Chapter 9: Special Relativity Introduction

Classical Mechanics

So far all you have learned about motion has been **Classical Mechanics**

All of Classical Mechanics flows from Newton's Second Law of Motion

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

Given knowledge a force $\vec{\mathbf{F}}$ in space acting on a particle then you can always find the acceleration of the particle in that space. Conversely, if you know the acceleration of a particle through a region of space, you can deduce the nature of the force acting on the particle. In fact this is how Newton himself deduced the Law of Universal Gravitation.

Inertial Systems

You have learned of the law of inertia, Newton's First Law. This law introduced the concept of Force while the Second Law quantified the use of force.
However, our experience tells us that there are same everyday examples where the Law of Inertia does not seem to work. Think of a car speeding around a turn. Objects inside the car tend to be pushed towards one side of the car, the side further away from the center of the turn. It might appear that such object accelerated inside the car with no apparent force acting on them. However, we state that a car making a turn, or an reference frame being accelerated, is a *non-inertial* frame.
Galileo and Newton came up with the concept of inertial frames of reference, meaning non-accelerated frames of reference. They stated the fundamental principle of Newtonian Mechanics
All inertial frames of reference are equivalent. The laws of mechanics may be derived in any inertial frame. There is no preferred inertial frame of reference.

Mechanics: The First Two Hundred Years

For more than 200 years Newton's Mechanics appeared to work beautifully. Nothing on Earth seemed to violate Newton's Laws as long as one remembered the principle of inertial frames. And almost everything moving in the skies was wonderfully consistent with Newton's Law of Gravity, except perhaps for a funny little wrinkle about the motion of Mercury . . . .

The First Postulate of the Theory of Special Relativity

The start of the relativity revolution

In 1905, Albert Einstein was a low-level technical expert in the Swiss patent office having had an undistinguished record at university while getting a physics degree. (He had learning disabilities.) In his spare time he pondered basic physics principles, and completely on his own developed the special theory of relativity which overthrew Newtonian mechanics.

In fact his special theory of relativity has only two simple postulates. The first of these was a generalization of the Galileo-Newton principle of inertial frames:

Special Relativity Postulate #1

All the Laws of Physics may be derived in any inertial frame. No law of physics will distinguish any preferred reference frame.

Background to the Special Relativity Postulate #1

At first sight, the Special Relativity Postulate #1 may seem a perfectly reasonable thing to say. After all, it simply says that not only can Mechanics be derived in any inertial frame, but also so too can the physics of Electricity, Magnetism, Optics, Thermodynamics, Sound, or any other branch of physics.

But in reality, the Special Relativity Postulate #1 was a major break with the conventional physics wisdom of the time. Physicists in the late nineteenth century had been searching for the one magic frame of reference called the *luminiferous ether* or *ether* for short.

The *ether* (which has nothing to do with the anesthetic of the same same) was the medium through which light traveled. There had to be such a medium because the Laws of Electricity and Magnetism singled out a special value of speed, the speed of light, at 3×10^8 meters/second. The simplest state of the laws of Electricity and Magnetism involved the used of the speed of light constant. But we have learned that speed can only be defined once one has defined a reference frame. Therefore, if there is a special speed then there must be a special reference frame.

This special reference frame for the transmission of light waves was called the *luminiferous ether*. It was the object of an intense search among scientists of the day, not unlike the search for the Northwest Passage to India which consumed Earth explorers of Newton's time.

The Search for the Ether

The Michaelson-Morely Experiment

Like the search for the Northwest Passage to India, the search for the ether was doomed because it did not exist. However, one could not know that at the start so one had to try the search.

The most precise and ingenious experiment designed to discover the ether frame of reference was by two physicists, an American named Albert Michaelson and an Englishman named Edward Morely. Michaelson grew up in Nevada before it became a state and is responsible for the development of the Michaelson interferometer, an incredibly precise instrument for measuring small differences in length or speed. For this experiment, even though it "failed" to measure the ether, Michaelson was the first American to win the Nobel prize.

The Michaelson Interferometer

The Michaelson Interferometer (see Fig. 9.3 on page 283) is a very simple device. A light source S is collimated to emit light through a half-silvered mirror M_0. Part of the light is travels through M_0 and goes also a distance d where it is also reflected back by another full mirror M_2. reflected at right angles and travels a distance d to a second full mirror M_1 where it is reflected back again the distance d. The other part of the light reflected at right angles and travels a distance d to a second full mirror M_1 where it is reflected back again the distance d. Both light rays then travel towards an observer who views the result through a special telescope T. The telescope will exhibit something called *interference fringes* which you'll learn more about next semester. The *interference fringes* occur because of the possibly different times of arrival of the light along the two sets of paths: 1) S–M_0–M_2–M_0–T, and 2) S–M_0–M_1–M_0–T.

Quantitative Analysis of the Interferometer

The quantitative analysis of the interferometer is very easy (and surprisingly omitted from the text.) It is very much like a problem of a boat crossing a flowing river of width d, and finding how much time it takes the both to make a round trip crossing as compared with just traveling upstream and downstream the same distance

The Search for the Ether by Michaelson and Morely

Quantitative Analysis of the Interferometer

Suppose that the apparatus is moving through the ether such that there is an *ether wind* of amount v in the direction from M_2 to M_0. In that case the speed of light will be $c - v$ on the way to M_2 and $c + v$ on the way back from M_2. On the other hand, in the transverse direction S–M_0–M_2–M_0–T, the light would have an effective speed of $\sqrt{c^2 - v^2}$. Hence the times for the two trips would be

$$t_1 = \frac{d}{c+v} + \frac{d}{c-v} = \frac{2d}{c^2 - v^2} = \frac{2d}{c^2(1 - v^2/c^2)}$$

$$t_2 = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - v^2/c^2}}$$

The time difference $\Delta t \equiv t_2 - t_1$ is derived as

$$\Delta t \equiv t_2 - t_1 = \frac{2d}{c}(\frac{1}{1 - v^2/c^2}) - \frac{2d}{c}(\frac{1}{\sqrt{1 - v^2/c^2}})$$

The telescope, via observation of the interference fringes, essentially tells one the time difference Δt.

Now the speed v would be the Earth's speed through the ether. One could not be sure of the direction of the Earth's motion through the ether. There's where Michealson's genius also came in again. He floated the whole apparatus in a pool of mercury such that it could be rotated level in any direction.

To give you an idea of the order of magnitude, suppose the Earth's speed through the ether was just the speed around the Sun $v = 3 \times 10^4$ m/s. The interferometer had a distance d of 11 meters. This works out to a time difference of $\Delta t \approx 7.0 \times 10^{-16}$ seconds, and the apparatus was sensitive down to the equivalent of 2.0×10^{-17} seconds !!

An Astonishing Null Result

As great as their apparatus and technical skills were, Michaelson and Morely were confounded when they measured no time difference. And no matter when they did the experiment: day or night; Winter, Spring, Summer, or Fall. It made absolutely no difference. They were never able to detect the *ether wind*, and consequently the existence of the special ether frame of reference for Optics and Electromagnetism.

Attempts to Explain Away the Michaelson-Morely Experiment
Lorentz and Fitzgerald get lucky

The Michaelson-Morely Experiment null result was very disturbing in its time. Various so-called *ad hoc* explanations were developed to explain away the results. One of these was that the Earth somehow manages to drag the ether along with it in orbit. However, other experiments using telescopes looking at light coming from star's proved that explanation wrong.

A second, even more on-the-spot sort of explanation was proposed independently by two European physicists G.F. Fitzgerald and H.A. Lorentz. They simply stated that any apparatus length contracts by a factor of $\sqrt{1 - v^2/c^2}$ in the direction of the ether motion. So instead of d one would have d' where

$$d' = d \times \sqrt{1 - v^2/c^2}$$

The factor $\sqrt{1 - v^2/c^2}$ became known as the Lorentz-Fitzgerald contraction, or more simply Lorentz contraction since Lorentz was the better known physicist. On its face, the Lorentz contraction idea might sound a bit silly and a bit too convenient. Moreover, it has no predictive capability. It explains away one discrepancy by postulating another. But it turned out to be quantitatively correct for a totally different reason and Lorentz contraction was immortalized.

Einstein Introduces the Special Relativity Postulate #2

Eventually it took Einstein's Special Relativity Postulate #2 to get physics back on track. Special Relativity Postulate #2 is startingly simple, but has drastic consequences:

The speed of light c is the same for all observers, independent of their own motion or the motion of the light source.

Right away, this saved the Physics of Electromagnetism and Optics because it gave a special place to the constant c. However, it destroyed the unspoken basis of Newton's mechanics, namely, that there was such a thing as absolute time. With Special Relativity Postulate #2, time became relative to the observer because that's the only way light could have the same speed for all observers. In other words, two observers in motion with respect to one another do not measure the same times for observed events. Absolute simultaneity was destroyed as a physics concept.

Special Relativity Consequences

Special relativity of motion is an amazingly simple theory. The two basic postulates of special relativity are easily understandably, but have startling consequences.

The Two Special Relativity Postulates

#1 *All the Laws of Physics may be derived in any inertial frame. No physics measurement will distinguish any preferred reference frame.*

#2 The speed of light c is the same for all observers, independent of their own motion or the motion of the light source.

These two postulates were introduced by Albert Einstein in 1905, and were based on his attempts to reconcile the theory of Electromagnetism as developed in the 1800s with the theory of Mechanics developed two hundred years earlier. In the end it was Newton's Mechanics which had to be revised and Electromagnetism which survived unscathed.
One of the historical mysteries of science is whether Albert Einstein was aware of the results of Michaelson and Morely which essentially are the best experimental proof of these two postulates. The best evidence is that he did not know about the results, somewhat surprising because Einstein knew just about everything in the physics of his time. Instead, he based is special relativity ideas more on the papers of Lorentz who was trying also to reconcile Electromagnetism and Classical Mechanics.

Gedanken Experiments to Disprove Absolute Time

The first most startling result of the theory of special relativity is the collapse of the idea of simultaneity, or equivalently of absolute time identical for all observers whether moving or at rest relative to one another.
Einstein conjured up a lot of so-called "thought experiments" (*gedanken* experiments in German) based on the two postulates to prove this effect, since it was so difficult to do true experiments at the time. Most of these experiments had to do with moving trains very common in Europe, although now days we can think of moving space ships.

Special Relativity Consequences

No Absolute Time

The first Gedanken experiment involves a moving train in which an observer O' (Liz) is seated exactly in the center. Two lightning bolts strike each end of the car, just as the observer O' passes another observer O (Mark) who happens to be standing alongside the train tracks. The light from the two lightning bolts reaches O at exactly the same time according to how O sees things. However, O' will see the light from the front of the car arriving before the light from the rear of the car. Hence, $O\prime$ will say that the one lightning bolt preceded the other.
But you might say O' is wrong because she is in a moving frame of reference. However, O' may not know she is in a moving frame of reference, and may never know in some cases of moving reference frames. In particular, O' cannot determine that she is moving by measuring a different value for the speed of light compared to what O will measure. Moreover, from her point of view it might be O who is moving backwards and therefore he got things mixed up in his frame. Hence, O''s viewpoint of non-simultaneous events is just as valid as O's viewpoint of simultaneous events.

Time Dilation

We saw above that two observers moving relative to one another may be in disagreement with regards to when two "events" are simultaneous. An "event" means something happened at a given spatial location (x, y, z), and a given time t. This suggests something might be happening to their clocks.
To show this is true, Einstein thought up of another experiment. He used again the same moving train with the same two observers O and O'. Observer O' shines a brief flash of light from the floor of the train car up to a mirror on the ceiling from which the light is reflected down to the floor again. Somehow O' manages to get the time difference between when the flash of light leaves the floor to when it returns. (That's the nice thing about Gedanken experiments, all the really hard technical parts are left out.) He calls this time $\delta t'$. And since she knows how far it is from floor to ceiling, distance d, she can use this time interval Δt to make a clock calibration.

Time Dilation

Time intervals as measured by two observers

Liz will measure a time interval:

$$\Delta t' = \frac{2d}{c}$$

This means that she will calculate the speed of light as:

$$\Longrightarrow c = \frac{2d}{\Delta t'}$$

Meanwhile, back alongside the tracks, observer O Mark sees things a little differently. In his view, the light flash has not simply traveled straight up and down, but rather has gone on two diagonal paths of length $c\Delta t/2$ which form the hypotenueses of two right triangles. Each of the triangles has a common height d, and an equal base length $v\Delta t/2$. Note that we are using a different symbol Δt for the time interval as measured by Mark.

So Mark will use the Pythagorean theorem to calculate that:

$$(c\Delta t/2)^2 = (v\Delta t/2)^2 + d^2$$

This he will measure Δt to be

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - v^2/c^2}}$$

Now we can relate Δt to $\Delta t'$ by substituting $\Delta t' = 2d/c$. This produces

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

The time $\Delta t'$ measured by Liz is shorter than the time Δt measured by Mark!

Now you see again the Lorentz contraction factor $1.0/\sqrt{1 - v^2/c^2}$, which occurs so often in relativity that it is given its own special symbol γ

$$\gamma = \frac{1.0}{\sqrt{1 - v^2/c^2}}$$

Time Dilation

The result of this thought experiment, based on the constancy of the speed of light, is the following:

Two observers moving relative to one another and measuring the time interval between two events will come up with different values for the time interval.

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t'$$

$$\gamma = \frac{1.0}{\sqrt{1 - v^2/c^2}}$$

Again, it makes no sense to ask "Who is right?, since they both are right in their own frames of reference.

Proper Time

There is an important difference between the two observers in the previous Gedanken experiment. Liz, in the moving train car, measured the start and stop times at the same space location. So her events can be written as (x'_1, y'_1, z'_1, t'_1) and (x'_2, y'_2, z'_2, t'_2) where $x'_1 = x'_2$, $y'_1 = y'_2$, $z'_1 = z'_2$, and $t'_2 = t'_1 + \Delta t'$

On the other hand Mark, standing alongside the tracks, saw the start and stop times occur at two different space locations. Suppose we take the x direction to be the one along which the train is moving with speed v. Then Mark's events are written as (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) where $x_1 = x_2 + v\Delta t$, $y_1 = y_2$, $z_1 = z_2$, and $t_2 = t_1 + \Delta t$.

The reference frame in which the interval between two events is measured at the same place defines the *proper time* interval for the events. The *proper time* is always measured with a single clock at rest in its own rest frame.

Proof of Time Dilation

Now Gedanken experiments are fine for illustrating a point, but in reality they prove nothing physically about the validity of the special relativity assumptions. Confirmation of the time dilation prediction would come only about forty years after the publication of the theory of special relativity.

Experimental Proof of Time Dilation

Muon Decay

The best known example of time dilation is the observation of high energy particles called *muons* coming down from the upper atmosphere onto the surface. A muon is nothing more than a "heavy" electron, about 200 times the mass of an electron. It is created in the upper atmosphere when atoms there are hit by high energy protons called cosmic rays which travel all through the universe.

The big difference, besides the mass, between an electron and a muon is that a muon is unstable. It will decay after about 2.2 microseconds into an electron plus a couple of neutrinos. For all practical purposes the muon is traveling near the speed of light, say $0.99c$ when it is created. Were it not for relativistic time dilation, then the muon would decay after just about 600 meters, and never reach the Earth's surface. However, because of time dilation, the muon's internal clock runs much slower. In fact, for a speed $v = 0.99c$ we have

$$\gamma = 1.0\sqrt{1 - v^2/c^2} \approx 7.1$$

Hence, to a stationary observer looking at the moving muon, the muon's lifetime will be more like 16 microseconds. And in 16 microseconds, the muon could make it a distance of 4800 meters. That's about 15,000 feet which is where the muons could be created in abundance.

In 1976, muons were created in the CERN accelerator laboratory and accelerated to a speed of $0.9994c$. This then gave a $\gamma \approx 30$, and the accelerated muons were observed to live about 30 times as long as muons created with near 0 speed.

Use of Real Clocks

Time dilation applies to real clocks as well. This was demonstrated in 1971 when four identical cesium atomic clocks were synchronized, and then two of them were flown in an airplanes, one going West and the other East. Relative to the ground based clocks, the eastward flying clock lost 59 nanoseconds, while the westward flying clock gained 273 nanoseconds. These results, taking into account that the Earth based clocks were also moving in space, were exactly consistent with the predictions of special relativity

The Twin Paradox

Soon after Einstein's publication of the theory of special relativity and its time dilation predictions, there came a serious objection which became know as the *Twin Paradox.* This scientific riddle was the subject of many theoretical papers, until the flight of the cesium clocks which resolved the issue once and for all.

The premise of the paradox is very simple. Two twins, Speedo and Goslo, are both 20 years old. Speedo builds herself a rocket ship and accelerates to near the speed of light on a journey to another star 30 light years away from Earth. She then returns home, having aged only about 10 years because of time dilation. However, on Earth about 80 years has elapsed and her brother is looking like George Burns just before he died.

The paradox is the following: Could we look at the situation from Speedo's point of view that it was Goslo who went ("backwards") at near the speed of light. After all, all motion is relative. In that case it should be Goslo who stayed younger and Speedo who aged.

Now both conclusions cannot be correct, and when the spaceship returns to the Earth someone must be older.

The resolution of the paradox is that the situation is not completely symmetrical. We can distinguish the motion of the two twins because one twin was accelerating (and decelerating) and the other was not. So the proper frame to analyze the motion is the inertial frame, which is Goslo's frame. He is the one who ages, just has it was for the stationary cesium clocks in the airplane experiment.

Length Contraction

We have seen that there is a *proper time* interval between two events taking place at the same physical location in a given reference frame. In any other reference frame moving with respect to this one, the measured time interval will be longer by the Lorentz factor γ

There is also the concept of a *proper length* for an object. The proper length is the length of the object as measured in the reference frame where the object is at rest. The length of the object as measured in any reference frame in which the object appears to be moving will be *contracted* by the same Lorentz factor. Hence we speak of *Lorentz contraction* of moving object:

$$L = \frac{L_P}{\gamma} = (1 - \frac{v^2}{c^2} L_P$$

Here L is the length of the object as measured in a frame in which the object appears to be moving, and L_P is the length as measured in the objects own rest frame. The length contraction takes place only along the direction of the motion.

The frame of reference for the proper time measurement is always different from the frame of reference for the proper length measurement.