### 7.1.3 Geometry of Horizontal Curves

The horizontal curves are, by definition, circular curves of radius R. The elements of a horizontal curve are shown in Figure 7.9 and summarized (with units) in Table 7.2.


Table 7.2 A summary of horizontal curve elements

| Symbol | Name | Units |
| :--- | :--- | :--- |
| PC | Point of curvature, start of horizontal curve |  |
| PT | Point of tangency, end of horizontal curve |  |
| PI | Point of tangent intersection |  |
| D | Degree of curvature | degrees per 100 feet of centerline |
| R | Radius of curve (measured to centerline) | feet |
| L | Length of curve (measured along centerline) | feet |
| $\Delta$ | Central (subtended) angle of curve, PC to PT | degrees |
| T | Tangent length | feet |
| M | Middle ordinate | feet |
| LC | Length of long chord, from PC to PT | feet |
| E | External distance | feet |

The equations 7.8 through 7.13 that apply to the analysis of the curve are given below.

$$
\begin{align*}
& \mathrm{D}=\frac{36,000}{2 \pi \mathrm{R}}=\frac{5729.6}{\mathrm{R}}  \tag{7.8}\\
& \mathrm{~L}=\frac{100 \Delta}{\mathrm{D}} \tag{7.9}
\end{align*}
$$

$$
\begin{align*}
& \mathrm{T}=\mathrm{R} \tan \frac{1}{2} \Delta  \tag{7.10}\\
& \mathrm{M}=\mathrm{R}\left(1-\cos \frac{1}{2} \Delta\right)  \tag{7.11}\\
& \mathrm{LC}=2 \mathrm{R} \sin \frac{1}{2} \Delta  \tag{7.12}\\
& \mathrm{E}=\mathrm{R}\left(\frac{1}{\cos \frac{1}{2} \Delta}-1\right) \tag{7.13}
\end{align*}
$$

## Example 7.5

A 7-degree horizontal curve covers an angle of $63^{\circ} 15^{\prime} 34^{\prime \prime}$. Determine the radius, the length of the curve, and the distance from the circle to the chord M .

## Solution to Example 7.5

Rearranging Equation 7.8, with $\mathrm{D}=7$ degrees, the curve's radius R can be computed. Equation 7.9 allows calculation of the curve's length $L$, once the curve's central angle is converted from $63^{\circ} 15^{\prime} 34^{\prime \prime}$ to 63.2594 degrees. The middle ordinate calculation uses Equation 7.11. These computations are shown below.

$$
\begin{aligned}
& \mathrm{R}=\frac{5729.6}{7}=818.5 \text { feet } \\
& \mathrm{L}=\frac{100 \times 63.2594^{\circ}}{7}=903.7 \text { feet } \\
& \mathrm{M}=818.5^{*}\left(1-\cos 31.6297^{\circ}\right)=121.6 \mathrm{feet}
\end{aligned}
$$

If metric units are used, the definition of the degree of the curve must be carefully examined. Because the definition of the degree of curvature D is the central angle subtended by a 100 -foot arc, then a "metric D " would be the angle subtended by a 30.5 -meter arc. The subtended angle $\Delta$ does not change, but the metric values of $\mathrm{R}, \mathrm{L}$, and M become

$$
\begin{aligned}
& \mathrm{R}=\frac{5729.6}{7 * 3.28}=249.55 \text { meters } \\
& \Delta=63.2594^{\mathrm{O}} ; \frac{1}{2} \Delta=31.6297 \\
& \mathrm{~L}=\frac{100^{*} 63.2594^{\circ}}{7 * 3.28}=275.52 \text { meters } \\
& \mathrm{M}=249.55^{*}\left(1-\cos 31.6297^{\circ}\right)=37.07 \text { meters }
\end{aligned}
$$

