8.4 Area Between Curves (with respect to x)

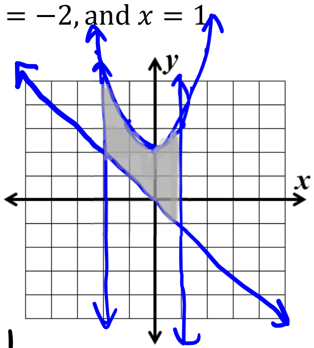
Calculus

Solutions

Practice

***Sketch* the graph of each equation, then set up the integral to find the area of the region bounded by the graphs. Do NOT evaluate, just set up the integral!**

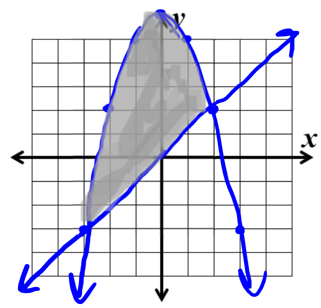
1. $f(x) = x^2 + 2,\ g(x) = -x,$ $x = -2$, and $x = 1$



$\int_{-2}^{1} (x^2+2) - (-x)\, dx$

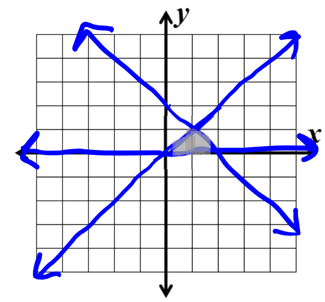
$\boxed{\int_{-2}^{1} (x^2+x+2)\, dx}$

2. $f(x) = 6 - x^2$ and $g(x) = x$



$\boxed{\int_{-3}^{2} (6-x^2-x)\, dx}$

3. $y = x,\ y = 2 - x,\ y = 0$



$\boxed{\int_{0}^{1} x\, dx + \int_{1}^{2} (2-x)\, dx}$

Find the area of the region bounded by the following graphs. Show your work.

4. $y = \frac{1}{x^2}$, $y = 0$, $x = 1$, $x = 5$

$\int_1^5 \frac{1}{x^2}\,dx = \int_1^5 x^{-2}\,dx$

$-\frac{1}{x}\Big|_1^5$

$-\frac{1}{5} - -\frac{1}{1}$

$-\frac{1}{5} + \frac{5}{5} = \boxed{4/5}$

5. $y = x^2$ and $y = x^3$

$x^2 = x^3$

$0 = x^3 - x^2$

$0 = x^2(x-1)$

$x=0 \quad x=1$

$(\frac{1}{2})^2 > (\frac{1}{2})^3$

$x^2 > x^3$ on $0<x<1$

$\int_0^1 (x^2 - x^3)\,dx$

$\frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1$

$(\frac{1}{3} - \frac{1}{4}) - (0)$

$\frac{4}{12} - \frac{3}{12} = \boxed{\frac{1}{12}}$

6. $y = \sqrt{x}, x = 0$ and $y = x - 2$

$\sqrt{x} = x - 2$

Calculator

$x = 4$

$\sqrt{1} > (1) - 2$

$\sqrt{x} > x - 2$ on $0<x<4$

$\int_0^4 (x^{\frac{1}{2}} - x + 2)\,dx$

$\frac{2x^{3/2}}{3} - \frac{x^2}{2} + 2x\Big|_0^4$

$[\frac{2}{3}(\sqrt{4})^3 - \frac{16}{2} + 8] - [0]$

$\frac{2}{3}\cdot 8 - 8 + 8 = \boxed{16/3}$

7. **Calculator active.** $y = e^{x^2} - 2$ and $y = \sqrt{4 - x^2}$

$e^{x^2} - 2 = \sqrt{4-x^2}$

$e^0 - 2 < \sqrt{4-0}$

used Calculator

$x = -1.137279 \rightarrow A$

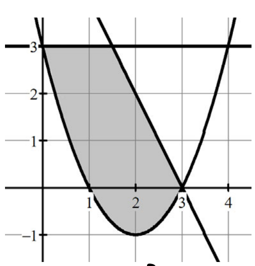
$x = 1.137279 \rightarrow B$

$\int_A^B [\sqrt{4-x^2} - (e^{x^2} - 2)]\,dx$

$\boxed{5.0495}$

Set up an integral(s) that represents the shaded region. Do not solve. Use a calculator if necessary to help find the lower and upper bounds.

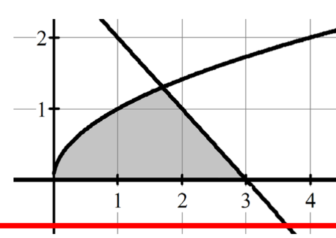
8. $y = x^2 - 4x + 3$, $y = 3$, and $y = 6 - 2x$



$\int_0^{1.5} (3 - (x^2 - 4x + 3))\,dx + \int_{1.5}^3 [(6-2x) - (x^2 - 4x + 3)]\,dx$

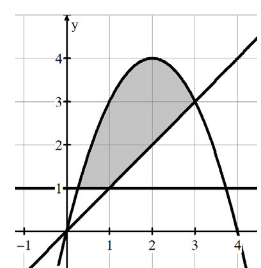
$\boxed{\int_0^{1.5} (-x^2 + 4x)\,dx + \int_{1.5}^3 (3 + 2x - x^2)\,dx}$

9. $y = \sqrt{x}$, $y = 0$, and $y = 3 - x$



$\boxed{\int_0^{1.69722} \sqrt{x}\,dx + \int_{1.69722}^3 (3 - x)\,dx}$

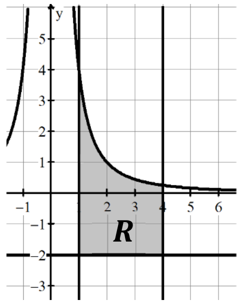
10. $y = 4x - x^2$, $y = 1$, and $y = x$



$\boxed{\int_{0.2679}^1 (4x - x^2 - 1)\,dx + \int_1^3 (3x - x^2)\,dx}$

Let R be the region bounded by the given curves as shown in the figure. If the line $x = k$ divides R into two regions of equal area, find the value of k

11. $y = \frac{4}{x^2}$, $y = -2$, $x = 1$, and $x = 4$



$$\int_1^k \frac{4}{x^2} - (-2)\,dx = \int_k^4 \frac{4}{x^2} - (-2)\,dx$$

$$-\frac{4}{x} + 2x \Big|_1^k = -\frac{4}{x} + 2x \Big|_k^4$$

$$\left[-\frac{4}{k} + 2k\right] - \left[-\frac{4}{1} + 2(1)\right] = \left[-\frac{4}{4} + 2(4)\right] - \left[-\frac{4}{k} + 2k\right]$$

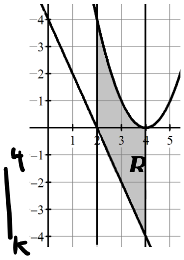
$$-\frac{4}{k} + 2k + 2 = 7 + \frac{4}{k} - 2k$$

$$-\frac{8}{k} + 4k = 5$$

Calculator

$$\boxed{k \approx 2.171}$$

12. $y = x^2 - 8x + 16$, $y = -2x + 4$, $x = 2$, and $x = 4$



$$x^2 - 8x + 16 - (-2x + 4) = x^2 - 6x + 12$$

$$\int_2^k (x^2 - 6x + 12)\,dx = \int_k^4 (x^2 - 6x + 12)\,dx$$

$$\frac{x^3}{3} - 3x^2 + 12x \Big|_2^k = \frac{x^3}{3} - 3x^2 + 12x \Big|_k^4$$

$$\left[\frac{k^3}{3} - 3k^2 + 12k\right] - \left[\frac{8}{3} - 3(4) + 24\right] = \left[\frac{64}{3} - 3(16) + 48\right] - \left[\frac{k^3}{3} - 3k^2 + 12k\right]$$

$$\frac{k^3}{3} - 3k^2 + 12k - \frac{8}{3} - 12 = \frac{64}{3} - \frac{k^3}{3} + 3k^2 - 12k$$

$$\frac{2}{3}k^3 - 6k^2 + 24k = 12 + \frac{72}{3}$$

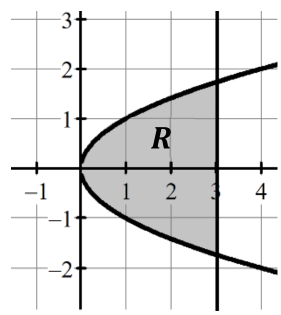
$$= 36$$

Calculator!

$$\boxed{k = 3}$$

13. $y = \sqrt{x}$, $y = -\sqrt{x}$, and $x = 3$

$$\sqrt{x} - -\sqrt{x} = 2\sqrt{x}$$



$$\int_0^k 2x^{\frac{1}{2}}\,dx = \int_k^3 2x^{\frac{1}{2}}\,dx$$

$$2x^{\frac{3}{2}}\left(\frac{2}{3}\right) \Big|_0^k = 2x^{\frac{3}{2}}\left(\frac{2}{3}\right) \Big|_k^3$$

$$\frac{4}{3}k^{\frac{3}{2}} - 0 = \frac{4}{3}\sqrt{3^3} - \frac{4}{3}k^{\frac{3}{2}}$$

$$\frac{8}{3}k^{\frac{3}{2}} = \frac{4}{3}\sqrt{27} \xrightarrow{\text{calculator}} \boxed{k \approx 1.8898}$$

Test Prep

8.4 Area Between Curves (with respect to x)

14. **Calculator active problem.** If $0 \le k \le \frac{\pi}{4}$ and the area under the curve $y = \sin x$ from $x = k$ to $x = \frac{\pi}{4}$ is 0.2, then what is the value of k?

$$\int_k^{\pi/4} \sin x = 0.2$$

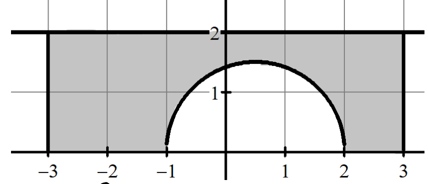
$$-\cos x \Big|_k^{\pi/4} = 0.2$$

$$-\cos\left(\frac{\pi}{4}\right) - -\cos k = 0.2$$

$$\cos k = 0.90710678$$

$$k = \cos^{-1}(0.90710678) \approx \boxed{0.434}$$

15. **Calculator active problem.** The shared region in the figure above is bounded by the graph $y = \sqrt{2 + x - x^2}$ and the lines $x = -3$, $x = 3$, and $y = 2$. What is the area of this region?



$A = 2 \cdot 6$

$A = 12$

$$12 - \int_{-1}^{2} \sqrt{2 + x - x^2}\,dx$$

$$\boxed{8.4657}$$