Managing Interest Rate Risk(II): Duration GAP and Economic Value of Equity

Pricing Fixed-Income Securities and Duration

The Relationship Between Interest Rates and Option-Free Bond Prices

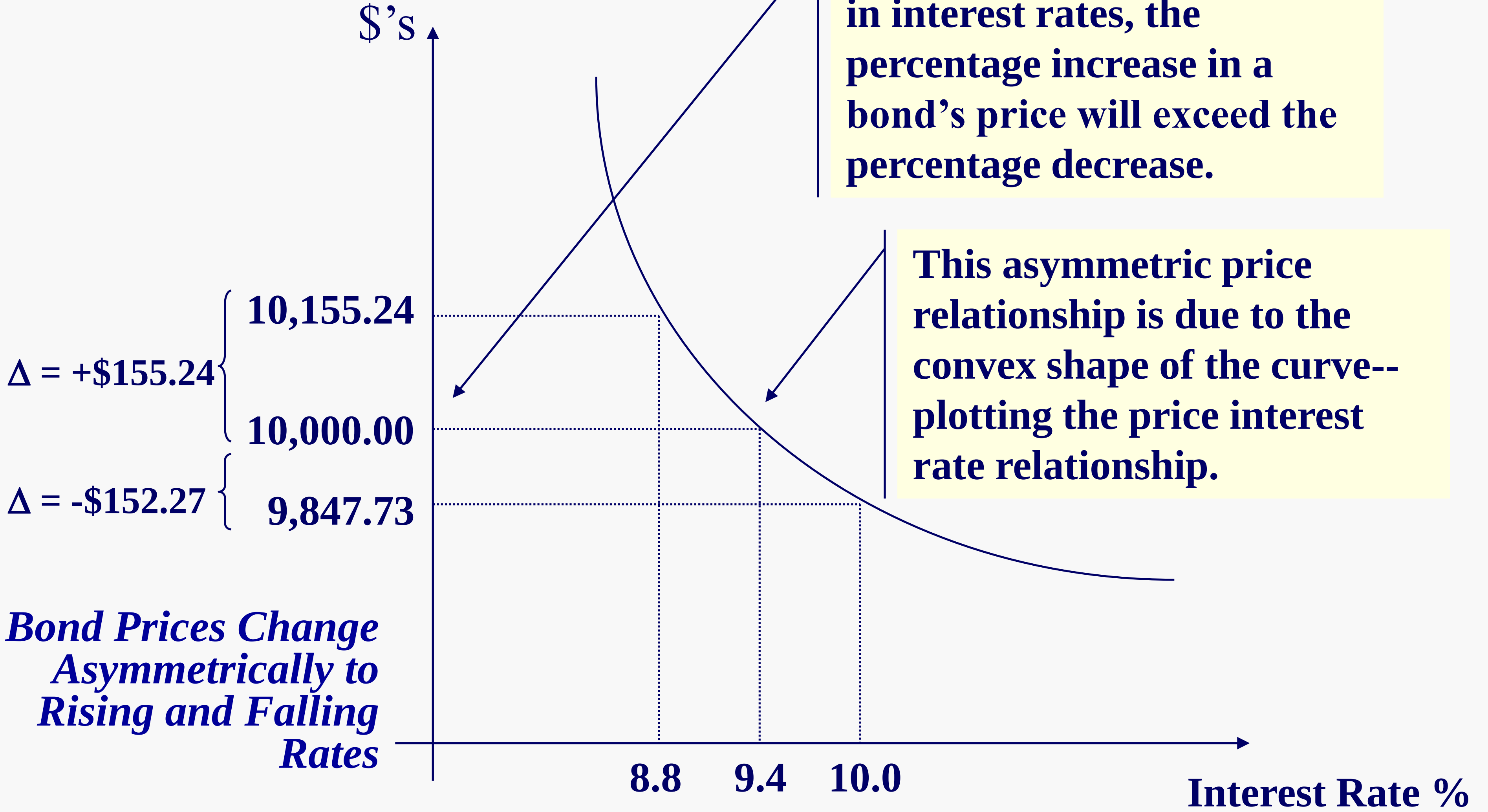
- Bond Prices
  - A bond's price is the present value of the future coupon payments (CPN) plus the present value of the face (par) value (FV)

$$\text{Price} = \frac{CPN_1}{(1+r)^1} + \frac{CPN_2}{(1+r)^2} + \frac{CPN_3}{(1+r)^3} + \ldots + \frac{CPN_n + FV}{(1+r)^n}$$

$$\text{Price} = \sum_{t=1}^{n} \frac{CPN_t}{(1+i)^t} + \frac{FV}{(1+i)^n}$$

- Bond Prices and Interest Rates are Inversely Related
  - Par Bond
    - Yield to maturity = coupon rate
  - Discount Bond
    - Yield to maturity > coupon rate
  - Premium Bond
    - Yield to maturity < coupon rate

Relationship between price and interest rate on a 3-year, $10,000 option-free par value bond that pays $470 in semiannual interest



For a given absolute change in interest rates, the percentage increase in a bond's price will exceed the percentage decrease.

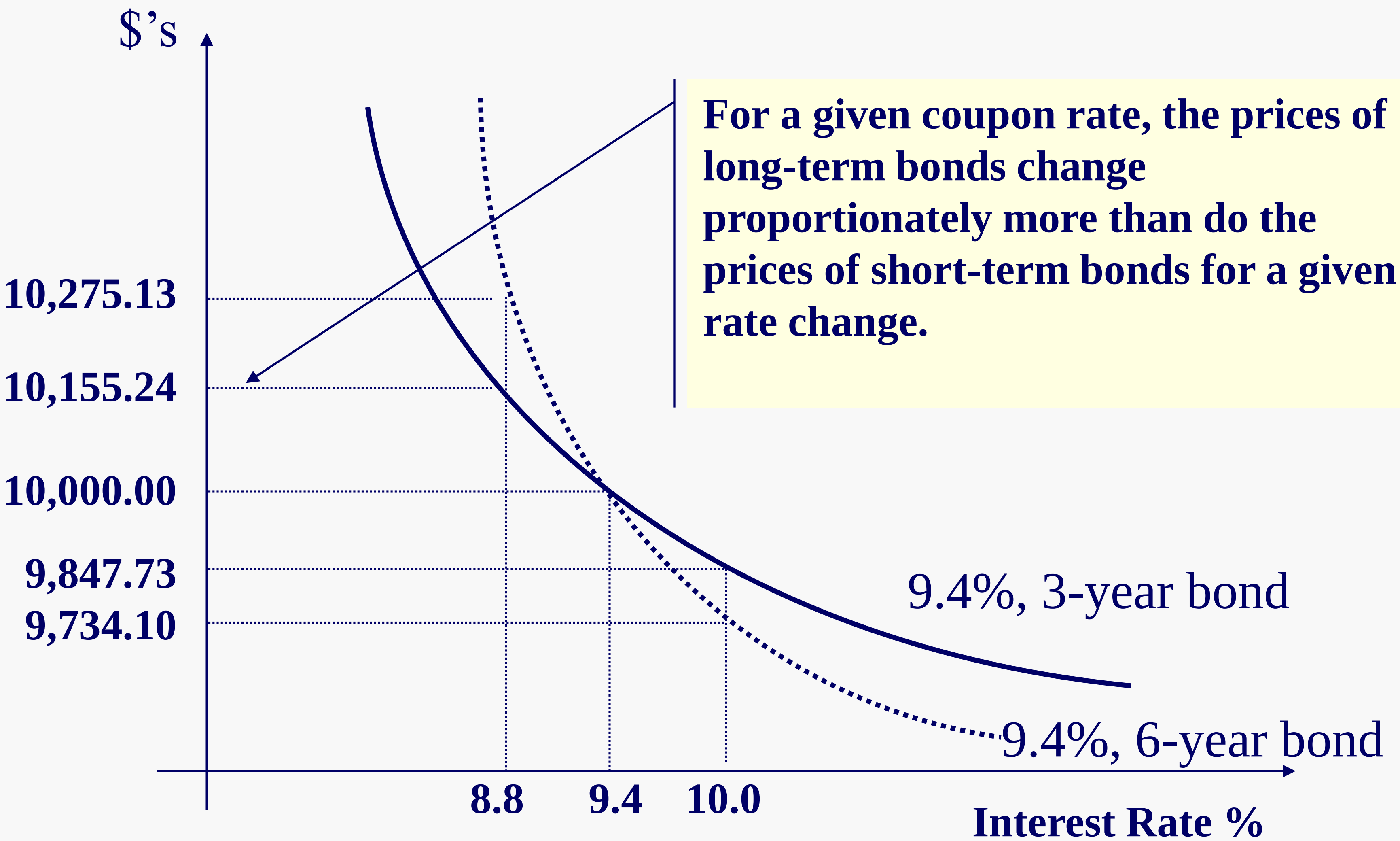
This asymmetric price relationship is due to the convex shape of the curve--plotting the price interest rate relationship.

Bond Prices Change Asymmetrically to Rising and Falling Rates

The Relationship Between Interest Rates and Option-Free Bond Prices

- Maturity Influences Bond Price Sensitivity
  - For bonds that pay the same coupon rate, long-term bonds change proportionally more in price than do short-term bonds for a given rate change.

The effect of maturity on the relationship between price and interest rate on fixed-income, option free bonds

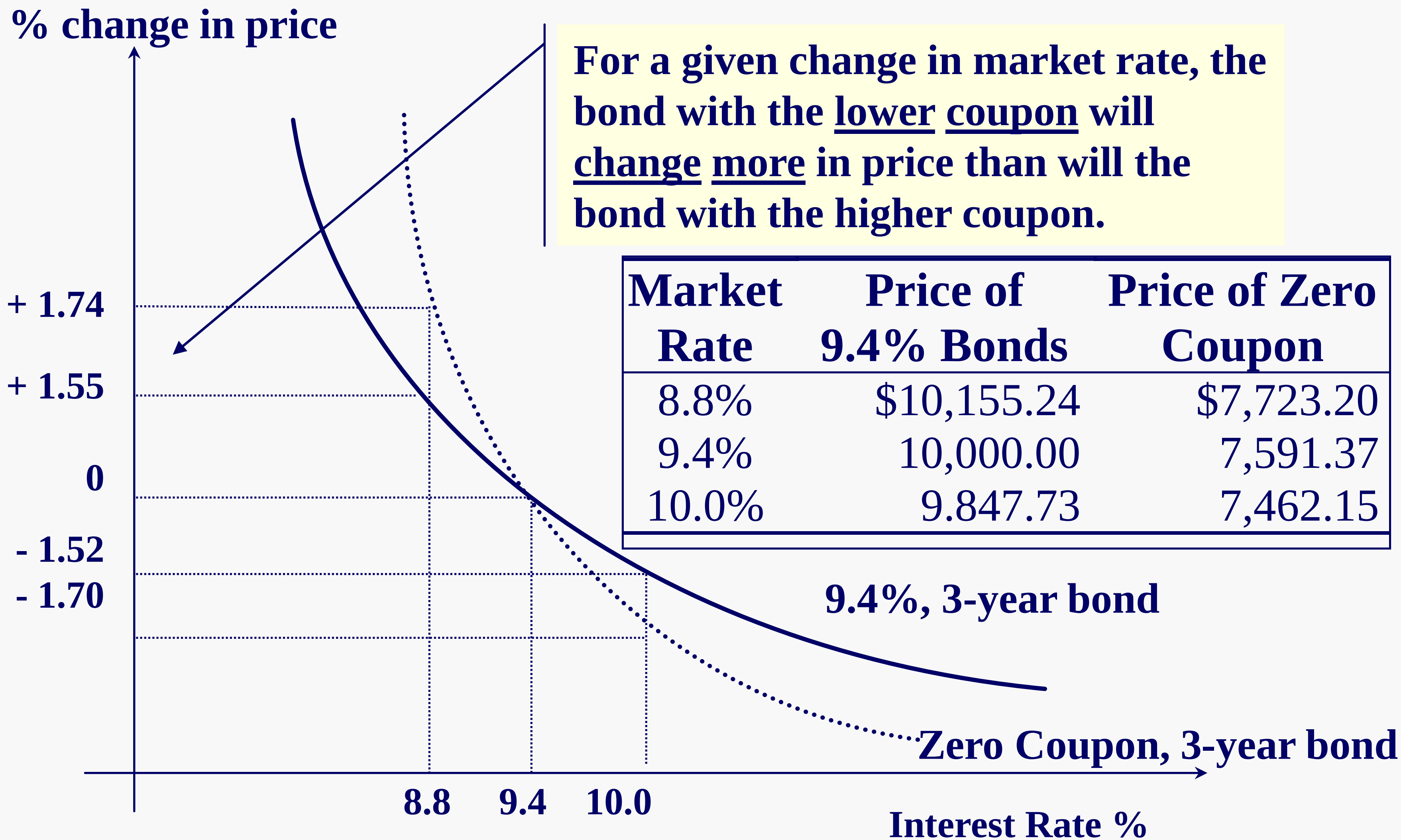


For a given coupon rate, the prices of long-term bonds change proportionately more than do the prices of short-term bonds for a given rate change.

The effect of coupon on the relationship between price and interest rate on fixed-income, option free bonds

For a given change in market rate, the bond with the lower coupon will change more in price than will the bond with the higher coupon.

Market Rate	Price of 9.4% Bonds	Price of Zero Coupon
8.8%	$10,155.24	$7,723.20
9.4%	10,000.00	7,591.37
10.0%	9.847.73	7,462.15



Duration and Price Volatility

- **Duration as an Elasticity Measure**
  - **Maturity simply identifies how much time elapses until final payment.**
  - **It ignores all information about the timing and magnitude of interim payments.**
  - **Duration is a measure of the *effective* maturity of a security.**
    - **Duration incorporates the timing and size of a security's cash flows.**
    - **Duration measures how price sensitive a security is to changes in interest rates.**
      - **The greater (shorter) the duration, the greater (lesser) the price sensitivity.**

Duration and Price Volatility

- **Duration as an Elasticity Measure**
  - **Duration is an approximate measure of the price elasticity of demand**

$$\text{Price Elasticity of Demand} = -\frac{\%\text{ Change in Quantity Demanded}}{\%\text{ Change in Price}}$$

Duration and Price Volatility

- **Duration as an Elasticity Measure**
  - **The longer the duration, the larger the change in price for a given change in interest rates.**

$$\text{Duration} \cong -\frac{\dfrac{\Delta P}{P}}{\dfrac{\Delta i}{(1+i)}}$$

$$\Delta P \cong -\text{Duration}\left[\frac{\Delta i}{(1+i)}\right]P$$

Duration and Price Volatility

- **Measuring Duration**
  - **Duration is a weighted average of the time until the expected cash flows from a security will be received, relative to the security's price**
- **Macaulay's Duration**

$$D = \frac{\displaystyle\sum_{t=1}^{k} \frac{CF_t(t)}{(1+r)^t}}{\displaystyle\sum_{t=1}^{k} \frac{CF_t}{(1+r)^t}} = \frac{\displaystyle\sum_{t=1}^{n} \frac{CF_t(t)}{(1+r)^t}}{\text{Price of the Security}}$$

Duration and Price Volatility

- **Measuring Duration**
  - **Example**
    - **What is the duration of a bond with a \$1,000 face value, 10% coupon, 3 years to maturity and a 12% YTM?**

$$D = \frac{\dfrac{100\times 1}{(1.12)^1} + \dfrac{100\times 2}{(1.12)^2} + \dfrac{100\times 3}{(1.12)^3} + \dfrac{1{,}000\times 3}{(1.12)^3}}{\sum_{t=1}^{3}\dfrac{100}{(1.12)^t} + \dfrac{1000}{(1.12)^3}} = \frac{2{,}597.6}{951.96} = 2.73 \text{ years}$$

Duration and Price Volatility

- **Measuring Duration**
  - **Example**
    - **What is the duration of a bond with a \$1,000 face value, 10% coupon, 3 years to maturity but the YTM is 5%?**

$$D = \frac{\frac{100*1}{(1.05)^1} + \frac{100*2}{(1.05)^2} + \frac{100*3}{(1.05)^3} + \frac{1,000*3}{(1.05)^3}}{1136.16} = \frac{3,127.31}{1,136.16} = 2.75 \text{ years}$$

Duration and Price Volatility

- **Measuring Duration**
  - **Example**
    - **What is the duration of a bond with a $1,000 face value, 10% coupon, 3 years to maturity but the YTM is 20%?**

$$D = \frac{\frac{100*1}{(1.20)^1} + \frac{100*2}{(1.20)^2} + \frac{100*3}{(1.20)^3} + \frac{1{,}000*3}{(1.20)^3}}{789.35} = \frac{2{,}131.95}{789.35} = 2.68 \text{ years}$$

Duration and Price Volatility

- Measuring Duration
  - Example
    - What is the duration of a zero coupon bond with a $1,000 face value, 3 years to maturity but the YTM is 12%?

$$D = \frac{\dfrac{1{,}000 * 3}{(1.12)^3}}{\dfrac{1{,}000}{(1.12)^3}} = \frac{2{,}135.34}{711.78} = 3 \text{ years}$$

    - By definition, the duration of a zero coupon bond is equal to its maturity

Duration and Price Volatility

- **Comparing Price Sensitivity**
  - **The greater the duration, the greater the price sensitivity**

$$\frac{\Delta P}{P} \cong -\left[\frac{\text{Macaulay's Duration}}{(1+i)}\right]\Delta i$$

$$\text{Modified Duration} = \frac{\text{Macaulay's Duration}}{(1+i)}$$

Duration and Price Volatility

- **Comparing Price Sensitivity**
  - **With Modified Duration, we have an estimate of price volatility:**

$$\%\,\text{Change in Price} = \frac{\Delta P}{P} \cong -\,\text{Modified Duration} * \Delta i$$

Measuring Interest Rate Risk with Duration GAP

- Economic Value of Equity Analysis
  - Focuses on changes in stockholders' equity given potential changes in interest rates
- Duration GAP Analysis
  - Compares the price sensitivity of a bank's total assets with the price sensitivity of its total liabilities to assess the impact of potential changes in interest rates on stockholders' equity.

Duration GAP

- Duration GAP Model
  - Focuses on managing the market value of stockholders' equity
    - The bank can protect EITHER the market value of equity or net interest income, but not both
    - Duration GAP analysis emphasizes the impact on equity
  - Compares the duration of a bank's assets with the duration of the bank's liabilities and examines how the economic value stockholders' equity will change when interest rates change.

Steps in Duration GAP Analysis

- Forecast interest rates.
- Estimate the market values of bank assets, liabilities and stockholders' equity.
- Estimate the weighted average duration of assets and the weighted average duration of liabilities.
  - Incorporate the effects of both on- and off-balance sheet items. These estimates are used to calculate duration gap.
- Forecasts changes in the market value of stockholders' equity across different interest rate environments.

Weighted Average Duration of Bank Assets

- **Weighted Average Duration of Bank Assets (DA)**

$$DA = \sum_{i}^{n} w_i Da_i$$

  - **Where**
    - **w_i = Market value of asset i divided by the market value of all bank assets**
    - **Da_i = Macaulay's duration of asset i**
    - **n = number of different bank assets**

Weighted Average Duration of Bank Liabilities

- **Weighted Average Duration of Bank Liabilities (DL)**

$$DL = \sum_{j}^{m} z_j Dl_j$$

  - **Where**
    - **z_j = Market value of liability j divided by the market value of all bank liabilities**
    - **Dl_j= Macaulay's duration of liability j**
    - **m = number of different bank liabilities**

Duration GAP and Economic Value of Equity

- Let MVA and MVL equal the market values of assets and liabilities, respectively.
- If: $\Delta EVE = \Delta MVA - \Delta MVL$

  and

  Duration GAP

$$DGAP = DA - (MVL/MVA)DL$$

- Then: $\Delta EVE = -DGAP\left[\frac{\Delta y}{(1+y)}\right]MVA$

  - where y = the general level of interest rates

- To protect the economic value of equity against any change when rates change , the bank could set the duration gap to zero:

Hypothetical Bank Balance Sheet

	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	$100				$ 100	
Earning assets						
3-yr Commercial loan	$ 700	12.00%	3	12.00%	$ 700	2.69
6-yr Treasury bond	$ 200	8.00%	6	8.00%	$ 200	4.99
Total Earn					$ 900	
Non-cash e					$ -	
Total assets					$ 1,000	2.88
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	$ 620	5.00%	1	5.00%	$ 620	1.00
3-yr Certificate of deposit	$ 300	7.00%	3	7.00%	$ 300	2.81
Tot. Int Bearing Liabs.	$ 920			5.65%	$ 920	
Tot. non-int. bearing	$ -				$ -	
Total liabilities	$ 920			5.65%	$ 920	1.59
Total equity	$ 80				$ 80	
Total liabs & equity	$ 1,000				$ 1,000	

$$D = \frac{\dfrac{84\times 1}{(1.12)^1} + \dfrac{84\times 2}{(1.12)^2} + \dfrac{84\times 3}{(1.12)^3} + \dfrac{700\times 3}{(1.12)^3}}{700}$$

Calculating DGAP

- DA
  - (\$700/\$1000)*2.69 + (\$200/\$1000)*4.99 = 2.88
- DL
  - (\$620/\$920)*1.00 + (\$300/\$920)*2.81 = 1.59
- DGAP
  - 2.88 - (920/1000)*1.59 = 1.42 years
    - What does this tell us?
      - The average duration of assets is greater than the average duration of liabilities; thus asset values change by more than liability values.

1 percent increase in all rates.

1	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	$ 100				$ 100	
Earning assets						
3-yr Commercial loan	$ 700	12.00%	3	13.00%	$ 683	2.69
6-yr Treasury bond	$ 200	8.00%	6	9.00%	$ 191	4.97
Total Earning Assets					$ 875	
Non-cash earning ass					$ -	
Total assets					$ 975	2.86
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	$ 620	5.00%	1	6.00%	$ 614	1.00
3-yr Certificate of deposit	$ 300	7.00%	3	8.00%	$ 292	2.81
Tot. Int Bearing Liabs.	$ 920			6.64%	$ 906	
Tot. non-int. bearing	$ -				$ -	
Total liabilities	$ 920			6.64%	$ 906	1.58
Total equity	$ 80				$ 68	
Total liabs & equity	$ 1,000				$ 975	

$$PV = \sum_{t=1}^{3} \frac{84}{1.13^t} + \frac{700}{1.13^3}$$

Change in the Market Value of Equity

$$\Delta EVE = -DGAP\left[\frac{\Delta y}{(1+y)}\right]MVA$$

- **In this case:**

$$\Delta EVE = -1.42\left[\frac{.01}{1.10}\right]\$1{,}000 = -\$12.91$$

Positive and Negative Duration GAPs

- Positive DGAP
  - Indicates that assets are more price sensitive than liabilities, on average.
    - Thus, when interest rates rise (fall), assets will fall proportionately more (less) in value than liabilities and EVE will fall (rise) accordingly.
- Negative DGAP
  - Indicates that weighted liabilities are more price sensitive than weighted assets.
    - Thus, when interest rates rise (fall), assets will fall proportionately less (more) in value that liabilities and the EVE will rise (fall).

DGAP Summary

DGAP Summary						
DGAP	Change in Interest Rates	Assets		Liabilities		Equity
Positive	Increase	Decrease	>	Decrease	→	Decrease
Positive	Decrease	Increase	>	Increase	→	Increase
Negative	Increase	Decrease	<	Decrease	→	Increase
Negative	Decrease	Increase	<	Increase	→	Decrease
Zero	Increase	Decrease	=	Decrease	→	None
Zero	Decrease	Increase	=	Increase	→	None

An Immunized Portfolio

- To immunize the EVE from rate changes in the example, the bank would need to:
  - decrease the asset duration by 1.42 years or
  - DA=DL* (MVL/MVA)
  - increase the duration of liabilities by 1.54 years
  - DL=DA / (MVL/MVA)

Immunized Portfolio

	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	$ 100				$ 100	
Earning assets						
3-yr Commercial loan	$ 700	12.00%	3	12.00%	$ 700	2.69
6-yr Treasury bond	$ 200	8.00%	6	8.00%	$ 200	4.99
Total Earning Assets	$ 900			11.11%	$ 900	
Non-cash earning asse	$ -				$ -	
Total assets	$ 1,000			10.00%	$ 1,000	2.88
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	$ 340	5.00%	1	5.00%	$ 340	1.00
3-yr Certificate of depos	$ 300	7.00%	3	7.00%	$ 300	2.81
6-yr Zero-coupon CD*	$ 444	0.00%	6	8.00%	$ 280	6.00
Tot. Int Bearing Liabs.	$ 1,084			6.57%	$ 920	
Tot. non-int. bearing	$ -				$ -	
Total liabilities	$ 1,084			6.57%	$ 920	3.11
Total equity	$ 80				$ 80	

$$\boxed{DGAP = 2.88 - 0.92\,(3.11) \approx 0}$$

Immunized Portfolio with a 1% increase in rates

1	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	$ 100.0				$ 100.0	
Earning assets						
3-yr Commercial loan	$ 700.0	12.00%	3	13.00%	$ 683.5	2.69
6-yr Treasury bond	$ 200.0	8.00%	6	9.00%	$ 191.0	4.97
Total Earning Assets	$ 900.0			12.13%	$ 874.5	
Non-cash earning asse	$ -				$ -	
Total assets	$ 1,000.0			10.88%	$ 974.5	2.86
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	$ 340.0	5.00%	1	6.00%	$ 336.8	1.00
3-yr Certificate of depos	$ 300.0	7.00%	3	8.00%	$ 292.3	2.81
6-yr Zero-coupon CD*	$ 444.3	0.00%	6	9.00%	$ 264.9	6.00
Tot. Int Bearing Liabs.	$ 1,084.3			7.54%	$ 894.0	
Tot. non-int. bearing	$ -				$ -	
Total liabilities	$ 1,084.3			7.54%	$ 894.0	3.07
Total equity	$ 80.0				$ 80.5	

Immunized Portfolio with a 1% increase in rates

- **EVE changed by only \$0.5 with the immunized portfolio versus \$25.0 when the portfolio was not immunized.**

Economic Value of Equity Sensitivity Analysis

- Effectively involves the same steps as earnings sensitivity analysis.
- In EVE analysis, however, the bank focuses on:
  - The relative durations of assets and liabilities
  - How much the durations change in different interest rate environments
  - What happens to the economic value of equity across different rate environments

Strengths and Weaknesses: DGAP and EVE-Sensitivity Analysis

- **Strengths**
  - **Duration analysis provides a comprehensive measure of interest rate risk**
  - **Duration measures are additive**
    - **This allows for the matching of total assets with total liabilities rather than the matching of individual accounts**
  - **Duration analysis takes a longer term view than static gap analysis**

Strengths and Weaknesses: DGAP and EVE-Sensitivity Analysis

- Weaknesses
  - It is difficult to compute duration accurately
  - “Correct” duration analysis requires that each future cash flow be discounted by a distinct discount rate
  - A bank must continuously monitor and adjust the duration of its portfolio
  - It is difficult to estimate the duration on assets and liabilities that do not earn or pay interest
  - Duration measures are highly subjective