Notes: Compound Interest

A common application of exponential growth is *compound interest.* Recall that simple interest is earned or paid only on the principal. **Compound interest** is interest earned or paid on *both* the principal and previously earned interest.

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A represents the balance after t years.

P represents the principal, or original amount.

r represents the annual interest rate expressed as a decimal.

n represents the number of times interest is compounded per year.

t represents time in years.

Reading Math

For compound interest

- *annually* means "once per year" (n = 1).
- *quarterly* means "4 times per year" (n = 4).
- *monthly* means "12 times per year" (n = 12).
- *Daily* usually means "365 times per year", or "366 times per year" during a leap year.

Ex 1: Write a compound interest function to model the situation. Then find the balance after the given number of years.

$1200 invested at a rate of 2% compounded quarterly; 3 years

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Step 1 Write the compound interest function for this situation.

$$= 1200\left(1 + \frac{0.02}{4}\right)^{4(3)}$$

Step 2: Substitute 1200 for P, 0.02 for r, and 4 for n, 3 for t.

$$= 1200(1 + 0.005)^{12}$$

Simplify.

$$= 1200(1.005)^{12}$$

$$\approx 1274.01$$

Use a calculator and round to the nearest hundredth.

The balance after 3 years is $1274.01.

Ex 2: Write a compound interest function to model the situation. Then find the balance after the given number of years.

$15,000 invested at a rate of 4.8% compounded monthly; 2 years

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Step 1 Write the compound interest function for this situation.

$$= 15000\left(1 + \frac{0.048}{12}\right)^{12(2)}$$

$$= 15000(1 + 0.004)^{24}$$

Step 2: Substitute 1200 for P, 0.02 for r, and 4 for n, 3 for t.

Simplify.

$$= 15000(1.004)^{24}$$

$$\approx 16508.22$$

Use a calculator and round to the nearest hundredth.

The balance after 2 years is $16,508.22.

Ex 3: Write a compound interest function to model the situation. Then find the balance after the given number of years.

$1200 invested at a rate of 3.5% compounded quarterly; 4 years

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Step 1 Write the compound interest function for this situation.

$$= 1200\left(1 + \frac{0.035}{4}\right)^{4(4)}$$

Step 2: Substitute 1200 for P, 0.02 for r, and 4 for n, 3 for t.

$$= 1200(1 + 0.00875)^{16}$$

Simplify.

$$= 1200(1.000875)^{24}$$

Use a calculator and round to the nearest hundredth.

$$\approx 1379.49$$

The balance after 4 years is $1379.49.

Ex 4: Write a compound interest function to model the situation. Then find the balance after the given number of years.

$4000 invested at a rate of 3% compounded monthly; 8 years

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Step 1 Write the compound interest function for this situation.

$$= 4000\left(1 + \frac{0.03}{12}\right)^{12(8)}$$

Step 2: Substitute 1200 for P, 0.02 for r, and 4 for n, 3 for t.

$$= 4000(1 + 0.0025)^{96}$$

Simplify.

$$= 4000(1.0025)^{96}$$

Use a calculator and round to the nearest hundredth.

$$\approx 5083.47$$

The balance after 8 years is $5083.47.

Ex 5: Write a compound interest function to model the situation. Then find the balance after the given number of years.

$4000 invested at a rate of 3% compounded monthly; 8 years

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Step 1 Write the compound interest function for this situation.

$$= 4000\left(1 + \frac{0.03}{12}\right)^{12(8)}$$

Step 2: Substitute 1200 for P, 0.02 for r, and 4 for n, 3 for t.

$$= 4000(1 + 0.0025)^{96}$$

Simplify.

$$= 4000(1.0025)^{96}$$

$$\approx 5083.47$$

Use a calculator and round to the nearest hundredth.

The balance after 8 years is $5083.47.