QUANTITATIVE PROBLEM-SOLVING

in Applied Sciences, Natural Sciences, Mathematics, and Commerce

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What is “quantitative problem-solving,” anyway?

This is a form of learning based on discovery: to solve the problem, you must both think and compute systematically.

It is different from both “exercise solving”, in which past routines are applied to solve similar problems, and the “trial and error” approach some use to match correct formula to problems.

A central idea in problem solving is the use of “concepts”, the fundamental general ideas on which other notions are built. In any subject, there are usually only a few basic concepts (sometimes expressed as formula) applied in a variety of ways. For example, basic concepts include limit of function in math, t- test in statistics, mole in chemistry, and liability in accounting. Identifying and deeply understanding key concepts, and developing an organizational structure to recall how they inter-relate, are essential to problem solving.

The “spiral of learning” occurs when basic concepts are used repeatedly to solve a variety of problems. The central concept is the core of the spiral, and various applications spin out from, and loop back to, that concept. Frequently re-visiting those basic concepts allows you to firmly fix them in your long-term memory, where they can be quickly recalled and applied.

People learn in different ways, and have different preferred styles of relating to their world, seeking sensory input, making information meaningful, and patterns of learning. It is very helpful to understand your own preferred learning style, and use methods that both mesh with and challenge your style. See the free “Index of Learning Styles” by Felder and Silverman and refer to our Working with Your Preferred Learning Style online module.

Self-reflection questions

Do you:

1. understand your own approach: strengths and weaknesses?
2. focus on concepts to increase understanding, and as an organizational framework?
3. learn material sequentially?
4. look for the “spiral of learning”: repetition and expansion of basic concepts?
5. develop a systematic, methodical approach, to talk yourself through each step?
6. compute accurately, and eventually… quickly
7. persist?
8. get help when needed?

What is YOUR approach to quantitative problem-solving?

Awareness of your attitudes and habits is a good starting point to see your strengths and areas to change. Take our Evidence-Based Components questionnaire to assess your approach.

Characteristics of expert problem-solvers

1. Attitude characteristics

- Optimistic: you believe “I can do it”
- Confident: the problem really does have a reasonable, but perhaps difficult, solution
- Willing to persevere: you aim for a complete and well-reasoned solution, not an immediate or superficial one
- Concern for accuracy in reading: you concentrate, re-read and paraphrase to increase understanding, and translate unfamiliar words or terms
- Concern for accuracy in thinking: you work at a moderate to slow pace initially, perform operations carefully, check answers periodically, and draw conclusions at the end not part way through.

2. Skill characteristics

- Systematic approach: you have a plan to follow, which
    - reduces the panic
    - allows you to monitor your thought processes
    - helps isolate errors in logic or computation
- Sound knowledge of basic concepts, which you mentally organize so you can recall and apply them
- Computational skill, at a good speed
- Habit of vocalizing or “thinking aloud”: you talk yourself through all thoughts
    - how to start the problem
    - steps to break problems into parts
    - decisions
    - analyses
    - conclusions
- Awareness of your own thought processes: What did I do or learn? How did I do or learn this? How effective was my process?

Typical characteristics of novice problem-solvers

1. You don’t believe that persistent analysis is essential, therefore your effort and motivation to persist is weak.

2. You are careless in your reasoning.

3. You don’t break problem into component parts and go step-by-step, therefore there are errors in logic and computation.

4. You focus on individual details, and don’t see how details relate to concepts. Therefore, every problem feels new…how overwhelming!

5. Formula-memorizing is the main strategy.

6. You get behind in your learning, and then sequential learning is hampered.

7. You lose confidence in your ability to solve problems, due to lack of success.

Strategies to improve problem-solving skills

1. Use time and resources effectively

- Work on courses regularly: keep up so you can build on past knowledge (sequential learning), and get help quickly for difficulties.
- Do all the questions assigned, rather than dividing questions among group members, as you will get more practice with the concepts your Professor expects you to know. Aim for accuracy, then speed. Start assignments at least a week ahead of the due date, so you have time for help if needed.
- Use study groups to compare completed solutions to assigned problems. Teaching someone is a very effective learning and study technique.
- Choose problems wisely: learn to apply a specific concept to solve a variety of related problems. Start with simpler ones, and work up. Identify the relevant concept and practice until you know when and how to apply it, i.e. you may not need to do all questions.
- Set a time limit: attempt a new problem every @ 15-20 minutes. If you can't complete a problem, check your "thinking strategies" and change to a new problem. Get help with the problems you couldn't complete, at tutorial, etc.
- Do some uncalculated solutions: If you are confident in your calculations-set up the solution but don't finish the calculation.
- Learn the necessary background and skills: find out from professor, course outline, etc. what the course involves and upgrade before the course begins if you don't feel confident about the prerequisites.
- Find and use help resources: use tutors, professors, TAs, friends, text, internet. For example: in accounting, economics, and finance texts, it is common to find examples that are quite similar to the problems at the end of the chapter. Work through the logic of the examples to develop a better understanding of how best to start the homework problems, if you run into trouble.

2. Develop strategies to organize your thinking

General problem-solving method

Use a methodical, thorough approach to solve problems logically from first principles. Refer to the self-assessment questionnaire by Woods et al. (2000) in this guide to remind yourself of target activities you need to focus on.

Steps:

- Engage with the problem

- Define and understand the problem- what is being asked? Express your thinking in several ways, such as verbally, graphically or pictorially, and finally mathematically
- Explore links between the current problem and related ones you have previously solved.
- Plan how you will solve the problem
- Do it √
- Evaluate your method and result, and revise as needed

Tool: General Problem Solving Strategy, Cognitive vs Metacognitive Questions.

Approaching practice problems for homework

Use homework as a learning tool; the important part isn't to get all the practice problems right (in fact, you probably won't, since it is new material!), but to pay attention to common patterns, themes, and areas where you will need to ask for clarification from the instructor.

Effective learning of the concepts and general methods will reduce the number of problems you may need to solve to feel confident in your knowledge and computations.

Tool: Problem Solving Homework Strategy, Diagnosing the Problem Questions.

Decision steps strategy

This strategy is a specific application of the General Problem Solving Strategy described above, and is suitable for use in statistics, accounting and other applied problem solving situations.

During the lecture or when reading course notes, focus on the process of solving the problem, instead of on the computation. When your professor is lecturing, listen to their comments on how steps are inked from one to another. This helps you identify the "decision steps" that lead to correct application of a concept. Ask yourself "Why did I move from this step to this step?"

Tools: Decisions Steps Strategy, and examples of Decision Steps in Calculus and Decision Steps for Rational Expressions.

View McMaster University's video: click Online Resources, scroll to "Math", select topic and format.

Quantitative concept summary

Concepts are general organizing ideas, are there are often very few of them taught in a course, along with their many applications (ie. the spiral of learning). Key concepts may be identified by:

- reading the learning objectives on the course outline or the course description,
- referring to the lecture outline to identify recurring themes,
- thinking about the common aspects of problems you are solving.

Learn and understand the small amount of information essential to each concept.

If in doubt, ask the professor what is important for you to "get".

Tools: Quantitative Concept Summary Strategy, Concept Summary form, an example of a Concept Summary for Ordinary Simple Annuities.

View a video about Concept Summaries at McMaster University. Click on Online Resources, scroll to "Math," then select desired topic and format.

Range of problems strategy

Exams will challenge you to apply your knowledge to new situations, so prepare by creating questions or problems that are slightly different in some variable from your homework problems.

Actively think about the range of problems that are associated with a concept. Think in terms of both

i. level of difficulty of the problems
ii. common kinds of difficult problems.

Use this to anticipate different kinds of difficult problems for exam preparation, and solve some practice problems to test yourself. This is an excellent activity for a study group.

Tool: Range of Problems Strategy.

View McMaster University's video: click Online Resources, scroll to "Math", select topic and format.

Some evidence-based components of expert problem-solving[1]

Observe yourself as you solve problems. Rate how often you DO any of the following. Progress toward internalizing these targets, aiming for doing these activities 80-100% of the time.

Targets for expert problem-solving	20%	40%	60%	80%	100%
1. I describe my thoughts aloud as I solve the problem.					
2. I occasionally pause and **reflect** about the process and what I have done.					
3. I don't expect my methods for solving problems to work equally well for others.					
4. I write things down to help overcome the storage limitations of short-term memory (where problem-solving takes place).					
5. I focus on accuracy and not on speed.					
6. I interact with others.[2]					
7. I spend time reading the problem.[3]					
8. I spend up to half the available time defining the problem.[4]					
9. When defining problems, I patiently build up a clear picture in my mind of the different parts of the problem and the significance of each part.[5]					
10. I use different tactics when solving exercises and problems.[6]					
11. I use an evidence-based systematic strategy (such as read, define the **stated** problem, explore to identify the **real** problem, plan, do it, look back). I am flexible in my application of the strategy.					
12. I monitor my thought processes about once per minute while solving problems.					

Source: Woods, D.R., Felder, R.M., Rugarcia, A., Stice, J.E. (2000). The Future of Engineering Education III: Developing Critical Skills. *Chemical Engineering Education*, 34 (2), 108-117.

[1] Problem-solving contrasts with exercise-solving. In exercise-solving, the solution methods are quickly apparent because similar problems have been solved in the past.

[2] An important target for team problem-solving

[3] Successful problem-solvers may spend **up to three times longer** than unsuccessful ones in reading problem statements.

[4] **Most mistakes are made in the definition stage**!

[5] The problem that is solved is not the textbook problem. Instead, it is your mental interpretation of that problem.

[6] Some tactics that are ineffective in solving problems include:

- trying to find an equation that includes precisely all the variables given in the problem statement, instead of trying to understand the fundaments needed to solve the problem
- trying to use solutions from past problems even when they don't apply
- trial and error

General problem-solving strategy

A systematic approach to problem solving helps the learner gain confidence, and is used consistently as a "blue print" by expert problem solvers as a way to be methodical, thorough and self-monitoring. This model is used in life generally, as well as in the sciences. The steps are not linear, and multiple processes are happening in your brain simultaneously, but the basic template hinges on effective questioning as you carry out various steps

1. Engage

Invest in the problem through reading about it and listening to the explanation of what is to be resolved. Your goal is to learn as much as you can about the problem before you begin to actually solve it, and to develop your curiosity (which is very motivating). Successful problem solvers spend two to three times longer doing this than unsuccessful problem solvers. Say "I want to solve this, and I can".

2. Define the stated problem…a challenging and time consuming task

- Understand the problem as it is given you, ie. "What am I asked to do?"
- Ask "What are the givens? the situation? the context? the inputs? the knowns? etc.
- Determine the constraints on the inputs, the solution and the process you can use. For example, "you have until the end of class to hand this solution in" is a time constraint.
- Represent your thinking conceptually first, by reading the problem, drawing a pictorial or graphic representation or mind map (see example attached), and then a relational representation.
- Then represent your thinking computationally, using a mathematical statement

3. Explore and search for important links between what you have just defined as a problem, and your past experience with similar problems. You will create a personal mental image, trying to discover the "real" problem. Ultimately, you solve your "best mental representation" of the problem.

- Guestimate an answer or solution, and share your ideas of the problem with others for added perspective.
- Self-monitoring questions include: What is the simplest view? Have I included the pertinent issues? What am I trying to accomplish? Is there more I need to know for an appropriate understanding?

4. Plan in an organized and systematic way

- Map the sub-problems
- List the data to be collected

- Note the hypotheses to be tested
- Self-monitoring questions include: What is the overall plan? Is it well structured? Why have I chosen those steps? Is there anything I don't understand? How can I tell if I'm on the right track?

5. Do it

- Self-monitoring questions include: Am I following my plan, or jumping to conclusions?
- Is this making sense?

6. Look back and revise the plan as needed. Significant learning can occur in this stage, by identifying other problems that use the same concepts (remember the spiral of learning?) and by evaluating your own thinking processes. This builds confidence in your problem solving abilities.

- Self-monitoring questions include: Is the solution reasonable? Is it accurate? (you will need to check your work to know this!) Does the solution answer the problem? How might I do this differently next time? How would I explain this to someone else? What other kinds of problems can I solve now, because of my success? If I was unsuccessful, what did I learn? Where did I go off track?

Based on D.R. Woods, "Problem–based Learning", 1994

Use cognitive and metacognitive questions to help you learn

Effective problem-solving requires thinking about how you think! It's helpful to know the difference between metacognitive strategies (i.e. "thinking about how you best learn mathematical concepts/skills") and cognitive strategies ("interacting with the specific information to understand it"). Next time you start to solve a problem, see if thinking through your responses to these questions can help you focus your efforts.

Metacognitive strategies

Advance organization	What's the purpose in solving this problem? What is the question? What is the information for?
Selective attention	What words or ideas cue the operation or procedure? Where are the data needed to solve the problem?
Organizational planning	What plan will help solve the problem? Is it a multi-step plan?
Self-monitoring	Does the plan seem to be working? Am I getting the answer?
Self-assessment	Did I solve the problem/answer the question? How did I solve it? Is it a good solution? If not, what else could I try?

Cognitive strategies

Elaborating prior knowledge	What do I already know about this topic or type of problem? What experiences have I had that are related to this? How does this information relate to other information?
Taking notes	What's the best way to write down a plan to solve the problem? Table, chart, list, diagram…
Grouping	How can I classify this information? What is the same and what is different (from other problems I have encountered, from other concepts in the class…)
Making inferences	Are there words I don't know that I must understand to solve the problem?
Using images	What can I draw to help me understand and solve the problem? Can I make a mental picture or visualize this problem?

Many students find these types of questions boring or irrelevant and simply want to blast through all the problems, but it's important to remember the actual purpose of solving problems (at least in homework, if not on a test): figuring out and then practicing new and different ways to solve a type of problem. The *process* is what matters, not getting the result as quickly as possible. Focusing on the process helps you to become more accurate and efficient, and it will save you time in the long run.

Approaching practice problems for homework

This strategy encourages a deep understanding of concepts and procedures in calculation. The time you spend on this will reduce the amount of time you may spend in "plug and chug" attempts to do the homework, and reduce the amount of time you will need for studying later on. Remember that the purpose of practice problems is to help you learn, not to get through all of them quickly – it is perfectly normal if you can't get the right answer on the first try. Think of them as experiments!

1. Prepare for the homework questions.
   - review class notes and understand the concepts in the examples. This might take 30 - 45 minutes.
   - write the first line of a sample problem, close the book, and work as far as you can without looking.
   - refer back to notes, and then again attempt sample
   - repeat over again until you can solve the sample problem both accurately and quickly.

   You will have memorized the rules in the process. This might take 1 hour.

2. Start the homework questions. Interrogate your problem solutions: ask questions about the problem and your method of solving it. E.g.
   - What are the givens? Can the givens be classified as Assets, Liabilities, Owner's Equity, Income, Expenses, etc? Is there any Depreciation?
   - What is required?
   - Can I diagram this?
   - What concepts are referred to? Theorems? Operations?
   - Is the problem similar to others I solved/How?
   - What more do I need to understand this?
   - Are there any "tricks" to the question? If so, how do I deal with them?

3. Keep track of problems you have trouble solving, isolate the particular difficulty, and get help to figure it out. Drill these problems until you are both accurate and fast in solving them.

Diagnose the problem and connect it to a misconception

Sooner or later, you will run into a practice problem that stumps you. This is actually a good thing! It allows you to refine your understanding of the material, so you'll be better prepared for the exam. At this point, it's helpful to diagnose *why* you don't understand this problem – what about your thought process isn't working?

Here are steps to follow for diagnosing a misconception:

1. Return to notes and review course material on the topic. Try sketching the overall concept or explaining it to someone else without looking at your notes. Is your sketch or explanation accurate?

2. Review your steps to the question. Look at each step individually: Was this step correct? Why did I do this part? (Think back to your sketch or explanation of the overall concept when trying to answer "why?").

3. When you have found the step where you first made an error, identify exactly why you made the error. Did you not read the question carefully? Did you use incorrect data? Did you misunderstand the purpose of the question? Did you misunderstand the concept?

4. Try to think of other approaches, or find a similar practice problem and see if you can mirror the steps. Ask, "Why is this step correct? How will I modify my Concept Summary, analogy, etc. of the concept in light of this new information?"

Inspired by Chapter 4: Misconceptions as Barriers to Understanding Science from Science Teaching Reconsidered, A Handbook *(1997).*

Try timing yourself for each problem. If you exceed your time limit (20 minutes or so?) for a particular question, do your best to determine what about the problem is troubling you and then bring it to your instructor as soon as possible to talk about it and learn a new approach.

Put a star next to this type of problem and be sure to practice this type again before any tests. This is exactly why practice problems are so helpful!

Decision step strategy: Applying the general method to a specific problem

Taken from: J. Fleet, F. Goodchild, R. Zajchowski, “Learning for Success”, 2006. See R. Zajchowski for a completed example.

Purpose:

To help learners focus on the process of solving problems, rather than on the mechanics of formula and calculations.

The focus is on correct application of concepts to specific situations. This strategy helps you to increase your awareness of the mental steps you make in problem solving, by “forcing” you to articulate your inner dialogue regarding procedure.

Method:

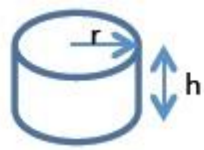
Identify the key decisions that determine what calculations to perform. In lecture, try to record the decision steps the professor uses but may not write down or post.

i. Analyze solved examples, using brief statements focusing on steps you find difficult:
   - What was done in this step?
   - How was it done; what formula or guideline was followed?
   - Why was it done?
   - Any spots or traps to watch out for?

ii. Test run the decision steps on a similar problem, and revise until the steps are complete and accurate.

Example: Decision steps in Calculus for max/min word problems

Problem: A peanuts manufacturer wishes to design a can to hold dry-roasted peanuts. The volume of the cylindrical can is 250 cm^3, and the circular top of the can is made from aluminum while the sides and the bottom are made from stainless steel. If aluminum is twice as expensive as stainless steel, what are the most economical dimensions of the can?

Steps	Solved example
1. Identify Quantity to be maximized/ minimized (Q)	C=Cost per can
2. Diagram when possible (including variables) a. Shapes (perimeter, area, volume) b. Equations to be graphed (axes, levels, distances)	 $V = 250\ cm^3$
3. Make equation for questions using terms from formulas a. Perimeter (P), surface area (S.A.), volume (V) b. Pythagorean relationship c. Sums, differences d. Cost of steel (k), overall cost (C) e. Distance between points - Define variables - Often need to combine equations	$V=\pi r^2 h = 250\ cm^3$ (1) S.A. $= 2(\pi r^2) + 2\pi rh$ (2) = (top+ bottom) + side $C = (2k)\ \pi r^2 + (k)(\ \pi r^2 + 2\pi rh)$ (3) = aluminum top + steel side and bottom
4. Substitute the given volume value into equation (1) to get h(r) and substitute into equation (3) to get C(r).	From (1): $h = \frac{250}{(\pi r^2)}$ From (3): $C = 3k(\pi r^2) + 2k\pi r(\frac{250}{\pi r^2})$ $C = 3k\pi r^2 + \frac{500k}{r}$
5. Set the 1st derivative of overall cost (C) with respect to radius to 0 to find the radius that gives the optimum overall cost	$\frac{dC}{dr} = 6k\pi r - \frac{500k}{r^2} = 0$ Cross out "k" in both terms since it is common in both, rearrange equation, and solve for r: r=2.98 cm and from (1): h=8.947cm
6. Check 2nd derivative to verify the values found for "r" and "h" indeed give a **minimum** cost. (if 2nd derive >0, min. cost is found; if 2nd derive <0, max cost is found)	$\frac{d^2C}{dr^2} = 6k\pi + \frac{500k}{r^3}$ Since the right hand side of the equation can never be negative, r=2.98cm gives the minimum cost.
7. State answer; watch significant figures	The most economical dimensions for the can are r=3.0 cm and h=8.9 cm.

The ‘what’ and the ‘how’

Note that these decision steps try to capture WHAT and especially HOW each step is carried out – including possible alternatives that can be tweaked so that the student is not left wondering how to make the decision needed. Most textbook steps tend to give the WHAT only. For example, these are steps from a calculus textbook:

1. Determine the quantity Q to be maximized or minimized
2. If possible, draw a figure illustrating the problem
3. Write an equation for Q in terms of another variable of the problem
4. Take the derivative of the function in step 3 … etc.

From Washington A.J. (2000). *Basic Technical Mathematics with Calculus* (7th ed.), Addison Wesley Longman.

Decision steps for rational expressions

Math 172. Used with permission.

1. Read question.

2. Make table:
    a. Identify cases (include a third case if total or difference of both cases)
    b. Put equation at top of table
    c. W = r x tor
    d. Total Cost= Cost/person x #of people

3. Fill in columns of table with knowns and unknowns:
    a. Use letters for formulas above for unknowns
    b. If two columns are filled, then do third by algebra
    c. **Watch**! Do previous step carefully!

4. Set up equation:
    a. Sum? Then add rates
    b. Difference? Then subtract rates
        i. **Watch**! Which rate is bigger? Then add to smaller

5. Solve resulting equation for one of the cases

6. Find answer for 'other' case

7. Check by substituting answer into its respective case

8. Write answer in appropriate format

Note:

1. Carefully following these steps should allow you to solve any problem of this kind. If these steps don't quite 'work' adjust them so that they do.
2. As you can see, good decision steps often explain HOW to do a complicated or new step quite carefully. They are much more than just a general approach e.g. "Read question, create table, set up and solve equations"

Good decision steps also can – and should – include some 'watch' steps to remind you to be careful in spots where it is easy to make careless errors.

Quantitative concept summary strategy

Taken from: Fleet, J., Goodchild, F. and Zajchowski, R., “Learning for Success”, 2006

See Camosun College faculty member Zack Zajchowki’s Resources web page for several completed examples.

Purpose

To provide a structure for organizing fundamental, general ideas. The mental work involved in constructing the summary helps clarify the basic ideas and shift the information from working memory to long-term memory. This is an excellent study tool, for quick review.

Method

The organizational elements are

i. Concept Title

You can identify key ideas by referring to the course outline, chapter headings in the text, lecture outline. Sometimes concepts are thought of individually, other times they are meaningfully grouped for better recall. Eg. Depreciation, Capital Cost Allowance, and Half-Year Rule; acid, base and PH.

i. Use **general categories** to organize material, and then add specific details as appropriate. Sample general categories may include:

- Allowable key formula- check summary page of text or ask professor
- Definitions- define every term, unit and symbol
- Additional important information- sign conventions, reference values, meaning of zero values, situations in which formula do not work, etc
- Simple examples or explanations- use your own words, diagrams, or analogies to deepen your thinking and check your understanding
- List of relevant knowns and unknowns- to help you know which concepts are associated with which problems, use crucial knowns to help distinguish among problems.

Quantitative concept summary

Concept Title:

Allowable Key Formula:

Definitions of each symbol, and its units:

Additional important information: (e.g. sign conventions, special characteristics, when concept doesn't work, special cases, etc.)

Simple examples, explanations, cases:

Relevant knowns, and unknowns: (and words/phrases from word problems that signal these)

Example: Concept summary for Ordinary Simple Annuities

Concept title: Ordinary Simple Annuities

1. Key allowable formula(s):
   - Ordinary annuity → payment at **end** of each payment period
   - Simple annuity → interest period = payment period

$$FV = \frac{PMT((1+i)^n - 1)}{i} \qquad PV = \frac{PMT\left(1-(1+i^{-n})\right)}{i}$$

PMT = ... (see formula sheet)
n = ... " "

2. Definition of each new symbol and its units:
   - PV is the present value of the annuity in $
   - FV is the future value of the annuity in $
   - PMT is the regular payment per period in $
   - n is the number of payments made
   - I is the interest rate per payment period

3. Additional important information: (sign conventions, special characteristics, reference & zero values, when concept does not work, special cases, etc.)
   - In problems check interest period – payment period & that payment is at the end of the period; otherwise, formulas need changing!
   - Be careful to find "i" **per period**
   - Watch signs on "n" value ! FV>PV and PMT is small in comparison
   - You **cannot** find "i" with formulas → a calculator is needed
   - Otherwise you can find any of the 4 symbols above if you know 3 others (e.g. find FV knowing PMT, i, n).

4. Simple examples of explanations:

   These formulas can "compress" an annuity into a value – either PV or FV. Mortgage is a simple example: I am loaned some money (PV) to buy a house. I pay "PMT" per month at i% for n payments (usually 25 years, and n = 300). Then I can find FV and I end up paying (knowing) PV, i, n, and I can find PMT.

5. Relevant knowns and unknowns: (words or phrases from word problems that signal these)

- To be an annuity, problem must say “annuity” or “series of payments” or “regular payments”, etc.
- To be an ordinary or a simple annuity, you need to check “pay at the end of the month or quarter”, etc. and that the pay period is the same as the interest period.
- PV is often “loan of…”; “price of…”
- FV is often “accumulates to …” or “how much after…”

Range of problems strategy: Common types of difficult problems

Taken from: J. Fleet, F. Goodchild, R. Zajchowski, *Learning for Success*, 2006

Expand your thinking in preparation for exams, where problems are not exactly the same as you have previously solved. Work from an existing problem, and make it more challenging by adding or changing:

Hidden knowns: needed information is hidden in a phrase or diagram Eg. “at rest” means initial v = 0 in physics.

Multipart-same concept: a problem may comprise 2 or more sub-problems, each involving the same concept. This type of problem can be solved only by identifying the given information in light of these sub-problems

Mulitpart-different concepts: same idea as above, but the sub-problems involve the use of different concepts

Multipart-simultaneous equations: same idea as above, but no single sub-problem can be solved by itself. You may have 2 unknowns and 2 equations or 3 unknowns and 3 equations, and you will need to solve them simultaneously, e.g. using substitution, comparison, addition and subtraction, matrices, etc.

Work backwards: some problems look different because to solve them you have to work in reverse order from problems you have previously solved

Letters only: when known quantities are expressed in letters, problems can look different. If you follow the decision steps, they are not usually as difficult.

“Dummy variables”: sometimes a quantity that you think should be a known is not specified because it is not really needed – that is, it cancels out. E.g. mass in work-energy problems, temperature in gas-law problems.

Red herrings, unnecessary information: a problem may give you more information than is needed, which is confusing if you think you should use everything provided.

Further resources

Online

NC State University's Index of Learning Styles, to assess preferred learning styles, and get additional information on interpretation of your profile.
Link: http://www.engr.ncsu.edu/learningstyles/ilsweb.html

McMaster University's academic resources website. There are 3 videos on Problem Solving illustrating general ideas (Problem Solver I), differences in applying concepts vs. formula chasing (Problem Solver II), and applying the Decision Steps strategy (Problem Solver III).
Link: http://csd.mcmaster.ca/academic

Richard Zajchowski's Resources web page, with examples of completed Concept Summaries, Decision Steps and other strategies.
Link: http://faculty.camosun.ca/zack/resources/

Books

Fleet, J, Goodchild, F, Zajchowski, R *Learning for Success: Effective strategies for students*, Thomson Nelson, 4th ed, 2006

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