

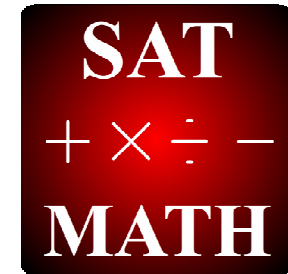
SAT Mathematics Review Geometry

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Failing to Prepare is Preparing to Fail!

Geometry and Measurement

Geometric Notation

\leftrightarrow
 BF *The line containing point B and F*

\overline{BF} *The line segment with endpoints B and F*

BF *The length of line segment BF*

\rightarrow
 BF *The ray starting at B and extending infinitely through F*

$\angle ABF$ *The angle formed by \overline{AB} and \overline{BF}*

$m\angle ABF$ *The measure of angle ABF*

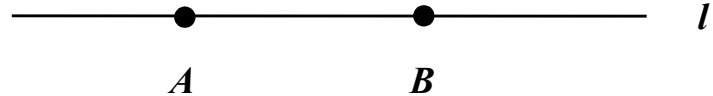
$\triangle ABF$ *The triangle with vertices A, B and F*

$ABFG$ *The quadrilateral with vertices A, B, F and G*

$\overline{AB} \perp \overline{FG}$ *AB is perpendicular to FG*

Geometry and Measurement

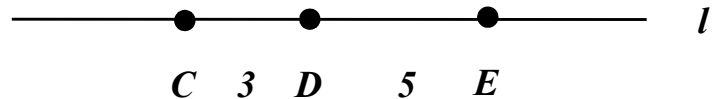
Points and Lines



unique line l , containing points A and B



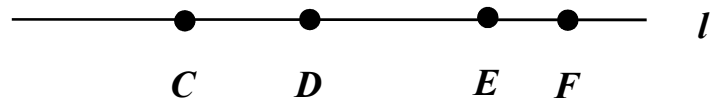
M is the midpoint of \overline{AB} , so $AM = MB$



$CD = 3$ and $DE = 5$, so $CE = 3+5 = 8$

Problem 1: A, B and C all lie on the same line l , if C is the midpoint of \overline{AB} and $AB = 12$, what is AC ?

Problem 2: On the line l above, if $CD = 4$, $EF = 2$ and $CF = 10$, what is the value of DE ?

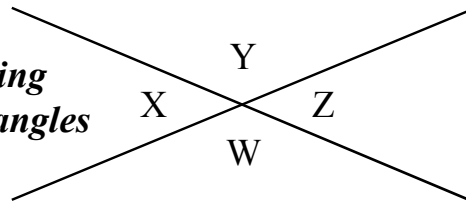


Geometry and Measurement

Angles in the Plane

Opposite angles formed by intersecting lines are equal and are called vertical angles

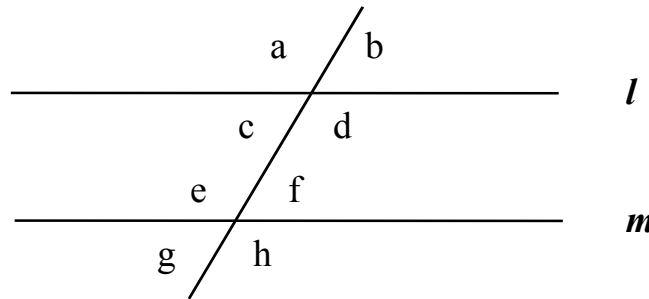
So, $X = Z$ and $W = Y$



Supplementary angles are Straight Angles and are equal to 180 degrees

*So, $X + W = 180^\circ$,
 $X + Y = 180^\circ$,
 $W + Z = 180^\circ$,
 $Y + Z = 180^\circ$*

Problem 1: *In the above diagram, if X is equal to 40 degrees, what is the value of W? What is the value of Z? What is the value of Y?*

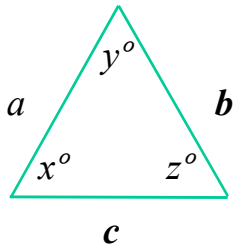


If two parallel lines (l and m) are intersected by a third line, the alternate interior angles are equal for example, e and d are alternate interior angles

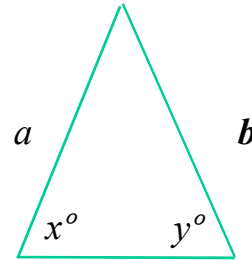
Problem 2: *In the above diagram, l and m are parallel, name all angles that are equal to angle d ? Name all angles that are supplementary to angle b ?*

Geometry and Measurement

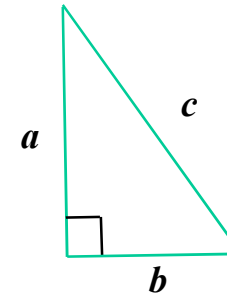
Triangles



Equilateral Triangle
equal sides ($a=b=c$)
equal angles ($x=y=z=60$)
Angles measure 60°



Isosceles Triangle
two equal sides ($a=b$)
two equal angles ($x=y$)



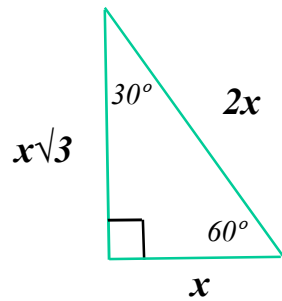
Right Triangle
one angle is 90° ($a = 90$)
two sides are perpendicular
 $a^2 + b^2 = c^2$ (Pythagorean Theorem)

Problem 1: If ABC is an Isosceles Triangle, such that $\angle ABC = \angle BAC$ and $m\angle ABC$ is 40° , what is the $m\angle ACB$?

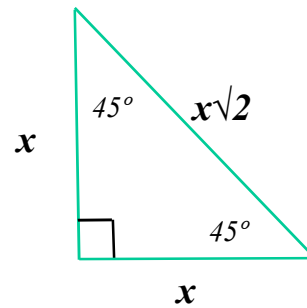
Problem 2: If ABC is a Right Triangle, such that $m\angle ABC$ is 35° , what is the $m\angle ACB$ if it is not 90° ?

Geometry and Measurement

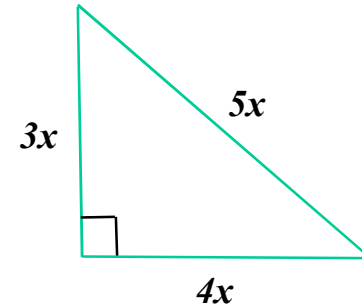
Triangles



30°-60°-90° Triangle



45°-45°-90° Triangle



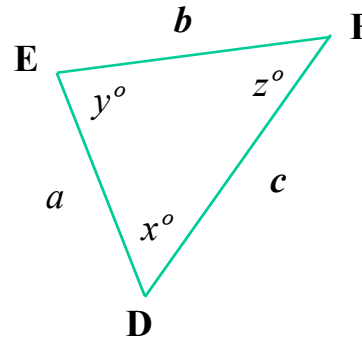
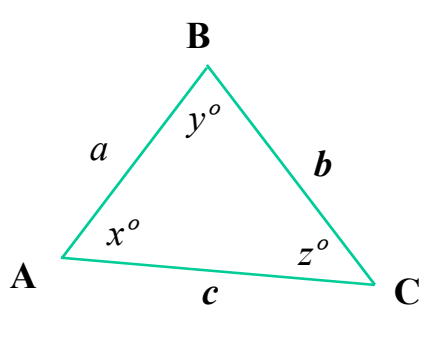
3-4-5 Triangle

Problem 1: If ABC is a Right Triangle, such that $m\angle ABC$ is 45° and $AC = 4$, what is the length of the longest side?

Geometry and Measurement

Congruent Triangles

triangles that have the same size and shape



$$\triangle ABC = \triangle DEF$$

$$AB = DE = a$$

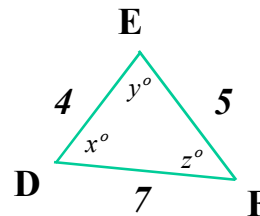
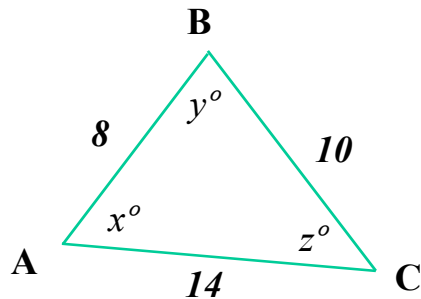
$$BC = EF = b$$

$$AC = DF = c$$

Problem 1: If ABC and DEF are congruent triangles, and $AB=5$ and $BC=15$, what is EF ?

Similar Triangles

*triangles that have the same shape
(corresponding angles are equal)*



$\triangle ABC$ and $\triangle DEF$
are similar triangles

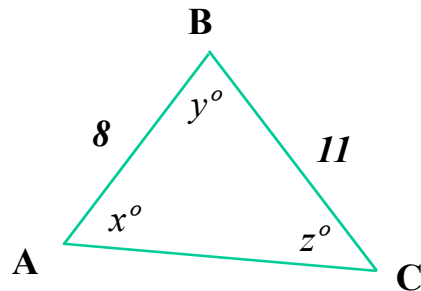
sides are proportional

Problem 2: If ABC and DEF are similar triangles, and $AB=5$, $BC=7$ and $DE=15$, what is EF ?

Geometry and Measurement

Triangle Inequality

The sum of the lengths of any two sides of a triangle is greater than the length of the third side



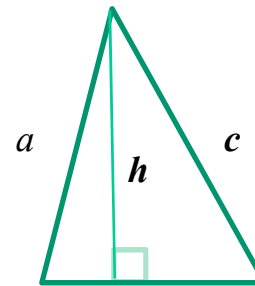
$$AC < 8 + 11$$

Problem 1: In ABC, AB = 3 and AC = 7, can BC be 4? Can BC be 12? What are the ranges of values of BC?

Triangle Perimeter and Area

*Perimeter = $b + a + c$
(sum of the three sides)*

$$\text{Area} = \frac{1}{2}bh$$

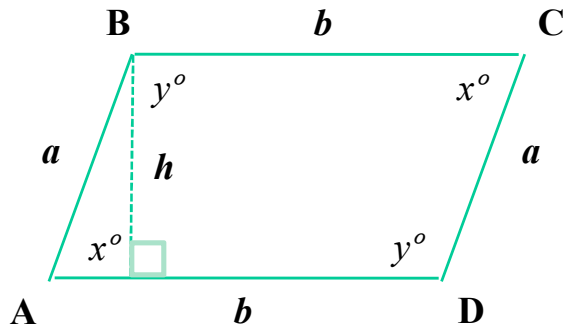


$b = \text{base}$

Problem 2: In the above triangle, if $a=6$, $b=4$, $c=7$ and $h=5$, what is the perimeter? What is the area?

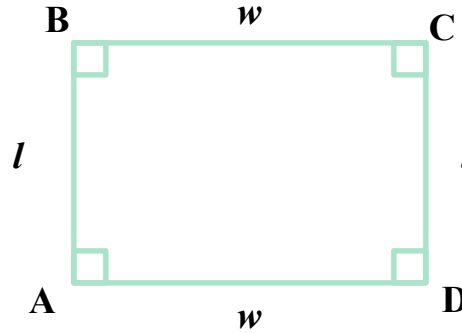
Geometry and Measurement

Quadrilaterals



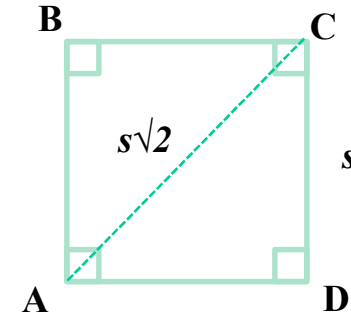
Parallelogram

opposite angles are equal
opposite sides are equal
 $P = 2a + 2b$
 $A = bh$



Rectangle

A parallelogram with right angles
 $P = 2w + 2l$
 $A = lw$
 $AC = BD = \sqrt{(l^2 + w^2)}$



Square

A rectangle with four equal sides
 $P = 4s$
 $A = s^2$
 $AC = BD = s\sqrt{2}$

Other Polygons

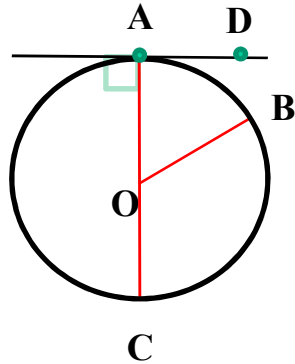
A Regular Polygon is a polygon with all sides and angles equal

Determine unknown angle and sides using triangles

<u>sides</u>	<u>interior angle sum</u>
3	180°
4	360°
5	540°
:	:
n	$180(n-2)^\circ$

Geometry and Measurement

Circles



O = Origin of Circle – the center

OA = **OB** = Radius of Circle

AB = Arc

AC = Diameter of Circle
(twice the radius **OA** or **OB**)

the line segment **AD** is tangent to the circle at point **A**.

AD touches the circle at only point **A**.

The **Diameter** of a circle is twice the **Radius** of the circle

$$d = 2r$$

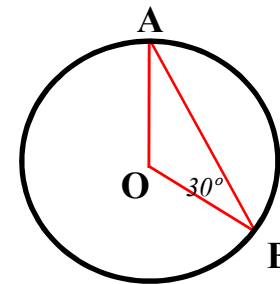
The **Circumference** of a circle is the distance around the circle –
it is analogous to perimeter of a polygon

$$C = \pi d = 2 \pi r$$

The **Area** of a circle is the amount of space within the circle –

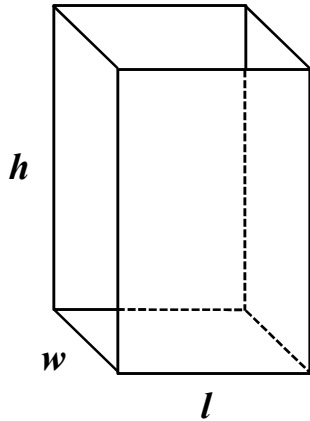
$$A = \pi r^2$$

Problem 1: Given a circle with center **O** and area 16π .
Points **A** and **B** are on the circle and angle **OBA** is 30° .
Find the length of line segment **AB**.

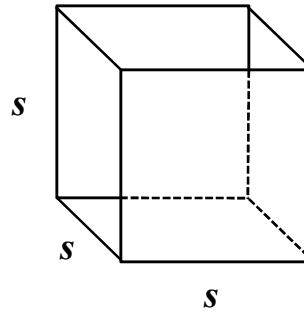


Geometry and Measurement

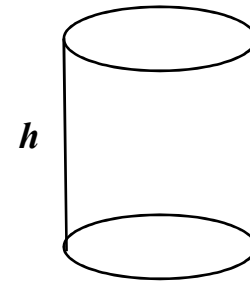
Solid Figures



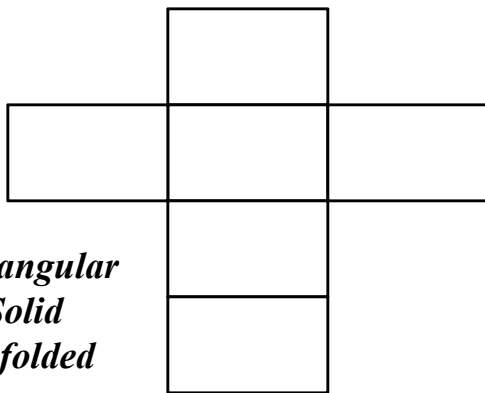
Rectangular Solid
Think of a Cardboard Box
 $V = lwh$
 $SA = 2lw + 2lh + 2wh$



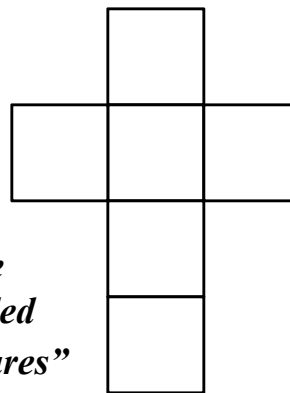
Cube
A Special Rectangular Solid in which $l=w=h=s$
 $V = s^3$
 $SA = 6s^2$



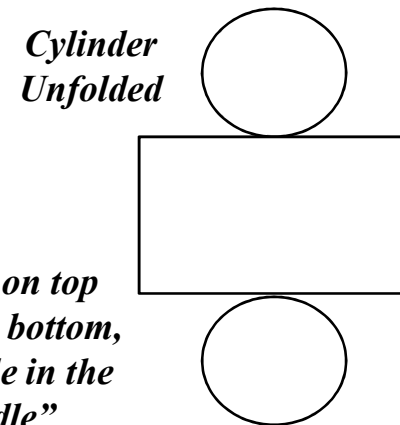
Cylinder
Think of a can of soup
 $V = \pi r^2 h$
 $SA = 2\pi r^2 + 2\pi rh$



Rectangular Solid Unfolded
“six rectangles”



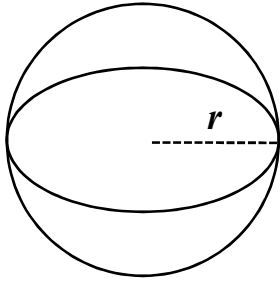
Cube Unfolded
“six squares”



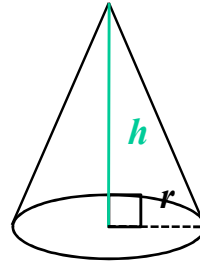
Cylinder Unfolded
*“circle on top
 circle on bottom,
 rectangle in the middle”*

Geometry and Measurement

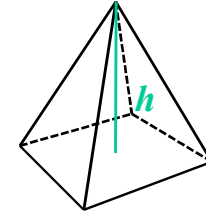
Solid Figures



Sphere
Think of a ball
All radii are equal



Cone
 $V = (1/3)\pi r^2 h$
Its Volume is 1/3 of a cylinder with the same height and base

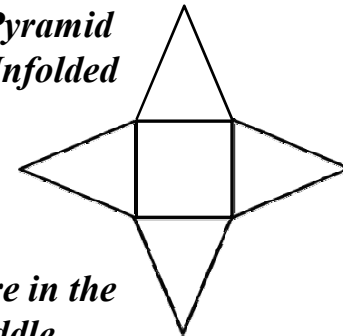


Pyramid
A square at the base
with four triangles
 $V = s^2 h / 3$

Problem 1: *If the volume of a cube is 125 in^3 , what is the length of a side? What the Surface Area of the cube?*

Problem 2: *If two cylinders have equal volume and the taller is four times higher than the shorter, what is the ratio of the radii?*

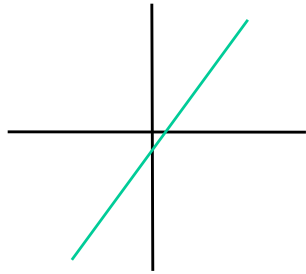
Pyramid Unfolded



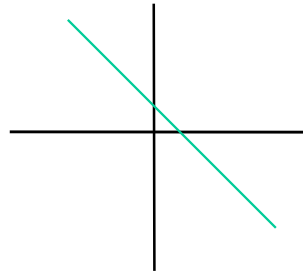
*“square in the middle,
with four congruent triangles”*

Geometry and Measurement

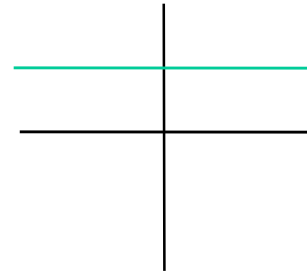
Coordinate Geometry



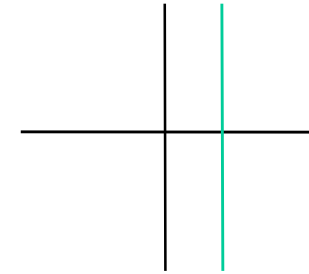
Positive Slope
 $y = 2x - 1$



Negative Slope
 $y = -2x + 2$



Zero Slope
 $y = 5$



Undefined Slope
 $x = 4$

Two lines are **parallel** when their slopes are the same
 $y = 2x + 3$ is parallel to $y = 2x - 7$ since the slope of both lines is 2

Two lines are **perpendicular** when their slopes are negative reciprocals OR the product of the slopes is -1
 $y = -2x + 3$ is perpendicular to $y = (1/2)x - 7$
since $(-2)(1/2) = -1$ OR (-2) is the negative reciprocal of $(1/2)$

Problem 1: Give a line that is parallel to the line $y = 3x - 4$. Give a line that is perpendicular to it.

Midpoint Formula

Given two points (x_1, y_1) and (x_2, y_2)
their midpoint is (x_m, y_m)
where $x_m = (x_1 + x_2)/2$ and $y_m = (y_1 + y_2)/2$

Distance Formula

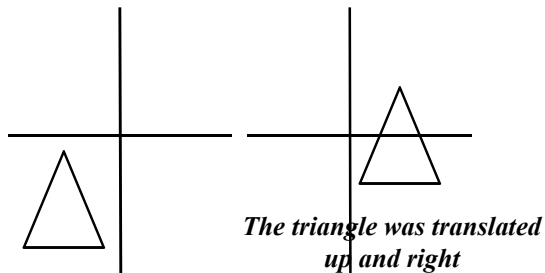
Given two points (x_1, y_1) and (x_2, y_2)
their distance is d
where $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Problem 2: What is the distance of the two points $(1, 4)$ and $(-1, -2)$? What is their midpoint?

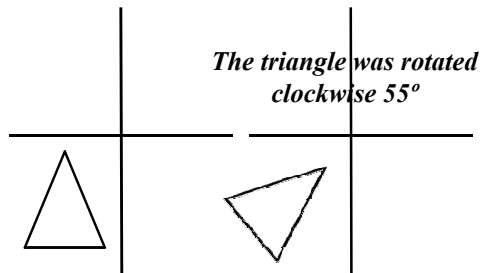
Problem 3: If $(3, 2)$ is the midpoint of two points, one being $(-1, -2)$, what is the other point?

Geometry and Measurement

Transformations

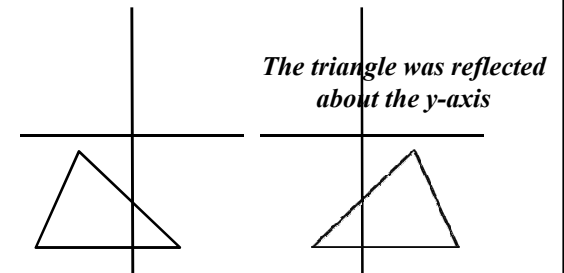


Translation
Moves up/down and left/right



Rotation
Rotates on a point
Not necessarily the center

counter-clockwise
clockwise



Reflection
Reflects along a line of symmetry

Problem 1: *If a clock is rotated 90 degrees clockwise, what number will be at the top?*

Problem 2: *If the triangle to the right is reflected about the y-axis, what are the new co-ordinates? If reflected about the x-axis, what are the new co-ordinates?*

Problem 3: *If the triangle to the right is translated 2 units up and 3 units left, what are the new co-ordinates? If then (after translation) it is reflected about the x-axis, what are the new co-ordinates?*

