# Probability Distributions: Discrete vs. Continuous

All probability distributions can be classified as **discrete probability distributions** or as **continuous probability distributions**, depending on whether they define probabilities associated with discrete variables or continuous variables.

## Discrete vs. Continuous Variables

If a variable can take on any value between two specified values, it is called a **continuous variable**; otherwise, it is called a **discrete variable**.

Some examples will clarify the difference between discrete and continuous variables.

- Suppose the fire department mandates that all fire fighters must weigh between 150 and 250 pounds. The weight of a fire fighter would be an example of a continuous variable; since a fire fighter's weight could take on any value between 150 and 250 pounds.
- Suppose we flip a coin and count the number of heads. The number of heads could be any integer value between 0 and plus infinity. However, it could not be any number between 0 and plus infinity. We could not, for example, get 2.5 heads. Therefore, the number of heads must be a discrete variable.

Just like variables, probability distributions can be classified as discrete or continuous.

## Discrete Probability Distributions

If a random variable is a discrete variable, its probability distribution is called a **discrete probability distribution**.

An example will make this clear. Suppose you flip a coin two times. This simple statistical experimentcan have four possible outcomes: HH, HT, TH, and TT. Now, let the random variable X represent the number of Heads that result from this experiment. The random variable X can only take on the values 0, 1, or 2, so it is a discrete random variable.

The probability distribution for this statistical experiment appears below.

| Number of heads | Probability |
|---|---|
| 0 | 0.25 |
| 1 | 0.50 |
| 2 | 0.25 |

following are discrete probability distributions.

- **Binomial probability distribution**
- Hypergeometric probability distribution
- Multinomial probability distribution
- Negative binomial distribution
- **Poisson probability distribution**

.

## Continuous Probability Distributions

If a random variable is a continuous variable, its probability distribution is called a **continuous probability distribution**.

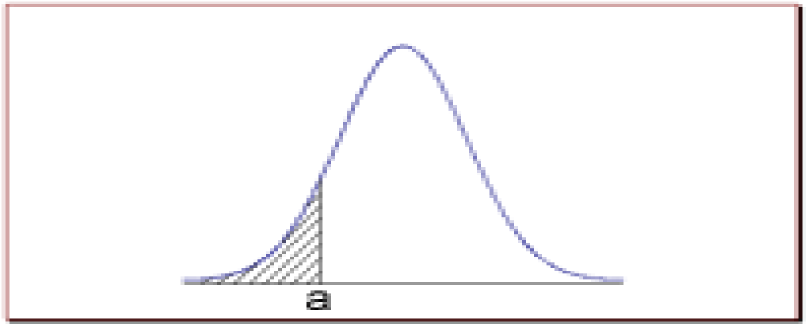
A continuous probability distribution differs from a discrete probability distribution in several ways.

- The probability that a continuous random variable will assume a particular value is zero.
- As a result, a continuous probability distribution cannot be expressed in tabular form.
- Instead, an equation or formula is used to describe a continuous probability distribution.

Most often, the equation used to describe a continuous probability distribution is called a **probability density function**. Sometimes, it is referred to as a **density function**, a **PDF**, or a **pdf**. For a continuous probability distribution, the density function has the following properties:

- Since the continuous random variable is defined over a continuous range of values (called the**domain** of the variable), the graph of the density function will also be continuous over that range.
- The area bounded by the curve of the density function and the x-axis is equal to 1, when computed over the domain of the variable.
- The probability that a random variable assumes a value between *a* and *b* is equal to the area under the density function bounded by *a* and *b*.

For example, consider the probability density function shown in the graph below. Suppose we wanted to know the probability that the random variable *X* was less than or equal to *a*. The probability that *X*is less than or equal to *a* is equal to the area under the curve bounded by *a* and minus infinity - as indicated by the shaded area.




**Note:** The shaded area in the graph represents the probability that the random variable *X* is less than or equal to *a*. This is a cumulative probability. However, the probability that *X* is *exactly* equal to *a*would be zero. A continuous random variable can take on an infinite number of values. The probability that it will equal a specific value (such as *a*) is always zero.

Following continuous probability distributions.

- Normal probability distribution
- Student's t distribution
- Chi-square distribution
- F distribution

## Standard Normal Distribution

The **standard normal distribution** is a special case of the normal distribution. It is the distribution that occurs when a normal random variable has a mean of zero and a standard deviation of one.

The normal random variable of a standard normal distribution is called a **standard score** or a **z-score**. Every normal random variable *X* can be transformed into a *z* score via the following equation:

$$z = (X - \mu) / \sigma$$

where *X* is a normal random variable, μ is the mean of *X*, and σ is the standard deviation of *X*

**Problem 1**

Molly earned a score of 940 on a national achievement test. The mean test score was 850 with a standard deviation of 100. What proportion of students had a higher score than Molly? (Assume that test scores are normally distributed.)

(A) 0.10
(B) 0.18

(C) 0.50
(D) 0.82
(E) 0.90

**Solution**

The correct answer is B. As part of the solution to this problem, we assume that test scores are normally distributed. In this way, we use the normal distribution as a model for measurement. Given an assumption of normality, the solution involves three steps.

- First, we transform Molly's test score into a z-score, using the z-score transformation equation.

$$z = (X - \mu) / \sigma = (940 - 850) / 100 = 0.90$$

- standard normal distribution table, we find the cumulative probability associated with the z-score. In this case, we find $P(Z < 0.90) = 0.8159$.

- Therefore, the $P(Z > 0.90) = 1 - P(Z < 0.90) = 1 - 0.8159 = 0.1841$.

Thus, we estimate that 18.41 percent of the students tested had a higher score than Molly.