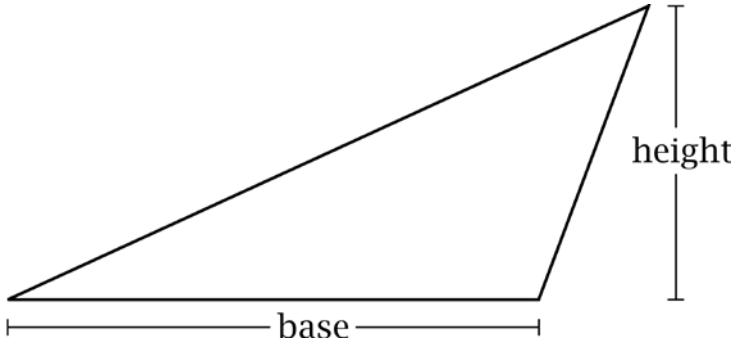
Height and base

Failure to conceptualise the meaning of height and base in 2 dimensional figures

- Ask the question “What happens when we turn the triangle around and thus choose a different height and base?”

The height is always perpendicular (at a right angle) to the base

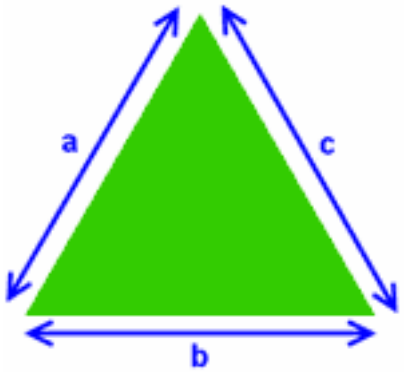




Measurement

Triangles

Finding the area of any triangle when given the lengths of all three of its sides.



Use "Heron's Formula" or sometimes referred to as 'Hero's Formula"

Heron's formula is named after Hero of Alexandria, a Greek Engineer and Mathematician in 10 - 70 AD.

Step 1: Calculate **"s"**

$s = \frac{1}{2}(a + b + c)$ half of the triangles perimeter

Step 2: Then calculate the **Area**:

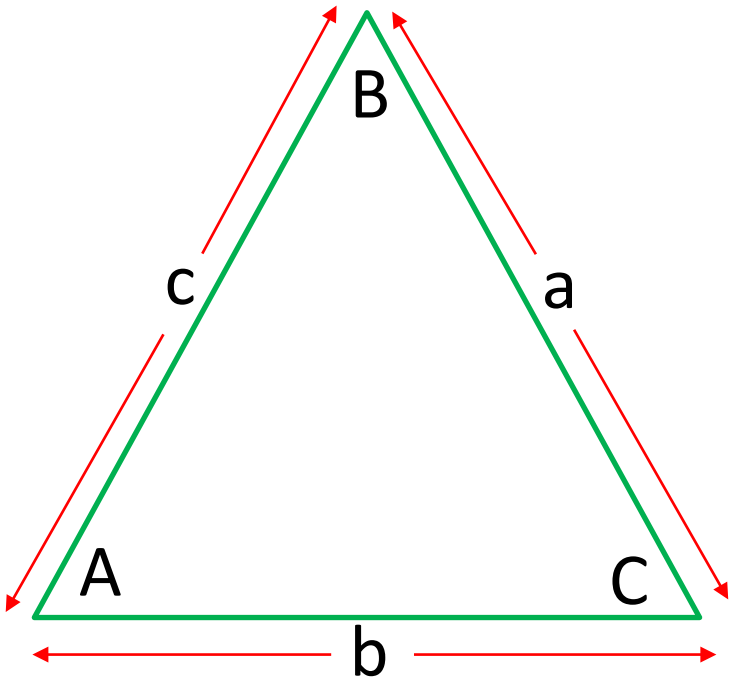
$A = \sqrt{s(s-a)(s-b)(s-c)}$

Level 10A
Establish the sine, cosine and area rules for any triangle and solve related problems

Measurement

Triangles

Finding the area of any triangle when given two sides and the included angle



Area = $\frac{1}{2}$ab sin C

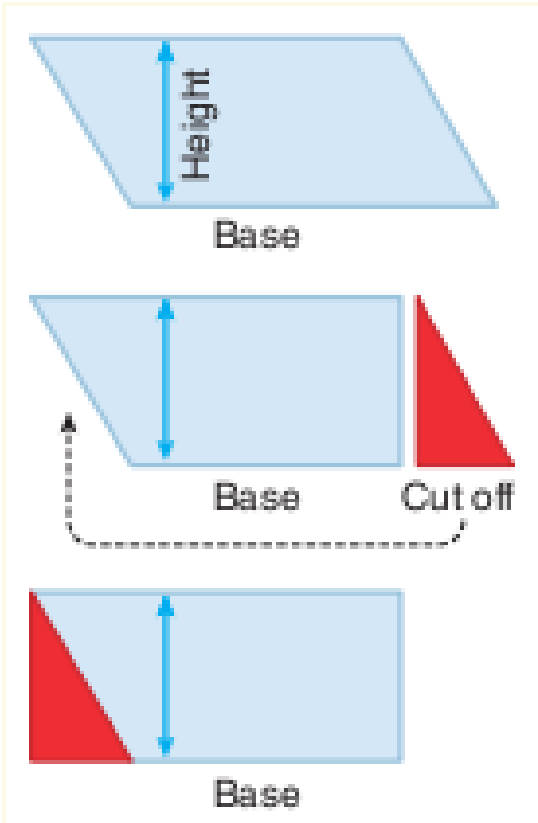
Area = $\frac{1}{2}$ bc sin A

Area = $\frac{1}{2}$ca sin B

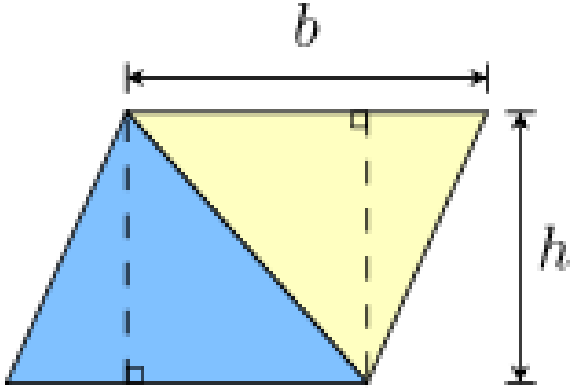
Level 10A
Establish the sine, cosine and area rules for any triangle and solve related problems

Measurement

Parallelogram



Two triangles will always form a parallelogram with the same base and height.

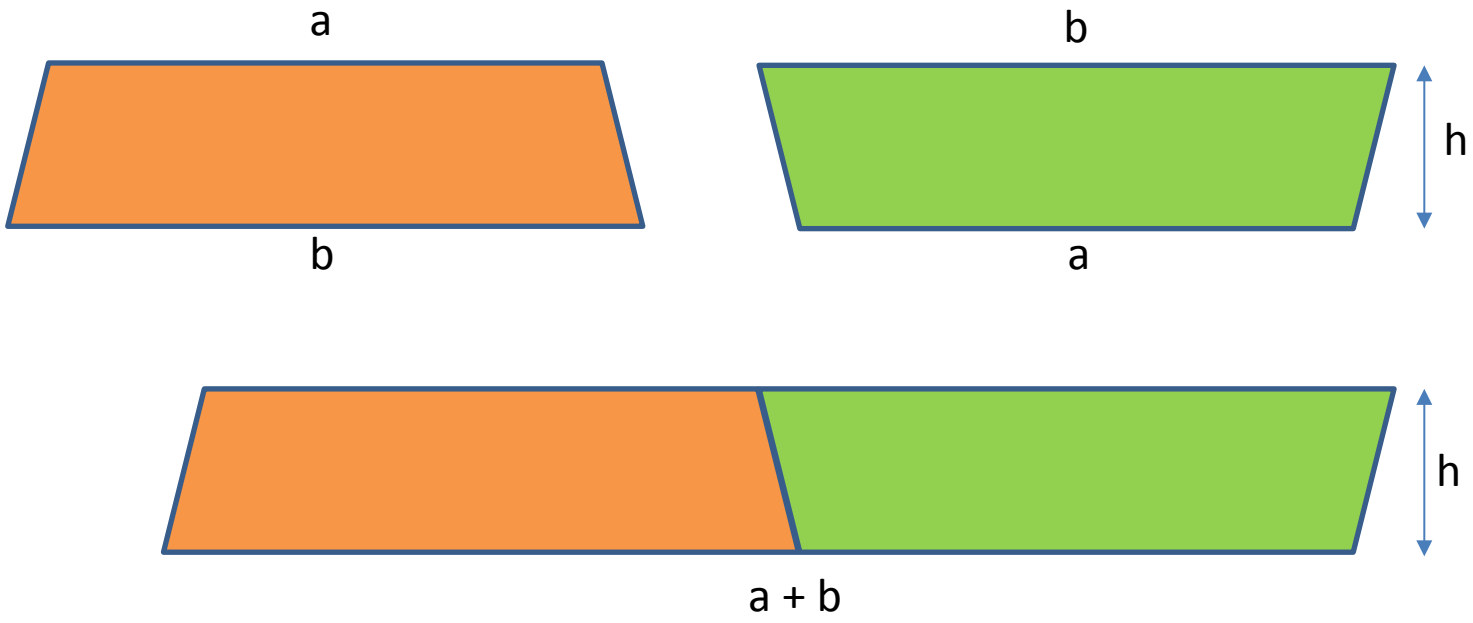


Transforming a parallelogram into a rectangle

Area = bh

Measurement

Trapezium



Two congruent trapezoids always make a parallelogram which helps explain the formula below

$$A = \frac{1}{2} \text{height} \times (a + b)$$

Level 8

Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites

Measurement

Circles

Draw four different size circles and label A, B, C, D
Measure the diameter and circumference for each circle
Fill in the following table

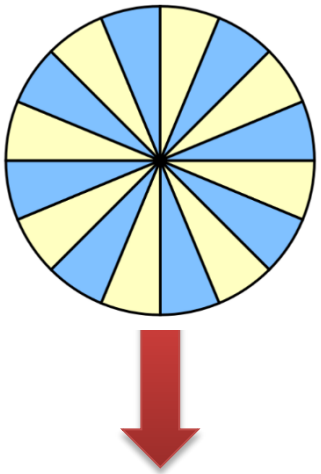
	Diameter (d)	Circumference (C)	$\frac{C}{d}$
A			
B			
C			
D			

What do you notice?

Level 8
Investigates the relationship between features of circles such as circumference, area, radius and diameter. Uses formulas to solve problems involving circumference and area

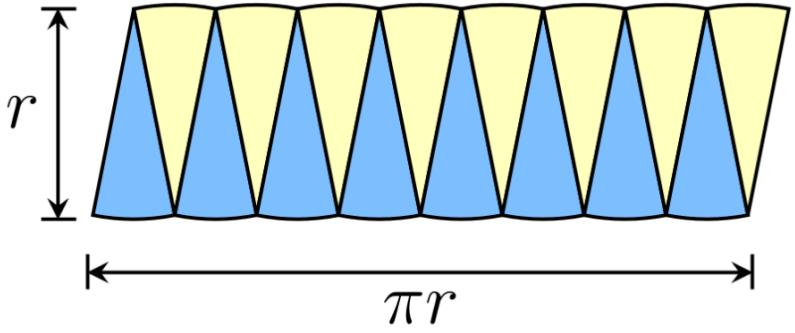
Measurement

Circles



C= 2πr

8 sectors can form a near parallelogram



πr

Area = πr x r

= πr²

Measurements

Capacity and Volume

Capacity is how much the container is able to hold

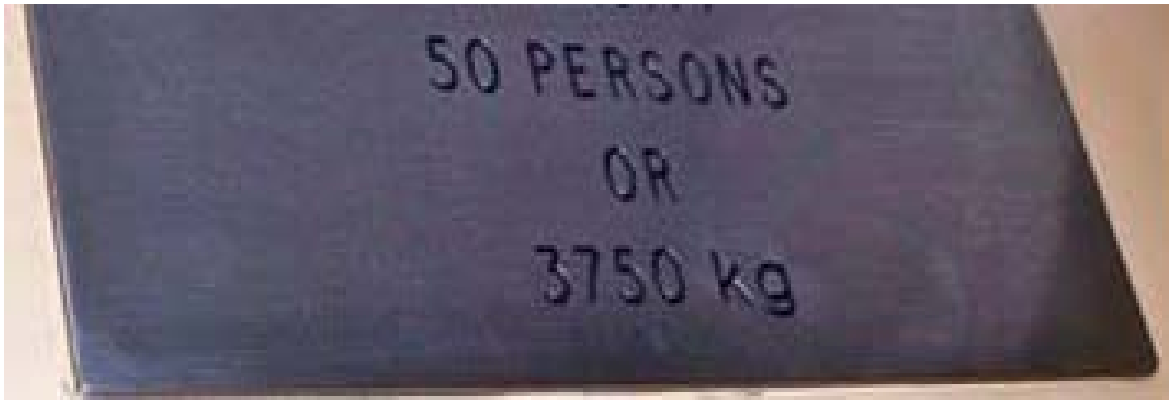
- How much wine can be stored?

Volume is the measure of the space taken up by something (this includes the keg itself)

Level 6
Connects volume and capacity and their units of measurement (e.g. recognise that 1mL is equivalent to 1cm^3)

Measurements

Capacity and Volume



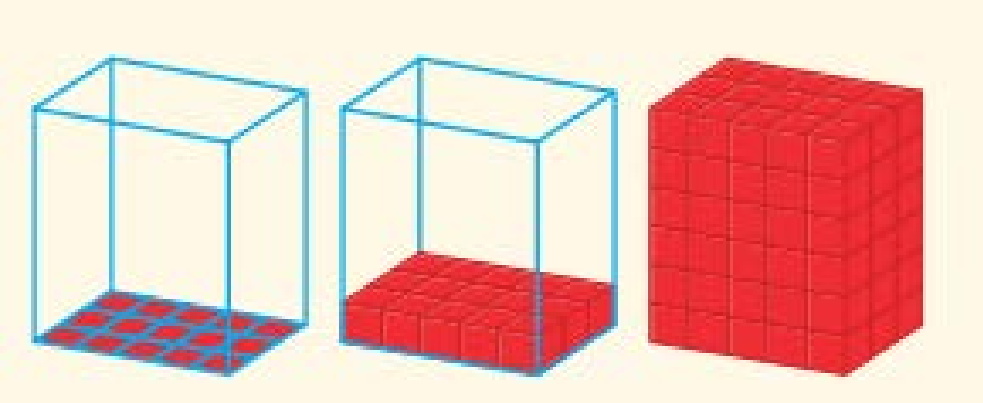
Does this show the volume or capacity of the lift?

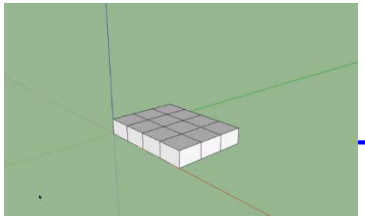
Do you measure the volume or capacity of a brick?

Measurement

Volume

Why is the volume of a prism equal to area of base multiplied by height?





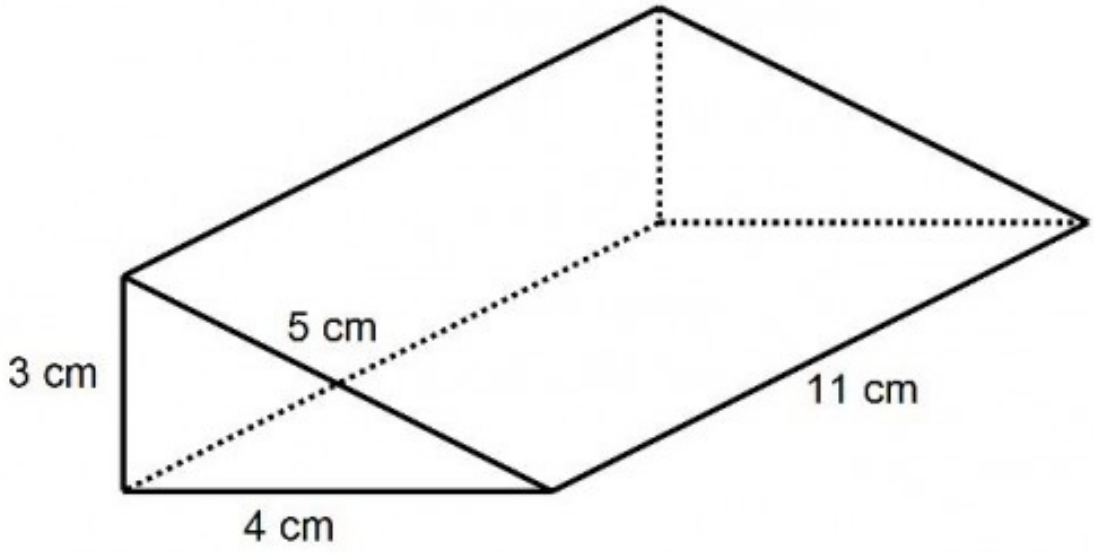
https://www.youtube.com/watch?v=xO-rfvp6uNY

Level 7
Calculates volumes of rectangular prisms

Measurement

Volume

Which is the base?



Level 8
Develops the formulas for volumes of rectangular and triangular prisms and prisms in general. Uses formulas to solve problems involving volume

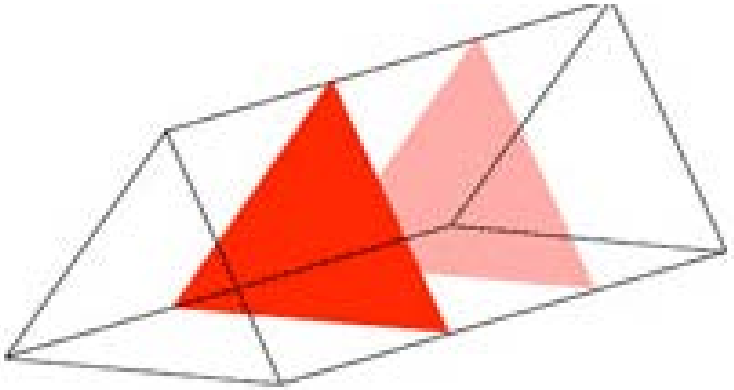
Measurement

Volume

Cross section

If you take a solid and slice it, then the face you create is called a cross-section and the area of the face is called the cross-sectional area.

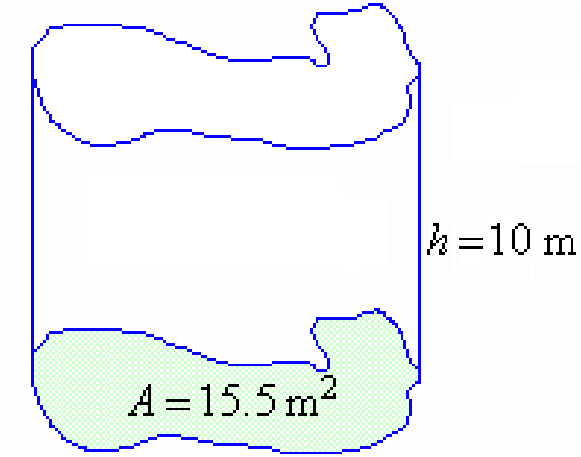
A prism is a solid with **straight** sides which has the same cross-sections.

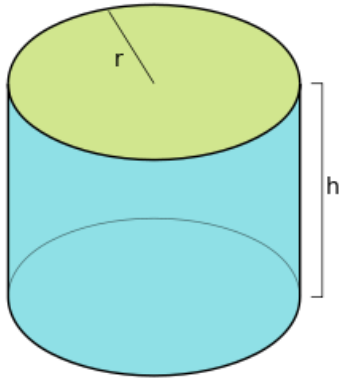


Measurement

Volume

Volume of a prism = Area of base (cross section) x height





Volume = Area of base x height

= 15.5 x 10

= 155 m^3

Volume = Area of base x height

= $\pi r^2 h$

Level 9

Calculates the surface area and volume of cylinders and solves related problems

Measurement

Prism and Pyramid

Investigation

How many times can a pyramid fit into a prism, both with the same base and height?

Volume of a pyramid = $\frac{1}{3}$ Area of base x height

Level 10A

Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids

Measurement

Total Surface Area

The total area of all surfaces of a three-dimensional object

The surface area of a tissue box (rectangular prism) is the area of all 6 faces added together

- concrete materials
- nets
- creation of nets

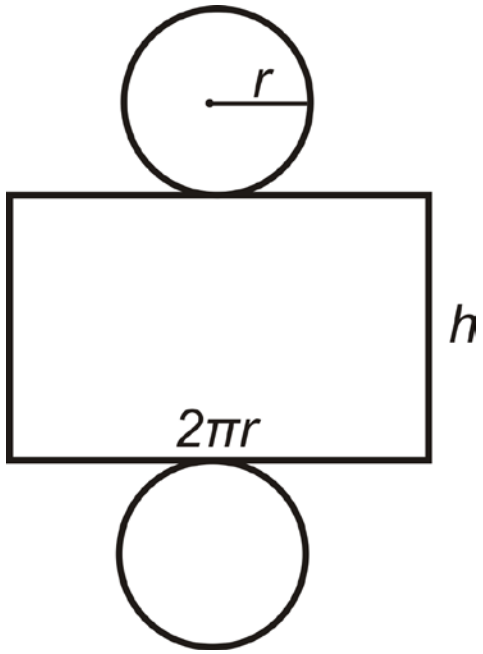
Level 9
Calculates the surface area and volume of cylinders and solves related problems

Measurement

Total Surface Area

Use of nets

What size label do I need for a can of soup?



Level 9 - Calculates the surface area and volume of cylinders and solves related problems

Measurement

Volume and Total Surface Area

Using 12 cubes construct four different arrangements

For each arrangement write the volume and the Total Surface Area

What do you notice?

Level 10
Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids

Measurement

Weight and Mass

Mass - amount of matter in an object

Weight - measure of the pull of gravity on an object

If you were to go to the moon would an object weigh less or more than the same object on Earth?

If you were to go to the moon would an objects mass change?

Measurement

Questions

AMSI

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