Chapter 2
Forward and Futures Prices

At the expiration date, a futures contract that calls for immediate settlement, should have a futures price equal to the spot price. Before settlement, futures and spot prices need not be the same. The difference between the prices is called the basis of the futures contract. It converges to zero as the contract approaches maturity. To understand how futures prices are established requires understanding the behavior of the basis.

The basis for a forward contract is defined in a similar way. Because of the marking-to-market feature of futures, there is no apparent reason to suspect that futures prices should equal forward prices. However, in some circumstances the prices are identical. The first part of this chapter investigates these conditions. The second part of this chapter takes a closer look at the determinants of the basis in a perfect market. In particular, we investigate the cost-of-carry model which quantifies the basis and provides an explicit model of futures prices. We explore other properties of futures prices, examine the relationship between futures prices and expected future spot prices and investigate the determinants of the volatility of futures prices.

The primary objectives of this chapter are:

- To explain the relationship between forward and futures prices;

- To examine how futures prices are established;

- To explain the determinants of the basis;

- To identify arbitrage opportunities if the futures price is not within prescribed ranges; and

- To establish whether futures prices provide useful forecasts of expected spot prices in the future.

The Basis

The basis is defined as the difference between the spot and futures price. Let $b(t)$ represent the size of the basis at date t, for a futures contract that settles at date T. Then,

$$b(t) = S(t) - F(t)$$

Example

(i) On October 12th an elevator operator buys corn from a farmer for \$2.06 per bushel. The November futures contract is \$2.09. The basis is -3 cents. The local grain elevator is said to be 3 cents under the November contract.

(ii) The New York Mercantile Exchange trades a futures contract on crude oil. The underlying grade of crude oil is West Texas Intermediate (WTI). Exhibit 1 shows a possible realization of the basis of WTI over the lifetime of a contract. Notice that the basis can change sign over time. As the settlement date approaches, the basis converges to zero.

Exhibit 1: Possible Dynamics of the Basis for a Futures Contract
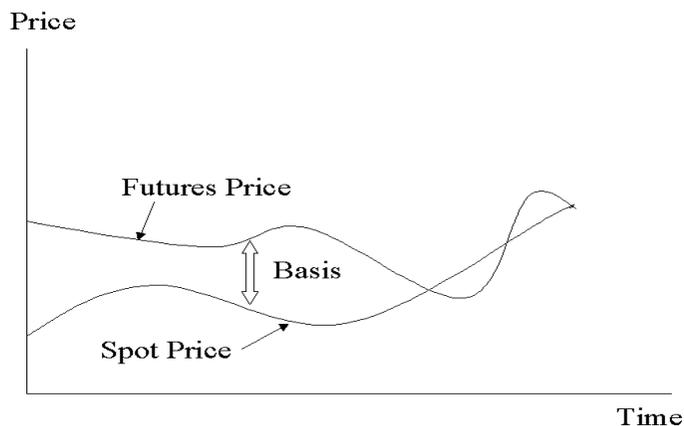
Example

On July 13th 1993 the spot price for corn was 234 cents. The following futures prices were observed.

Exhibit 2: Basis For Futures Contracts

Settlement Date	July	Sept	Dec	March 94	May	July	Sept	Dec
Futures Price	234 1/2	242	248 1/2	254 1/2	258 1/2	258 1/2	250	245 1/2
Basis	$-1/2$	-8	$-14\ 1/2$	$-20\ 1/2$	$-24\ 1/2$	$-24\ 1/2$	-16	$-11\ 1/2$

Notice that the basis for the July contract is close to zero. The basis for more distant contracts increase in the negative direction, and then decreases.

The above examples show that the basis can be positive or negative and can change direction over time. Further, at any point in time, the basis could be positive for some delivery months and negative for other delivery months. Understanding how futures prices are formed is equivalent to understanding how the basis is established. Before investigating the determinants of the basis, it is first worthwhile establishing the relationship between futures and forward prices. Indeed, our first task will be to show that under certain conditions these two prices are identical. If the appropriate conditions hold, the basis of a futures contract will equal the basis of a forward contract. This fact is useful because it allows us to ignore the marking-to-market feature in futures contracts and to quantify the basis by viewing the contract as a forward contract.

The Valuation of Forward and Futures Contracts

Assume that markets are perfect, with no taxes, transaction costs and margin requirements. The annualized risk free rate, r, is known and constant over time and borrowers and lenders earn the same rate. If $1.0 is invested at the riskless rate for 1 day then it grows to R where $R = e^{r\Delta t}$ and $\Delta t = 1/365$. Finally, let $B(t,T)$ represent the discount rate for $1.0 due at day T, viewed from day t. That is,

$$B(t,T) = e^{-r(T-t)\Delta t} = 1/R^{T-t}$$

The assumption that interest rates are known and constant over time may not be too severe for short term contracts on agricultural commodities and metals, but is not satisfactory for interest rate sensitive futures contracts. We shall therefore defer discussion of futures contracts on bonds to later. We assume that there are no delivery options in the futures contract. That is, we assume there is one deliverable grade and that delivery will take place at a specific settlement date. The underlying security may be a commodity such as corn or copper or a financial asset, like a stock or an index, depending on the situation. If the underlying instrument provides cash flows, the exact size and timing of these cash flows are assumed known.

The price of a forward contract at time t that calls for delivery of 1 unit of the commodity at time T is $FO(t)$. The price of a futures contract at time t, that calls for delivery at time

T is $F(t)$. The level of the basis at date 0 is $b(0)$ where

$$b(0) = S(0) - F(0)$$

Valuing Forward Contracts

Consider a trader who at time 0 enters into a long position in a forward contract, with settlement date T. Recall that this price is established so that the value of the contract is zero. Let V_0 be the value at time 0 of this contract. Then

$$V_0 = 0$$

Over time the value of this contract will change. Let V_t be its value at time t, where $0 \leq t \leq$ T. At the settlement date T, the value of the contract will be the difference between forward and settlement price. That is,

$$V_T = S(T) - FO(0)$$

To see this, note that the buyer is obliged to take delivery at price $FO(0)$. Since the spot price is $S(T)$, the value of this contract must be the difference.

We now consider the value of the contract at some intermediate time point $t, 0 < t < T$. Assume that at time t, the investor offsets the long commitment by selling a forward contract with price $FO(t)$. Clearly, the price of this new contract is set so that its value is zero. Hence, by entering into this position, the value of the trader's overall position remains unchanged at V_t. At time T, the buyer is obliged to take delivery at price $FO(0)$ and to sell at $FO(t)$. The date T cash flow is therefore $FO(t) - FO(0)$, which is actually known at date t.

Exhibit 3 summarizes the strategy.

Exhibit 3: Valuing Forward Contracts

Position	Value at date 0	Value at date t	Value at date T
Buy forward at date 0	0	V_t	$S(T) - FO(0)$
Sell Forward at date t	-	0	$-[S(T) - FO(t)]$
Value of strategy	0	V_t	$FO(t) - FO(0)$

Since this strategy guarantees a final cash flow at date T, its time t value should be the present value of the cash flow, where the discount rate is the riskless rate. Therefore, the fair date t value of the cash flow must be $[FO(t) - FO(0)]B(t,T)$. Hence, the value of the forward contract at date t must be

$$V_t = [FO(t) - FO(0)]B(t,T)$$

This leads to the following property.

Property 1

At time t, $0 \leq t \leq T$, the value of a forward contract entered into at time 0, is
$$[FO(t) - FO(0)]B(t,T)$$
where $FO(t)$ is the date t price of new forward contracts.

Example

A forward contract calling for delivery in six months was entered into at a forward price of $100. Assume two months later that a new forward contract with the same delivery date has a forward price of $110. By selling this forward contract, the investor is obliged to buy at $100 and sell at $110. Thus, the investor is guaranteed $10 in four months time. The current value of the net position is just the present value of $10. Given $T = 6$ months, $t = 2$ months, and the riskless rate is 12 percent, we have $B(t,T) = e^{-0.12 \times (4/12)} = 0.9607$, and $V_t = 10 \times 0.9607 = \$9.61$.

Valuing Futures Contracts

Let $F(0)$ be the current futures price for settlement at day T. Like forward contracts, the futures price is established so that the initial value of a futures contract is zero. Unlike forward contracts, futures contracts are marked to market daily. As futures prices change daily cash flows are made, and the contract rewritten in such a way that the value of future contracts at the end of each day remain zero.

Property 2

Futures contracts give the buyer the change in the futures price computed over every day up to the time the position is closed.

At the end of each day, after the contract is marked to market, the value of the contract is zero.

Example

Assume an investor buys a futures contract at $150. If at the end of the day the futures

price is \$156, the investor makes a \$6 profit, the contract is rewritten at a futures price of \$156, and the value of the contract is zero. If at the end of the second day the futures price is \$150, the investor loses \$6, and the contract is rewritten at a price of \$150. The accrued value of holding onto a futures contract for these two days is \$$(6R - 6)$. In contrast, if the futures price dropped \$6 on the first day, and then returned to its previous level on the second day, the two day accrued value on the futures position would be \$$(-6R + 6)$. The two numbers would only be equal if interest rates were zero. Notice that if interest rates were zero, then the value of holding onto a futures position for t days is just the change in the futures price over that period.

_____

The Relationship Between Forward and Futures Prices

Since forward and futures contracts are different there may be reason to think that forward and futures prices might not be equal. The next property summarizes their relationship.

Property 3

If interest rates are certain then futures and forward prices are equal.

To understand this, first consider forward and futures prices with one day to go to delivery. Consider a portfolio consisting of a long position in a forward and a short position in a futures contract. The initial investment is zero. At the end of the day, the value of the forward contract is $S(T) - FO(T - 1)$ and the value of the short position in the futures is $F(T - 1) - S(T)$. The net portfolio value, $F(T - 1) - FO(T - 1)$ is certain. Since the initial cost of this portfolio was zero, to avoid riskless arbitrage the terminal value must be zero. Hence $F(T - 1) = FO(T - 1)$. That is, with one day to go futures prices equal forward prices.

Now consider the case with two days to go. Consider the strategy of buying 1 forward contract and selling $B(T - 1, T)$ futures contract. Since interest rates are constant the value $B(T - 1, T)$ is known at date $T - 2$ and is $1/R$. Again the initial investment is zero. At the end of the day, the value of the forward contract is given by the change in forward prices discounted from the settlement date, namely, $[FO(T - 1) - FO(T - 2)]B(T - 1, T)$, and the profit on each futures contract is $F(T - 2) - F(T - 1)$. Hence, at the end of the day, the value of this strategy is:

$$[FO(T - 1) - FO(T - 2)]B(T - 1, T) + [F(T - 2) - F(T - 1)]B(T - 1, T)$$
$$= [FO(T - 1) - FO(T - 2) + F(T - 2) - F(T - 1)]B(T - 1, T)$$
$$= [F(T - 2) - FO(T - 2)]B(T - 1, T).$$

Viewed from date $T - 2$ this value is known. Again, to avoid riskless arbitrage, this value must therefore be zero. Hence, with 2 days to go, futures and forward prices must be equal.

If interest rates are deterministic, this argument can be repeated successively to demonstrate that at all points in time futures and forward prices are equal. The result can be obtained only because the number of futures contracts to sell for every forward bought can be established. At time $T - 2$, for example, the strategy required selling $B(T - 1, T)$ futures contracts. If interest rates are uncertain the value $B(T - 1, T)$ is not known at date $T - 2$ and the appropriate hedge cannot be established.

———

Example: Replicating a Forward Contract with Futures.

(i) Assume interest rates are constant, and over any day an investment of \$1.0 grows to \$$R$ where $R = 1.0005$. Now consider a futures and forward contract that has 3 days to go to settlement. The forward and futures prices are both set at \$1000.0. After 1 day the prices change to 1200; after 2 days prices are at 1500, and the settlement price is 1600. The 3 day profit on the forward position is \$600. The profit on the futures is $200R^2 + 300R + 100 = \$603.5$

Now consider the replicating strategy just discussed. With 3 days to go buy $1/R^2$ futures contracts and hold them for one day. With 2 days to go buy $1/R$ futures contracts and hold them for one day. Finally, with 1 day to go buy 1 futures contract. The accrued profit from this strategy is

$$\frac{200}{R^2}R^2 + \frac{300}{R}R + 100 = 600$$

Hence, this rollover position in futures produces the same cash flows as a forward contract. To reemphasize, this replication can only work if interest rates are certain. If they are not certain, then establishing how many futures contracts to purchase would not be possible. Finally, this example confirms that futures prices must equal forward prices. If, the forward price with 3 days to go was \$1100 then an astute investor, would sell the contract and initiate the futures rollover strategy, that replicates the forward. Regardless of what prices occur over the remaining time horizon, the investor is guaranteed a \$100 profit.

(ii) Assume a firm sold twenty forward contracts on crude oil to a corporate customer. The delivery period is in five years. Assume the five year interest rate is 10 percent. The firm decides to hedge this risk by purchasing futures contracts. Assume the deliverables on the futures and forward contracts are the same. The initial number of futures to purchase equals $20B(0, T) = 20e^{-0.10 \times 5} = 12.13$. As time advances, the hedge needs to be adjusted. For example, assume that with 2 years to go, interest rates are 8 percent. Then the hedge consists of $20e^{-0.08 \times 2} = 17.04$ futures. As the settlement date gets closer, the number of futures to sell converges to 20.

———

Property 4

(i) If futures prices are positively correlated with interest rates then futures prices will exceed forward prices.

(ii) If futures prices are negatively correlated with interest rates, then futures prices will be lower than forward prices.

(iii) If futures prices are uncorrelated with interest rates, then futures prices will equal forward prices.

When interest rates are uncertain, there is no reason for forward prices to equal futures prices. Consider the case when interest rates are not certain and it is known that futures prices and interest rates tend to move in the same direction. Then the long position knows that cash flows generated from an increase in futures prices can more likely be invested at high interest rates, while losses derived from a falling futures price, can more likely be financed at a lower interest rate. In this case the long position in a futures contract is at an advantage relative to an otherwise identical forward contract. Of course, the short position is at a disadvantage, and for a fair transaction to be made one would anticipate that the short position would require a higher price than the forward price. Indeed, to entice traders into selling a futures contract, the futures price must be set higher than the forward price. Similarly, for the case where futures prices and interest rates move in opposite directions, profits from rising futures prices will be invested in a falling interest rate environment, while losses will be financed by borrowing at higher rates. In this case the futures holder is at a disadvantage relative to a forward holder. To entice investors into holding futures then, the futures price must be set lower than the forward price.

In summary, due to interest rate uncertainty, the setting of futures prices may differ from that of forward prices. Indeed, for a futures contract the total cash flow, together with accrued interest depends not only on the behavior of the future spot price but also on the joint behavior of the underlying price with interest rates.

———————

Example

Consider a futures contract on a particular bond. If interest rates move down and then up, the price of the bond will increase, then decrease. The long position will make money in the first day, and invest it at a low interest rate. In contrast, the short position will loose money in the first day but be able to finance this loss at a lower rate. On the second day, the short wins and invests the proceeds at a high rate, while the long looses and has to finance the loss at a higher rate.

Relative to a forward contract, the advantage rests with the short position. Of course, the long position realizes this and requires the futures price to be set a bit lower so as to compensate for this disadvantage. Clearly, the magnitude of this compensation, depends on the sensitivity of bond prices to the daily interest rate. We shall have much more to say about this relationship in future chapters.

In general, differences between futures and forward prices for short term contracts with settlement dates less than 9 months tend to be very small. That is, the daily marking-to-market process appears to have little effect on the setting of futures and forward prices. Moreover, if the underlying asset's returns are not highly correlated with interest rate changes, then the marking-to-market effects are small even for longer term futures. Only for longer term futures contracts on interest sensitive assets will the making-to-market costs be significant. As a result, many studies analyze futures contracts as if they were forwards. In the rest of this chapter we establish pricing relationships for forward contracts, and then use property 4 to make statements about future prices.

Pricing of Forward Contracts on Storable Commodities

Consider an investor who borrows funds to purchase a commodity and caries it over the period $[0, T]$. Assume all dollars required to carry the inventory are borrowed. The amount owed at date T is the initial pricipal, S(0), the accrued interest on the principal, AI(0,T), say, and the future value of all expensed items relating to the inventory, $\pi(0, T)$, say. These include physical storage insurance charges. Some commodities that are stored may provide cash flows over the period prior to the delivery date. For example, if the commodity is a Treasury bond, coupons may be paid, or if it is a stock, then dividends may be paid out. Assume that all of these intermediate cash flows are invested at the riskless rate, until date T. These accrued benefits, G(0,T) say, are then used to reduce the amount owed. Let C(0,T) represent the net funds owed at date T. Then:

$$C(0, T) = S(0) + AI(0, T) + \pi(0, T) - G(0, T) \tag{1}$$

If the trader could arrange to sell the commodity forward for a price that exceeds the net amount owed, then arbitrage free profits could be made.

Example

Consider a trader who borrows $206,000$ to purchase $100,000$ bushels of corn. The corn is stored for use in 3 months time. Storage and insurance charges of $200 are paid at the beginning of each month. These charges are also financed by borrowing. Interest expenses are 10% per year continuously compounded.

The principal amount, together with accrued interest is:

$$S(0) + AI(0, T) = S(0)e^{rT} = (206000)e^{0.10(3/12)} = \$211,215$$

The accrued storage cost is

$$\pi(0,T) = 200e^{r(3/12)} + 200e^{r(2/12)} + 200e^{r(1/12)} = \$610.$$

Since the commodity provides no cash flow over the period, $G(0,T) = 0$ and hence the total amount owed in three months is

$$C(0,T) = S(0) + AI(0,T) + \pi(0,T) = \$211,825.$$

This quantity is known at time 0. If the trader could sell the corn forward for a price that exceeds the net amount owed, then arbitrage free profits can be locked in.

_____

The relationship of forward prices to spot prices prior to settlement is determined by the above *cost of carry* argument. Exhibit 4 shows the profits from the cost of carry strategy.

Exhibit 4: Cost-of Carry Strategy

Position	Cash Flow at date 0	Cash Flow at Date T
Borrow $S(0)$ to Buy the commodity	$S(0)$ $-S(0)$	Pay back loans $-C(0,T)$
and sell forward	-	Sell commodity for Forward Price, $FO(0)$.
Net Cash Flow	0	$FO(0) - C(0,T)$

Hence, to avoid riskless arbitrage, the terminal value should be nonpositive. That is:

$$FO(0) - C(0,T) \leq 0.$$

Substituting the individual components, we have:

$$FO(0) \leq S(0) + AI(0,T) + \pi(0,T) - G(0,T) \tag{2}$$

Property 5

Arbitrage opportunities arise if the forward (futures) price is too high relative to the spot price. In particular, the forward (futures) price should always be bounded above by the spot price plus the net carry charge to the delivery date. That is,

$$FO(0) \leq S(0) + AI(0,T) + \pi(0,T) - G(0,T)$$

Example

In the previous example, the trader purchased $200,000$ bushels at a cost of \$2.06 per bushel. We had $C(0, T) = \$211,825$. If the trader can sell forward at a price that exceeds $211,825/200,000 = \$2.118$, then arbitrage free profits can be locked in.

We have seen that the forward price is bounded above according to the cost of carry. If the forward price equals this upper bound, then the forward price is said to be at *full carry*. In practice the forward price may not be at full carry. This is investigated next.

Reverse Cash and Carry Arbitrage

Consider a commodity, such as a financial asset, that can be sold short at the current market price. Recall that when an asset is sold short, the stock is borrowed and sold. Initial funds are received equal to the market price. If the price drops, the trader can purchase it at the lower price, return the stock, and profit from the difference. Of course, if the stock pays a dividend in the interim, the short seller is responsible for the cash flow.

Now consider the strategy of selling the commodity short, investing the proceeds at the riskless rate, and buying the forward contract. At expiration the asset sold short is returned by accepting delivery on the forward contract. In addition, if any dividends occurred over the period, then the short seller borrows these funds. Let $G(0, T)$ represent the total amount owed from the dividend payments at date T. At date T the trader owes the asset and G(0,T) dollars. The investor pays the forward price, $FO(0)$ dollars to receive the commodity, and uses this transaction to cover the short position. In addition, the investor pays the debt of $G(0, T)$ dollars. Exhibit 5 shows the cash flows from this strategy.

Exhibit 5: Reverse Cash and Carry

Position	Cash Flow at date 0	Cash Flow at Date T
Sell commodity short, invest proceeds at riskless rate	$S(0)$ $-S(0)$	Receive $S(0) + AI(0, T)$
and buy forward	-	Take delivery of asset and return it: $-[FO(0) + G(0, T)]$
Net Cash Flow	0	$S(0) + AI(0, T) - FO(0) - G(0, T)$

Since there is no initial net investment, and since the cash flow at date T is certain, to avoid riskless arbitrage, the final net cash flow should be nonpositive. That is

$$S(0) + AI(0,T) - FO(0) - G(0,T) \leq 0$$

or

$$FO(0) \geq S(0) + AI(0,T) - G(0,T)$$

The reverse cash and carry strategy leads to a lower bound on forward prices.

Property 6

For a commodity that can be sold short, arbitrage opportunities arise if the forward (futures) price is too low relative to the spot price. In particular, the forward (futures) price should always be bounded below by the spot price plus accrued interest. That is,

$$FO(0) \geq S(0) + AI(0,T) - G(0,T)$$

For most commodities that can be sold short, the storage costs are negligible. In this case the lower bound equals the upper bound and forward prices are fully determined by the cost of carry model.

$$FO(0) = S(0) + AI(0,T) - G(0,T)$$

Property 7

For a commodity that has negligible storage costs and can readily be sold short, forward and futures prices should be set at full carry.

Actually, the ability to sell the commodity short is not really essential to prevent forward prices falling below the spot plus carry charge. To see this, consider a trader who owns the commodity and plans on holding it for some time. If the forward price dropped below the spot plus carry charge the investor would sell the commodity, invest the proceeds at the riskless rate and buy the forward contract. At the settlement date the investor would take delivery of the asset. In effect the investor would find this strategy cheaper than storing inventory. If the commodity was in ample supply other traders who held excess inventory would initiate the same strategy. Their activity would continue until forward prices would rise to a level that reflected full carry. This argument suggests that to avoid

riskless arbitrage, forward prices on commodities that are abundantly available and storable should reflect full carry charges.

Property 8

If a commodity is in ample supply, then forward and futures prices should reflect full carry charges. That is,

$$FO(0) = S(0) + AI(0, T) - G(0, T)$$

If the underlying commodity provides no cash flows and incurs no storage charges, then the carry charge just reflects the opportunity cost of funds and:

$$FO(0) = S(0) + AI(0, T) = S(0)R^T$$

If the time to delivery, T, is measured in years, rather than days, then the above forward price can be expressed as

$$FO(0) = S(0)e^{rT}$$

If storage charges are incurred continuously and are proportional to the price of the commodity then the effective cost of carry is increased from the rate r to $r + u$ and

$$FO(0) = S(0)e^{(r+u)T}$$

where u is the storage cost per year expressed as a proportion of the spot price. For storable commodities such as financial securities that do not provide cash flows, and for investment commodities, such as gold, the net cost of carry is usually positive and forward prices will typically lie above spot prices. A market of this type is called a *contango* market.

Finally, if the underlying commodity pays a continuous dividend yield at rate d, the net cost rate of carry in equation is reduced from $r + u$ to $r + u - d$ and

$$FO(0) = S(0)e^{(r+u-d)T}$$

(i) Forward Contract on Gold

Consider a forward contract on gold. For this product the bulk of the carrying charge is interest expense with a negligible mount required for insurance and storage. Assume the spot price of gold is $305. Interest rates are 5 percent and the contract settles in one year. From the cost of carry model, the theoretical futures price is

$$FO(0) = 305e^{0.05} = \$320.64$$

The further the delivery date the greater the interest expense, so typically forward prices exceed spot prices by an amount that increases with maturity.

Forward Contract on a Dividend Paying Stock

Assume the spot price of a stock is $100, a dividend of $5.0 is due in $t_1 = 0.5$ years and interest rates are 5%. Since there are no storage costs, the forward price of a contract that settles in $T = 1$ year is

$$
\begin{aligned}
FO(0) &= S(0)e^{rT} - d_1 e^{r(T-t_1)} \\
&= 100e^{0.05} - 5e^{0.05(0.5)} \\
&= \$100.00
\end{aligned}
$$

Forward Contracts on Foreign Exchange

Assume one British Pound costs $1.60. A one year forward contract is purchased on the pound. The short position has to deliver one pound in exchange for $FO(0)$ dollars. To establish what the theoretical forward price should be assume that interest rates in the US are $r = 5\%$ while British interest rates are $r_F = 8\%$. In order to apply the cost of carry model, we recognise that the storage cost,u,is zero and the dividend yield on the foreign exchange, $d = r_F = 8\%$. Specifically, the purchased pound can be invested at a guaranteed rate of 8% in a British bank. Hence

$$
FO(0) = S(0)e^{(r-r_F)T} = 1.60e^{(0.05-0.08)} = 1.5527
$$

Lets consider this case more carefully. The cost of carry model requires borrowing S(0) dollars to buy one pound. The pound is invested for a year and grows to $1e^{r_F} = 1.0833$ pounds. Assume, at date 0 the trader sold forward 1.0833 pounds at a guaranteed dollar price. The forward price is the price of one British pound, expressed in dollars. The seller of the contract has to deliver FO(0) pounds in exchange for one dollar. Hence each pound is delivered at an exchange rate of $\frac{1}{FO(0)}$ dollars. Since the number of pounds to transact is known, at date 0, and is 1.0833, the total guaranteed dollar value of selling forward is $1.0833/FO(0)$ dollars.

An alternative riskless investment is to place $1.0 into a riskless US account. The value a year later is $e^r = \$1.051$. Hence, to avoid riskless arbitrage, $1.051 = 1.0833/FO(0)$, from which, FO(0) = 1.5527.

Futures Contracts on Price Weighted Stock Indices

Consider a forward or futures contract on a price weighted index, $I(t)$, which is computed by taking the average of the prices of two stocks, A and B, say, and multiplying the resulting value by a multiplier, m. That is

$$
I(t) = \frac{[S_A(t) + S_B(t)]}{2} \times m.
$$

Assume stock A pays a dividend of d_A at time t_A and B pays d_B at time t_B. A futures contract with delivery date T trades on the index. At the settlement date, T, futures prices equal spot prices. In this case, $F(T) = I(T)$. For index futures, settlement is made in cash, rather than by delivering the appropriate amount of stocks in the index. In order to establish the fair futures price prior to the settlement date, we can apply the cost of carry model to this problem.

Consider the strategy of purchasing the index with borrowed funds and selling the futures contract. Unfortunately, the index cannot be bought. However, it is possible to buy $m/2$ shares of A and $m/2$ shares of B. If this is done, then the price of the portfolio should perfectly correlate with the index. However, while the index does not pay dividends the underlying portfolio does. These dividends will be used to reduce the cost of carry and hence will effect the price of the futures contract.

Exhibit 6 summarizes the transactions associated with this strategy. All dividends are invested at the riskless rate until time T.

Exhibit 6

The Cost-of-Carry Model for Futures

	Initial Value	Final Value
Borrow $I(0)$ dollars	$-I(0)$	$-I(0)e^{rT}$
Buy $m/2$ shares of A	$mS_A(0)/2$	$mS_A(T)/2+ me^{r(T-t_A)}d_A/2$
Buy $m/2$ shares of B	$mS_B(0)/2$	$mS_B(T)/2+ me^{r(T-t_B)}d_B/2$
Sell Forward contract	0	$F(0) - I(T)$

To avoid riskless arbitrage the value at time T of this strategy must be nonpositive. That is $V(T) \leq 0$. Therefore we have

$$F(0) - I(T) + \frac{m}{2}[S_A(T) + S_B(T)] + \frac{m}{2}[d_A e^{r(T-t_A)} + d_B e^{r(T-t_B)}] - I(0)e^{rT} \leq 0$$

or

$$F(0) \leq I(0)e^{rT} - \frac{m}{2}[d_a e^{r(T-t_A)} + d_b e^{r(T-t_B)}]$$

If the futures price exceeded the right hand side then an arbitrage opportunity exists. If the futures price is not at full carry than a reverse cash and carry strategy can be created to capture riskless returns. Specifically, the futures contract is purchased and $m/2$ shares of both A and B are sold short, with the proceeds invested at the riskless rate. Since the investor is responsible for the dividend payments, payments are made from the riskless investment. The cash flows from this strategy are shown in Exhibit 7.

Exhibit 7:
The Reverse Cash and Carry Model for Futures

	Initial Value	Final Value
Sell $m/2$ shares of A and	$-\frac{m}{2}[S_A(0)]$	$-\frac{m}{2}[S_A(T)]$ - $\frac{m}{2}[e^{r(T-t_a)}d_A]$
Invest Proceeds in Bonds	$\frac{m}{2}[S_A(0)]$	$\frac{m}{2}[e^{rT}S_A(0)]$
Sell $m/2$ shares of A and	$-\frac{m}{2}[S_B(0)]$	$-\frac{m}{2}[S_B(T)]$ - $\frac{m}{2}[e^{r(T-t_b)}d_B]$
Invest Proceeds in Bonds	$\frac{m}{2}[S_B(0)]$	$\frac{m}{2}[e^{rT}S_B(0)]$
Buy Forward Contract	0	$I(T) - F(0)$

The initial value of the portfolio is 0. Hence, to avoid riskless profits $V(T) \leq 0$, which implies

$$F(0) \geq I(0)e^{rT} - \frac{m}{2}[d_A e^{r(T-t_a)} + d_b e^{r(T-t_b)}]$$

The above results imply that a futures contract on an index, that corresponds to traded securities, should be at full carry.

_____

Example: Stock Index Arbitrage with a Price Weighted Index

A stock index is computed as the price weighted average of two stocks, A and B. A futures contract trades on the index. The contract settles in 60 days. Current interest rates are 10%. The following information is available.

	A	B
Price	\$100	\$200
Dividend	1.50	2.00
Time to Dividend	10 days	15 days

The index price, $I(0)$, is computed as $I(0) = (100 + 200)/2 = 150$. To compute the theoretical futures price, we first compute the total carry charge from purchasing the stocks underlying the index with borrowed funds. The initial investment is \$300. The amount owed in 60 days is

$300e^{0.10(60/365)} - 1.50e^{0.10(50/365)} - 2.00e^{0.10(45/365)} = \$301.42$.

The futures price is expressed in units which is related to the actual index price. In this example the index value is one half the total value of the stock prices in the index. Hence, the theoretical futures price is 150.71.

Say the actual futures price is 153.71. Then an index arbitrage opportunity would become available. In particular, the arbitrageur would sell two overpriced futures contract, and initiate the program trade for the two underlying assets. The funds for the purchase of the assets would be obtained by borrowing. The sequence of events are shown in Exhibit 8.

Exhibit 8: Pricing Stock Index Futures

	Cash Market	Futures Market
Day 0	Borrow $300, and buy A and B.	Sell 2 overpriced futures at futures price of 153.71.
Day 10	Receive Dividend. Invest it for 50 days at 10%.	
Day 15	Receive Dividend. Invest it for 45 days at 10%.	
Day 60	Sell A and B for $S_A(T) + S_B(T)$ and repay the net amount owed which is 301.42.	At settlement, futures price is $[S_A(T) + S_B(T)]/2$. Profit on the sale of two futures contracts is $307.42 - [S_A(T) + S_B(T)]$.
	Profit $= S_A(T) + S_B(T) - 301.42$	Profit $= 307.42 - [S_A(T) + S_B(T)]$

Total profit is therefore $307.42 - 301.42 = \$6.0$, which is captured regardless of the future index level.

Pricing Futures Contracts on Value Weighted Indices

Consider the PR3 Index. It is a value weighted index of 3 stocks, A, B and C. The current information on the stocks are shown in Exhibit 9.

Exhibit 9:
Composition of the PR3 Index

Stock	A	B	C
Price	40	35	25
Shares Outstanding	1 million	2 million	2 million
Market Value	40 million	70 million	50 million
dividend	$0.5	$0.5	-
Time to dividend	10 days	12 days	-

Given the market values, the composition of the index is $40/160 = 4/16$ in stock A, $70/160 = 7/16$ in B and $50/160 = 5/16$ in C. The current index is 400. Consider a futures contract

on this index. The multiplier of the contract is 500. Therefore, the contract controls $400 \times 500 = \$200,000$. Of this amount $(4/16)200,000 = \$50,000$ is in $A, (7/16)200,000 = \$87,500$ is in B, and the remaining $(5/16)200,000 = \$62,500$ in C. The initial replicating portfolio consists $50,000/40 = 1,250$ shares of $A, 87,500/35 = 2,500$ shares of B, and $62,500/25 = 2500$ shares of C.

Assume the futures contract settles in 30 days, and interest rates are 10%. The calculations for the theoretical futures price are shown below.

Stock	A	B	C
Borrow Funds	50,000	87,500	62,500
Number of Shares Bought	1,250	2,500	2,500
Dividend Income Received and reinvested at 10% until settlement date	628.43	1256.18	-

The amount owed after 30 days is $200,000e^{-0.10(30/365)} = 201,650.61$. The net amount owed, after adjusting for dividends, is therefore $201,650.61 - 1884.61 = 199,766$. The theoretical futures price is therefore $199,766/500 = \$399.53$.

_____

The stock index arbitrage strategy just described ensures that futures prices do not deviate to far from the implied net cost of carry. However, in the analysis we made several simplifying assumptions. First, we assumed that transaction costs could be ignored. While computer entry systems, such as DOT, has made the trading of portfolios more efficient, stock index arbitrageurs estimate the total round trip costs to be at least 0.5% of the portfolio value. Second, the assumption that index prices and replicating portfolio prices are exactly equal is not precise. Third, the size and timing of all dividends on all stocks in the index was assumed to be known with certainty. While this assumption may be valid for some short term contracts, any uncertainty in dividends will impact the arbitrage scheme. In addition, the reverse cash-and-carry strategy requires selling short stocks in the index. Such a strategy implicitly assumed that the proceeds of the sales from selling stock short would be immediately available for investment at the riskless rate. In fact, margin is required for short sales, and these funds may not be available to the investor. Moreover, short selling all stocks in the index may be difficult or may take some time. Specifically, stocks can only be sold short after an uptick in the stock price. Finally, certain rules, such as circuit breakers, can impede arbitrage.

As a result of the above real world considerations, the futures price on an index may not be at full net carry. Indeed, there exist price bands such that no riskless arbitrage strategies exist as long as the futures price trades within the band. Only when the futures price moves outside this range will it become possible for arbitrageurs to initiate cash-and-carry or reverse cash-and-carry arbitrage. Implementing systems that actually monitor the stock index futures-stock portfolio relationship requires that these features be considered.

For futures prices, the cost-of-carry model, modified for real world imperfections such as transaction charges, works extremely well. Exhibit 9 shows the futures prices and spot prices. Around the futures prices are theoretical price bounds generated using the cost-of-carry model. If the actual futures price drifted out the theoretical band, arbitrage between the two markets would be possible. Notice that the actual futures price almost never deviates outside the band.

Convenience Yields on Commodities

So far we have assumed that the commodity could be sold short or that it was in abundant supply. In this case, to avoid arbitrage, the forward price would reflect the full carry charge. If short sales are not possible then the reverse cash and carry arbitrage cannot be initiated. In this case the futures price has no lower no-arbitrage bound and prices may not be set at full carry. Of course, if the underlying commodity is in ample supply, producers with large inventories may temporarily relinquish their inventories and purchase forward contracts, and their activity will prevent prices deviating significantly from the full carry price. However, if the commodity is not abundant, then traders may be reluctant to temporarily relinquish their inventories, and futures prices may not reflect full carry.

Examples of commodities that should reflect full carry include all securities that are held for investment purposes such as stocks, bonds and certain precious metals such as gold. The supply of these securities may fluctuate slightly over time, but these fluctuations are small compared to outstanding inventories. The majority of commodities, including all agricultural commodities, such as corn and wheat, and metals, such as copper and zinc, are held for consumption purposes. The supply and demand for these commodities fluctuates over time and inventories expand and shrink in response. For some commodities, such as agricultural commodities, production is seasonal, but consumption is fairly steady. For other goods, such as heating oil, consumption is seasonal while production is continuous. In times of scarcity, users of such commodities may be reluctant to relinquish their inventories on a temporary basis. By having inventory readily available, production processes can be maintained despite local shortages or profits can be generated from local price variations that arise when shortages emerge. The owner of inventory has valuable claims that provide the right to liquidate inventory, contingent on price and demand fluctuations. By temporarily relinquishing the inventory these claims are lost. The benefits from owning the physical commodity provides the owner with a yield that is called the *convenience yield.* The size of the convenience yield will clearly depend on factors such as the current aggregate levels of inventory, the current supply and demand characteristics and upon projected future supplies and demands. In addition the convenience yield will depend on risk-reward preferences, with greater inventories being required as risk aversion in the economy increases. Let $k(0,T)$ represent the size of the convenience yield, measured in dollars, accrued up to time T. In general, then, the futures price on a commodity can be written as

$$F(0) = S(0) + AI(0,T) + \pi(0,T) - G(0,T) - K(0,T)$$

where $AI(0,T)$ and $\pi(0,T)$ are the interest expenses on the initial borrowed funds and the future value of all expensed storage and insurance costs, $G(0,T)$ is the future value of all explicit cash flows derived from the commodity, and $K(0,T)$ is the accrued convenience yield over the period.

Property 9

For commodities that possess a convenience yield, forward and futures prices are not at full carry.

Recall, if storage costs were assumed to be continuous and proportional to the spot price, then, with no dividends, or convenience yields, forward prices are given by:

$$FO(0) = S(0)e^{(r-u)T}$$

If the convenience yield per unit time is also a constant proportion of spot price, then we could define κ as the net convenience yield. Then forward prices are linked to spot prices by:

$$F0(0) = S(0)e^{(r-\kappa)T}$$

If the continuous convenience yield is larger than the storage cost, κ is positive. If $\kappa > r$, then forward prices will be lower than the spot price.

If the forward price is observable the above equation can be solved to obtain the *implied* convenience yield. Specifically, we obtain

$$\kappa = [\ln[S(0)/F(0)] + rT]/T$$

The implied convenience yield provides the forward market's consensus forecast of the convenience yield.

Example

The futures prices of a storable commodity are shown below. The carry charge for the commodity is 1% of the spot price per year ($u = 0.01$) and interest rates are 9%. The current time is April, 1 month prior to the May settlement date. The spot price is 1.96.

Settlement Date	May	July	Sept	Dec	March
Futures Price	1.95	1.92	1.87	1.89	1.89

The annualized convenience yields are shown below.

Settlement Date	May	July	Sept	Dec	March
Upper Bound (using cost-of-carry model)	1.976	2.0096	2.043	2.095	2.148
Convenience Yield	16.13	18.24	21.28	15.45	13.96

Notice that by buying a July contract and selling a September contract, the trader is committing to purchase at \$1.92 and sell 2 months later at \$1.87. Using the cost-of -carry model we have

$$F_T(t) = F_S(t)e^{(r+u-\kappa)(T-S)}$$

where S is the near term futures delivery date (July), T is the far term delivery date date (September), and κ is the annualized convenience yield over the time period $[S, T]$. The implied convenience yield over this period can be computed. For example, the annualized implied convenience yield over the July-September period is 25.83%. The implied convenience yields over the successive time periods are shown below

Time Period	April-May	May - July	July-Sept	Sept-Dec	Dec-March
Implied Convenience Yield	16.13	19.29	25.83	5.74	10.0

Given the implied convenience yields it appears that the benefits of holding inventories are greatest over the July-September time period and are least beneficial over the September-December period.

_____

The Term Structure of Futures Prices and Basis Risk

Futures prices for consumption commodities could increase or decrease as settlement dates increase. A downward sloping term structure of forward prices is especially likely if demand for the commodity is very high and current supplies are limited. In this case the convenience yield derived by having the commodity in inventory is extremely high. The term structure of futures prices need not be a monotone function. This often occurs when the futures price for the last delivery month of one marketing year, with its own supply and demand mechanisms, are quite different from the futures price for the first delivery month in the next marketing year. The basis of some contracts may be negative while those of other contracts could be positive. For example if a crop is in abundant supply from May to September then futures prices over this period could be monotone increasing, reflecting full carry. However, from September to December a sharp drop off in prices could

occur relecting the fact that future yields are uncertain and shortages might occur, and convenience yields are high.

In general, the magnitude of the basis is determined by the carry charge to the settlement date and the convenience yield. If these charges remain stable over time, then the basis should converge smoothly to zero. However, if the carry charge or convenience yields change unexpectedly, then the basis will deviate from its smooth path towards zero. The fact that the basis can deviate from its projected path introduces basis risk. We shall return to basis risk in the next chapter.

Futures Prices for Non-Storable Commodities

The cost-of-carry model requires that the underlying commodity is storable. We now consider the behavior of prices on non storable commodities. As an example, consider the futures price of electricity. The futures price cannot be directly linked to the spot price through the cost-of-carry model because electricity cannot be stored. In this section we investigate how the futures price is determined. To make matters specific, assume that the market's consensus was that the future spot price at the delivery date would be $100. More precisely, based on all information available, the expected future spot price is $100. If the futures price were significantly lower than $100 (say $90), then some speculators would consider buying the contract with the expectation of making *a* $10 profit. As the discrepancy between the two prices increases, their participation would increase. Indeed, their very activity would prevent prices deviating "too far" from the expected price.

If there is terrific uncertainty about the futures price, then the speculator may not purchase the futures contract, even if its price is below the expected futures price. If, for example, the futures price were $99, then speculators may not pursue the $1 expected profit. Indeed, to entice speculators to bear the risk of a position in the futures, they must be compensated with an appropriate risk premium. As a result, a simple relationship between futures prices and expected future spot prices may not exist.

Speculators, Hedgers and Futures Prices

Speculators are traders that are prepared to bear risk in return for which they expect to achieve an appropriate compensation. As a result, speculators will only buy (sell) futures contracts, if they expect futures prices will increase (decrease). Hedgers, on the other hand, are prepared to pay a premium to lay off unwanted risk to speculators. If hedgers, in aggregate, are short, then speculators are net long, and, in this case the futures price will be set below the future expected spot price. This situation is referred to as *normal backwardation*. In contrast, if hedgers are net long, speculators will be net short, and futures prices will be set above the the expected future spot price. This situation is known as *contango*.

Of course, the net position of hedgers may change over time. When the contract begins trading, the hedgers may be net short. For example, when a corn futures contract is introduced, farmers may attempt to lock in prices for their crop by selling short. In such

a case, the futures price lies below the expected future spot price. Over time, the hedgers gradually offset their positions, and food processors, for example, begin to lock in prices by buying long. As the hedging imbalance moves from net short to net long, the futures price moves from being below the expected spot price to being above the expected spot price.

A price process is said to be a *martingale* if the expectation of its future value equals its current value. If futures prices were martingales, then,

$$E_0\{F(t)\} = F(0) \text{ for all } 0 \le t \le T$$

In particular, with $t = T$,

$$E_0\{F(T)\} = E_0\{S(T)\} = F(0)$$

Hence, if futures prices are martingales then the futures price is an unbiased estimator of the expected future spot price. If, on the other hand, if the market is a normal backwardation or contango market than prices will not be martingales.

From the above discussion there is no real reason to suspect that futures prices should be martingales. Nonetheless, a substantial number of statistical studies have been performed to test the martingale hypothesis. Many of the early tests were unable to reject this hypothesis. One reason for this is that the tests lacked statistical power, and hence required overwhelming evidence to reject. Nonetheless, these early results have spawned many additional studies, some of which have shown that futures prices may not provide good estimates of future spot prices.

The Volatility of Futures Prices

The current futures price reflects current information about the spot price at delivery time. As information is revealed, futures prices will change. The volatility of price changes clearly is related to the quality and quantity of information revealed over time. If information is revealed more quickly as the delivery date approaches, then one might expect futures prices to show increasing volatility as the maturity approaches. Empirical studies have shown this type of behavior holds for metal futures, such as gold and copper. For other commodities, the volatility of prices may be seasonal. This is particularly true if important information is revealed over short periods of time. For example, rainfall considerations at crucial points in time clarify the supply of agricultural products, and the severity of a winter month clarifies the demand for heating oil. Such seasonality in prices should be less pronounced for financial contracts. However, even there seasonal variability has been uncovered. For example, the volatility of some financial products seem to be larger on Mondays, then on other days of the week. Volatility also seems to be related to volume. Namely, as uncertainty unfolds, volume of trade accelerates in response to expanding volatility.

Conclusion

This chapter has developed the basic principles for pricing futures and forward contracts. Throughout this chapter we have assumed that interest rates are constant. As a result,

futures prices and forward prices are identical. This does not mean that the two contracts are equivalent. However, when interest rates are certain, the payoffs from forward contracts can be replicated by a dynamic trading strategy involving futures. In the next chapter we shall see how futures and forward contracts can be used by traders to offset unwanted risks.

The cost-of-carry model and the reverse cash-and-carry model show how futures prices must be linked to their underlying spot prices. In the development of these pricing relationships we assumed that markets were perfect. Of course this is not the case. Transaction costs, unequal borrowing and lending rates, margins and restrictions on short selling and limitations to storage are four imperfections that can affect the pricing relationships. Nonetheless, the cost-of-carry model does capture the most important components of the basis. For commodities that cannot be stored, the expected futures price framework provides a procedure for thinking about futures prices. Finally, the volatility of prices is the greatest in time periods in which information dissemination on the underlying commodity is the most rapid.

In this chapter we have seen how the prices of futures contracts are closely linked to the prices of their underlying commodities. Futures prices react rationally to information that is revealed about the underlying commodity. In the next chapter we shall see how hedgers can use these contracts to manage price risk.

References

Cox, J., J. Ingersoll, and S. Ross "The Relation Between Forward and Futures Prices" *Journal of Financial Economics*, 9, Dec. $1981, 321 - 346$

Fama, *E* and K. French "Commodity Futures Prices: Some Evidence on Forecast Power, Premiums,and the Theory of Storage", *Journal of Business,*Vol.$60, 1987, 55 - 73$.

Jarrow, R. and G.Oldfield, " Forward Contracts and Futures Contracts", *Journal of Financial Economics,* 9, Dec. $1981373 - 382$.

Kane, E. "Market Incompleteness and Divergences between Forward and Futures Interest Rates", *Journal of Finance,* Vol 35, May $1980, 221 - 234$

Kolb R. "Understanding Futures Markets", Kolb Publishing, 1990.

Park, H. and A. Chen " Differences Between Futures and Forward Prices: A Further Investigation of Marking to Market Effects", *Journal of Futures Markets,* Vol 5, Feb.$1985, 77 - 88$

Rendleman, R. and C. Carabini, "The Efficiency of T-Bill Futures Markets", *Journal of Finance,*Vol 44, Sept. $1979, 895 - 914$

Richard S. and M. Sundaresan, "A Continuous Time Equilibrium Model of Forward Prices and Futures Prices in a Multigood Economy", *Journal of Financial Economics,* Dec. $1981, 347 - 371$.

Samuelson P. "Proof that Properly Anticipated Prices Fluctuate Randomly", *Industrial Management Review,* Vol $6, 1965, 41 - 49$.

Turnovsky, S. "The Determination of Spot and Futures Prices with Storage Commodities", *Econometrica,* Vol. $51, 1983, 1363 - 1387$.

Working, H. "The Theory of Price of Storage ", *American Economic Review* $39, 1949, 1242-1262$.

Exercises

(1) In January, firm ABC enters into a long position in a forward contract with firm DEF as a counterparty. The contract requires delivery in nine months time. The forward price is set at $100 per unit and 1000 units are involved in the transsaction. Three months later DEF is offering 6 month contracts at a forward price of $80. At this time ABC realizes that it does not need the underlying commodity in the future and is keen to negotiate a price with DEF to terminate the contract. If interest rates at 10% per year, continuously compounded,what is the fair compensation ABC should pay DEF?

(2) A farmer currently holds 5000 bushels of corn. The local mill is offering a price of 2.18a$ bushel. Currently, a 3 month futures contract is trading at $2.24. The farmer is considering selling to the local mill or holding the corn in inventory and selling a futures contract. The farmer can store and insure the corn at a total cost of 1 cent per bushel per month. Payment for this cost is due up front, at date 0. Interest rates are 10%, continuously compounded. Which alternative should the farmer pursue? Are there any additional factors that need to be considered?

(3) Develop a pricing relationship that links the nearby futures price of gold to a more distant futures price. Should this relationship stay stable over time? If the futures contract was a commodity such as wheat, which is produced seasonally, but consumed steadily,would the relationship be stable over time? Explain.

(4) A forward contract is entered into on a non dividend paying stock. The stock price is $100, and the settlement date is 1 year. The interest rate is 10%.
(a) Compute the forward price.
(b) After 7 months, the stock price is $60. What is the value of the forward contract?

(5) The spot price of gold is $400. Interest rates are 10%.
(a) Use the cost of carry model to establish the futures price of a 1 year contract.
(b) If transaction costs are introduced into the analysis, then the cost of carry model and the reverse cash and carry model arguements need to be modified. In this case bounds on the futures price can be obtained. Reconsider the above problem under the assumption that the transaction cost for buying or selling an ounce of gold is $1.0 and the round trip futures trading cost is $25 per contract. In particular, develop the appropriate bounds on the futures price. (Note, that a round trip cost, is the total commission cost associated with buying and then selling a contract, or selling then buying the contract. Assume this cost is paid up front.
(c) Reconsider the above problem but now add in another market imperfection. Specifically, assume the borrowing rate is 10%, but the lending rate is only 8%. Establish the new bounds on the futures price.

(6) Assume gold trades at $400.$A$1 year futures contract has a price of $420. What is the implied cost of carry rate?

(7) A stock pays a $1 dividend in 3 months time, and a second $1 in 6 months. The stock

price is $40. The riskfree rate is 10%.

(a) Compute the fair forward price of a contract that requires delivery in 7 months time.

(b) Four months later, the stock price is $30. What is the new forward price and what is the value of the original forward contract.

(8) The current price of silver is $8 per ounce. Storage costs are $0.10 per ounce per year. Payments are expected in two installments, half the total cost now, with the remaining balance due in 6 months. Interest rates are constant at 8%. Compute the fair futures price.

(9a) If the futures price of a commodity is greater than the spot price during the delivery period, there is an arbitrage opportunity. Is this true? If so, construct an appropriate strategy.

(b) If the futures price of a commodity is lower than the spot price during the delivery period, there is an arbitrage opportunity. Is this true? If so, construct an appropriate strategy.