Problem-Solving Strategies: Guess and Check, Work Backward

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CONCEPT **1** Problem-Solving Strategies: Guess and Check, Work Backward

Learning Objectives

- Read and understand given problem situations.
- Develop and use the strategy: guess and check.
- Develop and use the strategy: work backward.
- Plan and compare alternative approaches to solving problems.
- Solve real-world problems using selected strategies as part of a plan.

Introduction

In this chapter, we will continue using our problem solving plan to solve real-world problems. In this section, you will learn about the methods of **Guess and Check** and **Working Backwards.** These are very powerful strategies in problem solving and probably the most commonly used in everyday life. Let's review our problem-solving plan.

Step 1

Understand the problem.

Read the problem carefully. Once the problem is read, list all the components and data that are involved. This is where you will be assigning your variables

Step 2

Devise a plan – Translate

Come up with a way to solve the problem. Set up an equation, draw a diagram, make a chart or construct a table as a start to solving your problem.

Step 3

Carry out the plan – Solve

This is where you solve the equation you came up with in Step 2.

Step 4

Look – Check and Interpret

Check to see if you used all your information and that the answer makes sense.

Let's now apply this plan to a few problems.

Read and Understand Given Problem Situations

The most difficult parts of problem-solving are most often the first two steps in our problem solving plan. First, you need to read the problem and make sure you understand what you are being asked. Then devise a strategy that uses the information you have been given to arrive at a solution.

Let's look at a problem without solving it. We will read through the problem and list the information we have been given and what we are trying to find. We will then try to devise a strategy for solving the problem.

Example 1

A book cost $18 if bought online and $22.50 if bought at the store. The bookstore sold 250 books and took in $4995. How many books were bought online and how many were bought in the store?

Problem set-up:

Step 1

Understand

A book bought online is $18

A book bought at the store is $22.50

The total takings equal $4995

The total number of books sold equals 250

How many books were bought online and how many books were bought in the store?

Step 2

Strategy

Total takings = Total for online sales + Total for in-store sales.

$4995 = $18 (number of books sold online) + $22.50 (number of books sold in-store)

Number of books sold online + Number of books sold in the store = 250 books.

We can guess values for each category and see which of them will give the correct answers.

Develop and Use the Strategy: Guess and Check

The strategy for the method "Guess and Check" is to guess a solution and use the guess in the problem to see if you get the correct answer. If the answer is too big or too small, then make another guess that will get you closer to the goal. You continue guessing until you arrive at the correct solution. The process might sound like a long one, however the guessing process will often lead you to patterns that you can use to make better guesses along the way.

Here is an example of how this strategy is used in practice.

Example 2

Nadia takes a ribbon that is 48 inches long and cuts it in two pieces. One piece is three times as long as the other. How long is each piece?

Solution

Step 1

Understand

We need to find two numbers that add to 48. One number is three times the other number.

Step 2

Strategy

We guess two random numbers, one three times bigger than the other and find the sum.

If the sum is too small we guess larger numbers and if the sum is too large we guess smaller numbers.

Then, we see if any patterns develop from our guesses.

Step 3

Apply Strategy/Solve

Guess	5 and 15	the sum is $5 + 15 = 20$	which is too small
Guess bigger numbers	6 and 18	the sum is $6 + 18 = 24$	which is too small

However, you can see that the answer is exactly half of 48.

Multiply 6 and 18 by two.

Our next guess is	12 and 36	the sum is $12 + 36 = 48$	This is correct.

Answer The pieces are 12 inches and 36 inches long.

Step 4

Check

$12 + 36 = 48$	The ribbon pieces add up to 48 inches.
$36 = 3(12)$	One piece is three times the length of the other piece.

The answer checks out.

Develop and Use the Strategy: Work Backward

The "Work Backward" method works well for problems in which a series of operations is applied to an unknown quantity and you are given the resulting number. The strategy in these problems is to start with the result and apply the operations in reverse order until you find the unknown. Let's see how this method works by solving the following problem.

Example 3

Anne has a certain amount of money in her bank account on Friday morning. During the day she writes a check for $24.50, makes an ATM withdrawal of $80 and deposits a check for $235. At the end of the day she sees that her balance is $451.25. How much money did she have in the bank at the beginning of the day?

Solution:

Step 1

Understand

We need to find the money in Anne's bank account at the beginning of the day on Friday.

She took out $24.50 and $80 and put in $235.

She ended up with $451.25 at the end of the day.

Step 2

Strategy

From the unknown amount we subtract $24.50 and $80 and add $235. We end up with $451.25.

We need to start with the result and apply the operations in reverse.

Step 3

Apply Strategy/Solve

Start with $451.25. Subtract $235 and add $80 and then add $24.50.

$$451.25 - 235 + 80 + 24.50 = 320.75$$

Answer Anne had $320.75 in her account at the beginning of the day on Friday.

Step 4

Check

Anne starts with	$320.75
She writes a check for $24.50	$320.75 − $24.50 = $296.25
She withdraws $80	$296.25 − $80 = $216.25
She deposits $235	$216.25 + $235 = $451.25

The answer checks out.

Plan and Compare Alternative Approaches to Solving Problems

Most word problems can be solved in more than one way. Often one method is more straight forward than others. In this section, you will see how different approaches compare for solving different kinds of problems.

Example 4

Nadia's father is 36. He is 16 years older than four times Nadia's age. How old is Nadia?

Solution

This problem can be solved with either of the strategies you learned in this section. Let's solve the problem using both strategies.

Guess and Check Method

Step 1

Understand

We need to find Nadia's age.

We know that her father is 16 years older than four times her age. Or $4 \times (\text{Nadia}[U+0080][U+0099]\text{s age}) + 16$

We know her father is 36 years old.

Step 2

Strategy

We guess a random number for Nadia's age.

We multiply the number by 4 and add 16 and check to see if the result equals to 36.

If the answer is too small, we guess a larger number and if the answer is too big then we guess a smaller number.

We keep guessing until we get the answer to be 36.

Step 3

Apply strategy/Solve

Guess Nadia's age   10 $4(10) + 16 = 56$ which is too big for her father's age

Guess a smaller number   9 $4(9) + 16 = 52$ which is too big

We notice that when we decreased Nadia's age by one, her father's age decreased by four.

We want the father's age to be 36, which is 16 years smaller than 52.

This means that we should guess Nadia's age to be 4 years younger than 9.

Guess   5 $4(5) + 16 = 36$ This is the right age.

Answer Nadia is 5 years old.

Step 4

Check

Nadia is 5 years old. Her father's age is $4(5) + 16 = 36$. This is correct.

The answer checks out.

Work Backward Method

Step 1

Understand

We need to find Nadia's age.

We know her father is 16 years older than four times her age. Or $4 \times (\text{Nadia's age}) + 16$

We know her father is 36 years old.

Step 2

Strategy

Nadia's father is 36 years old.

To get from Nadia's age to her father's age, we multiply Nadia's age by four and add 16.

Working backwards means we start with the father's age, subtract 16 and divide by 4.

Step 3

Apply Strategy/Solve

Start with the father's age   36

Subtract 16 $36 - 16 = 20$

Divide by 4 $20 \div 4 = 5$

Answer Nadia is 5 years old.

Step 4

Check

Nadia is 5 years old. Her father's age is: $4(5) + 16 = 36$. This is correct.

The answer checks out.

You see that in this problem, the "Work Backward" strategy is more straight-forward than the Guess and Check method. The Work Backward method always works best when we perform a series of operations to get from an unknown number to a known result. In the next chapter, you will learn algebra methods for solving equations that are based on the Work Backward method.

Solve Real-World Problems Using Selected Strategies as Part of a Plan

Example 6

Nadia rents a car for a day. Her car rental company charges $50 per day and $0.40 per mile. Peter rents a car from a different company that charges $70 per day and $0.30 per mile. How many miles do they have to drive before Nadia and Peter pay the same price for the rental for the same number of miles?

Solution Let's use the Guess and Check method.

Step 1

Understand

Nadia's car rental costs $50 plus $0.40 per mile.

Peter's car rental costs $70 plus $0.30 per mile.

We want to know how many miles they have to drive to pay the same price of the rental for the same number of miles.

Step 2

Strategy

Nadia's total cost is $50 plus $0.40 times the number of miles.

Peter's total cost is $70 plus $0.30 times the number of miles.

Guess the number of miles and use this guess to calculate Nadia's and Peter's total cost.

Keep guessing until their total cost is the same.

Step 3

Apply Strategy/Solve

Guess	50 *miles*		
Check	$50 + \$0.40(50) = \$70$	$70 + \$0.30(50) = \$85$	too small
Guess	60 *miles*		
Check	$50 + \$0.40(60) = \$74$	$70 + \$0.30(60) = \$88$	too small

Notice that for an increase of 10 miles, the difference between total costs fell from $15 to $14.

To get the difference to zero, we should try increasing the mileage by 140 miles.

Guess	200 *miles*		
Check	$50 + \$0.40(200) = \$130$	$70 + \$0.30(200) = \$130$	correct

Answer: Nadia and Peter each have to drive 200 miles to pay the same total cost for the rental.

Step 4

Check

Nadia	$50 + \$0.40(200) = \$130$	Peter	$70 + \$0.30(200) = \$130$

The answer checks out.

Lesson Summary

The four steps of the **problem solving plan** are:

- **Understand the problem**
- **Devise a plan – Translate**
- **Carry out the plan – Solve**
- **Look – Check and Interpret**

Two common problem solving strategies are:

- **Guess and Check**

Guess a solution and use the guess in the problem to see if you get the correct answer. If the answer is too big or too small, then make another guess that will get you closer to the goal.

- **Work Backward**

This method works well for problems in which a series of operations is applied to an unknown quantity and you are given the resulting number. Start with the result and apply the operations in reverse order until you find the unknown.

Review Questions

1. Finish the problem we started in Example 1.
2. Nadia is at home and Peter is at school which is 6 miles away from home. They start traveling towards each other at the same time. Nadia is walking at 3.5 miles per hour and Peter is skateboarding at 6 miles per hour. When will they meet and how far from home is their meeting place?
3. Peter bought several notebooks at Staples for $2.25 each and he bought a few more notebooks at Rite-Aid for $2 each. He spent the same amount of money in both places and he bought 17 notebooks in total. How many notebooks did Peter buy in each store?
4. Andrew took a handful of change out of his pocket and noticed that he was only holding dimes and quarters in his hand. He counted that he had 22 coins that amounted to $4. How many quarters and how many dimes does Andrew have?
5. Anne wants to put a fence around her rose bed that is one and a half times as long as it is wide. She uses 50 feet of fencing. What are the dimensions of the garden?
6. Peter is outside looking at the pigs and chickens in the yard. Nadia is indoors and cannot see the animals. Peter gives her a puzzle He tells her that he counts 13 heads and 36 feet and asks her how many pigs and how many chickens are in the yard. Help Nadia find the answer.
7. Andrew invests $8000 in two types of accounts. A savings account that pays 5.25% interest per year and a more risky account that pays 9% interest per year. At the end of the year he has $450 in interest from the two accounts. Find the amount of money invested in each account.
8. There is a bowl of candy sitting on our kitchen table. This morning Nadia takes one-sixth of the candy. Later that morning Peter takes one-fourth of the candy that's left. This afternoon, Andrew takes one-fifth of what's left in the bowl and finally Anne takes one-third of what is left in the bowl. If there are 16 candies left in the bowl at the end of the day, how much candy was there at the beginning of the day?
9. Nadia can completely mow the lawn by herself in 30 minutes. Peter can completely mow the lawn by himself in 45 minutes. How long does it take both of them to mow the lawn together?

Review Answers

1. 140 online sales and 110 in-store sales.
2. 37.9 minutes 2.2 miles from home
3. 8 notebooks at Staples and 9 notebooks at Rite-Aid
4. 12 quarters and 10 dimes
5. 10 feet wide and 15 feet long
6. 5 pigs and 8 chickens
7. $7200 in the savings account and $800 in the high-risk account
8. 48 candies
9. 18 minutes

Texas Instruments Resources

In the CK-12 Texas Instruments Algebra I FlexBook, there are graphing calculator activities designed to supplement the objectives for some of the lessons in this chapter. See http://www.ck12.org/flexr/chapter/9612.