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Math

Reteach Workbook

PUPIL EDITION
Grade 6

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California Edition

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Estimate with Whole Numbers

You can use compatible numbers to estimate a quotient. **Compatible numbers** are helpful to use because they divide without a remainder, are close to the actual numbers, and are easy to compute mentally.

Oakdale Middle School is collecting recycled cans. The school has set a goal of collecting 2,788 cans. There are 38 homerooms in the school. About how many cans should each homeroom collect for the school to reach its goal?

Because you are asked "about how many," an estimate is appropriate for the answer. Use compatible numbers to estimate.

Step 1: Look at the actual numbers that make up the problem. Think about numbers that are close to the real numbers that will divide without a remainder.

$2{,}788 \div 38$ → $2{,}800 \div 40$

Step 2: Divide. → $2{,}800 \div 40 = 70$

So, each homeroom should collect about 70 cans.

Complete to show how compatible numbers are used to estimate the quotient.

1. $2{,}615 \div 47 \rightarrow 2{,}500 \div \underline{\quad} = \underline{\quad}$ 2. $3{,}104 \div 62 \rightarrow 3{,}000 \div \underline{\quad} = \underline{\quad}$

3. $3{,}591 \div 88 \rightarrow \underline{\qquad} \div 90 = \underline{\quad}$ 4. $4{,}733 \div 74 \rightarrow 4{,}800 \div \underline{\quad} = \underline{\quad}$

5. $7{,}105 \div 77 \rightarrow \underline{\qquad} \div 80 = \underline{\quad}$ 6. $5{,}511 \div 62 \rightarrow 5{,}400 \div \underline{\quad} = \underline{\quad}$

7. $15{,}843 \div 381 \rightarrow 16{,}000 \div \underline{\quad} = \underline{\quad}$ 8. $20{,}972 \div 287 \rightarrow 21{,}000 \div \underline{\quad} = \underline{\quad}$

9. $29{,}100 \div 307 \rightarrow \underline{\qquad} \div 300 = \underline{\quad}$ 10. $95{,}347 \div 795 \rightarrow 96{,}000 \div \underline{\quad} = \underline{\quad}$

Use compatible numbers to estimate the quotient.

11. $434 \div 68$ ____   12. $394 \div 5$ ____   13. $448 \div 15$ ____   14. $986 \div 102$ ____

15. $627 \div 89$ ____   16. $554 \div 63$ ____   17. $293 \div 31$ ____   18. $705 \div 97$ ____

19. $1{,}246 \div 43$ ____   20. $2{,}779 \div 28$ ____   21. $3{,}896 \div 38$ ____   22. $7{,}164 \div 78$ ____

Use Addition and Subtraction

Before you add or subtract, you can estimate to check for reasonableness.

Add. 599 + 462 + 630 + 348

Round to the nearest hundred.

599	→	600
462	→	500
630	→	600
+ 348		+ 300
		2,000

Now find the exact sum.

599
462
630
+ 348
2,039

Because 2,039 is close to the estimate of 2,000, the answer is reasonable.

Subtract. 1,347 − 482

Round to the nearest hundred.

1,347	→	1,300
− 482	→	− 500
		800

Now find the exact difference.

1,347
− 482
865

Because 865 is close to the estimate of 800, the answer is reasonable.

Complete each estimate. Then find the exact sum or difference.

1.

1,325	→	1,000
3,882	→	4,000
+ 12,356	→	+ 12,000

2.

948	→	950
− 63	→	− 60

3.

84,638	→	80,000
− 36,903	→	− 40,000

4.

524	→	500
2,833	→	2,800
109	→	100
+ 326	→	+ 300

Solve by using addition and subtraction. Estimate first.

5. The weight of the steel in the Statue of Liberty is 125 tons. The total weight of the statue is 225 tons. How much do the nonsteel materials weigh?

6. In a recent year, 19,410 U.S. students were studying in Britain, 7,872 in France, 4,715 were in Mexico, and 1,257 were studying in China. How many U.S. students were studying abroad in those four countries?

Use Multiplication and Division

Mr. Rivera's class has collected 1,272 rocks. The students are using egg cartons to hold the rocks. If a dozen rocks fit in each carton, how many cartons are needed for the collection?

Paul did some calculations to solve this problem. This is what he did.

$$
\begin{array}{r}
106 \\
12\overline{)1{,}272} \\
-12 \\
\hline
07 \\
-\ 0 \\
\hline
72 \\
-72 \\
\hline
0
\end{array}
$$

← (1 × 12)

← (0 × 12)

← (6 × 12)

- Since 12 is greater than 1, the first digit appears in the hundreds place. Paul divided the 12 hundreds.

- Paul brought down the 7 tens. Since 12 > 7, he wrote 0 in the quotient.

- Paul brought down the 2 ones. He divided the 72 ones.

The class needs 106 egg cartons to hold the collection.

Multiply or divide.

1. 399
      ×171

2. 824
      × 32

3. 1,440
      ×   78

4. 17$\overline{)3{,}553}$

5. 35$\overline{)7{,}210}$

6. 28$\overline{)1{,}736}$

7. 38$\overline{)11{,}666}$

8. 46$\overline{)5{,}060}$

9. 68$\overline{)34{,}204}$

10. 32$\overline{)13{,}472}$

11. 43$\overline{)21{,}973}$

12. 52$\overline{)12{,}740}$

Problem Solving Strategy

Predict and Test

Looking for clues in a problem can help you find its answer. You can use the clues to help you guess and check different answers until you find the right one.

Valley Middle School is holding a canned food drive. Sixth-grade students have collected 150 more cans than seventh-grade students. Together, the students in both grades have collected a total of 530 cans. How many cans did the sixth graders collect? How many cans did the seventh graders collect?

Step 1: Think about what you know.
• You are asked to find the number of cans collected by each grade.
• You know the total number of cans collected and how many more cans the sixth graders collected than the seventh graders.

Step 2: Plan a strategy to solve.
• Use the *predict and test* strategy.
• Use these clues: total cans collected is 530; the difference between amounts collected by sixth and seventh graders is 150.

Step 3: Solve.
• Use a table to record your predictions and tests. Try to predict in an organized way to help you get closer to the exact answer.

PREDICT		TEST		
Sixth Graders	Seventh Graders	Clue 1: The sum is 530.	Clue 2: The difference is 150.	
330	200	$330 + 200 = 530$ ✓	$330 - 200 = 130$ ⊗	← Difference is too low.
350	180	$350 + 180 = 530$ ✓	$350 - 180 = 170$ ⊗	← Difference is too high.
340	190	$340 + 190 = 530$ ✓	$340 - 190 = 150$ ✓	← Both clues are satisfied.

Use the strategy *predict and test* with a table to help you solve.

1. In the problem above, what if the sixth graders had collected 120 more cans than the seventh graders? How many cans would each grade have collected?

2. Tony collected 85 cans of either soup or fruit. He collected 15 more cans of soup than of fruit. How many cans of soup did he collect? How many cans of fruit did he collect?

Algebra: Use Expressions

Writing *numerical expressions* and *algebraic expressions* requires translating words into numbers and symbols. You can do this by looking for key words.

Addition	Subtraction	Multiplication	Division
sum	difference	product	quotient
increase	decrease	factors	equally divided
more than	less than	times	divided by
plus	minus	multiplied by	

- Write a numerical expression for "nine increased by seven squared."

  nine increased by seven squared

  9   + 7^2

  A numerical expression contains only numbers.

- Write an algebraic expression for "nine increased by a number, *n*."

  nine increased by a number, *n*

  9   +   *n*

  An algebraic expression contains a variable.

Identify each as a *numerical expression* or *algebraic expression*. Explain your answer.

1. $3 + 6$ **2.** $9 - d$ **3.** $24 \div 12$

_____ _____ _____

_____ _____ _____

Write a numerical or algebraic expression for the word expression. Remember to look for key words.

4. twenty-four less than a number, *k*   **5.** twelve increased by seventeen

_____ _____

6. the product of seven and twenty   **7.** thirty-eight divided by a number, *m*

_____ _____

Algebra: Mental Math and Equations

You can use the number facts you know to help solve equations.
Remember, you can use fact families to find missing numbers.

• Solve the equation $8 + x = 12$ by using mental math.

Related fact: $12 - 8 = 4$

So, $8 + 4 = 12$.

$8 + x = 12$

$\quad x = 4$ *The solution is 4.*

Check to be sure your answer is correct.

$\quad 8 + 4 = 12$ *Replace x with 4.*

$\quad\quad 12 = 12$ *8 + 4 is equal to 12.*

• Solve the equation $y \times 7 = 35$ by using mental math.

Related fact: $35 \div 7 = 5$

So, $5 \times 7 = 35$.

$y \times 7 = 35$

$\quad\quad y = 5$ *The solution is 5.*

Check to be sure your answer is correct.

$\quad 5 \times 7 = 35$ *Replace y with 5.*

$\quad\quad 35 = 35$ *5 × 7 is equal to 35.*

• Solve the equation $m - 9 = 8$ by using mental math.

Related fact: $9 + 8 = 17$.

So, $17 - 9 = 8$.

$m - 9 = 8$

$\quad m = 17$ *The solution is 17.*

Check to be sure your answer is correct.

$\quad 17 - 9 = 8$ *Replace m with 17.*

$\quad\quad 8 = 8$ *17 − 9 is equal to 8.*

• Solve the equation $d \div 6 = 8$ by using mental math.

Related fact: $8 \times 6 = 48$

So, $48 \div 6 = 8$.

$d \div 6 = 8$

$\quad\quad d = 48$ *The solution is 48.*

Check to be sure your answer is correct.

$\quad 48 \div 6 = 8$ *Replace d with 48.*

$\quad\quad 8 = 8$ *48 ÷ 6 is equal to 8.*

Solve each equation by using mental math.

1. $a - 5 = 9$

Related fact:

$9 + 5 =$ _____

The solution is _____

2. $\dfrac{k}{5} = 5$

Related fact:

$5 \times 5 =$ _____

The solution is _____

3. $3r = 18$

Related fact:

$18 \div 3 =$ _____

The solution is _____

4. $10 + f = 15$

_____

5. $n \div 12 = 3$

_____

6. $8s = 64$

_____

7. $w - 20 = 10$

_____

8. $n \times 9 = 81$

_____

9. $x + 9 = 16$

_____

10. $\dfrac{m}{10} = 10$

_____

11 $g - 30 = 6$

_____

12. $16 = c \div 100$

_____

Use the Properties

One way to find a sum or product mentally is to use a number property.

Commutative Property
Numbers can be added in any order
without changing the sum.
$45 + 29 + 55 = 29 + 45 + 55$
         ↑ ↑
    Order has been changed.

Associative Property
Addends can be grouped differently.
The sum is always the same.
$(45 + 29) + 55 = 29 + (45 + 55)$
             ↑   ↑
    Grouping has been changed.

Numbers can be multiplied in any order
without changing the product.
$5 \times 13 \times 8 = 5 \times 8 \times 13$
         ↑ ↑
    Order has been changed.

Factors can be grouped differently.
The product is always the same.
$(5 \times 13) \times 8 = 5 \times (8 \times 13)$
             ↑   ↑
    Grouping has been changed.

Distributive Property
$25 \times 23 = 25 \times (20 + 3) = (25 \times 20) + (25 \times 3)$
      ↑   ↑   ↑   ↑
    Product of a number   Sum of two products
    and a sum

---

1. Complete to show how to find the sum. Name the reason for each step.

$19 + 45 + 21 + 5 = 19 + \underline{\quad+\quad} + 5$ → __Commutative Property__

$= (\underline{\quad\quad}) + (\underline{\quad\quad})$ → _____

$= \underline{\quad+\quad}$ → _____

$= \underline{\quad\quad\quad}$

Add. Use mental math.

2. $16 + 9 + 24$

3. $33 + 26 + 17 + 44$

4. $21 + 14 + 29 + 36$

_____   _____   _____

Complete to show how to use the Distributive Property to find each product.

5. $8 \times 14 = 8 \times (\underline{\quad} + 4)$

$= (8 \times \underline{\quad}) + (8 \times \underline{\quad})$

$= \underline{\quad} + \underline{\quad}$

$= \underline{\quad\quad\quad}$

6. $9 \times 34 = 9 \times (30 + \underline{\quad})$

$= (9 \times \underline{\quad}) + (9 \times \underline{\quad})$

$= \underline{\quad} + \underline{\quad}$

$= \underline{\quad\quad\quad}$

Exponents

Powers of numbers can be written in exponent form. An *exponent* shows how many times a number called the *base* is used as a factor.

exponent

$10^5 = \underbrace{10 \times 10 \times 10 \times 10 \times 10}_{} = 100{,}000$

base   factors

10 used as a factor 5 times

exponent

$2^6 = \underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2}_{} = 64$

base   factors

2 used as a factor 6 times

Joan is asked to express 64 using an exponent and the base 4.

Step 1 The base is 4. So, Joan must find equal factors.

$$8 \times 8 \qquad = 64$$
$$4 \times 2 \times 2 \times 4 \quad = 64$$
$$4 \times \quad 4 \quad \times 4 \quad = 64$$

Step 2 Joan writes the base. Then she counts how many times it is used as a factor.

$4^3 \leftarrow$ used as a factor 3 times
↑
base

So, $64 = 4^3$.

Write in exponent form.

1. $4 \times 4 \times 4$ _____

2. $3 \times 3 \times 3 \times 3 \times 3$ _____

3. $10 \times 10 \times 10 \times 10 \times 10 \times 10$ _____

4. $30 \times 30 \times 30 \times 30$ _____

Express with an exponent and the base given.

5. 36, base 6

6. 32, base 2

7. 81, base 3

_____

_____

_____

8. 625, base 5

9. 343, base 7

10. 128, base 2

_____

_____

_____

11. 512, base 8

12. 1,000,000, base 10

13. 121, base 11

_____

_____

_____

Algebra: Order of Operations

When an expression involves more than one operation, you use the order of operations to evaluate it.

1. Operate inside parentheses.
2. Clear exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

What is the value of $2 \times 3 + (6 - 2) \div 2$?

Step 1 Simplify inside parentheses.
Step 2 There are no exponents.

Step 3 Multiply and divide in order from left to right.

Step 4 Add and subtract in order from left to right.

$$2 \times 3 + \underline{(6 - 2)} \div 2$$
$$\downarrow$$
$$2 \times 3 + \quad 4 \quad \div 2$$
$$\downarrow \qquad\qquad \downarrow$$
$$6 \quad + \qquad 2$$
$$\downarrow$$
$$8$$

Evaluate the expression.

1. $5 + 4 + 7 \times 3$

$5 + 4 +$ _____

_____ + _____

_____

2. $28 \div 4 - (4 + 3)$

$28 \div 4 -$ _____

_____ − _____

_____

3. $2^2 + 16 - 8$

_____ $+ 16 - 8$

_____ $- 8$

_____

4. $(6 + 4)^2 \times 8$

(_____$)^2 \times 8$

_____$\times 8$

_____

Find the value of each expression. From the box at the right, choose the letter that corresponds to the value.

5. $(5 \times 6) \div (4 + 2)$ _____

6. $10 + 15 - 9 \times 2$ _____

7. $3^2 + 1 - (18 - 9)$ _____

8. $(4 + 8) \times 3 \div 6 + 10$ _____

9. $2 + 5^2 - (4 + 3)$ _____

10. $4 \div 2 \times 6 - 3 + 3^2$ _____

A	1
B	5
C	7
D	10
E	16
F	18
G	20
H	32

Problem Solving Skill

Sequence and Prioritize Information

Brian wants to bake some cookies for his class party. He knows he will need to mix the ingredients, preheat the oven, assemble the ingredients, cool the cookies, and bake the cookies, but he is not sure how to begin. The order in which Brian does the steps is important. You can help Brian decide what to do.

Here are the steps he needs to do:

Mix the ingredients.
Preheat the oven.
Assemble the ingredients.
Cool the cookies.
Bake the cookies.

In order to help Brian, write the steps on 5 index cards, with one step on each card.

Preheat the oven.
Assemble the ingredients.
Mix the ingredients.
Cool the cookies.
Bake the cookies.
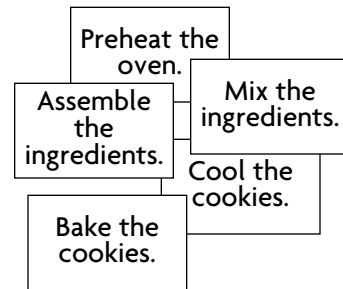
Now put the steps in order, by changing the order of the index cards until the sequence of steps makes sense.

_____

_____

Write the steps in order.

1. To make mashed potatoes:
   • Drain the potatoes.
   • Mash the potatoes.
   • Peel the potatoes.
   • Add butter and milk, and stir.
   • Cook the potatoes.

   1._____

   2._____

   3._____

   4._____

   5._____

2. To fly to another city:
   • Get on the plane.
   • Buy a ticket.
   • Check in at the gate.
   • Drive to the airport.

   1._____

   2._____

   3._____

   4._____

3. To find the value of $(3 \times 5) + 2 - 8$:
   • Subtract 8.
   • Multiply 3 by 5.
   • Add 2.
   • What is your answer? _____

   1._____

   2._____

   3._____

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Name _____

Represent, Compare, and Order Decimals

The numbers you use every day are part of the decimal system. To find the value of a number, look at the digits and the position of each digit.

A place-value chart can help you.

Knowing place values is useful when comparing decimal numbers. Mindy is asked to list 18.3, 17.8, and 24.1 in order from greatest to least.

Place Value										
Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten-Thousandths

↑ decimal point

Step 1 Mindy compares the first two numbers.

- She starts at the left. Both numbers have the digit 1 in the tens place.

  1̲8.3 ⟷ 1̲7.8

- So, Mindy looks at the digits in the ones place. The first number has the digit 8 in the ones place, while the second number has the digit 7.

  18̲.3 ⟷ 17̲.8

- Since 8 > 7, 18.3 > 17.8.

Step 2 Mindy compares the third number to the greatest number so far, the first number.

  2̲4.1 ⟷ 1̲8.3

- The third number has the digit 2 in the tens place, while the first number has the digit 1 in the tens place.

- Since 2 > 1, 24.1 > 18.3.

Using what she has discovered, Mindy makes the list: 24.1; 18.3; 17.8.

Give the position and the value of each underlined digit.

1. 30.1̲94

   a. position _____

   b. value _____

2. 4,082,113.72̲3

   a. position _____

   b. value _____

Compare the numbers. Write <, >, or = in the ◯.

3. 9.03 ◯ 0.93

4. 0.210 ◯ 0.012

5. 8.241 ◯ 8.24

Write the numbers in order from least to greatest.

6. 42.05; 45.02; 40.52

7. 19.7; 19.007; 19.07

8. 0.59; 0.95; 0.6

_____ _____ _____

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Problem Solving Strategy: Make a Table

Putting data in numerical order in a table can often help you determine the greatest or the least piece of data.

The areas of some sports fields are given below.

Basketball . . . . . . .427 square yards   Ice hockey . . . . .2,222 square yards
Football . . . . .6,400 square yards   Tennis (doubles) . . . . . .312 square yards

The area of an Olympic swimming pool is 5,135 square yards less than the greatest area above and 953 square yards greater than the least area. What is the area of an Olympic swimming pool?

Step 1 Think about what you know and what you are asked to find.
- You know the sizes of different sports fields.
- You need to find the area of the pool.

Step 2 Plan a strategy to solve.
- Use the strategy *make a table* to order the data.
- Use the table entries to find the area of the pool.

Step 3 Carry out the strategy.

The greatest area is 6,400 square yards and the least area is 312 square yards. Use the greatest area to find the area of an Olympic swimming pool.

Tennis (doubles)	312 square yards
Basketball	427 square yards
Ice hockey	2,222 square yards
Football	6,400 square yards

$$6,400 - 5,135 = 1,265$$

So, the area of an Olympic pool is 1,265 square yards.

Now find the difference between the area of an Olympic swimming pool and the least area to check your answer.

$$1,265 - 312 = 953$$ The answer checks.

Solve the problem by making a table.

1. The areas of some other sports fields are boxing, 44 yd^2; fencing, 33 yd^2; judo, 306 yd^2; karate, 75 yd^2; and kendo, 132 yd^2. The area used for wrestling is 10 yd^2 greater than the least area above. How does the area for wrestling compare to the area for karate?

   _____

2. Baseball fields are not a standard size. However, the bases on the field are the corners of a square whose area is 5,500 yd^2 less than the area of a football field and 867 yd^2 greater than the area for a fencing match. What is the area of this square?

   _____

Estimate with Decimals

When you estimate with decimals, you want to get an idea of the size of the result. Working with whole numbers can help you estimate quickly.

Estimate 19.7 + 40.13 + 100.4.

Step 1 Round each number to the nearest whole number.

$$19.7 \rightarrow 20$$
$$40.13 \rightarrow 40$$
$$100.4 \rightarrow 100$$

Step 2 Add the rounded numbers.

$$20 + 40 + 100 = 160$$

So, a good estimate for the sum is 160.

Estimate $78.31 − $49.47.

Step 1 Round to the nearest ten.

$$\$78.31 \rightarrow \$80$$
$$\$49.47 \rightarrow \$50$$

Step 2 Subtract the rounded numbers.

$$\$80 - \$50 = \$30$$

So, a good estimate for the difference is $30.

Estimate 62.88 × 28.97

Step 1 Round to the nearest ten.

$$62.88 \rightarrow 60$$
$$28.97 \rightarrow 30$$

Step 2 Multiply the rounded numbers.

$$60 \times 30 = 1,800$$

So, a good estimate for the product is 1,800.

Estimate 54.67 ÷ 8.56.

Step 1 Find compatible numbers close to those given.

$$54.67 \rightarrow 54$$
$$8.56 \rightarrow 9$$

Step 2 Divide the compatible numbers.

$$54 \div 9 = 6$$

So, a good estimate for the quotient is 6.

Estimate.

1. 19.82 + 51.5 _____
2. 149.2 ÷ 23.8 _____
3. 784.49 − 610.88 _____
4. 39.66 × 6.75 _____
5. 1003.2 − 796.1 _____
6. 7.86 + 10.03 _____
7. 98.15 × 8.23 _____
8. 82.88 ÷ 9.31 _____
9. 38.8 × 9.12 _____
10. 161.10 ÷ 7.84 _____
11. 108.46 + 392.54 _____
12. 80.55 − 67.86 _____
13. 57.93 × 21.5 _____
14. 119.4 ÷ 42.3 _____
15. 53.3 + 39.2 _____
16. 48.28 ÷ 6.82 _____
17. 28.7 × 61.75 _____
18. 982.3 − 498.7 _____
19. 411.9 + 298.34 + 128.6 _____
20. $49.28 + $32.61 + $18.95 _____

Decimals and Percents

If a figure is divided into 100 equal parts, each part represents $\frac{1}{100}$, 0.01, or 1% of the whole.

The shaded squares in the 10 × 10 grid at the right represent $\frac{54}{100}$, 0.54, or 54% of the whole square.
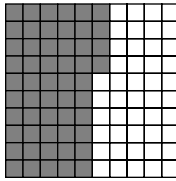
Percent can also represent value.

One hundredth of the value of a dollar is 1% of its value. So, one cent is 1% of one dollar.

$0.43 represents 0.43, or 43% of one dollar.

$0.70 represents 0.70, or 70% of one dollar.

Write the decimal and percent for the amount of money as part of one dollar.

1.

2.

3.

_____   _____   _____

Write the corresponding decimal or percent.

4. 75%   **5.** 6%   **6.** 0.15   **7.** 0.40

_____   _____   _____   _____

8. 71%   **9.** 0.08   **10.** 98%   **11.** 0.12

_____   _____   _____   _____

Add and Subtract Decimals

Ian is buying school supplies.
Find the total cost.

Binder	$3.49	Paper	$2.79
Pen	$0.88	Highlighter	$0.98

Step 1: List the items in a column.
Remember to align the decimal points.

$3.49
0.88
2.79
+ 0.98

Step 2: Write the decimal point for the answer.
Place it directly under the other points.

Step 3: Add as you would whole numbers.
Remember to regroup if needed.

$3.49
0.88
2.79
+ 0.98
$8.14

The total cost of Ian's school supplies is $8.14.

Now suppose Ian pays for his supplies with a $20 bill.
Find the amount of change he should receive.

Step 1: Write the numbers in a column.
Remember to align the decimal points.

$20.00
− 8.14

Step 2: Write the decimal point for the answer.
Then subtract as you would with whole
numbers, regrouping as necessary.

$20.00
− 8.14
$11.86

Ian should receive $11.86 in change.

Add or subtract.

1. $75.50 − $47.86

2. 347.9 − 69.38

3. 81.42 − 57.932

_____

4. 2.89 + 1.65 + 3.86

5. 4.62 + 7.89 + 9.17

6. 2.891 + 3.006 + 2.861

_____

7. 18.21 + 6.85 + 2.77

8. 4.0689 − 1.0791

9. 2.478 + 6.811 + 7.222

_____

10. 6.42 + 5.1 + 0.28

11. 3.016 − 1.2173

12. 38.2 + 5 + 6.83

_____

Multiply Decimals

Marty worked 26.5 hours this week. He earns $6.40 per hour. How much money did Marty earn this week?

To solve, you need to find the product 26.5×6.40.

Step 1: Multiply as you would with whole numbers.

$$\begin{array}{r} \$6.40 \\ \times\ 26.5 \\ \hline 169600 \end{array}$$

Step 2: Count the number of decimal places in the factors.

$$\begin{array}{r} \$6.40\ \rightarrow\ 2\ \text{decimal places} \\ \times\ 26.5\ \rightarrow\ 1\ \text{decimal place} \\ \hline 169600\ \rightarrow\ 3\ \text{decimal places} \end{array}$$

Step 3: Starting at the right side of the answer, count over that number of places. This is where the decimal point is placed.

$$\begin{array}{r} \$6.40 \\ \times\ 26.5 \\ \hline \$169.600 \end{array}$$

Marty earned $169.60 this week.

Complete to solve.

1. Sean worked 34 hours this week. He earns $9.25 an hour. How much money did Sean earn this week?

   a. What multiplication problem will you use? _____

   b. How many decimal places are in the factors? _____

   c. How much did Sean earn? _____

2. Denise earns $4.65 an hour. Last week she worked 17 hours. How much did she earn last week?

   a. What multiplication problem will you use? _____

   b. How many decimal places are in the factors? _____

   c. How much did Denise earn? _____

Multiply.

3. 23.1×5.7

4. 28.44×7

5. 25.4×24.55

_____   _____   _____

6. 16.6×0.24

7. 101.01×8.8

8. 2.32×11.48

_____   _____   _____

Divide with Decimals

Mary has saved $0.35 each day. She now has a total of $9.10. How long has Mary been saving?

To solve, you need to divide 9.10 by 0.35.

Step 1: Make the divisor a whole number by multiplying the divisor by a multiple of 10. Multiply the dividend by the same multiple of 10.

$$0.35\overline{)9.10} \qquad \rightarrow \qquad 35\overline{)910}$$
$$\times\,100 \quad \times\,100$$

Step 2: Place the decimal point in the quotient directly above the decimal point in the dividend. Divide as you would with whole numbers.

$$
\begin{array}{r}
26. \\
35\overline{)910.} \\
-70 \\
\hline
210 \\
-210 \\
\hline
0
\end{array}
$$

Since the remainder is 0, the answer is a whole number. You do not need to show the decimal point.

Mary has been saving for 26 days.

Complete to solve.

1. Dusty bought 7 movie tickets for his friends. The tickets cost a total of $34.65. How much does each friend owe him for the cost of one ticket?

   a. What division problem will you use to solve? _____

   b. Do you need to multiply to make the divisor a whole number? _____

   c. How do you decide where to place the decimal point in the

      quotient? _____

   d. How much does each friend owe? _____

Place the decimal point in the quotient.

2. $131.52 \div 6.4 = 2055$

3. $50.085 \div 10.6 = 4725$

4. $1936.95 \div 2.22 = 8725$

_____   _____   _____

Find the quotient.

5. $16.88 \div 5$

6. $81.9 \div 18$

7. $332.8 \div 40$

_____   _____   _____

8. $118 \div 12.5$

9. $203.205 \div 5.7$

10. $421.155 \div 14.7$

_____   _____   _____

Problem Solving Skill: Interpret the Remainder

When you solve a problem using division, there is often a remainder. When there is a remainder, you must decide what it means for the problem. You need to determine whether the solution to the problem is

- the quotient without the remainder,
- the next whole number greater than the quotient,
- the next whole number less than the quotient, or
- the remainder.

A real-estate agent ordered 190 doughnuts for an open house. The Donut Shoppe packed the order in boxes of 12 doughnuts. How many boxes were needed?

Step 1: Think about what you know and what you are asked to find.

- You know the number of doughnuts ordered and the number that were put in each box.
- You are asked to find the number of boxes that were needed.

Step 2: Decide on a plan to solve.

- Since the same number of doughnuts were put into each box, use division to solve the problem.
- Examine the quotient and remainder and decide how they relate to the problem.

Step 3: Carry out the plan.

$$12 \overline{)190} = 15r10$$

The quotient 15 means that 15 boxes were filled. The remainder 10 means that there were 10 doughnuts left after the last full box was packed.

So, the Donut Shoppe needed 15 + 1, or 16 boxes in all, to pack the agent's 190 doughnuts.

Interpret the remainder to solve.

1. A school received a shipment of 166 new social studies textbooks. The textbooks had been packed into cartons with 12 books in each full carton. How many cartons were delivered to the school?

2. In the school science laboratory, students generally work in groups of 4. Extra students are divided among the groups to make some groups of 5. If there are 27 students in a class, how many groups will have 5 students?

Algebra: Decimal Expressions and Equations

You speak and write using words, for example

the number of days in a week times the number of weeks

Some words can be expressed as numbers or numerical expressions.

days × weeks
  7 × 5

An algebraic expression uses a letter to represent a number. For example, if the number of weeks is written as the letter w, the algebraic expression is $7 \times w$.

It can also be written as $7w$.

To evaluate the expression $7w$ to find the number of days in 8 weeks, write $7 \times 8 = 56$.

Match the algebraic expression with the words.

1. number of inches in a foot times the number of feet   ____

2. number of items in a dozen plus some more items   ____

3. sum of money divided among several people   ____

4. total amount of money less the amount spent   ____

> **A** $12 + s$
>
> **B** $\$25 - d$
>
> **C** $12n$
>
> **D** $\$25 \div p$

Solve using mental math.

5. How much is $6y$ if $y = 3$? _____

6. How much is $8 + r$ if $r = 9$? _____

7. How much is $24 \div t$ if $t = 3$? _____

8. How much is $35 - a$ if $a = 15$? _____

9. How much is $4.5 - q$ if $q = 1.9$? _____

10. How much is $5b$ if $b = 1.1$? _____

11. How much is $3.6 \div z$ if $z = 3$? _____

12. How much is $2.4 + c$ if $c = 4.2$? _____

Evaluate each expression.

13. $5m$ for $m = 3.3$

14. $w \div 2$ for $w = 8.4$

15. $y + 12.4$ for $y = 5.2$

_____

_____

_____

16. $7.4 - b$ for $b = 1.3$

17. $6x$ for $x = 2.5$

18. $m - 4.6$ for $m = 8.9$

_____

_____

_____

Name _____

Samples

A sample is a part of a population. Samples are used when it would be too time-consuming or too expensive to survey an entire population.

A population may be people, such as all the people in a city, or it may be objects, such as all the books in a bookstore.

A sample is chosen using one of several methods. Three such methods are:

• **Random sample:** Each person or object in a population has the same chance of being chosen.

• **Convenience sample:** Those people or objects that are most readily available are chosen.

• **Systematic sample:** A person or object is chosen randomly and then a pattern is used, such as every fifth person, to choose others.

In general, it is best to use a random sample, since it is most representative of an entire population. It is also important to use the largest sample possible, since the larger the sample, the closer the results will represent the entire population.

---

1. Carole is surveying members of her community asking them to name their favorite restaurant. Determine the type of sample. Write *convenience, random,* or *systematic.*

   a. She stands outside a local mall and asks people leaving the mall to name their favorite restaurant.

   _____

   b. She opens the telephone book, randomly chooses a name, and then chooses every tenth name that begins with the same letter.

   _____

   c. She knows that every family in her community is listed in the telephone book. She calls 5 people randomly chosen from each page of the book.

   _____

2. Evan is on a committee to determine if the players in the Tri-Town Soccer League want to extend the season. Tell whether he should survey *all the players* or use *a sample* in each situation.

   a. There are 40 players in the league. _____

   b. The players are difficult to reach since they live in many areas. _____

   c. There are 650 players in the league. _____

Bias in Surveys

A sample is **biased** if individuals or groups from the population are not represented in the sample.

Linda wanted to survey the 125 sixth-grade students at her school to find out the number of hours they spent reading each week. She made a list of four sampling methods she could use.

- randomly survey 5 students
- randomly survey 15 sixth-grade boys
- randomly survey 13 students at her school
- randomly survey 15 sixth-grade students

She decided that the first method would not include a large-enough sample. The second method would not include sixth-grade girls. The third method was biased because it would not be exclusively for sixth graders. Linda decided to use the last method. It would contain a large-enough sample, and every member of the sixth grade would have an equal chance of being selected.

---

1. Marian wants to survey the 58 sixth-grade teachers at Oak Park School to find out the average number of hours they spend grading papers. She makes a list of four sampling methods she could use. Circle the sampling method that is not biased. Then explain how the other three methods are biased.

a. randomly survey all teachers who have taught for more than 10 years

b. randomly survey 10 male teachers at the school

c. randomly survey 8 sixth-grade teachers

d. randomly survey 8 math teachers at the school

_____

_____

2. Dan wants to survey the 230 members of a golf club to find out the average number of hours they play golf each week. He makes a list of four sampling methods he could use. Circle the sampling method that is not biased. Then explain how the other three methods are biased.

a. randomly survey all club members who have won tournaments

b. randomly survey 30 club members

c. randomly survey all members who have ever shot a hole-in-one

d. randomly survey 25 members under the age of 30

_____

_____

Problem Solving Strategy: Make a Table

Sometimes, when you have a lot of data, you do not need to know about each value. It may be enough to know how many values fit into a particular category. When this is the case, a tally table can help you solve problems.

Mr. Franks corrected the test papers of the 29 students in his 3rd period class. The scores (out of 100) are given below. How many more students had a score below 90 than had a score of 90 or above?

73, 83, 85, 92, 93, 85, 89, 91, 99, 80, 80, 84, 76, 78, 80, 93, 90, 88, 98, 82, 100, 78, 67, 88, 98, 94, 90, 76, 73

Step 1: Think about what you know and what you are asked to find.
- You know all the grades that the students in the 3rd period class received. You do not know who received each grade, but that information is not needed in this situation.
- You are asked to find the difference between the number of students who scored less than 90 and the number who scored 90 or above.

Step 2: Plan a strategy to solve.
- Use the strategy *make a table*
- Organize the data into a tally table. Make only as many rows as you need to solve the problem. For this problem, you need 2 rows: *less than 90* and *90 or above.*

Step 3: Solve.
- Carry out the strategy.

Test Scores	
Less than 90	ℍℍ ℍℍ ℍℍ ⦀⦀⦀
90 or above	ℍℍ ℍℍ ⏐

Count the tallies and find the difference. $18 - 11 = 7$

So, 7 more students had a score below 90 than had a score of 90 or above.

Use the data below and the strategy *make a table* to help you solve 1–2.

The students in a sixth-grade class were surveyed about the number of minutes they spend on the Internet on a typical day. The results of the survey are given below.

15, 0, 20, 15, 30, 0, 0, 0, 15, 20, 60, 45, 20, 10, 0, 30, 0, 0, 90, 45, 0, 10, 0, 20

1. How many more students said they spend some time on the Internet each day than said they do not spend any time on the Internet?

2. Of those students who use the Internet, how many more of them spend less than 30 minutes online than spend at least 30 minutes online?
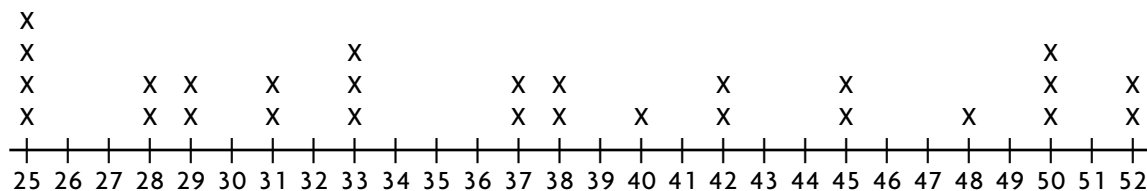
Frequency Tables and Line Plots

A local fitness center wants to survey 50 adults to find out if they
exercise daily. The age of each person who said *yes* was recorded. Use the
data below to make a line plot.

Ages of Adults Who Said *Yes*						
28	33	45	25	50	42	33
25	48	31	37	28	25	50
42	29	45	50	38	31	29
38	52	40	33	25	37	52

Step 1: Draw a horizontal line.
Step 2: On the line, write numerical values for the ages, using vertical
tick marks.
Step 3: Plot the data by drawing an x on the line plot for each value,
or person's age, in the table.

```
X
X                           X                                                        X
X       X X     X   X           X X         X       X                     X       X
X       X X     X   X           X X     X   X       X           X   X     X
+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52
```

For Exercises 1–2, use the data in the tables to make a line plot.

1. A local bookstore wants to survey 50 students to find out if they buy at
least one book a month. The surveyor recorded the age of each student
who said *yes.*

Ages of Students Who Said *Yes*						
13	12	16	15	11	14	15
11	16	14	14	13	15	11
14	15	11	12	16	11	13

2. The 20 students in science class take a test. Here are their results.

Science Test Scores				
76	82	85	95	98
92	78	76	90	85
88	95	74	78	76
74	85	92	82	88

Measures of Central Tendency

The test scores of a math class are shown below. Find the mean, median, and mode for the data.

Test Scores									
78	81	93	91	100	100	81	78	98	100

Mean: Find the sum of the scores and divide by the number of scores.

$$78 + 81 + 93 + 91 + 100 + 100 + 81 + 78 + 98 + 100 = 900$$
$$900 ÷ 10 = 90$$

Median: Arrange the scores in order from least to greatest. Find the number that is in the middle. Since there is an even number of scores, find the mean of two middle numbers.

78   78   81   81   <u>91   93</u>   98   100   100   100

92←The mean of 91 and 93 is 92.

Mode: Find the number or numbers that appear most often.

100

So, the mean test score is 90. The median score is 92. The mode score is 100.

Find the mean, median, and mode for each set of data.

1.

Test Scores									
85	77	91	97	95	95	79	83	95	93

mean: _____

median: _____

mode: _____

2.

Golf Scores					
88	76	92	91	69	76

mean: _____

median: _____

mode: _____

3.

Number of School Days								
Japan	England	Israel	Germany	Netherlands	USA	Thailand	Sweden	Canada
243	200	216	210	200	180	200	180	180

mean: _____   median: _____   mode: _____

Outliers and Additional Data

The measures of central tendency help you describe a set of data using only one number. Three measures of central tendency are the *mean*, the *median*, and the *mode*. When additional data values are added to a set, one or more of these measures may change.

Jenna has the five items shown below in her shopping cart.

She can describe the average amount in the containers using each of the measures of central tendency.

Mean: $(8 + 6 + 16 + 12 + 12) \div 5 = 54 \div 5 = 10.8$ ounces
Median: 6 8 **12** 12 16 → 12 ounces
Mode: 12 ounces

When Jenna adds the container of juice at the right to her cart, one or more of the measures of central tendency may change

Mean: $(8 + 6 + 16 + 12 + 12 + 64) \div 6 = 118 \div 6 \approx 19.7$ ounces
Median: 6 8 **12** **12** 16 64 → $(12 + 12) \div 2 = 12$ ounces
Mode: 12 ounces
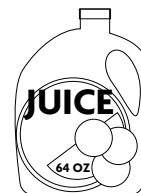
When the large container was added to the others in the cart, only the mean changed. The median and mode remained the same as before.

When new data values are added to a set, often the mean changes the most because the mean is found by adding all the data values.

The median and mode usually are not affected as greatly by the addition of an *outlier,* a data value very different from the other data values in a set.

The table below shows how much money Garrett spent buying house plants. Use the data for 1–2.

Date	March 1	March 7	March 15	March 24
Amount Spent	$10.00	$20.00	$15.00	$15.00

1. Find the mean, median, and mode of the amounts.

2. On April 3, Garrett bought a large plant that cost $60. What are the new mean, median, and mode for the costs of the five plants?

Data and Conclusions

You must consider many things when you are deciding whether a conclusion based on a set of data is valid. Here are four considerations.

- You must know the population that you are interested in. A population may be people, such as the students in your school or class, or it may be objects, such as the cars produced by a factory in one week.

- You must know whether that population was surveyed. For example, if you are interested only in adults between the ages of 25 and 40, how do you know that only people of this age were surveyed?

- You must know whether each survey question was unbiased. Were people led to believe that you wanted one answer rather than another?

- Were the people or objects selected randomly so that every member of the population had an equal chance of being chosen?

Diana's school has 986 students attending it. She asked 80 randomly selected students in the school the question "What kind of pet or pets do you have?" The results of her survey were as follows:

    26% said they had a cat.

    23% said they had a dog.

    12% said they had a pet other than a cat or dog.

She concluded that percentages close to these would apply to all the students in her school. Her conclusion was valid because:

- She wanted to find out data about the entire population of her school.
- She surveyed the population she was interested in.
- She asked the question in a fair and unbiased way.
- The sample was large enough to be representative of the students in her school and the students sampled were selected randomly.

Write *yes* or *no* to tell whether Diana's conclusions would have been valid if she had used each of these surveying methods. Explain your answer.

1. Diana asked some students who attend her school, as well as some students who attend another middle school.

_____

_____

2. Diana asked the first 80 students on the daily attendance sheet, which has students' names listed alphabetically.

_____

_____

Make and Analyze Graphs

At Fox Run School, the sixth-grade and seventh-grade classes are competing to see which class can collect more aluminum cans.
The bar graphs show the results for the first three weeks.

Grade 6 Weekly Totals

Grade 7 Weekly Totals
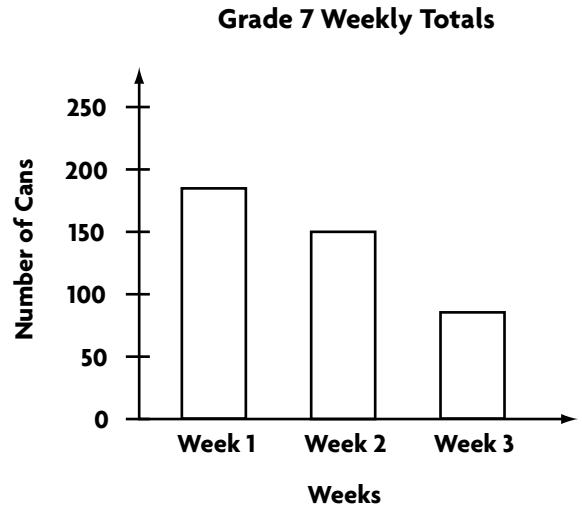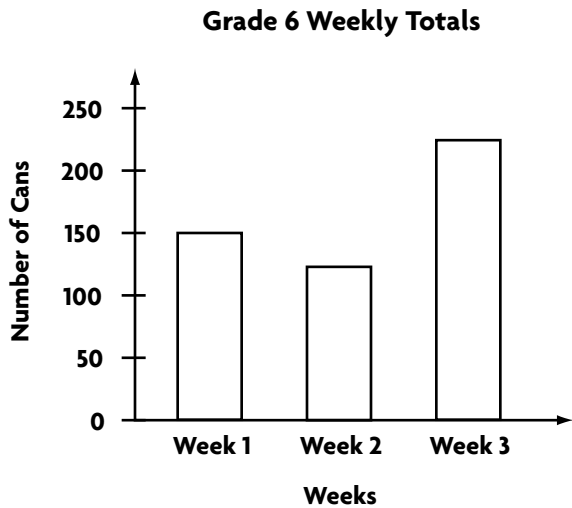
To compare the two classes' results more easily, you can show all the data in one graph. The following graph is a double-bar graph.

Weekly Totals
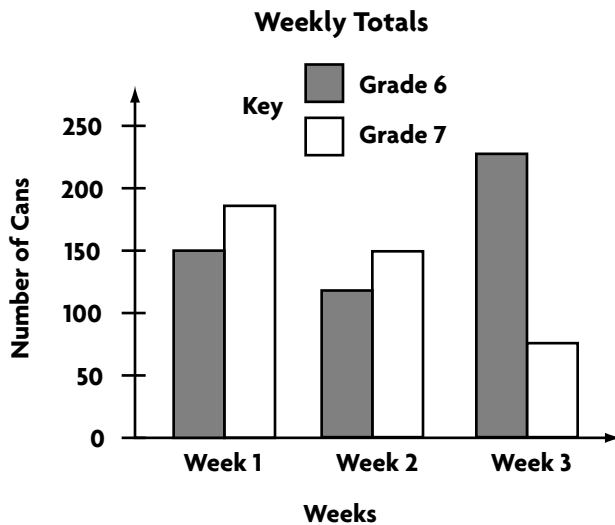
The key tells which bar represents the sixth grade and which represents the seventh grade. A bar graph that shows two or more sets of data is called a multiple-bar graph.

For Exercises 1–3, use the double-bar graph above.

1. How do you know which bar refers to which grade? _____

2. Which grade collected more cans in week 1? in week 2? in week 3?

_____

3. Which grade collected more cans altogether? _____

Find Unknown Values

Line graphs often show a pattern. You can use a line graph to estimate unknown values by extending the graph.

Shauna swims laps in a pool to train for a race. She swims at a rate of 25 m per min. The table below shows how far she swims.

Time (min)	1	2	3	4	5
Distance (m)	25	50	75	100	125

Make a line graph from the data in the table. Use the graph to estimate how many minutes it will take Shauna to swim 150 m.

Step 1 Use the data to draw the graph.

Step 2 Extend the graph until it crosses the 150 m line. Look directly down to the horizontal axis and estimate the value along that axis. If Shauna continues to swim at this rate, it will take her about 6 min to swim 150 m.

Shauna's Race Training
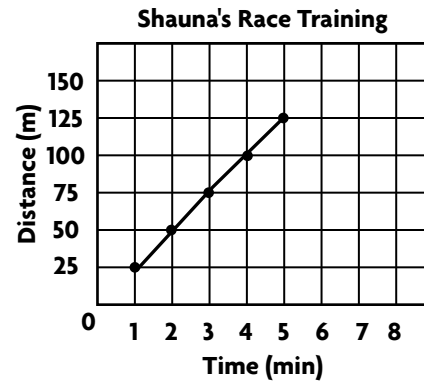
You can also use logical reasoning and arithmetic. Since 150 m = 100 m + 50 m, add the number of minutes she takes to swim those two distances: 4 min + 2 min = 6 min.

The formula $d = rt$, where d is the distance, r is the rate, and t is the time, can also be used to find the time Shauna needs to swim 150 m.

$d = rt$ Write the formula.
$150 = 25t$ Substitute the values you know into the formula.
$6 = t$ Solve to find the value of t.

So, it will take Shauna 6 min to swim 150 m.

A bicycle racer has averaged 20 mi per hr during the first 4 hr of a race.

Time (hr)	1	2	3	4
Distance (mi)	20	40	60	80

1. Make a line graph. Use the graph to estimate how long it will take the

   racer to ride 100 mi. _____

2. Use logical reasoning and arithmetic to find how long it will take the

   racer to ride 100 mi. _____

3. Use the formula $d = rt$ to find how long it will take the racer to ride 140 mi.

   _____

Stem-and-Leaf Plots and Histograms

A histogram is a type of bar graph. Histograms are different from bar graphs in two ways.

• The bars are side by side, not spaced apart.

• Each bar refers to an interval, not a single item.

All of the students in Mr. Higgins' physical education class ran the 100-yard dash. This histogram shows the students' times to the nearest second.

100-Yard Dash Times

Frequency (Number of Runners)

Runners' Times in Seconds
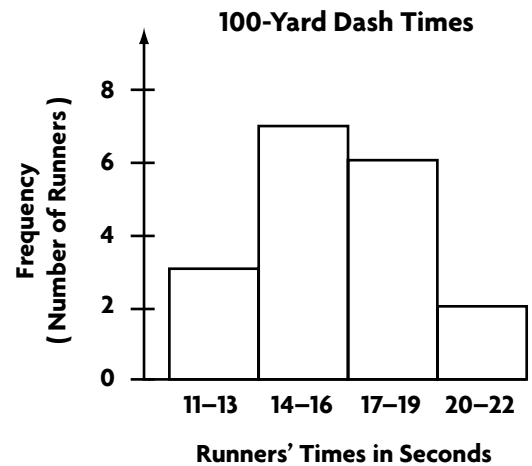
Complete.

1. The number of runners with times of 17–19 sec was _____.

2. _____ students had times of 11–13 sec.

There were 18 students who recorded how many baskets they made in 30 sec. Their results are in the following tables.

Student	1	2	3	4	5	6	7	8	9
Baskets made	5	8	17	13	15	5	19	7	7

Student	10	11	12	13	14	15	16	17	18
Baskets made	8	9	6	14	7	16	12	12	10

3. Use the data above to complete the histogram.

0–5   6–10   11–15   16–20

4. Complete the stem-and-leaf plot of the *Baskets Made* data.
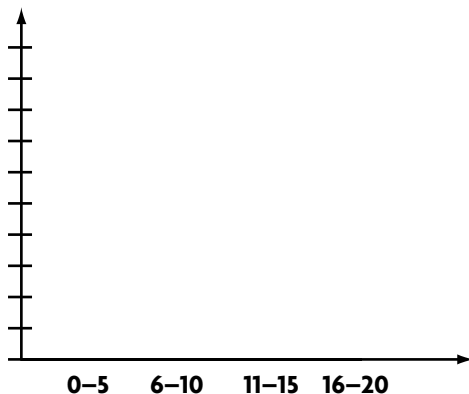
Stem	Leaves
0	5
1	

Key: 0 | 5 = 5

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Box-and-Whisker Graphs

The owner of a gift shop made a box-and-whisker graph to represent the number of customers who came into the shop each day.
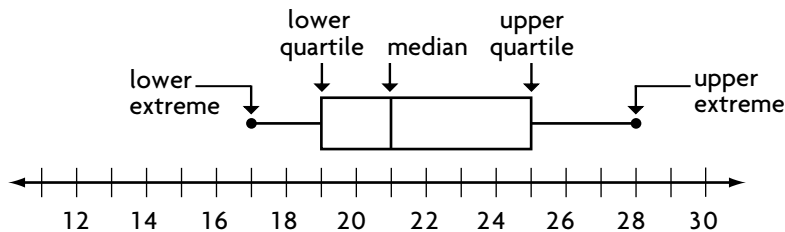
The least number in a box-and-whisker graph is called the **lower extreme.** It is the black dot at the end of the left whisker. In this graph, the lower extreme represents the least number of customers to enter the shop in a single day. The lower extreme is 17.

The greatest number in a box-and-whisker graph is called the **upper extreme.** It is the black dot at the end of the right whisker. In this graph, the upper extreme represents the greatest number of customers to enter the shop in one day. The upper extreme is 28.

The range is the difference between the upper and lower extremes. The range of this set of data is 11.

The median of a set of data is the middle number when all numbers are arranged in numerical order. The median of this set is 21.

Another store owner also made a box-and-whisker graph of the customers who entered her store daily.
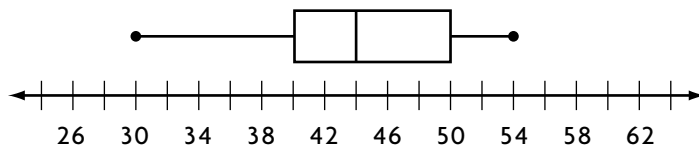
1. What is the lower extreme? _____

2. What is the upper extreme? _____

3. What is the lower quartile? _____

4. What is the upper quartile? _____

5. What is the range of this set of data? _____

6. What is the median for this set of data? _____

Analyze Graphs

Graphs are sometimes drawn in order to mislead the reader. In order for a graph to provide information honestly, it must meet several requirements. One requirement is that the scale must be accurate.

Compare Prices!

Stereo City placed an advertisement for a stereo system in a newspaper. The advertisement included the graph at the right, which compares its price for the system to the price of the system at Speaker Town.

In the graph, the bar for Speaker Town appears to be twice the height of the bar for Stereo City. Some readers might think this means Speaker Town's price is twice that of Stereo City's. The scale, however, shows that Speaker Town's price for the stereo system is not twice the price at Stereo City. The actual difference between the two prices is only $380 − $340, or $40.
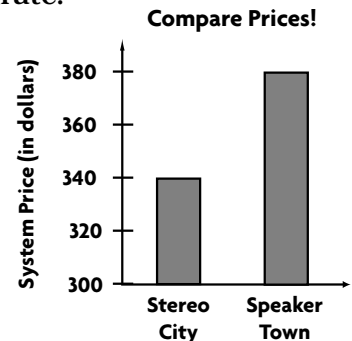
In order to give a true representation of a set of data, any scale used on a graph should follow these rules:

• The scale should begin with zero.
• An interval that makes sense for the data that is shown in the graph should be chosen.
• Every interval on the scale must be the same size.

Customer Loyalty

A consumer research company conducted a survey at both Stereo City and Speaker Town. At each store, customers who had just purchased an item were asked if they would return to the store for their next electronics purchase. The results of the survey are shown in the graph.
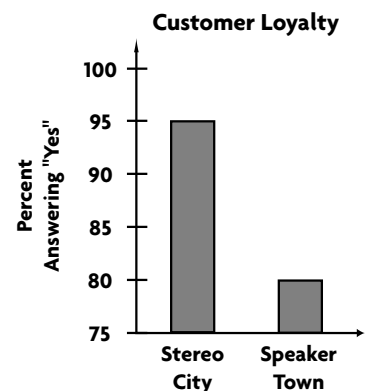
1. What percent of Stereo City's customers said they would make their next electronics purchase there?

_____

2. What percent of Speaker Town's customers said they would make their next

electronics purchase there? _____

3. About how many times as high is the bar for Stereo City as the bar for

Speaker Town?_____

4. How can you change the graph so that it is not misleading?

_____

Divisibility

The rules for divisibility are:

A number is divisible by:	If:
2	the last digit is 0, 2, 4, 6, or 8.
3	the sum of its digits is divisible by 3.
4	the number formed by the last two digits is divisible by 4.
5	the last digit is 0 or 5.
6	it is divisible by 2 and by 3.
8	the number formed by the last three digits is divisible by 8.
9	the sum of its digits is divisible by 9.
10	the last digit is 0.

To determine whether 3,882 is divisible by 3, follow these steps:

Step 1 Find the sum of the digits of 3,882:

$$3 + 8 + 8 + 2 = 21$$

Step 2 Decide whether the sum, 21, is divisible by 3. Since 3 divides 21 evenly (with no remainder), 21 is divisible by 3. So, 3,882 is *divisible* by 3.

To determine whether 3,882 is divisible by 9, decide whether 21, the sum of the digits, can be divided evenly by 9. Since 21 ÷ 9 = 2 r3, 3,882 is *not divisible* by 9.

To determine whether 7,032 is divisible by 4, follow these steps:

Step 1 Identify the number formed by the last two digits: 32.

Step 2 Decide whether 32 is divisible by 4. Since 4 divides 32 evenly (with no remainder), 32 is *divisible* by 4. So, 7,032 is divisible by 4.

To determine whether 7,032 is divisible by 8, decide whether 032 (the last three digits) is a number divisible by 8. Since 032 ÷ 8 = 4, the number 7,032 is *divisible* by 8.

Determine whether each number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

1. 146   **2.** 369   **3.** 195   **4.** 284

_____ _____ _____ _____

5. 444   **6.** 512   **7.** 788   **8.** 612

_____ _____ _____ _____

9. 2,865   **10.** 4,470   **11.** 6,048   **12.** 3,240

_____ _____ _____ _____

25

Prime Factorization

When you write a composite number as the product of prime factors, you have found the **prime factorization** of the number. A factor tree can help you find the prime factors of a composite number.

What is the prime factorization of 36?

Step 1 Choose any two factors of 36. Draw two lines from 36. Write one factor at the end of each line.

36
4   9
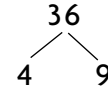
Step 2 Look at the factors. Are they prime? composite? Since they are composite, you must continue to find more factors.

36
4   9 ← factors not prime
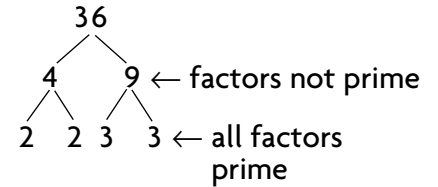
Step 3 Look at the new bottom row of factors. Are they prime? composite? Since they are prime, you have found the prime factors of 36.

2   2 3   3 ← all factors prime

You can write the prime factorization of 36 as $2 \times 2 \times 3 \times 3$, or $2^2 \times 3^2$.

Complete each factor tree to find the prime factors of the number.

1.   70
2   _____
_____   _____

Prime factorization:

$2 \times$ _____ $\times$ _____

2.   63
3   _____
_____   _____
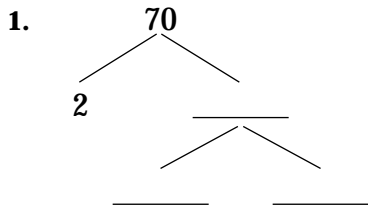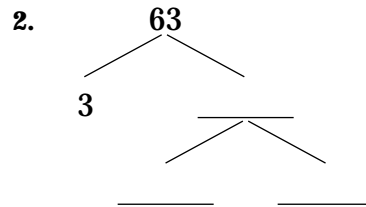
Prime factorization:

$3 \times$ _____ $\times$ _____ or _____ $\times$ _____

3.   54
2   _____
3   _____
_____   3
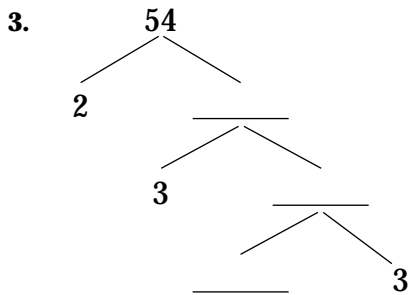
Prime factorization:

$2 \times 3 \times$ _____ $\times 3$ or $2 \times$ _____

4.   200
2   _____
2   _____
2   _____
_____   _____
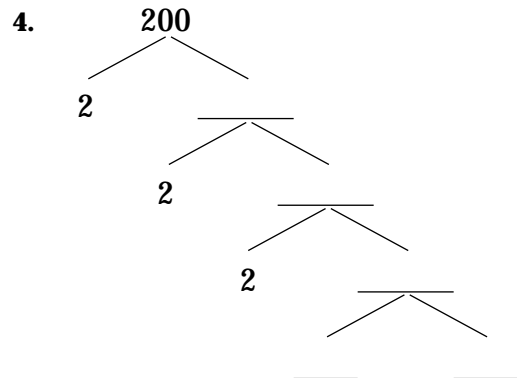
Prime factorization:

$2 \times 2 \times 2 \times$ _____ $\times$ _____

or _____ $\times$ _____

Least Common Multiple and Greatest Common Factor

The greatest common factor, or GCF, of two numbers is the largest common factor of both numbers. You can use prime factors to find the GCF of two numbers.

What is the GCF of 28 and 36?

Step 1 Use factor trees to find the prime factors of the numbers.
$28 = 2 \times 2 \times 7$
$36 = 2 \times 2 \times 3 \times 3$

28
4   7
2   2

36
4   9
2   2   3   3
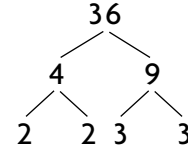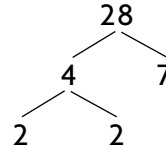
Step 2 Find the prime factors that are in both trees.

2 and 2

Step 3 Multiply the common factors.

$2 \times 2 = 4$

Using this method, you discover that the GCF of 28 and 36 is 4.

To find the least common multiple, or LCM, of two numbers, you can list the multiples of each number. The smallest number in both lists is the LCM.

To find the LCM for 8 and 12:

8 → 8, 16, 24, 32, . . .

12 → 12, 24, 36, 48, . . .

The LCM for 8 and 12 is 24.

Complete to find the GCF of 12 and 72.

1. Use factor trees to find the prime factors of the numbers.

12

72
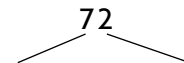
2. Find the common prime factors. _____

3. Multiply the common factors. _____

4. The GCF of 12 and 72 is _____ .

Find the GCF of each pair of numbers.

5. 6 and 15 _____   **6.** 18 and 81 _____   **7.** 24 and 84 _____

Find the LCM of each pair of numbers.

8. 9 and 12 _____   **9.** 20 and 15 _____   **10.** 18 and 24 _____

Problem Solving Strategy: Make an Organized List

Organizing data into a list is one way of making sure that you have considered every possibility for a given situation.

Two trains that ran in Great Britain in the 1930s were the Night Scotsman and the Bournemouth Belle. They both ran between London and other cities. The Night Scotsman left London every 4 days and the Bournemouth Belle left every 6 days. If they both left London on Thursday, March 1, on which other dates in March did they leave London on the same day?

Step 1 Think about what you know and what you are asked to find.

- You know how often each train left from London:
  The Night Scotsman left every 4 days.
  The Bournemouth Belle left every 6 days.
  You know that they both left London on Thursday, March 1.

- You are asked to find the other dates in March on which both trains left London on the same day.

Step 2 Plan a strategy to solve.

- Use the strategy *make an organized list.*

- Make a list of the multiples of 4 and the multiples of 6.

- Use the common multiples to determine the other dates in March on which both trains left London on the same day.

Step 3 Solve.

- Carry out the strategy. Make lists of the multiples.

    multiples of 4: 4, 8, **12**, 16, 20, **24**, 28, 32, …

    multiples of 6: 6, **12**, 18, **24**, 30, 36, 42, …

    The common multiples of 4 and 6 that are less than 31
    (the number of days in March) are 12 and 24.

So, the trains both left London together 12 days after March 1 and again 24 days after March 1. So, the dates they left on the same day were:

    1 + 12, or March 13   and   1 + 24, or March 25.

Solve the problem by making *an organized list*.

1. A bus company has routes between Chicago and several other cities. Every 5 hours a bus leaves from Chicago for Detroit and every 3 hours another bus leaves for Cleveland. If buses leave for both cities at 6:00 A.M., when is the next time two buses will leave together for these two cities?

2. Some buses have seats for 40 people. Other buses can seat only 36. One of each type of bus is filled with families. There are no empty seats and all the families are the same size. What is the greatest number of members that each family can possibly have?

Equivalent Fractions and Simplest Form

When the numerator and denominator of a fraction have no common factors other than 1, the fraction is in **simplest form**. You can use a GCF to write a fraction in simplest form.

What is the simplest form of $\frac{32}{56}$?

Step 1 Find the GCF of 32 and 56 by listing the factors of each. The GCF is 8.

32: 1, 2, 4, <u>8</u>, 16, 32
56: 1, 2, 4, 7, <u>8</u>, 14, 28, 56

Step 2 Divide the numerator and denominator by the GCF.

$$\frac{32}{56} = \frac{32 \div 8}{56 \div 8} = \frac{4}{7}$$

So, $\frac{4}{7}$ is the simplest form of $\frac{32}{56}$.

Complete to find the simplest form of $\frac{88}{104}$.

1. Find the GCF of 88 and 104.   88: _____

   The GCF is _____.   104: _____

2. Divide the numerator and denominator by the GCF. ⟶ $\frac{88 \div \Box}{104 \div \Box} = \frac{\Box}{\Box}$

3. So, _____ is the simplest form of $\frac{88}{104}$.

Complete to find the simplest form of $\frac{78}{120}$.

4. Find the GCF of 78 and 120.   78: _____

   The GCF is _____.   120: _____

5. Divide the numerator and denominator by the GCF. ⟶ $\frac{78 \div \Box}{120 \div \Box} = \frac{\Box}{\Box}$

6. So, _____ is the simplest form of $\frac{78}{120}$.

Find the GCF of the pair of numbers. Then write the fraction in simplest form.

7. 4, 10; $\frac{4}{10}$ 8. 8, 12; $\frac{8}{12}$ 9. 18, 36; $\frac{18}{36}$ 10. 21, 60; $\frac{21}{60}$

_____   _____   _____   _____

Mixed Numbers and Fractions

A mixed number is made up of two parts: a whole number and a fraction.

- You can rewrite a mixed number as an equivalent fraction. This type of fraction will have a numerator that is greater than the denominator.
- You can also rewrite a fraction with a numerator greater than the denominator as a mixed number.

Write $4\frac{2}{3}$ as an equivalent fraction.	Write $\frac{19}{5}$ as a mixed number.
Step 1 Multiply the whole number by the denominator of the fraction. $4 \times 3 = 12$	**Step 1** Divide the numerator by the denominator. $19 \div 5 = 3 \text{ r}4$
Step 2 Add the numerator. This sum is the numerator of the equivalent fraction. $12 + 2 = 14$	**Step 2** Use the remainder as the numerator in the fraction part of the mixed number. The denominator is the divisor.
Step 3 Use the same denominator to write the equivalent fraction. $\frac{14}{3}$	$3\frac{4}{5}$ ←remainder ←divisor

---

Complete.

1. Write $7\frac{3}{5}$ as a fraction.

Step 1 Multiply the whole number by the

_____.

$7 \times \boxed{} = \boxed{}$

Step 2 Add the _____. This

sum is the _____ of
the equivalent fraction.

$\boxed{} + \boxed{} = \boxed{}$

Step 3 Use the same _____
to write the equivalent fraction.

$\dfrac{\boxed{}}{\boxed{}}$

Write the mixed number as a fraction.

2. $5\frac{1}{3}$ _____ **3.** $4\frac{1}{5}$ _____ **4.** $5\frac{2}{7}$ _____ **5.** $6\frac{1}{3}$ _____ **6.** $7\frac{4}{5}$ _____

Write the fraction as a mixed number.

7. $\frac{17}{3}$ _____ **8.** $\frac{19}{6}$ _____ **9.** $\frac{37}{8}$ _____ **10.** $\frac{52}{9}$ _____ **11.** $\frac{69}{11}$ _____

Compare and Order Fractions

To compare fractions without using a number line, you can use a
flowchart approach. Here are the steps for comparing $\frac{1}{3}$ and $\frac{2}{5}$.

Step 1 Are the denominators the same? *If YES, then the fraction with the greater numerator is the greater fraction. If NO, go on.*	$\frac{1}{3} \quad \frac{2}{5}$ Since the denominators are not the same, go on to the next step.
Step 2 Are the numerators the same? *If YES, then the fraction with the smaller denominator is the greater fraction. If NO, go on.*	$\frac{1}{3} \quad \frac{2}{5}$ Since the numerators are not the same, go on to the next step.
Step 3 Find the least common denominator of the two fractions.	multiples of 3: 3, 6, 9, 12, **15**, 18, . . . multiples of 5: 5, 10, **15**, 20, 25, . . . 15 is the least common multiple of 3 and 5. So, it is the least common denominator of the fractions.
Step 4 Write fractions equivalent to the original fractions using the common denominator.	$\frac{1}{3} \times \frac{5}{5} = \frac{5}{15} \quad \frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$
Step 5 Compare the fractions using the common denominator.	$\frac{5}{15} < \frac{6}{15}$
Step 6 Write the comparison of the original fractions in the same order.	$\frac{1}{3} < \frac{2}{5}$

Compare the fractions. Write $<$, $>$, or $=$ in each $\bigcirc$.

1. $\frac{4}{7} \bigcirc \frac{4}{9}$

2. $\frac{3}{5} \bigcirc \frac{2}{5}$

3. $\frac{3}{5} \bigcirc \frac{7}{8}$

4. $\frac{8}{10} \bigcirc \frac{4}{5}$

5. $\frac{2}{3} \bigcirc \frac{5}{6}$

6. $\frac{1}{6} \bigcirc \frac{3}{11}$

7. $\frac{6}{15} \bigcirc \frac{2}{5}$

8. $\frac{6}{11} \bigcirc \frac{5}{9}$

9. $\frac{2}{7} \bigcirc \frac{1}{3}$

10. $\frac{3}{5} \bigcirc \frac{6}{9}$

11. $\frac{7}{11} \bigcirc \frac{5}{12}$

12. $\frac{9}{10} \bigcirc \frac{7}{8}$

13. $\frac{2}{5} \bigcirc \frac{1}{3}$

14. $\frac{1}{8} \bigcirc \frac{2}{11}$

15. $\frac{4}{7} \bigcirc \frac{8}{14}$

16. $\frac{9}{14} \bigcirc \frac{9}{10}$

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Fractions, Decimals, and Percents

When you need to change a fraction to a decimal, you can use division.
Write the fraction $\frac{3}{5}$ as a decimal.

Step 1 Set up a division problem, dividing
the numerator by the denominator.
$5\overline{)3}$

Step 2 Place a decimal point after the
numerator. Write a zero.
$5\overline{)3.0}$

Step 3 Divide as you would with whole numbers.
$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ 3\ 0 \\ \hline 0 \end{array}$

So, written as a decimal, $\frac{3}{5} = 0.6$. Recall that 0.6 is a terminating decimal because it ends after the tenths place.

When you need to change a decimal to a fraction, use place value.

Change 0.364 to a fraction.

Step 1 Identify the place value of the last digit
in the decimal number.
$\begin{array}{c} 0.364 \\ \uparrow \\ \text{thousandths} \end{array}$

Step 2 Use the place value of the last digit
as the denominator.
$\frac{364}{1,000}$

So, $0.364 = \frac{364}{1,000}$.

Answer the questions to change the fraction to a decimal.

1. $\frac{1}{4}$

   a. What division problem will you use?

   _____

   b. What is the quotient?

   _____

2. $\frac{3}{8}$

   a. What division problem will you use?

   _____

   b. What is the quotient?

   _____

Use place value to write the decimal as a fraction.

3. 0.6 _____ **4.** 0.92 _____ **5.** 0.48 _____ **6.** 0.137 _____

Estimate Sums and Differences

Some problems need only an estimated answer, not an exact answer.
You can estimate with fractions by rounding them to 0, $\frac{1}{2}$, or 1.

Estimate. $\frac{4}{5} + \frac{5}{12}$

Step 1: Look at the first fraction.
- Is 4 much less than 5? No.
- Is 4 about half of 5? No.
- Is 4 almost the same as 5? Yes.
- Since the numerator is almost the same as the denominator, round the fraction to 1.

Step 2: Look at the second fraction.
- Is 5 almost the same as 12? No.
- Is 5 about half of 12? Yes.
- Since the numerator is about half of the denominator, round the fraction to $\frac{1}{2}$.

Step 3: Add the rounded fractions to get an estimated answer. $1 + \frac{1}{2} = 1\frac{1}{2}$

Follow the same steps to estimate a difference.

Estimate. $\frac{8}{9} - \frac{1}{10}$

Step 1: Look at the first fraction.
- Is 8 much less than 9? No.
- Is 8 about half of 9? No.
- Is 8 almost the same as 9? Yes.
- So, round the fraction to 1.

Step 2: Look at the second fraction.
- Is 1 much less than 10? Yes.
- So, round the fraction to 0.

Step 3: Find the difference between the rounded fractions to get an estimated answer. $1 - 0 = 1$

Estimate the sum or difference.

1. $\frac{6}{7} - \frac{1}{3}$ **2.** $\frac{11}{12} + \frac{4}{7}$ **3.** $\frac{1}{9} + \frac{5}{8}$ **4.** $\frac{8}{11} - \frac{1}{15}$ **5.** $\frac{13}{14} - \frac{11}{12}$

_____   _____   _____   _____   _____

6. $\frac{5}{6} + \frac{1}{7}$ **7.** $\frac{7}{8} - \frac{4}{9}$ **8.** $\frac{3}{5} + \frac{1}{2}$ **9.** $\frac{11}{12} - \frac{15}{16}$ **10.** $\frac{3}{7} - \frac{1}{8}$

_____   _____   _____   _____   _____

Add and Subtract Fractions

To add or subtract unlike fractions, first change them to equivalent fractions with the same denominator.

Find the sum. $\frac{1}{4} + \frac{2}{3}$

Step 1: Find the LCM of the denominators. The LCM of 4 and 3 is 12.
So, the LCD of $\frac{1}{4}$ and $\frac{2}{3}$ is 12.

Step 2: Multiply to write equivalent fractions, using the LCD.

$$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Step 3: Add the numerators. Write the sum over the denominator.

$$\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$ ← Remember, keep the denominator the same.

Step 4: Write the answer as a fraction or as a mixed number in simplest form.

$\frac{11}{12}$ is already in simplest form.

$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$

Follow the same steps to subtract unlike fractions.

Complete to find each sum. Write the answer in simplest form.

1. Find. $\frac{2}{5} + \frac{3}{10}$

The LCM of 5 and 10 is 10.

So, $\frac{2}{5} + \frac{3}{10} = \frac{\square}{10} + \frac{3}{10} = \frac{\square}{10}$.

2. Find. $\frac{7}{8} + \frac{2}{3}$

The LCM of 8 and 3 is 24.

So, $\frac{7}{8} + \frac{2}{3} = \frac{\square}{24} + \frac{\square}{24} = \frac{\square}{24} = 1\frac{\square}{24}$.

3. Find. $\frac{1}{4} + \frac{5}{6}$

The LCM of 4 and 6 is 12.

So, $\frac{1}{4} + \frac{5}{6} = \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{12} = 1\frac{\square}{12}$.

4. Find. $\frac{1}{6} + \frac{3}{8}$

The LCM of 6 and 8 is 24.

So, $\frac{1}{6} + \frac{3}{8} = \frac{\square}{24} + \frac{\square}{24} = \frac{\square}{24}$.

Find the sum or difference. Write the answer in simplest form.

5. $\frac{2}{7} + \frac{1}{2}$

6. $\frac{11}{12} - \frac{7}{8}$

7. $\frac{4}{5} + \frac{1}{3}$

8. $\frac{3}{4} - \frac{1}{5}$

_____   _____   _____   _____

Add and Subtract Mixed Numbers

Zack is working on a science project. He needs a $1\frac{1}{8}$-ft piece of wire and a $2\frac{1}{4}$-ft piece of wire. Zack wants to know the total amount of wire he needs for the project. He decides to make a diagram to find the total.

| 1 | $\frac{1}{8}$ | ← $1\frac{1}{8}$

Step 1 Zack draws a diagram that represents each piece of wire.

| 1 | $\frac{1}{4}$ | ← $2\frac{1}{4}$

| 1 |
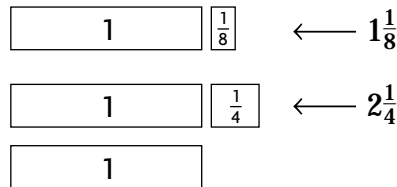
Step 2 Zack combines the whole numbers. He draws equivalent fractions with the LCD of 8 to combine the fractions.

| 1 | $\frac{1}{8}$ $\frac{1}{4}$ |

| 1 |

| 1 |
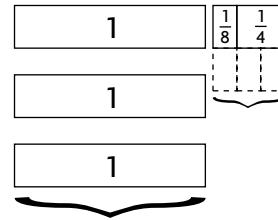
$1 + 2 = 3$ $\qquad \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

Step 3 Then Zack adds the whole numbers and the fractions.

$3 + \frac{3}{8} = 3\frac{3}{8}$

So, Zack needs $3\frac{3}{8}$ ft of wire.

Draw a diagram to find each sum or difference.

1. $3\frac{1}{4} + 2\frac{1}{2} = $ _____

2. $2\frac{5}{6} - 1\frac{1}{3} = $ _____

3. $1\frac{3}{4} + 2\frac{1}{8} = $ _____

4. $3\frac{4}{5} - 1\frac{1}{2} = $ _____

Subtract Mixed Numbers

Mrs. Ruiz buys $4\frac{1}{2}$ lb of apples. She uses $1\frac{2}{3}$ lb to bake apple tarts. How many pounds of apples are left?

Step 1   The LCD of $\frac{1}{2}$ and $\frac{2}{3}$ is 6.
Rename the fractions using the LCD.

$$4\frac{1}{2} = 4\frac{3}{6}$$
$$-1\frac{2}{3} = 1\frac{4}{6}$$

Step 2   Since you can't subtract $\frac{4}{6}$ from $\frac{3}{6}$, rename $4\frac{3}{6}$.

Think: $4\frac{3}{6} = 3 + \frac{6}{6} + \frac{3}{6} = 3\frac{9}{6}$

Now, subtract the fractions. Then subtract the whole numbers.

$$4\frac{1}{2} = 4\frac{3}{6} = 3\frac{9}{6}$$
$$-1\frac{2}{3} = 1\frac{4}{6} = 1\frac{4}{6}$$
$$\overline{\phantom{-1\frac{2}{3} = 1\frac{4}{6} =} 2\frac{5}{6}}$$

So, $4\frac{1}{2} - 1\frac{2}{3} = 2\frac{5}{6}$

Mrs. Ruiz has $2\frac{5}{6}$ lbs of apples left.

Find the difference. Write the answer in simplest form.

1. $6\frac{2}{5}$
$\underline{-3\frac{7}{10}}$

2. $4\frac{1}{6}$
$\underline{-2\frac{3}{4}}$

3. $9\frac{3}{7}$
$\underline{-5\frac{1}{2}}$

4. $11\frac{1}{3}$
$\underline{-7\frac{8}{9}}$

5. $10\frac{3}{8}$
$\underline{-3\frac{1}{2}}$

6. $6\frac{1}{10}$
$\underline{-4\frac{4}{5}}$

7. $9\frac{1}{8}$
$\underline{-4\frac{3}{4}}$

8. $12\frac{2}{5}$
$\underline{-6\frac{1}{2}}$

9. $8\frac{2}{3}$
$\underline{-5\frac{5}{6}}$

Problem Solving Strategy: Draw a Diagram

Some problems become clearer when you draw a diagram to represent the information you are given.

Ms. Kaminsky is teaching her class a game in which students stand in a circle. Two girls stand together, then 1 boy, then 2 girls, then 1 boy, and so on. Altogether, 7 boys are standing in the circle along with all the girls in the class. If there are 27 students in the class, how many boys are not participating?

Step 1: Think about what you know and what you are asked to find.

- You know that all the girls in the class are in the circle.
- You know that there are 7 boys in the circle.
- You know that for every 2 girls in the circle, there is 1 boy.
- You are asked to find how many boys in the class are not in the circle.

Step 2: Plan a strategy to solve.

- Use the strategy *draw a diagram.*
- Draw what you know from the problem.

Step 3: Solve.

- Carry out the strategy.

The diagram shows that if 7 boys are in the circle, then there must be 14 girls.

$7 + 14 = 21$ and $27 - 21 = 6$.

So, 6 boys in the class are not participating.

Use the strategy *draw a diagram* to help you solve.

1. In another game, the arrangement around the circle is reversed. Two boys stand together, then 1 girl, then 2 boys, then 1 girl, and so on. In this version of the game, what is the greatest number of students in Ms. Kaminsky's class that can participate at one time?

2. The desks are arranged in rows of 6 from the front of the room to the back. In all, there are 5 rows. Arthur sits in the fourth seat of the first row. Andrea sits 3 rows over and 1 seat back. Charles sits 3 seats in front of Andrea. Juan sits 2 rows over and 2 seats farther back than Charles. Where is Juan compared to Arthur?

Estimate Products and Quotients

You can often estimate the product or quotient of two mixed numbers by rounding each of them to the nearest whole number. Using a number line may help you round in the appropriate direction.
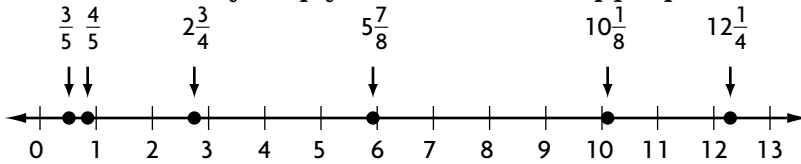
Use rounding to multiply $2\frac{3}{4} \times 10\frac{1}{8}$

Step 1 Round each mixed number to the nearest whole number.

$$2\frac{3}{4} \times 10\frac{1}{8}$$
$$\downarrow \qquad \downarrow$$
$$3 \times 10$$

Step 2 Multiply the rounded values.

$$3 \times 10 = 30$$

The product is about 30.

Use rounding to divide $12\frac{1}{4} \div 5\frac{7}{8}$

Step 1 Round each mixed number to the nearest whole number.

$$12\frac{1}{4} \div 5\frac{7}{8}$$
$$\downarrow \qquad \downarrow$$
$$12 \div 6$$

Step 2 Divide the rounded values.

$$12 \div 6 = 2$$

The quotient is about 2.

When a fraction is between 0 and 1, round it to 0, $\frac{1}{2}$, or 1, whichever is closest. Remember that you cannot divide by 0.

Use rounding to multiply $\frac{4}{5} \times 10\frac{1}{8}$

$$\frac{4}{5} \times 10\frac{1}{8} \rightarrow 1 \times 10$$

The product is about 1×10, or 10.

Use rounding to divide $12\frac{1}{4} \div \frac{3}{5}$

$$12\frac{1}{4} \div \frac{3}{5} \rightarrow 12 \div \frac{1}{2}$$

The quotient is about $12 \div \frac{1}{2} = 12 \times 2$, or 24.

Complete the estimation of each product or quotient.

1. Estimate $15\frac{1}{8} \times 1\frac{7}{8}$

$$15\frac{1}{8} \times 1\frac{7}{8} \rightarrow \underline{\qquad} \times \underline{\qquad}$$

$$= \underline{\qquad}$$

2. Estimate $24\frac{5}{6} \div 4\frac{4}{5}$

$$24\frac{5}{6} \div 4\frac{4}{5} \rightarrow \underline{\qquad} \div \underline{\qquad}$$

$$= \underline{\qquad}$$

3. Estimate $\frac{3}{5} \times 48$

$$\frac{3}{5} \times 48 \rightarrow \underline{\qquad} \times \underline{\qquad}$$

$$= \underline{\qquad}$$

4. Estimate $59\frac{7}{8} \div 15\frac{1}{8}$

$$59\frac{7}{8} \div 15\frac{1}{8} \rightarrow \underline{\qquad} \div \underline{\qquad}$$

$$= \underline{\qquad}$$

Name _____

Multiply Fractions

Bo knows that $\frac{2}{3}$ of the students in his class play soccer. Of those students, $\frac{1}{6}$ are in the school band. Bo wants to know what fraction of his class play soccer and are in the school band.

Step 1 Write a multiplication sentence.　　　$\frac{2}{3} \times \frac{1}{6} = \blacksquare$

Step 2 Multiply the numerators.　　　　　　$\frac{2 \times 1}{3 \times 6} = \frac{2}{18}$
　　　　　Multiply the denominators.

Step 3 Divide the numerator and　　　　　　$= \frac{2 \div 2}{18 \div 2}$
　　　　　denominator by the GCF, 2.

Step 4 Write the product in simplest form.　　$= \frac{1}{9}$

So, $\frac{1}{9}$ of Bo's class play soccer and are in the school band.

Complete to find each product.

1. $\frac{3}{4} \times \frac{2}{9} = \dfrac{3 \times \boxed{}}{\boxed{} \times 9}$

$= \dfrac{6}{\boxed{}}$

$= \dfrac{6 \div \boxed{}}{\boxed{} \div \boxed{}}$

$= \dfrac{\boxed{}}{\boxed{}}$

2. $\frac{1}{8} \times \frac{2}{3} = \dfrac{1 \times 2}{8 \times \boxed{}}$

$= \dfrac{\boxed{}}{24}$

$= \dfrac{\boxed{} \div \boxed{}}{24 \div \boxed{}}$

$= \dfrac{\boxed{}}{\boxed{}}$

Use GCFs to simplify the fractions. Write the product in simplest form.

3. $\frac{5}{6} \times \frac{2}{3}$　　　　**4.** $\frac{3}{4} \times \frac{5}{12}$　　　　**5.** $\frac{5}{9} \times \frac{3}{10}$　　　　**6.** $\frac{3}{4} \times \frac{6}{7}$

_____　　　　_____　　　　_____　　　　_____

7. $\frac{4}{5} \times \frac{5}{8}$　　　　**8.** $\frac{5}{6} \times \frac{12}{25}$　　　　**9.** $\frac{2}{3} \times \frac{9}{20}$　　　　**10.** $\frac{3}{8} \times \frac{4}{15}$

_____　　　　_____　　　　_____　　　　_____

RW46　Reteach

Name _____

Multiply Mixed Numbers

Barbara bought $2\frac{1}{4}$ dozen donuts. Her family ate $\frac{2}{3}$ of them. How many dozen did her family eat?

Step 1 What is $\frac{2}{3}$ of $2\frac{1}{4}$?

Find $\frac{2}{3} \times 2\frac{1}{4}$.

$\frac{2}{3} \times 2\frac{1}{4} = \blacksquare$

Step 2 Write the mixed number as a fraction.

$\frac{2}{3} \times 2\frac{1}{4} = \frac{2}{3} \times \frac{9}{4}$

Step 3 Use the GCF to simplify.
The GCF of 2 and 4 is 2.
The GCF of 3 and 9 is 3.

$= \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}}$

Step 4 Multiply.

$= \frac{1}{1} \times \frac{3}{2} = \frac{3}{2} = 1\frac{1}{2}$

So, Barbara's family ate $1\frac{1}{2}$ dozen.

Find the product. Write it in simplest form.

1. $\frac{2}{5} \times 1\frac{2}{3}$

_____

2. $4\frac{1}{2} \times 6\frac{2}{3}$

_____

3. $2\frac{1}{5} \times 3\frac{1}{8}$

_____

4. $\frac{8}{9} \times 3\frac{3}{4}$

_____

5. $1\frac{3}{7} \times 1\frac{5}{9}$

_____

6. $2\frac{1}{3} \times 1\frac{5}{7}$

_____

7. $\frac{5}{8} \times 4\frac{4}{5}$

_____

8. $2\frac{3}{4} \times 1\frac{3}{5}$

_____

9. $1\frac{1}{3} \times 5\frac{3}{4}$

_____

10. $2\frac{4}{9} \times 1\frac{1}{2}$

_____

11. $1\frac{2}{5} \times \frac{5}{8}$

_____

12. $3\frac{5}{6} \times 2\frac{1}{4}$

_____

13. $1\frac{1}{7} \times \frac{9}{10}$

_____

14. $\frac{4}{7} \times 2\frac{3}{8}$

_____

15. $\frac{5}{7} \times 4\frac{1}{5}$

_____

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Name _____

Divide Fractions and Mixed Numbers

Beth is working on a science project. She needs $\frac{2}{3}$-yd pieces of wire for the project. She bought a 6-yd piece of wire at the hardware store. How many $\frac{2}{3}$-yd pieces can she cut from this piece?

Step 1 Write a division sentence to find this amount.

$$\frac{6}{1} \div \frac{2}{3} = \blacksquare$$

Step 2 Use the reciprocal of the divisor to write a multiplication problem.

$$\frac{6}{1} \div \frac{2}{3} = \frac{6}{1} \times \frac{3}{2}$$

Think: the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Step 3 Simplify.

$$= \frac{\overset{3}{\cancel{6}}}{1} \times \frac{3}{\underset{1}{\cancel{2}}}$$

Step 4 Multiply.

$$= \frac{3}{1} \times \frac{3}{1} = \frac{9}{1} = 9$$

So, Beth can cut 9 pieces of wire.

Find the quotient. Write it in simplest form.

1. $8 \div \frac{3}{4}$

2. $\frac{5}{9} \div \frac{2}{3}$

3. $2\frac{4}{5} \div \frac{2}{3}$

_____

_____

_____

4. $2\frac{1}{10} \div \frac{3}{5}$

5. $12 \div \frac{4}{5}$

6. $2\frac{5}{8} \div \frac{3}{4}$

_____

_____

_____

7. $4\frac{2}{5} \div 1\frac{2}{3}$

8. $2\frac{1}{5} \div 1\frac{3}{10}$

9. $3\frac{2}{7} \div 1\frac{6}{7}$

_____

_____

_____

10. $5\frac{5}{6} \div 3\frac{1}{3}$

11. $6\frac{1}{2} \div 2\frac{3}{4}$

12. $\frac{5}{12} \div \frac{5}{8}$

_____

_____

_____

13. $27 \div \frac{3}{8}$

14. $\frac{4}{7} \div \frac{1}{2}$

15. $\frac{7}{8} \div \frac{1}{4}$

_____

_____

_____

Problem Solving Skill: Choose the Operation

It is often helpful to think about the kind of answer you need to solve a problem before you decide which operation or operations to use. Here are some different problem types and the operations used to solve them.

- **Add to combine two or more like measures, such as length or weight.**

  Lolly put $3\frac{1}{2}$ lb of sugar into a can containing 6 lb of sugar. The total amount of sugar in the can is $3\frac{1}{2} + 6$, or $9\frac{1}{2}$ lb.

- **Subtract to take away an amount or to compare like measures.**

  Perry spilled $2\frac{1}{3}$ lb of flour out of a 10-lb bag. The flour remaining in the bag weighs $10 - 2\frac{1}{3}$, or $7\frac{2}{3}$ lb. There is $7\frac{2}{3} - 2\frac{1}{3}$, or $5\frac{1}{3}$ lb more flour in the bag than was spilled.

- **Multiply to combine a number of equal measures or to calculate a new type of measure.**

  A square $3\frac{1}{2}$ yd on each side has a perimeter of $4 \times 3\frac{1}{2}$, or 14 yd. The square also has an area of $3\frac{1}{2} \times 3\frac{1}{2}$, or $12\frac{1}{4}$ yd^2.

- **Divide to determine how many parts of equal size are in a measure or to determine the size of several equal parts.**

  A $9\frac{1}{2}$ in. long board is cut into 4 equal lengths. Each piece has a length of $9\frac{1}{2} \div 4$, or $2\frac{3}{8}$ in. If a 6 ft board is cut in $1\frac{1}{2}$ ft lengths, there will be $6 \div 1\frac{1}{2}$, or 4 pieces.

Name the operation you would use to solve the problem. Then solve it.

1. Orange juice comes in 1-gal (128-oz) containers. William uses $12\frac{1}{2}$ oz of orange juice to make his favorite fruit smoothie. How many smoothies should he be able to make from one container of orange juice?

   _____

   _____

2. William spills about $3\frac{1}{2}$ oz of liquid each time he makes a smoothie. He makes an average of 44 smoothies each day. How many ounces of liquid would he spill on a typical day?

   _____

   _____

Name _____

Algebra: Fraction Expressions and Equations

Algebraic expressions can have fractions in them. Also, fractional values can replace a variable in the expression.

Let's explore the fraction expressions $x + \frac{2}{5}$ and $\frac{3}{4}y$ for several values of the variables x and y.

x	$x + \frac{2}{5}$
$\frac{1}{5}$	$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$
1	$1 + \frac{2}{5} = 1\frac{2}{5}$
10	$10 + \frac{2}{5} = 10\frac{2}{5}$

y	$\frac{3}{4}y$
$\frac{1}{2}$	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
2	$\frac{3}{4} \times 2 = 1\frac{1}{2}$
10	$\frac{3}{4} \times 10 = 7\frac{1}{2}$

Notice that the expression $x + \frac{2}{5}$ adds $\frac{2}{5}$ to any value of x, and that the expression $\frac{3}{4}y$ multiplies any value of y by $\frac{3}{4}$.

When solving an equation involving fractions and fractional expressions, often you can use your number sense to decide what value for the variable makes the equation true.

- Solve the equation $x + \frac{2}{5} = \frac{3}{5}$.

  Ask yourself: *What number can I add to $\frac{2}{5}$ to get $\frac{3}{5}$?*

  You can add $\frac{1}{5}$ to $\frac{2}{5}$ to get $\frac{3}{5}$. So, $x = \frac{1}{5}$.

- Solve the equation $\frac{3}{4}y = \frac{30}{4}$.

  Ask yourself: *What can I multiply $\frac{3}{4}$ by to get $\frac{30}{4}$?*

  You can multiply $\frac{3}{4}$ by 10 to get $\frac{30}{4}$. So, $y = 10$.

Evaluate the expression.

1. $x + \frac{1}{10}$ for $x = \frac{1}{2}$ _____

2. $\frac{1}{8}y$ for $y = \frac{4}{9}$ _____

3. $\frac{1}{3} - z$ for $z = \frac{1}{4}$ _____

Use mental math to solve the equation.

4. $y + \frac{1}{2} = \frac{5}{2}$ _____

5. $\frac{1}{10}m = 2$ _____

6. $\frac{1}{3}t = 1$ _____

7. $\frac{1}{4}x = 2$ _____

8. $n + \frac{3}{8} = \frac{1}{2}$ _____

9. $w - \frac{3}{10} = \frac{1}{5}$ _____

10. $\frac{5}{6} - x = \frac{3}{4}$ _____

11. $t - \frac{1}{3} = 1$ _____

12. $a + \frac{1}{3} = 7$ _____

Name _____

Understand Integers

A diver started out at the bottom of the ocean, 250 feet below sea level. He came to the surface and then climbed a hill 300 feet above sea level. How can you represent these numbers?

Integers are numbers that can show opposite directions.

250 feet below sea level is $^-250$ ft.   300 feet above sea level is $^+300$ ft.

Every integer, except zero, has an opposite.
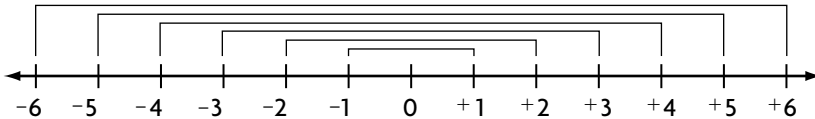
The absolute value of a number is used to show how far a number is from 0.

Find $|^-2|$: This is read as the absolute value of negative 2.
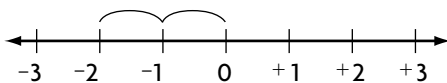
So, $|^-2| = 2$.

Write the opposite integer.

1. $^-8$ **2.** $^+7$ **3.** $^+11$ **4.** $^-9$ **5.** $^-14$ **6.** $^+17$

_____   _____   _____   _____   _____   _____

7. $^-3$ **8.** $^+12$ **9.** $^+23$ **10.** $^-30$ **11.** $^+33$ **12.** $^-50$

_____   _____   _____   _____   _____   _____

Find the absolute value.

13. $|^-6|$ **14.** $|^+6|$ **15.** $|^-8|$ **16.** $|^-21|$ **17.** $|^+13|$ **18.** $|^-26|$

_____   _____   _____   _____   _____   _____

19. $|^-45|$ **20.** $|^+56|$ **21.** $|^-77|$ **22.** $|^+92|$ **23.** $|^-345|$ **24.** $|^+880|$

_____   _____   _____   _____   _____   _____

Rational Numbers

Between any two rational numbers, you can always find other rational numbers.

Fractions or Mixed Numbers

Find a rational number between $3\frac{3}{4}$ and $3\frac{7}{8}$.

- Use a common denominator to write equivalent fractions. 8 is a common denominator of 4 and 8.

$$\frac{3}{4} = \frac{6}{8} \qquad \frac{7}{8}$$

- There are no eighths between $\frac{6}{8}$ and $\frac{7}{8}$. Use a greater common denominator. Try 16.

$$\frac{3}{4} = \frac{12}{16} \qquad \frac{7}{8} = \frac{14}{16}$$

$$3\frac{3}{4} = 3\frac{12}{16} \qquad 3\frac{7}{8} = 3\frac{14}{16}$$

$3\frac{13}{16}$ is between $3\frac{12}{16}$ and $3\frac{14}{16}$.

So, $3\frac{13}{16}$ is between $3\frac{3}{4}$ and $3\frac{7}{8}$.
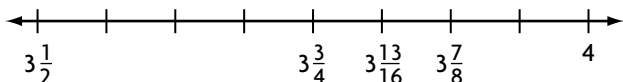
Decimals

Find a rational number between 1.3 and 1.4.

- Add a zero to each number.   1.3 = 1.30   1.4 = 1.40

- Find a number between the two numbers.   1.31, 1.32, 1.33, 1.34, 1.35, 1.36, 1.37, 1.38, and 1.39 are between 1.30 and 1.40.

So, 1.31, 1.33, 1.36, and 1.38 are some of the numbers between 1.3 and 1.4.
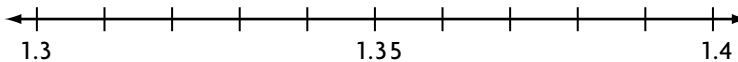
Find a rational number between the two given numbers.

1. $\frac{1}{2}$ and $\frac{3}{4}$

2. $\frac{1}{8}$ and $\frac{1}{4}$

3. $1\frac{1}{3}$ and $1\frac{2}{3}$

4. $^{-}3\frac{1}{2}$ and $^{-}3\frac{3}{4}$

_____

_____

_____

_____

5. 3.6 and 3.7

6. 1.2 and 1.3

7. $^{-}$5.9 and $^{-}$6

8. 2.11 and 2.12

_____

_____

_____

_____

9. $5\frac{1}{4}$ and $5\frac{3}{4}$

10. $^{-}$2.75 and $^{-}$2.76

11. $\frac{7}{10}$ and 0.8

12. $1\frac{1}{10}$ and 1.2

_____

_____

_____

_____

Compare and Order Rational Numbers

You can compare rational numbers in decimal or fraction form.

Using Decimals

Compare $\frac{7}{25}$ and 0.35.

- Write the number that is not a decimal in decimal form.

$$\frac{7}{25} = 25\overline{)7.00} \quad {}^{0.28}$$

- Compare decimals using place value.

$$0.28 < 0.35$$

So, $\frac{7}{25} < 0.35$.

Using Fractions

Compare $\frac{1}{4}$ and 0.3.

- Write the number that is not a fraction in fraction form.

$$0.3 = \frac{3}{10}$$

- Rewrite the fractions with the same denominator. Use 20 as a common denominator.

$$\frac{1}{4} = \frac{5}{20} \qquad \frac{3}{10} = \frac{6}{20}$$

- Compare the two fractions.

$$\frac{5}{20} < \frac{6}{20}$$

So, $\frac{1}{4} < 0.3$.

Compare. Write < or >.

1. $\frac{2}{5}$ _____ 0.2

2. 0.65 _____ $\frac{2}{3}$

3. 8.9 _____ $8\frac{4}{5}$

4. $^{-}4\frac{1}{8}$ _____ $^{-}4.3$

Compare the rational numbers and order them from least to greatest.

5. 4.2, 2.4, $\frac{8}{2}$, $\frac{2}{5}$

6. $\frac{7}{5}$, $\frac{2}{3}$, 0.2, 0.8

7. 0.1, 0.6, 0.9, 0, $\frac{1}{9}$

_____

_____

_____

8. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{8}$, 0.1

9. $^{-}$1.4, $^{-}$1.5, 1.2, $\frac{6}{4}$

10. 2.1, $^{-}$3.8, $\frac{10}{2}$, $\frac{-3}{4}$

_____

_____

_____

11. 5.8, 4.9, 5.7, $2\frac{3}{5}$, 0.58

12. $^{-}$5.6, $^{-}$4.62, 2.34, $^{-}$4.68

_____

_____

Problem Solving Strategy: Use Logical Reasoning

When a problem presents a lot of information, logical reasoning can often be used in combination with organizing the information in a table.

Chris, Keiko, Rosa, and Jamal are officers in their school's student government. One of them is president, one is vice president, one is secretary, and one is treasurer. Rosa is the president. Chris is not the treasurer. Keiko is the vice president. What office does Jamal hold?

Step 1: Think about what you know and what you are asked to find.

You know: the names of the officers and the offices;

Rosa is the president, Chris is not the treasurer, and Keiko is the vice president.

You are asked to find the office held by Jamal.

Step 2: Plan a strategy to solve.

- Use the strategy *use logical reasoning.*
- Organize the information in a table, using one at a time.

Step 3: Solve.

- Since Rosa is president, put a Y for *yes* in the box where the Rosa column and the President row cross. Put an N for *no* in the other boxes in this row and column, since Rosa cannot hold any other office and nobody else can be president.

- Since Chris is not the treasurer, put an N where the Chris column and the Treasurer row cross.

- Since Keiko is the vice president, put a Y where the Keiko column and the Vice President row cross. Put Ns in the other boxes in this row and column.

- The only office left for Chris is secretary. Put a Y where the Chris column and the Secretary row cross.

- Therefore, Jamal must be the treasurer.

	Chris	Keiko	Rosa	Jamal
President	N	N	Y	N
Vice President	N	Y	N	N
Secretary	Y	N	N	N
Treasurer	N	N	N	Y

Solve the problem using logical reasoning.

1. The Raiders, Rangers, Cougars, and Lions are in one division of a baseball league. Currently, the Raiders are ahead of the Rangers and Cougars. The Raiders and Cougars are behind the Lions. The Cougars are in last place. Who is in first place?

2. José, Cody, and Sid are students. One of them is in sixth grade, one is in seventh, and one is in eighth. The seventh-grader and José walk to school together. Sid plays ball with the eighth-grader. Cody is in sixth grade. Who is in seventh grade?

Algebra: Add Integers

You can use a number line to help you add integers.

Find the sum $^+4 + {}^-8$.

• Draw a number line.
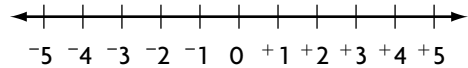
• Start at 0. Move 4 spaces to the right to show $^+4$.
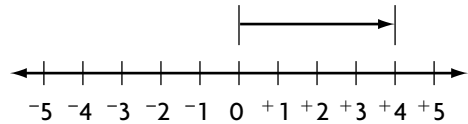
• From $^+4$, move 8 spaces to the left to show $^-8$.

So, $^+4 + {}^-8 = {}^-4$.
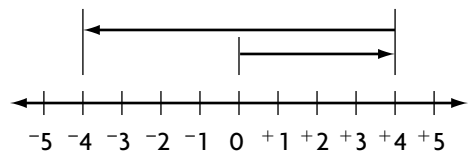
Find the sum $^-3 + {}^-5$.

• Draw a number line.
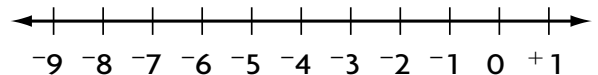
• Start at 0. Move 3 spaces to the left to show $^-3$.
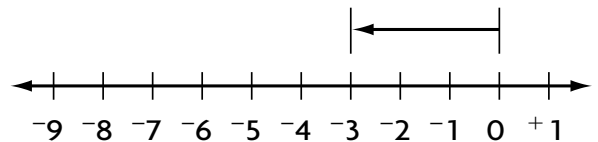
• From $^-3$, move 5 spaces to the left to show $^-5$.

So, $^-3 + {}^-5 = {}^-8$.
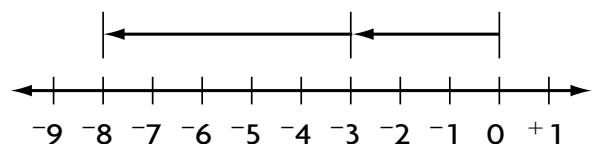
Use a number line to find each sum.

1. $^+7 + {}^-4 = $ _____

2. $^-8 + {}^-2 = $ _____

3. $^+5 + {}^-3 = $ _____

4. $^-6 + {}^-3 = $ _____

5. $^+5 + {}^-8 = $ _____

6. $^-4 + {}^-9 = $ _____

7. $^+7 + {}^-2 = $ _____

8. $^-3 + {}^+5 = $ _____

9. $^-1 + {}^-6 = $ _____

10. $^-2 + {}^+6 = $ _____

11. $^+8 + {}^-3 = $ _____

12. $^-2 + {}^-5 = $ _____

Algebra: Subtract Integers

In New York City, the 9:00 A.M. temperature reading was ⁻5°C. By noon, the temperature had dropped 3°C. What was the temperature reading at noon?

Find ⁻5 − ⁺3.

You can find the difference of two integers by adding the opposite of the integer you are subtracting.

The opposite of ⁺3 is ⁻3.
⁻5 − ⁺3 becomes ⁻5 + ⁻3.

• Draw a number line.

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1
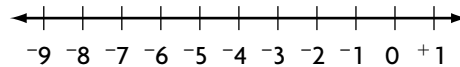
• Start at 0. Move 5 spaces to the left to show ⁻5.

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1
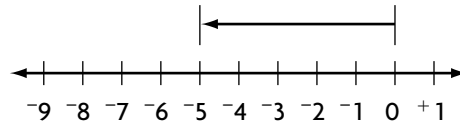
• From ⁻5, move 3 spaces to the left to add ⁻3.
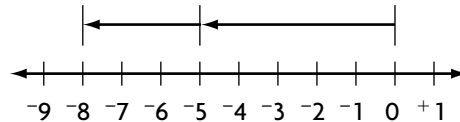
So, at noon the temperature was ⁻8°C.

⁻9 ⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1

Rewrite the subtraction problem as an addition problem.

1. ⁻4 − ⁻3 **2.** ⁺8 − ⁻8 **3.** ⁺5 − ⁺9 **4.** ⁻6 − ⁺4

_____   _____   _____   _____

Use a number line to find the difference.

5. ⁺6 − ⁺9 = _____ **6.** ⁻5 − ⁻4 = _____ **7.** ⁺7 − ⁻5 = _____

8. ⁻8 − ⁺6 = _____ **9.** ⁺6 − ⁺4 = _____ **10.** ⁻9 − ⁺5 = _____

11. ⁻3 − ⁻7 = _____ **12.** ⁺1 − ⁺8 = _____ **13.** ⁺2 − ⁻7 = _____

14. ⁻7 − ⁺5 = _____ **15.** ⁺9 − ⁺6 = _____ **16.** ⁻6 − ⁺4 = _____

17. ⁻4 − ⁻8 = _____ **18.** ⁺2 − ⁺9 = _____ **19.** ⁺4 − ⁻3 = _____

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Algebra: Multiply Integers

You can use patterns to think about multiplying integers.

Look at the patterns.

$$4 \times 2 = 8$$
$$4 \times 1 = 4$$
$$4 \times 0 = 0$$
$$4 \times {}^-1 = {}^-4$$
$$4 \times {}^-2 = {}^-8$$

$${}^-2 \times 4 = {}^-8$$
$${}^-2 \times 2 = {}^-4$$
$${}^-2 \times 0 = 0$$
$${}^-2 \times {}^-2 = 4$$
$${}^-2 \times {}^-4 = 8$$

When you multiply a positive integer by a negative integer, the product is a negative integer.

When you multiply two negative integers, the product is a positive integer.

Find the product $^-8 \times 9$.

• Multiply as with whole numbers. The product of a negative integer and a positive integer is a negative integer.

$$^-8 \times 9 = {}^-72$$

Find the product $^-6 \times {}^-4$.

• Multiply as with whole numbers. The product of two negative integers is a positive integer.

$$^-6 \times {}^-4 = 24$$

Find the product.

1. $4 \times {}^-8$ **2.** $^-7 \times {}^-7$ **3.** $^-4 \times 5$ **4.** 9×10

_____   _____   _____   _____

5. $10 \times {}^-4$ **6.** $^-11 \times {}^-10$ **7.** $^-15 \times 3$ **8.** $^-12 \times {}^-6$

_____   _____   _____   _____

9. $^-11 \times 5$ **10.** $8 \times {}^-12$ **11.** $^-5 \times {}^-15$ **12.** $^-9 \times 8$

_____   _____   _____   _____

13. $6 \times {}^-30$ **14.** $^-16 \times {}^-2$ **15.** $^-8 \times 14$ **16.** $20 \times {}^-20$

_____   _____   _____   _____

17. 10×12 **18.** $8 \times {}^-11$ **19.** $^-9 \times {}^-15$ **20.** $^-13 \times 3$

_____   _____   _____   _____

Algebra: Divide Integers

Multiplication and division are inverse operations. You can think
of related multiplication problems to divide integers. Look at the
problems below.

Division	**Multiplication**
$20 \div 4 = 5$	$4 \times 5 = 20$
$^-20 \div ^-4 = 5$	$^-4 \times 5 = ^-20$
$^-20 \div 4 = ^-5$	$4 \times ^-5 = ^-20$
$20 \div ^-4 = ^-5$	$^-4 \times ^-5 = 20$

When you divide two negative integers, the quotient is positive.

When you divide a positive integer by a negative integer, the
quotient is negative.

When you divide a negative integer by a positive integer, the
quotient is negative.

Find $^-18 \div 6$.

• Divide as with whole numbers. $^-18 \div 6 = ^-3$
  A negative integer divided by a positive
  integer is a negative integer.

Find $^-28 \div ^-7$.

• Divide as with whole numbers. $^-28 \div ^-7 = 4$
  A negative integer divided by a negative
  integer is a positive integer.

Find the quotient.

1. $^-15 \div 5$ **2.** $12 \div ^-3$ **3.** $^-24 \div ^-6$ **4.** $30 \div 6$

_____   _____   _____   _____

5. $^-32 \div 8$ **6.** $^-40 \div ^-10$ **7.** $^-45 \div ^-9$ **8.** $^-72 \div ^-8$

_____   _____   _____   _____

9. $^-400 \div 20$ **10.** $^-84 \div ^-14$ **11.** $72 \div ^-6$ **12.** $^-120 \div 40$

_____   _____   _____   _____

13. $75 \div ^-5$ **14.** $60 \div 12$ **15.** $^-125 \div 5$ **16.** $^-100 \div ^-10$

_____   _____   _____   _____

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Combine Operations with Integers

When you use the order of operations to evaluate an expression, you may need to read through the expression several times. Each time you read the expression, think "what is the next operation I should perform?"

Find the value of the expression.

$$^-5 + 6(^-2 - 4) \times 8 \div 2$$

Step 1 Read through the expression. Keep the order of operations in mind. Look to see whether there is any part of the expression inside parentheses that must be simplified.

$^-5 + 6(\mathbf{^-2 - 4}) \times 8 \div 2$	Work inside parentheses first.
$^-5 + 6(^-6) \times 8 \div 2$	

Step 2 Read through the new expression. Think about the order of operations. Look for multiplications or divisions and do them from left to right. There are three.

$^-5 + \mathbf{6(^-6)} \times 8 \div 2$	First multiply: $6(^-6)$
$^-5 + \mathbf{(^-36)} \times \mathbf{8} \div 2$	Then multiply the result by 8.
$^-5 + \mathbf{(^-288)} \div \mathbf{2}$	Then divide the result by 2.
$^-5 + (^-144)$	

Step 3 Read through the new expression. There is only one operation left to perform: addition.

$^-5 + (^-144)$	Add the two remaining integers.
$^-149$	

Evaluate the expression.

1. $^-9 + 6 \div ^-2 - 5$

2. $(7 \times 2) - ^-8 \div ^-4$

3. $1 - 15 \times 3 + 5$

_____

4. $^-10 - (^-2 \div 2) + 3$

5. $(^-15 \div 3) + (15 \div ^-3)$

6. $5^2 - (7 + ^-4) \times 2$

_____

7. $18 \div 2 + 9 \div ^-3$

8. $(6 + ^-4) + 4^2 \div 8$

9. $12 - 10 \times ^-2 \div 4$

_____

Write Expressions

Writing *numerical expressions* and *algebraic expressions* requires translating words into numbers and symbols. You can do this by looking for key words.

Addition	Subtraction	Multiplication	Division
sum	difference	product	quotient
increase	decrease	factors	equally shared
more than	less than	times	divided by
plus	minus	multiplied by	

- Write a numerical expression for "nine increased by seven squared."

    nine increased by seven squared

    9　　　+　　　7^2

    A numerical expression contains only numbers.

- Write an algebraic expression for "nine increased by a number, *n*."

    nine increased by a number, *n*

    9　　　+　　　*n*
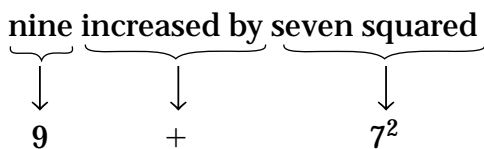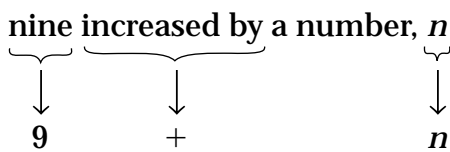
    A variable is a letter or symbol that represents an unknown number.

    An algebraic expression contains a variable.

Identify each as a *numerical expression* or *algebraic expression*. Explain your answer.

1. $12 + 8$　　　　　　　**2.** $m - 6$　　　　　　　**3.** $x \div 9$

_____　_____　_____

_____　_____　_____

Write an algebraic expression for the word expression. Remember to look for key words.

4. thirty more than a number, *n*　　　　**5.** *t* increased by nineteen

_____　　　　_____

6. the product of *y* and twenty　　　　**7.** sixty divided by a number, *m*

_____　　　　_____

Evaluate Expressions

Evaluating expressions is like substituting players in sports. In place of a variable, you must substitute a numerical value.

Evaluate $12 \div 4 + k$, for $k = 16$.

Order of Operations
1. Operate inside parentheses.
2. Clear exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Step 1

To evaluate $12 \div 4 + k$, replace the variable with the numerical value.

$$12 \div 4 + \ k$$
$$\updownarrow$$
$$12 \div 4 + 16$$

Step 2

Use the order of operations to simplify.

$12 \div 4 + 16$ No parentheses
                No exponents

$12 \div 4 + 16$ Divide: $12 \div 4 = 3$

       $3 + 16$ Add: $3 + 16 = 19$

          19

So, the answer is 19.

Use the order of operations to tell which operation you would perform first, second, and third. Find the value.

1. $4 \times 6 + 7 \times 2$

2. $30 \div 3 - 2 \times 4$

3. $12 + 6 \div 2 \times 3$

4. $^-3 + 12 \times 6 - 5$

5. $^-6 - 5 + 12 - 3$

6. $^-2 \times {}^-4 \div 2 \times 3$

Evaluate the expression for the given value of the variable.

7. $x - 13$, for $x = 20$

8. $y - 30$, for $y = 10$

9. $26 + m - 14$, for $m = 7$

10. $18 + a$, for $a = {}^-39$

11. $4 + k - 16$, for $k = {}^-5$

12. $j \div 6 + 8$, for $j = 42$

13. $5 \times (p - 15)$, for $p = {}^-21$

14. $a^2 - {}^-3$, for $a = {}^-2$

15. $n^2 + 8 \times 2$, for $n = 3$

Expressions with Squares and Square Roots

You already know pairs of operations that are opposites.

Addition and subtraction are opposites.
Addition and subtraction undo each other.
$25 + 10 = 35$, and $35 - 10 = 25$

Multiplication and division are opposites.
Multiplication and division undo each other.
$7 \times 9 = 63$, and $63 \div 9 = 7$

In the same way, squaring a number and finding the square root are also opposites.

$5^2 = 5 \times 5 = 25$ When you multiply 5 by itself, you are squaring it.
$\sqrt{25} = 5$ When you find the square root of 25, you are finding the number that can be multiplied by itself to give a product of 25.

Keep these relationships in mind when you evaluate expressions with squares and square roots.

Evaluate	$23 + \sqrt{16} \cdot (3 + 2^2)$	
	$23 + \sqrt{16} \cdot (3 + 4)$	Work inside parentheses first.
	$23 + \sqrt{16} \cdot (7)$	Continue within the parentheses.
	$23 + 4 \cdot (7)$	Evaluate $\sqrt{16}$.
	$23 + 28$	Multiply.
	51	Add.

Evaluate the expression.

1. $6 + 9^2 - 2$

_____

2. $24 + (\sqrt{25} + 4^2)$

_____

3. $\sqrt{36} + 9^2 - 2 \cdot \sqrt{4}$

_____

4. $^-2 \cdot (3^2 - \sqrt{9})$

_____

5. $\sqrt{81} - 6 + 3 \cdot (4^2)$

_____

6. $26 + 4 \cdot \sqrt{49} - 6^2$

_____

7. $^-5 + 4 \cdot \sqrt{9}$

_____

8. $\sqrt{100} - 8^2 + 2 \cdot (4^2)$

_____

9. $3^2 - \sqrt{81} + 5 \cdot (2^2)$

_____

10. $\sqrt{64} + 9^2 - 3 \cdot (5^2)$

_____

Connect Words and Equations

You can write an algebraic equation for a word sentence by reading the sentence as though it were an equation.

Example 1

The price of a ticket plus $3.95 for snacks equals $10.20.

- Read the sentence aloud, listening for where you will place the variable.
- The only unknown part of this equation is the price of a ticket. Let p represent the price of a ticket.

Read as	The price	plus $3.95	equals	$10.20
Then	p	$+$ 3.95	$=$	10.20

So, the equation is $p + 3.95 = 10.20$.

Example 2

Nine-year-old Sara is one half the age of her sister, Carmen.

- In this example, you must make two decisions: where to place the variable and where to place the equal sign.
- The only unknown is Carmen's age, so let C represent her age.
- As you read the sentence aloud, think about which word or words could be replaced by the word *equals*. The word *is* represents the equal sign.

Nine-year-old Sara is one half the age of her sister, Carmen.

$$9 \qquad = \qquad \tfrac{1}{2} \qquad \times \qquad C$$

So, the equation is $9 = \frac{1}{2} \times C$ or $9 = \frac{C}{2}$.

Write an equation for the word sentence.

1. 8 more than a number is 23.

_____

2. $^-2$ times a number is $^-40$.

_____

3. One third of a number is 16.

_____

4. A number divided by 5 is 7.

_____

5. 12 less than a number is 19.

_____

6. 8 times a number is 56.

_____

Name _____

Solve Addition Equations

An equation tells you that two quantities are equal. You can think of an equation as a balance scale where the values on each side of the scale balance each other.

You can write an algebraic equation for a word sentence by reading the sentence as though it were an equation.
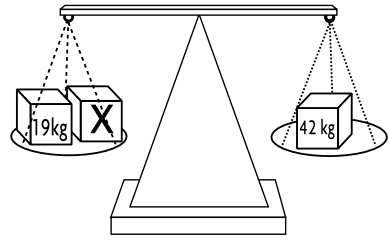
$$x + 19 = 42$$

When x is alone on one side of the scale, and the scale balances, the value on the other side is the solution of the equation. So, you want to get x alone on one side of the scale.

$$
\begin{aligned}
x + 19 &= 42 \\
-\ 19\ &-\ 19 \\
\hline
x\quad &= 23
\end{aligned}
$$
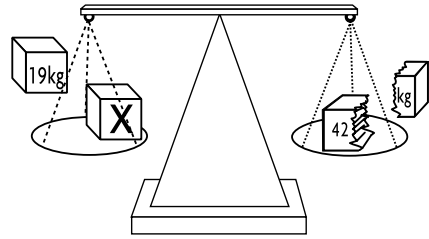
So, the solution to the equation $x + 19 = 42$ is $x = 23$. You can check the solution by replacing the variable in the equation with the value of x.

$$
\begin{aligned}
x + 19 &= 42 \\
23 + 19 &= 42 \\
42 &= 42
\end{aligned}
$$

Solve and check.

1. $d + 17 = 22$ **2.** $w + 6 = 19$ **3.** $18 + y = 32$

4. $46 = t + 27$ **5.** $x + 3.6 = 10.1$ **6.** $17.2 = a + 9.7$

7. $p + 63 = 91$ **8.** $8\frac{1}{2} + k = 15$ **9.** $18 = h + 6\frac{1}{3}$

10. $5.3 + m = 10.0$ **11.** $75.3 = 32.4 + b$ **12.** $9\frac{1}{4} = a + 3\frac{1}{3}$

Solve Subtraction Equations

Addition and subtraction are *inverse operations* because what one operation does, the other one can undo.

$8 + 3 = 11$

$11 - 3 = 8$ ← Subtraction undoes addition.

You can use the idea of inverse operations to solve subtraction equations.

Solve $x - 3 = 9$.

Read the equation as "a number x minus three equals nine." Since 3 is subtracted from x, *add* 3 to each side of the equation to solve for x.

$$\begin{array}{r} x - 3 = \ 9 \\ + 3 \quad +3 \\ \hline x = 12 \end{array}$$

Solve the equation. Show your steps.

1. $x - 5 = 16$

2. $19 = b - 9$

3. $y - 38 = 6$

4. $7 = a - 15$

5. $t - 16 = 24$

6. $52 = p - 42$

7. $y - 2 = 19$

8. $23 = w - 26$

9. $k - 36 = 21$

10. $d - 2.5 = 6$

11. $h - 3.7 = 2.5$

12. $f - 11.9 = 7.3$

13. $r - 3.9 = 7.8$

14. $13.5 = z - 6.5$

15. $m - 17.8 = 11.3$

16. $e - \frac{1}{4} = \frac{3}{4}$

17. $c - \frac{1}{3} = 2\frac{1}{3}$

18. $3\frac{1}{2} = k - 1\frac{1}{4}$

Solve Multiplication and Division Equations

The expression $6b$ means $6 \times b$.

So, if $b = 8$, $6b$ does *not* mean 68. It means 6×8.

The expression $\frac{x}{5}$ means $x \div 5$.

Multiplication and division are inverse operations.

• Use division to solve a multiplication equation.

• Use multiplication to solve a division equation.

To solve $6p = 18$,
divide both sides by 6.

$$6p = 18$$
$$\frac{6p}{6} = \frac{18}{6}$$
$$p = 3$$

Always check your solution.

$$6p = 18$$
$$6 \times \mathbf{3} = 18 \quad \text{Replace } p \text{ with 3.}$$
$$18 = 18 \; \checkmark \text{ The solution checks.}$$

To solve $\frac{y}{5} = 7$,
multiply both sides by 5.

$$\frac{y}{5} = 7$$
$$5 \times \frac{y}{5} = 7 \times 5$$
$$y = 35$$

Always check your solution.

$$\frac{y}{5} = 7$$
$$\frac{\mathbf{35}}{5} = 7 \quad \text{Replace } y \text{ with 35.}$$
$$7 = 7 \quad \checkmark \text{ The solution checks.}$$

Write the expression without using a multiplication symbol.

1. $8 \times b$ _____

2. $m \times 4$ _____

3. $15 \times x$ _____

4. $a \times 9$ _____

Use inverse operations to solve. Check your solution.

5. $4x = 16$

6. $7x = 14$

7. $4s = 12$

8. $3y = 9$

_____

_____

_____

_____

9. $\frac{x}{4} = 2$

10. $\frac{y}{3} = 4$

11. $\frac{s}{2} = 1$

12. $\frac{x}{5} = 6$

_____

_____

_____

_____

13. $32 = 4n$

14. $\frac{a}{5} = 3$

15. $8 = \frac{n}{8}$

16. $6m = 66$

_____

_____

_____

_____

Use Formulas

If the temperature is 30°, is it hot or is it cold? The answer depends on whether the temperature is measured in degrees Celsius (°C) or degrees Fahrenheit (°F).

- 30°F is cold enough for a major snowstorm. Water freezes at 32°F.

- 30°C is a typical temperature on a hot summer afternoon. At 30°C, you might feel like going swimming.

To convert from °C to °F, use the formula below.

$F = (\frac{9}{5} \times C) + 32$

To convert from °F to °C, use the formula below.

$C = \frac{5}{9} \times (F - 32)$

°F   °C
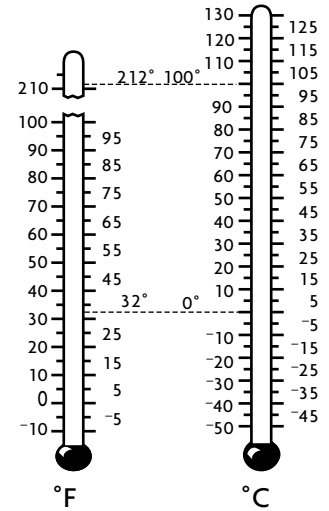
Complete.

1. Convert 30°C to degrees Fahrenheit.

$F = (\frac{9}{5} \times C) + 32$

$= (\frac{9}{5} \times \underline{\hspace{1cm}}) + 32$

$= \underline{\hspace{1.5cm}} + 32$

$= \underline{\hspace{1.5cm}}$ °F

2. Convert 59°F to degrees Celsius.

$C = \frac{5}{9} \times (F - 32)$

$= \frac{5}{9} \times (\underline{\hspace{1.5cm}} - 32)$

$= \frac{5}{9} \times \underline{\hspace{1.5cm}}$

$= \underline{\hspace{1.5cm}}$ °C

Convert the temperature to degrees Fahrenheit. Round the answer to the nearest degree.

3. 15°C _____   **4.** 80°C _____   **5.** 100°C _____   **6.** 42°C _____

7. 25°C _____   **8.** 32°C _____   **9.** 71°C _____   **10.** 19°C _____

Convert the temperature to degrees Celsius. Round the answer to the nearest degree.

11. 41°F _____   **12.** 50°F _____   **13.** 200°F _____   **14.** 89°F _____

15. 212°F _____   **16.** 38°F _____   **17.** 53°F _____   **18.** 69°F _____

Problem Solving Strategy: Work Backward

Jayne is going to hike the Appalachian Trail. To prepare, she bought 10 lb of hiker's trail mix, a stuff sack for $14.60, and a water filter for $59.99. She spent a total of $91.49. How much did the trail mix cost per pound?

Step 1 Write the information you know.

total amount spent ($91.49)

cost of two items purchased ($14.60 and $59.99)

want to find the cost of the trail mix and then the price per pound

Step 2 Plan a strategy to solve. Use the strategy *work backward.*

• Write an equation using the information in the problem.

cost of trail mix ?	+	stuff sack ($14.60)	+	water filter ($59.99)	=	total spent ($91.49)

• Decide how you can find the missing information. Write the information in reverse order with inverse operations.

total spent − water filter − stuff sack = cost of trail mix
  $91.49 − $59.99 − $14.60 =   ?

cost of trail mix ÷ 10 = price per pound of trail mix

Step 3 Solve. Carry out the strategy.

$91.49 − $59.99 − $14.60 = $16.90

$16.90 ÷ 10 = $1.69

The trail mix cost $1.69 per lb.

Use the strategy *work backward* to help you solve.

1. Marco bought 5 yd of fabric to make kites. He also paid $24.89 for rods and $17.00 for string. Marco spent $98.49 in all. What was the cost of the fabric per yard?

2. The print store charges $0.55 for each transparency made plus a $25.00 service charge. Sarah spent $135 on her project. How many transparencies did she make?

3. If you add 7 to Jason's age, then multiply by 5, the result is 105. How old is Jason?

4. If you subtract 10 from Sylvia's age, then multiply by 4, the result is 216. How old is Sylvia?

Name _____

Points, Lines, and Planes

Here is a drawing of the front wall in a typical classroom. It shows a chalkboard, flag, bulletin board, clock, and poster.
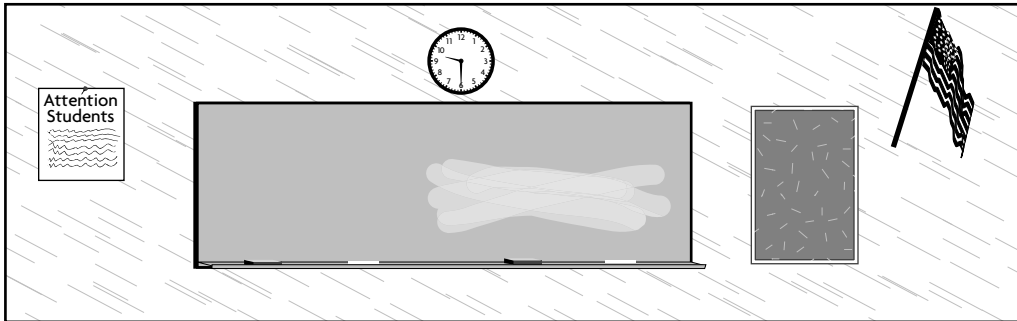
Here are some geometric figures. The arrowhead means the figure goes on forever in that direction.
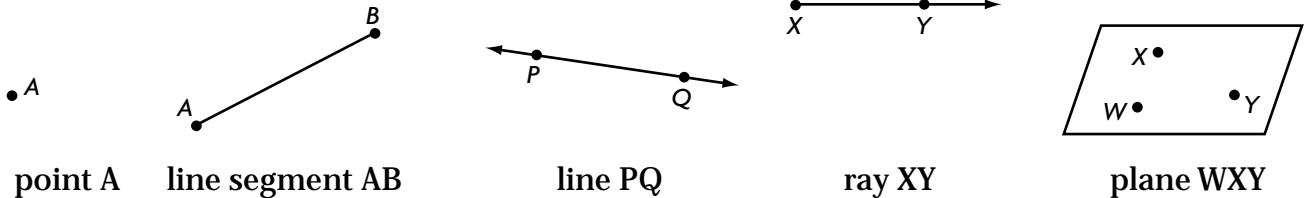
point A   line segment AB   line PQ   ray XY   plane WXY

Name the geometric figure suggested by each item on the classroom wall.

1. the flag pole _____

2. where the hands of the clock meet _____

3. the "Attention Students" poster _____

4. the chalkboard _____

5. where the flag pole is fastened to the wall _____

6. a piece of chalk _____

7. a corner of the bulletin board _____

8. the left edge of the chalkboard _____

9. a hand on the clock _____

10. the chalk ledge _____

Angle Relationships

Find the measure of angles ∠2, ∠3, and ∠4.

4
3 1   m∠1 = 110
2
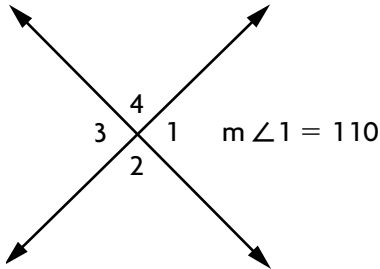
Definitions

Angles are formed when two lines intersect. The opposite angles, called *vertical angles,* have the same measure.

Supplementary angles are two angles that form a 180° angle.

Step 1 Use the definition of *vertical angles.* Since ∠3 is opposite ∠1, then m∠3 = m∠1. So ∠3 = 110°.

Step 2 Use the definition of *supplementary angles* to find the measure of ∠4.

∠4

∠3 = 110°   ∠1 = 110°

∠2

Angles 3 and 4 are supplementary angles. Their sum equals 180°. To find the measure of ∠4, subtract the measure of ∠3 from 180°.

180° − 110° = 70°   So, m∠4 = 70°.
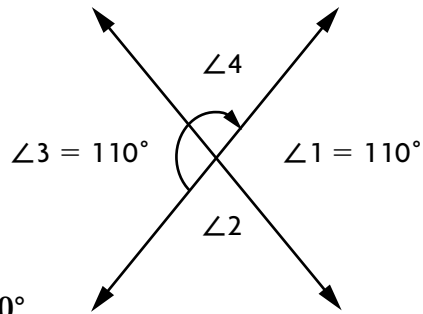
Step 3 Use the definition of *vertical angles* to find the measure of ∠2.

Since ∠4 is opposite ∠2, then m∠4 = m∠2. So m∠2 = 70°.

The angle measures are m∠2 = 70°, m∠3 = 110°, and m∠4 = 70°.

Find the unknown measures in the following figures.

1.

80° | ?

2.

?   30°

3.

162°   ?

4.

90°
45°   ?

5.

? 81°

6.

60° ? 60°
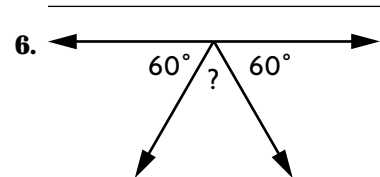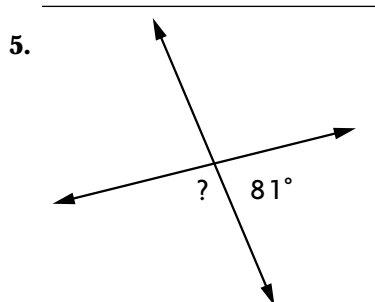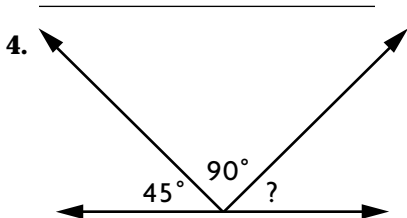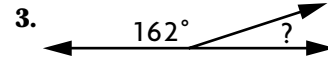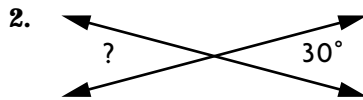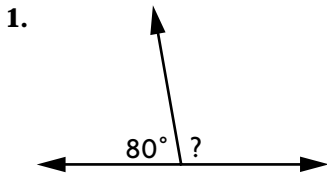
Classify Lines

Learning math vocabulary is easier if you connect it to everyday language.

Explain what is meant by each underlined word in the following statements.

• There is a light at the intersection.

_____

• Be sure the edges of the picture frame are perpendicular.

_____

• Some people find parallel parking difficult.

_____

Here are the math definitions for these words.

Intersecting lines are lines that cross at exactly one point.
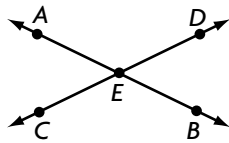
Lines *CD* and *AB* intersect at point *E*.

Perpendicular lines are lines that intersect to form 90° angles, or right angles.
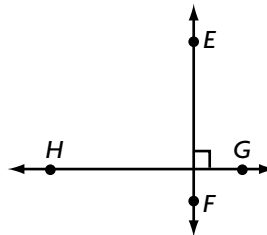
Lines *EF* and *GH* are perpendicular.

Parallel lines are lines that are always the same distance apart.
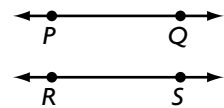
$\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$

Lines *PQ* and *RS* are parallel.

For Exercises 1–5, use the figure at the right.

1. Name a line that is parallel to $\overleftrightarrow{AD}$. _____

2. Name a line that is perpendicular to $\overleftrightarrow{BC}$. _____

3. Name two lines that intersect $\overleftrightarrow{AD}$. _____

4. Name a line that intersects $\overleftrightarrow{BC}$ but is not perpendicular

   to $\overleftrightarrow{BC}$. _____

5. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ intersect at what point? _____
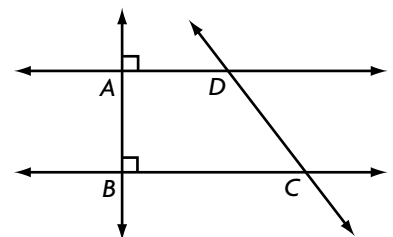
Triangles

There are several facts that you will need to use in your study
of triangles:

• A right angle has a measure of 90°.

• A straight angle has a measure of 180°.

• The sum of the measures of the angles of a triangle is 180°.

Draw any right triangle.
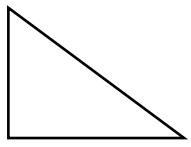
Fold each of the acute angles onto the
right angle. The sum of the two acute
angles should match the right angle.
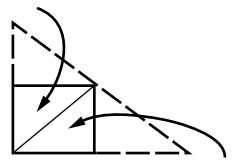
Since the right angle has a measure of 90°, the sum of the two acute angles also is 90°.
The sum of the measures of the three angles is 180°.

Triangles can be classified according to the measures of their angles.

• In an **acute triangle,** all angles will be less than 90°.

• In an **obtuse triangle,** there will be one angle greater than 90°.

• In a **right triangle,** there will be one 90° angle.

Find the measure of the angle and classify the triangle.

1.
? 65° 54°

_____

2.
25° ? 116°

_____

3.
? 38°

_____

4.
? 112° 35°

_____

5.
61° ? 52°

_____

6.
? 68° 68°

_____
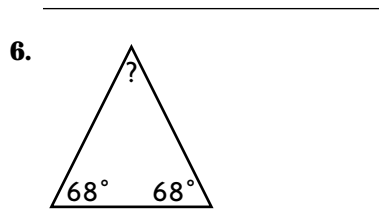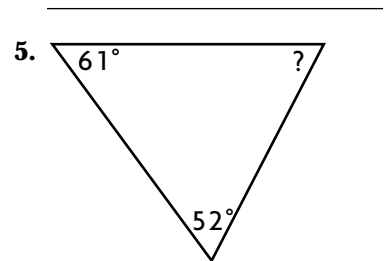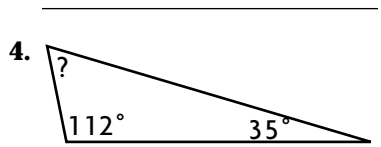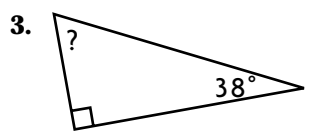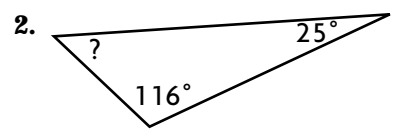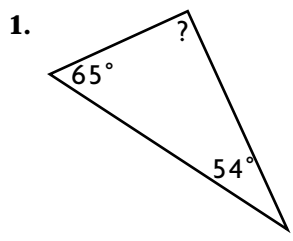
Problem Solving Strategy: Find a Pattern

Juan has a piece of land that he wants to separate into sections. He wants to be able to keep the animals he raises apart from one another. The sections can be different sizes to hold different-size animals.

What is the greatest number of sections Juan can make if he builds only fences that are straight and he uses 6 fences?

Step 1 Think about what you know and what you are asked to find.

- You know that the sections do not have to be the same size.
- You know that Juan will use only straight fences.
- You are asked to find the greatest number of sections he can make from 6 fences.

Step 2 Plan a strategy to solve.

- Use the strategy *find a pattern.*
- Look for a pattern as the number of fences increases, starting with 1 fence.
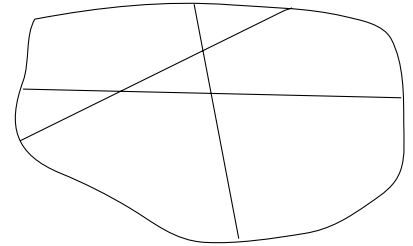
Step 3 Solve.

- Carry out the strategy.

  Make a table. Use the data in the table to look for a pattern.

Number of Fences	Number of Sections	Increase
1	2	
2	4	2
3	7	3
4	11	4

The pattern appears to be that for each fence Juan adds, he increases the number of sections by one more than the previous increase.

So, for 5 fences, Juan can add 5 sections to the 11 he has, for a total of 16. For 6 fences, he can add 6 sections to 16, for a total of 22.

Use the strategy *find a pattern* to help you solve.

1. The town library has a front stairway with 5 steps. The first step is made from 6 stone blocks. Each step uses 3 more stone blocks than the previous step. How many blocks are used in the stairway?

2. A display of cans in a store window has 9 cans on the bottom row. The number of cans decreases by 1 with each row. The top row is 1 can. How many cans are in the entire display?

Quadrilaterals

You have seen that quadrilaterals can be related to one another by use of a Venn diagram.

The relationship between quadrilaterals can also be shown by another type of diagram.

Quadrilaterals

Parallelograms   Trapezoids

Rectangles   Rhombuses
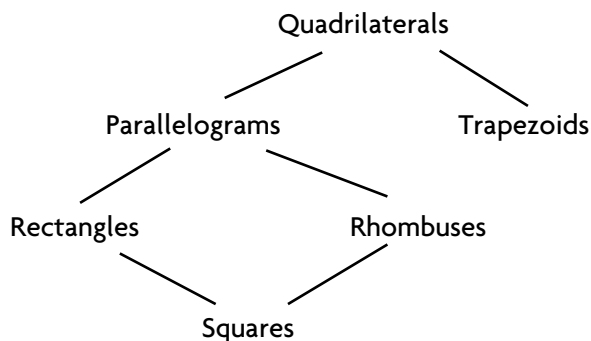
Squares

The diagram shows the following:

- All the figures are quadrilaterals.
- All rectangles are parallelograms.
- All rhombuses are parallelograms.
- All squares are rectangles.
- All squares are rhombuses.
- A trapezoid is neither a parallelogram nor a rectangle, rhombus, or square.

Use the diagram at the top of the page. Complete each sentence using *must be, can be,* or *cannot be.*

1. a rhombus _____ a square.

2. a rectangle _____ a parallelogram.

3. a trapezoid _____ a rhombus.

4. a parallelogram _____ a rhombus.

5. a rectangle _____ a square.

6. a square _____ a rhombus.

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Draw Two-Dimensional Figures

In order to draw a two-dimensional figure, use the following steps:

• Recall the properties of the figure you are going to draw.

• Decide on the order in which you will use those properties.

Remember, when you draw a figure on dot paper, you may extend the sides past the point where they intersect.

Draw a parallelogram with no right angles.

Step 1 Recall the properties of the figure.

• A parallelogram is a quadrilateral.

• Opposite sides are congruent.

• Opposite sides are parallel.
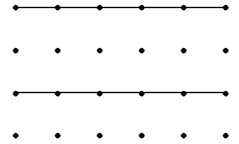
Step 2 Decide on the order in which you will use these properties.

• On dot paper, draw two parallel line segments. It is not important for them to be the same length.

• Now, draw the second pair of sides, making them parallel.
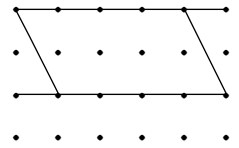
Draw the figure. Use square dot paper or isometric dot paper.

1. an acute scalene triangle

2. a pentagon with three congruent sides

3. a quadrilateral with four congruent angles and four congruent sides

4. a hexagon with all sides congruent

5. a triangle with one obtuse angle and two congruent sides

6. a quadrilateral with two pairs of congruent sides and a right angle

Circles

To better understand a circle, draw one. You can construct a circle using just a pencil and a ruler.

|←— 2 cm —→|
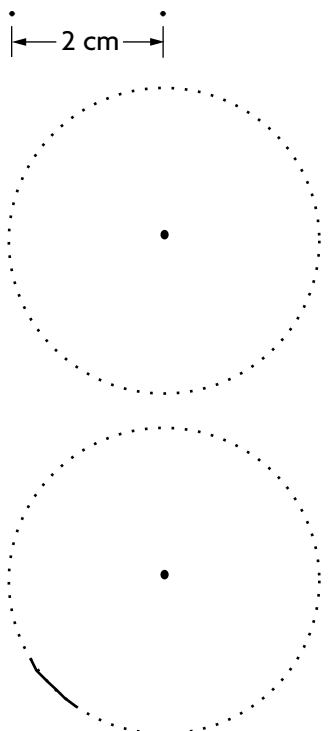
- Draw a dot and measure a length of 2 cm from the dot. Mark another dot 2 cm from the first one.

- Continue to measure 2 cm lengths from the first dot you made. Mark each location with another dot. You'll need to make at least 20 marks.

- As you fill in the spaces, you will notice that the dots are becoming closer and closer together. If you continued to fill in the spaces between the dots with more dots, the line segments connecting them would get so short that eventually you could not see them. When the space between the marks was completely filled, you would have a circle.

- A **circle** is all the points in a plane that are the same distance form one point, the center.
- The lengths that you measured are all **radii** of the circle.
- The distance from one side of the circle to the other, through the center, is the **diameter** of the circle.
- The distance between any two dots along the circle is an **arc.**

Use the circle you drew to answer 1–3.

1. How many points do there seem to be around the circle?

_____

2. How many radii do there seem to be around the circle?

_____

3. How many diameters do there seem to be around the circle?

_____

Types of Solid Figures

A **polyhedron** is a solid figure with flat faces that are polygons. A polyhedron is named by the shape of its base.

A **prism** is a polyhedron with two congruent and parallel bases. In a prism the lateral faces between the bases are rectangles.

Name this figure.

Think: All the faces are flat. All the faces are polygons.
There are two congruent and parallel bases.
The base is a triangle, and the lateral faces are rectangles.

So, the figure is a triangular prism.
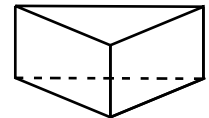
triangular prism

A **pyramid** is a polyhedron that has one base, and the lateral faces are triangles that meet at a vertex.

Name this figure.

Think: All the faces are flat. All the faces are polygons.
The lateral faces are triangles that meet at a vertex.
The base is a square.

So, the figure is a square pyramid.
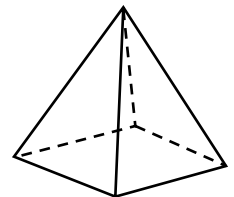
square pyramid

Name each solid figure.

1.

_____

2.

_____

3.

_____

4.

_____

5.

_____

6.

_____

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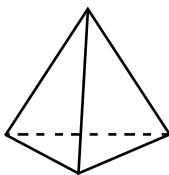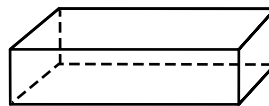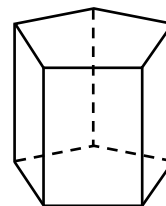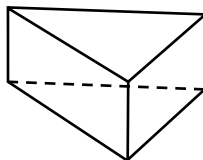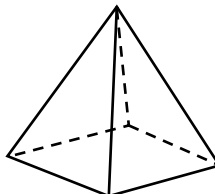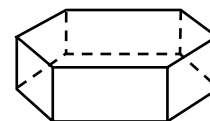
Different Views of Solid Figures

Solid figures can look different when you view them from the front, top, and bottom.

Look at the rectangular prism at the right.

If you look at the prism only from the top, you would see a rectangle.
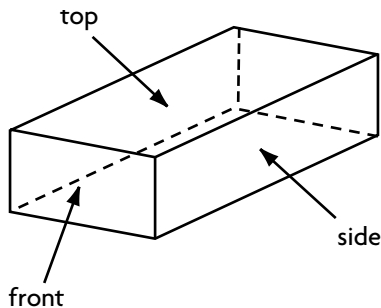
If you look at the prism only from the front, you would see a rectangle.
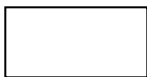
If you look at the prism only from the side, you would see a rectangle.
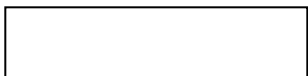
Name the figure you would see from each view of the solid figure.

1. Top: _____

Front: _____

Side: _____
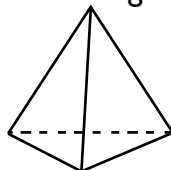
2. Top: _____

Front: _____

Side: _____
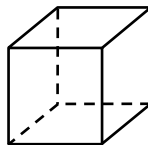
3. Top: _____

Front: _____

Side: _____
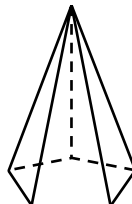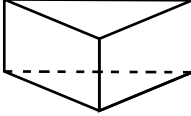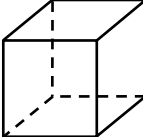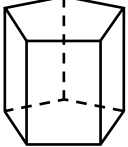
Problem Solving Strategy: Solve a Simpler Problem

Rhea is building models of prisms, using balls of clay for vertices and straws for edges. How many balls of clay and how many straws will she need to make a prism whose base has 8 sides?

Sometimes it helps you solve a more difficult problem by solving a similar, but simpler, problem first.

You can *solve a simpler problem* by thinking about the numbers of vertices and edges on prisms whose bases have 3, 4, and 5 sides.

Number of sides on base	3	4	5
Number of vertices	3 + 3, or 6	4 + 4, or 8	5 + 5, or 10
Number of edges	3 + 3 + 3, or 9	4 + 4 + 4, or 12	5 + 5 + 5, or 15

From the pattern in the table, you can see that for a prism whose base has 8 sides, the number of vertices is 8 + 8, or 16, and the number of edges is 8 + 8 + 8, or 24.

So, Rhea will need 16 balls of clay and 24 straws.

Solve by first solving a simpler problem.

1. Suppose Rhea is building a model of a prism whose base has 16 sides. How many balls of clay and how many straws will she need? _____

2. Greg wants to make a model of a pyramid whose base has 5 sides. He will use balls of clay for the vertices and toothpicks for the edges. How many toothpicks will he need? _____

3. Terri is making edible prisms. She uses cheese cubes for vertices and carrot sticks for edges. How many cheese cubes and carrot sticks will she need to make a prism whose base has 14 sides? _____

Name _____

Ratios and Rates

You can use a ratio to compare two numbers.

Two squares are shaded, and four are unshaded.
You can use a ratio to compare the shaded
squares to the unshaded squares.
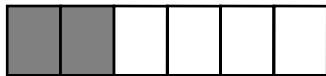
The ratio of shaded squares to unshaded squares is two to four. This
ratio can be written in three different ways:

$$2 \text{ to } 4 \qquad 2{:}4 \qquad \frac{2 \leftarrow \text{first term}}{4 \leftarrow \text{second term}}$$

Now, divide the figure in half.
The amount of shading hasn't changed.
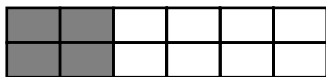
The ratio of shaded parts to unshaded parts is now $\frac{4}{8}$.
$\frac{2}{4}$ and $\frac{4}{8}$ are **equivalent ratios,** since both ratios name the same
comparison of shaded to unshaded parts.

For any ratio, if you multiply or divide both terms by the same
number, the result is an equivalent ratio.

$$\frac{2}{4} \rightarrow \frac{2 \times 2}{4 \times 2} = \frac{4}{8} \qquad\qquad \frac{2}{4} \text{ and } \frac{4}{8} \text{ are equivalent.}$$

$$\frac{15}{10} \rightarrow \frac{15 \div 5}{10 \div 5} = \frac{3}{2} \qquad\qquad \frac{15}{10} \text{ and } \frac{3}{2} \text{ are equivalent.}$$

Write the ratio in three ways.

1. five to eleven _____

2. twelve to six _____

3. twenty-five to eleven _____

4. thirty to thirteen _____

Write three equivalent ratios.

5. $\frac{3}{6}$ _____

6. $\frac{7}{1}$ _____

7. $\frac{4}{16}$ _____

8. $\frac{3}{5}$ _____

9. $\frac{15}{20}$ _____

10. $\frac{8}{12}$ _____

Find the missing term that makes the ratios equivalent.

11. $\frac{2}{6} = \frac{4}{\square}$ _____

12. $\frac{3}{1} = \frac{\square}{4}$ _____

13. $\frac{16}{8} = \frac{\square}{2}$ _____

14. $\frac{4}{5} = \frac{8}{\square}$ _____

15. $\frac{5}{6} = \frac{\square}{12}$ _____

16. $\frac{3}{8} = \frac{\square}{16}$ _____

17. $\frac{3}{7} = \frac{\square}{21}$ _____

18. $\frac{25}{55} = \frac{5}{\square}$ _____

19. $\frac{3}{4} = \frac{30}{\square}$ _____

Problem Solving Strategy

Write an Equation

An equation with a ratio on each side is a **proportion**. For example,
$\frac{1}{2} = \frac{5}{10}$ and $\frac{1}{n} = \frac{3}{9}$ are proportions.

When the proportion has a variable for one of the terms, you can solve the proportion by cross-multiplying. Study this problem.

Example: A car travels 75 mi on 3 gal of gasoline.
How far can it go on 8 gallons?

• Write a proportion. You can write the ratios in several ways:

$$\frac{mi}{mi} = \frac{gal}{gal} \text{ or } \frac{gal}{mi} = \frac{gal}{mi} \text{ or } \frac{mi}{gal} = \frac{mi}{gal}$$

Compare miles to gallons.

$$\frac{75 \text{ mi}}{3 \text{ gal}} = \frac{n \text{ mi}}{8 \text{ gal}} \leftarrow \begin{array}{l} \text{Let the variable stand} \\ \text{for the unknown distance.} \end{array}$$

• Solve the proportion by cross-multiplying.

$$\frac{75}{3} \bowtie \frac{n}{8}$$
$$3n = 75 \times 8$$
$$3n = 600 \leftarrow \text{Divide by 3.}$$
$$n = 200$$

The car can go 200 mi on 8 gal of gasoline.

Solve the problem by writing an equation.

1. You can bicycle 5 mi in 40 min. At that rate, how long does it take to travel 18 mi?

_____

2. Six cartons of bread weigh 192 lb. How much do 10 cartons weigh?

_____

3. A car travels 360 mi on 12 gal of gasoline. How far can it go on 15 gal?

_____

4. A plane travels 200 mi in 30 min. At that rate, how long does it take to travel 900 mi?

_____

5. A straight fence requires 4 posts for every 50 ft of fence. How many posts are required for 325 ft of fence?

_____

6. Eight candles cost $2.00. What is the cost of 36 candles?

_____

Name _____

Algebra: Ratios and Similar Figures

Two shapes that are similar have corresponding sides and
corresponding angles. Triangles *DEF* and *ABC* are similar.
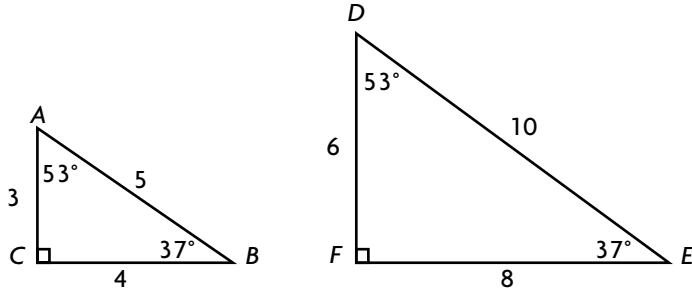
Corresponding Sides

$\overline{AB}$ corresponds to $\overline{DE}$.

$\overline{BC}$ corresponds to $\overline{EF}$.

$\overline{CA}$ corresponds to $\overline{FD}$.

The corresponding sides have the
same ratio.

$$\frac{AB}{DE} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{BC}{EF} = \frac{4}{8} = \frac{1}{2}$$

$$\frac{CA}{FD} = \frac{3}{6} = \frac{1}{2}$$

Corresponding Angles

$\angle A$ corresponds to $\angle D$.

$\angle B$ corresponds to $\angle E$.

$\angle C$ corresponds to $\angle F$.

The corresponding angles are equal.

$\angle A = 53°$ and $\angle D = 53°$

$\angle B = 37°$ and $\angle E = 37°$

$\angle C = 90°$ and $\angle F = 90°$

Name the corresponding sides and angles. Write the ratio of the
corresponding sides in simplest form.

1.
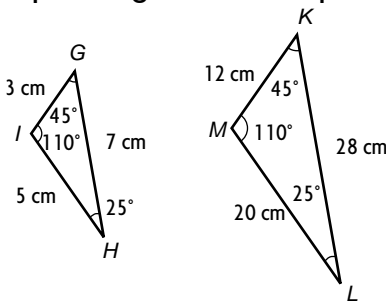
2.

_____

_____

_____

_____

_____

_____

_____

_____
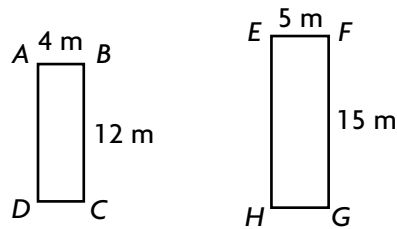
Name _____

Algebra: Proportions and Similar Figures

There are objects that can't be measured by climbing them. To measure these objects, you can use indirect measurement.

Look at the drawing of a tree and its shadow. You may not be able to climb the tree, but you can measure the shadow.

h

50 ft

You could also measure your own height and your shadow.

5 ft
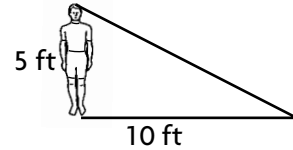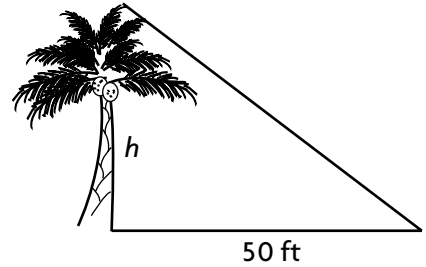
10 ft

Using similar figures, you can find the height of the tree. Follow these steps.

Step 1

Write ratios using corresponding sides.

$$\frac{\text{your shadow}}{\text{tree's shadow}} = \frac{10 \text{ ft}}{50 \text{ ft}}$$

$$\frac{\text{your height}}{\text{tree's height}} = \frac{5 \text{ ft}}{h}$$

Step 2

Set the ratios equal to each other.

$$\frac{10 \text{ ft}}{50 \text{ ft}} = \frac{5 \text{ ft}}{h}$$

Step 3

Solve the proportion.

$$\frac{10 \text{ ft}}{50 \text{ ft}} = \frac{5 \text{ ft}}{h}$$

$$10 \times h = 50 \times 5$$

$$\frac{10h}{10} = \frac{250}{10}$$

$$h = 25$$

So, the tree is 25 ft tall.

Use the two similar right triangles to write a proportion. Then solve for h.

1.

h

15 ft

6 ft

3 ft

2.

h

28 ft

7 ft

4 ft

3.

h

88 in.

52 in.

44 in.

_____   _____   _____

Name _____
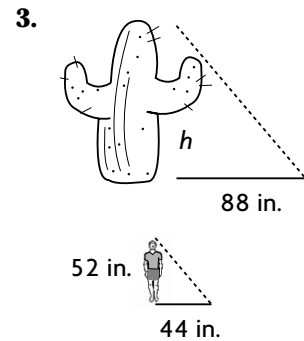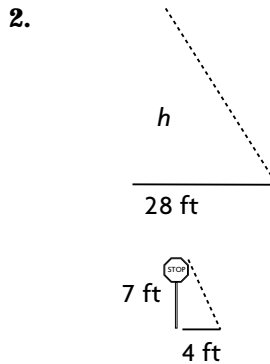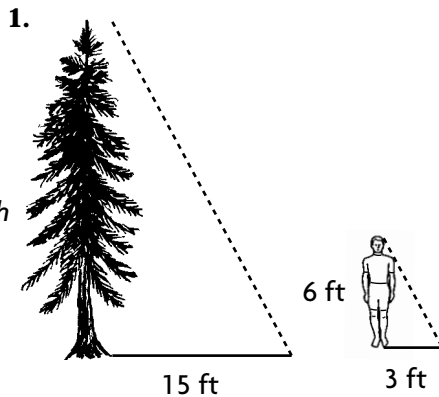
Algebra: Scale Drawings

Tom's backyard is a rectangle 100 ft long and 50 ft wide. Measure the rectangles below. Which of them can be a scale drawing of Tom's backyard? What is the scale?

a.

b.

c.
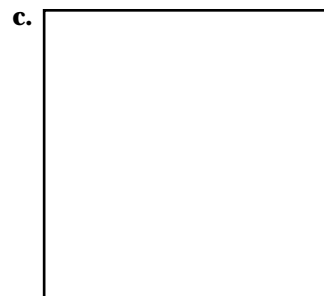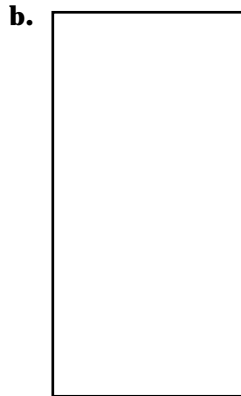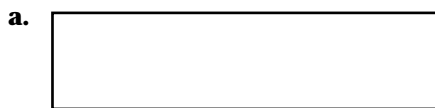
Only figure **b** can be a scale drawing of Tom's backyard. Figure **a** is too narrow, and figure **c** is too square. Tom's backyard is twice as long as it is wide, and so is figure **b.**

In a scale drawing, the shape is the same as the actual object. The drawing is smaller or larger.

Since figure **b** is 1 in. wide and Tom's yard is 50 ft wide, the drawing has a scale of 1 in. = 50 ft.

Two trees in the yard are $1\frac{1}{2}$, or 1.5, in. apart on the drawing. How far apart are the actual trees? Write a proportion.

$$\frac{1}{50} = \frac{1.5}{d} \quad \leftarrow \left\{ \begin{array}{l} \text{Compare inches to feet.} \\ \text{The scale is } \frac{1 \text{ in.}}{50 \text{ ft}}. \\ \text{The distance between the trees is } d \text{ ft.} \end{array} \right.$$

$$d \times 1 = 50 \times 1.5$$

$$d = 75$$

The trees are 75 ft apart.

Find the unknown dimension.

1. scale: 1 in. = 2 ft

drawing length: 5 in.

actual length: _____ ft

2. scale: 1 in. = 6 ft

drawing length: 2 in.

actual length: _____ ft

3. scale: 1 in. = 20 ft

drawing length: _____ in.
actual length: 100 ft

4. scale: 10 cm = 3 mm

drawing length: 5 cm

actual length: _____ mm

Name _____

Algebra: Maps

A map is a scale drawing of a region. You use the scale on a map to write a proportion in the same way you use the scale in a scale drawing.

How far is it from point A to point B?

The distance between the points on the map is 2 in. You know the scale, so you can write a proportion.

$\dfrac{\text{in.}}{\text{mi}} \rightarrow \dfrac{1}{5} = \dfrac{2}{n} \leftarrow \dfrac{\text{in.}}{\text{mi}}$

$1 \times n = 2 \times 5$

$n = 10$

The distance from point A to point B is 10 mi.

Scale : 1 in. = 5 mi

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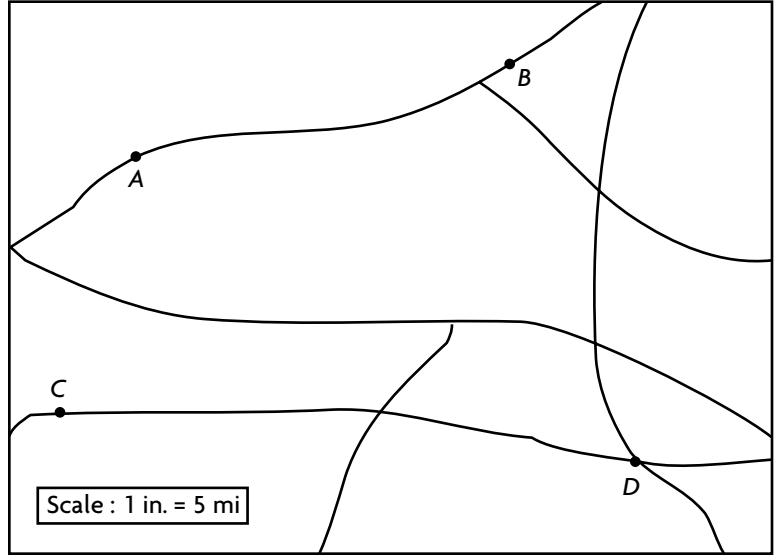
1. On the map above, the distance from point C to point D is 3 in. What is the actual distance from point C to point D?

   a. Complete the proportion.

   $\dfrac{1}{5} = \dfrac{\square}{n}$

   b. Solve the proportion. $n =$ _____

   c. So, the distance from point C to point D is _____.

Write and solve a proportion to find the actual distance.
Use the scale 1 in. = 5 mi.

2. map distance: 4 in.

3. map distance: $2\frac{1}{2}$ in.

_____

4. map distance: 6 in.

5. map distance: $7\frac{1}{2}$ in.

_____

Percent

What percent of the figure is shaded?

To find out, make a fraction.

$\dfrac{9}{30}$ ← 9 parts are shaded.
← There are 10 × 3, or 30, equal parts in all.

Now, write the fraction as a decimal by
dividing 30 into 9.

$$
\begin{array}{r}
0.30 \\
30\overline{)9.00} \\
-9\,0 \\
\hline
00
\end{array}
$$
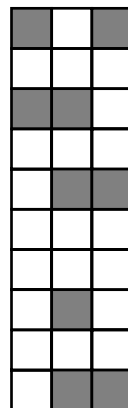
Finally, write the decimal as a percent.

0.30 = 30%   ← Move the decimal point
2 places to the right.

So, 30% of the figure is shaded.

Tell what percent of the figure is shaded.

1.

_____

2.

_____

3.

_____

4.

_____

5.

_____

6.

_____

7.

_____

8.

_____

9.

_____

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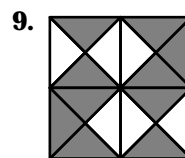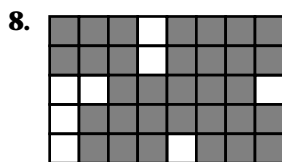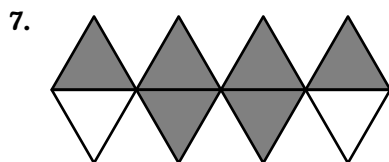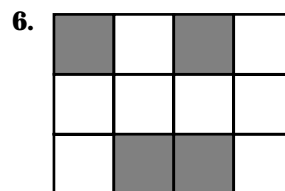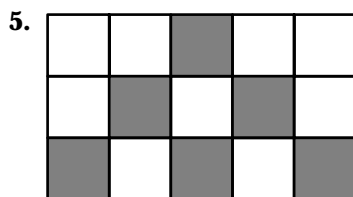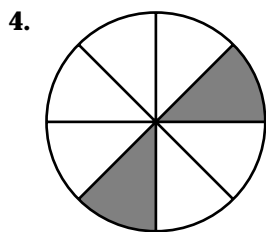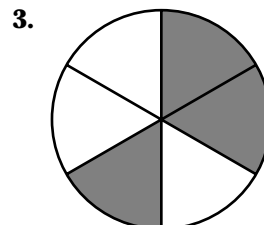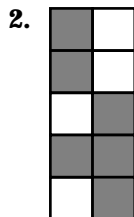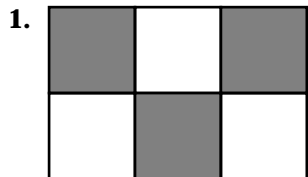
Percents, Decimals, and Fractions

Percent means "per 100." So, 25% means $\frac{25}{100}$.

A **percent** is the ratio of a number to 100. You can write any percent as a fraction or decimal.

- To write a percent as a fraction, put the percent over 100 and drop the % sign. Write the fraction in simplest form.

  $60\% = \frac{60}{100} = \frac{3}{5}$

- To write a percent as a decimal, move the decimal point two places to the left. Drop the percent sign. Remember, for a whole-number percent, the decimal point is to the right. So, 85% = 85.0%.

  $85\% = 0.85$

You can write decimals and fractions as percents.

- To write a decimal as a percent, move the decimal point two places to the right. Add a % sign.

  $0.255 = 25.5\%$

- To write a fraction as a percent, first write the fraction as a decimal. Then write the decimal as a percent.

  $\frac{5}{8} = 0.625$ ← Divide the numerator by the denominator.

  $= 62.5\%$

Complete the table.

	Fraction	Decimal	Percent
1.	$\frac{1}{2}$	0.5	
2.		0.2	20%
3.	$\frac{3}{4}$		
4.		0.9	
5.			15%
6.	$\frac{3}{8}$		

Estimate and Find Percent of a Number

Jill surveyed 20 of her classmates about creating a reading period at school. She found that 55% of those surveyed favored having a special time for reading during the day. How many of her 20 classmates favored a reading period, or what is 55% of 20?

Solving percent problems can be done using this formula and equivalent fractions.

$$\frac{\%}{100} = \frac{\text{part}}{\text{whole}}$$

Step 1

Fill in the information in the formula. % = 55 and whole = 20 students

$$\frac{55}{100} = \frac{\text{part}}{20}$$

Step 2

Find what 100 has to be divided by to give a result of 20.

Think: 100 ÷ 20 = 5.

Step 3

Divide 55 by 5 also.

$$\frac{55 \div 5}{100 \div 5} = \frac{11}{20}$$

So, 11 students favor a special reading time.

Use the formula to find the percent of the number.

1. 15% of 20

$$\frac{15}{100} = \frac{\square}{\square}$$

2. 30% of 50

$$\frac{\square}{100} = \frac{\square}{\square}$$

3. 44% of 25

$$\frac{\square}{100} = \frac{\square}{\square}$$

4. 20% of 5

$$\frac{\square}{100} = \frac{\square}{\square}$$

5. 20% of 50

$$\frac{\square}{\square} = \frac{\square}{\square}$$

6. 30% of 25

$$\frac{\square}{\square} = \frac{\square}{\square}$$

7. 40% of 5

$$\frac{\square}{\square} = \frac{\square}{\square}$$

8. 60% of 20

$$\frac{\square}{\square} = \frac{\square}{\square}$$

9. 80% of 50

_____

10. 65% of 20

_____

11. 72% of 50

_____

12. 86% of 25

_____

13. 20% of 15

_____

14. 40% of 20

_____

15. 80% of 9

_____

16. 55% of 40

_____

17. 85% of 40

_____

18. 75% of 20

_____

19. 35% of 60

_____

20. 74% of 55

_____

21. 75% of 80

_____

22. 45% of 60

_____

23. 85% of 100

_____

24. 25% of 25

_____

Name

Discount and Sales Tax

Fred wants to buy a new shirt. He found one on sale at the local store. How much will Fred pay for the shirt?

Shirt Sale
Regular Price
$25.60
30% OFF

To find the sale price of the shirt, follow these steps:

Step 1: Change 30% to a decimal.

$30\% = 0.30$

Remember, move the decimal point two places to the left.

Step 2: Multiply the percent, as a decimal, by the regular price.

$\$25.60 \times 0.30 = \$7.68$

Round to the nearest cent if needed.

Step 3: Subtract the discount from the regular price.

$\$25.60 - \$7.68 = \$17.92$

So, Fred will pay $17.92 for the shirt.

Find the amount of discount.

1. regular price: $16.00

30% off

_____

2. regular price: $27.50

SAVE 40%

_____

3. regular price: $39.00

60% discount

_____

4. regular price: $85.50

50% off

_____

Find the sale price.

5. regular price: $26.50

Discount 10%

_____

6. regular price: $47.80

70% off

_____

7. regular price: $86.00

SAVE 25%

_____

8. regular price: $184.00

SALE 90% off

_____

Simple Interest

Kirk wanted to open a savings account with the $2,000 he had earned over the summer. If the bank's simple interest rate is 5%, how much will he have in 1 year?

The money Kirk put in the bank is called **principal**. The money the bank pays Kirk is called **interest**.

Follow these steps to find the interest Kirk will have after 1 year.

Step 1: Change 5% to a decimal.

5% = 0.05

Remember, move the decimal point two places to the left.

Step 2: Multiply the percent, as a decimal, by the principal.

$2,000 × 0.05 = $100.00

Round to the nearest cent if needed.

Step 3: Multiply by the number of years.

$100.00 × 1 = $100.00

So, Kirk will receive $100.00 in interest for 1 year.

Since Kirk earned $100 in interest, at the end of the year he will have $2,100. This is his principal plus the interest earned.

Find the simple interest.

	Principal	Yearly Rate	Interest for 1 Year	Interest for 2 Years
1.	$90	4%		
2.	$225	3.8%		
3.	$480	5%		
4.	$750	4.8%		
5.	$2,600	5.4%		
6.	$3,400	7%		
7.	$4,900	8%		
8.	$6,800	8.6%		
9.	$12,000	7.75%		
10.	$15,500	9.5%		

Theoretical Probability

To find the probability, write a ratio.

Probability: $\dfrac{\text{number of favorable outcomes}}{\text{number of possible equally likely outcomes}}$

---

1. You have a spinner numbered 1 to 5. What is the probability of spinning a 2?

   a. How many possible outcomes are there? List them.

   _____

   b. How many favorable outcomes are there? _____

   c. You can write as a fraction the probability of spinning a 2.

   $P(2) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible equally likely outcomes}}$

   Write the probability as a fraction. _____
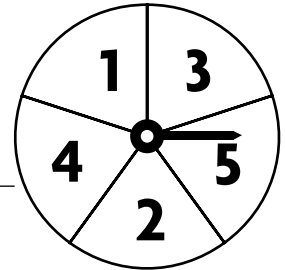
2. Each letter of the word *FRACTION* is written on a card and placed in a bag. What is the probability of choosing a *C*?

   a. How many possible outcomes are there? List them.

   _____

   b. How many favorable outcomes are there? List them. _____

   c. You can write the probability of choosing a *C* as a fraction.

   $P(C) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible equally likely outcomes}}$

   Write the probability as a fraction. _____
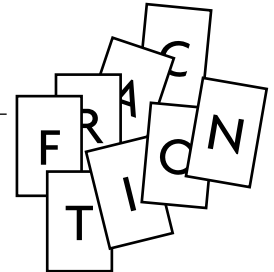
3. Each letter of the word *NUMBER* is written on a card and placed in a bag. What is the probability of choosing a vowel?

   a. How many possible outcomes are there? List them.

   _____

   b. How many favorable outcomes are there? List them.

   _____

   c. You can write the probability of choosing a vowel as a fraction.

   $P(\text{vowel}) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible equally likely outcomes}}$

   Write the probability as a fraction in simplest form. _____

Problem Solving Skill: Too Much or
Too Little Information

Some problems seem confusing because you are given more information than you need. One way to help understand the problem is to first circle the question asked. Second, underline only the information you need to answer the question. Third, solve the problem.

Read the following problem.

Christine tries calling 5 friends to ask about her homework. She knows that she must do problems 5–12, but she cannot remember the page number. She knows it is either page 150, 152, 154, or 156. Her friends aren't home, so she guesses. What is the probability that she will do the problems on the correct page?

Step 1 Circle the question asked.

You are asked to find the probability that she will do the problems on the correct page.

Step 2 Underline the information you need.

You know the homework is on one of these pages: 150, 152, 154, or 156.

You know she guesses.

Step 3 Solve the problem.

The probability that she will choose the correct page is $\frac{1}{4}$.

Write if each problem has too much, too little, or the right amount of information. Then solve the problem if possible, or write what information is needed to solve it.

1. Corey is waiting at a bus stop for the Number 3 bus. If all the buses that use the bus stop are equally likely to come along, what is the probability that the next bus will be the Number 3 bus?

2. Twelve boys and 13 girls answered a survey about their favorite food. Fifteen said pizza, 6 said French fries, and the rest said hamburgers. What is the probability that a student picked at random did not say pizza?

_____

_____

Experimental Probability

You have a box containing thousands of radish seeds. How many of them will sprout? You can experiment to find out.

Suppose you plant 20 of the seeds, and 15 of them sprout.

$$\frac{15 \text{ seeds sprout}}{20 \text{ seeds in all}} \rightarrow \frac{15}{20} = \frac{3}{4}$$

You might conclude, based on your experiment, that about $\frac{3}{4}$ of the seeds will sprout.

So, if you select a seed at random from the box, the **experimental probability** that it will sprout is $\frac{3}{4}$.

Mel spins a spinner numbered 1 to 5. The table below shows the results of 25 spins.

Number	1	2	3	4	5
Frequency	4	2	6	8	5

1. What is the experimental probability of spinning a 4?

   a. How many times did Mel spin a 4? _____

   b. How many times did Mel spin in all? _____

   c. You can write the experimental probability as a fraction.

   $$\text{Experimental probability} = \frac{\text{number of times outcome occurs}}{\text{total number of trials}}$$

   Use the formula to write the experimental probability of

   spinning 4 as a fraction in simplest form. _____

2. Suppose Mel spins the spinner 50 more times. How often can he expect to spin a 3?

   a. Begin by finding the experimental probability of spinning a 3.

   How many times did Mel spin a 3? _____

   b. How many times did Mel spin? _____

   c. Use the formula to write the experimental probability as a fraction in

   simplest form. _____

   d. Multiply 50 spins by the experimental probability.

   _____

   e. How many times can Mel expect to spin a 3? _____

Problem Solving Strategy: Make an Organized List

When you read a problem, you need to decide on a problem solving strategy to solve it. One problem solving strategy that often works well when you want to find combinations of things is *Make a List.*

Juan is planning a camping trip. He can go in June, July, or August to Yosemite, Yellowstone, or Grand Teton. How many different camping trips involving one place and one month are possible?

Step 1 Think about what you want to find and what facts are given.

- You are asked to find how many different camping trips involving one place and one month are possible.

- There are 3 months in which Juan can go and 3 different locations.

Step 2 Decide on a strategy for solving.

- Since you need to match months and locations in order to find the number of camping trips, you can use the strategy *Make an Organized List.*

Step 3 Solve.

- Follow through with the strategy. List the 3 months and pair each one with the three camping trips. Then count the number of choices in your list.

June/Yosemite	July/Yosemite	August/Yosemite
June/Yellowstone	July/Yellowstone	August/Yellowstone
June/Grand Teton	July/Grand Teton	August/Grand Teton

There are 9 possible choices for Juan's camping trip.

Solve the problem by making an organized list.

1. Craig and his friends want to go to a movie. There are 3 movies playing. They each begin at 12:30 P.M., 6:00 P.M., 8:30 P.M., and 10:30 P.M. How many different movie choices do Craig and his friends have?

2. Mr. Campo is taking a train to visit a friend. He can take a local train or an express train. They both leave the station at the same times: 11:30 A.M., 4:30 A.M., and 8:00 A.M. How many choices does Mr. Campo have?

3. Ms. Jackson sells sweat suits in her sporting goods store. They come in either a medium or large size. Both sizes come in red, navy, green, or black. How many different choices of a sweat suit does Ms. Jackson offer?

4. Liz wants to buy a bouquet of flowers. She has her choice of tulips, roses, or carnations. She can get each kind of flower in red, white, or yellow. She can have a fancy vase or a plain vase. How many choices does Liz have?

Compound Events

A compound event includes 2 or more simple events. One way to find the number of possible outcomes for a compound event is by using the **Fundamental Counting Principle:**

> To find the total number of possible outcomes, multiply the number of possible outcomes of each event.

Tiwa is buying a pizza. She has the choice of 2 sizes, 2 crusts, and 4 toppings. How many choices of size, crust, and topping does she have?

Event 1		**Event 2**		**Event 3**	
Sizes		Crusts		Toppings	
2	×	2	×	4	= 16

Tiwa has 16 choices.

Use the Fundamental Counting Principle to find the number of possible outcomes for each situation.

1. a choice of 3 salads and 4 dressings

2. a choice of 6 ice-cream flavors and 5 toppings

3. Mark's family is buying a new car. They want a station wagon or a sedan. The colors they like are black, white, and burgundy, with either a black or gray interior. How many choices do they have?

4. Carla wants to buy a sandwich. She can have either white or whole wheat bread. She has her choice of 4 fillings with either mayonnaise or mustard. How many sandwich choices does she have?

5. The Sideline Cafe offers 10 different sandwiches, 3 different salads, and 4 different soups. How many different sandwich-salad-soup combinations does the cafe offer?

6. Miko is buying fabric to make curtains. She has a choice of 8 patterns in 12 colors. The fabric comes in cotton or rayon. How many fabric choices does she have?

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Independent and Dependent Events

To determine if an event is an independent event or a dependent event, ask yourself if the first event affects the second event.

Independent Events	**Dependent Events**
If the outcome of the first event doesn't affect the outcome of the second event, the events are **independent** events.	If the outcome of the first event affects the outcome of the second event, the events are **dependent** events.
To find the probability of two independent events happening, multiply the probability of the first event by the probability of the second event.	To find the probability of two dependent events happening, multiply the probability of the first event by the probability of the second event.

If A and B are independent events, then $P(A, B) = P(A) \times P(B)$	If A and B are dependent events, then $P(A, B) = P(A) \times P(B \text{ after } A)$

A bag contains 6 lettered tiles labeled *E, V, E, N, T,* and *S.* Without looking, Emilio selects a tile and replaces it. Then he selects a second tile. What is the probability of Emilio selecting an *E* and then a *T*?

$$\text{Find } P(E, T)$$

first event: $P(E) = \frac{2}{6} = \frac{1}{3}$

second event: $P(T) = \frac{1}{6}$

$P(E, T) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

The probability of Emilio selecting an E, replacing it, and then selecting a T is $\frac{1}{18}$.

A bag contains 6 lettered tiles labeled *E, V, E, N, T,* and *S.* Without looking, Emilio selects a tile and keeps it. Then he selects a second tile. What is the probability of Emilio selecting an *E* and then a *T*?

$$\text{Find } P(E, T)$$

first event: $P(E) = \frac{2}{6} = \frac{1}{3}$

second event: $P(T \text{ after } E) = \frac{1}{5}$

$P(E, T) = \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

The probability of Emilio selecting an E, keeping it, and then selecting a T is $\frac{1}{15}$.

A jar contains cards with all the letters of the word "ALGEBRA." Without looking, you take a card out of the jar and replace it before selecting again. Find the probability of each event. Then find the probability assuming the card is not replaced after each selection.

1. P(L, R) **2.** P(A, E) **3.** P(A, A) **4.** P(G, A)

_____   _____   _____   _____

5. P(R, not A) **6.** P(B, L or R) **7.** P(A, not R) **8.** P(R, vowel)

_____   _____   _____   _____

Make Predictions

A population is a group being studied. A sample is a representative part of that group. You can use the results of a sample to make predictions about a population.

A random sample of 100 sixth graders showed that 20 of them choose green as their favorite color. What is the probability that a randomly selected sixth grader chooses green as his or her favorite color?

Step 1   Write a ratio: $\dfrac{20 \text{ students prefer green}}{100 \text{ students in sample}}$

Step 2   Write in simplest form: $\dfrac{20}{100} = \dfrac{2}{10} = \dfrac{1}{5}$

The probability that a sixth grader chooses green as his or her favorite color is about $\dfrac{1}{5}$

There are 240 sixth graders at Lincoln Middle School. Predict about how many of them would choose green as their favorite color.

Step 1   Use the ratio to write a proportion: $\dfrac{1}{5} = \dfrac{n}{240}$

Step 2   Find the cross products: $1 \times 240 = 5 \times n$
$240 = 5n$

Step 3   Solve the equation: $\dfrac{240}{5} = \dfrac{5n}{5}$
$48 = n$

So, about 48 sixth graders would choose green as their favorite color.

Use Data For 1–3, use the table. The table shows the favorite music indicated by a random sample of 100 sixth graders from Thorton Middle School.

FAVORITE MUSIC	
Music	**Number of Students**
pop	40
classical	15
rock	25
country	20

1. If there are 280 sixth graders at Thorton Middle School, about how many would prefer pop music?

_____

2. If there are 420 sixth graders at Thorton Middle School, about how many would prefer a kind of music other than pop music?

_____

3. If there are 250 sixth graders at Thorton Middle School, about how many would prefer either pop music or country music?

_____

Algebra: Customary Measurements

Ricki needs to buy 90 in. of wire for a science project. The store sells the wire by the foot. How many feet of wire does Ricki need?

Divide to change smaller units to larger units.

90 in. = ▇ ft

Think: 12 in. = 1 ft

Divide 90 by 12.

So, 90 in. = $7\frac{1}{2}$ ft.

$$7\frac{6}{12}, \text{ or } 7\frac{1}{2}$$
$$12\overline{)90}$$
$$\underline{-84}$$
$$6$$

Ricki needs $7\frac{1}{2}$ ft of wire.

Juan has 9 qt of punch. He wants to give each classmate 1 c. How many classmates can he serve?

Multiply to change larger units to smaller units.

9 qt = ▇ c

Think: 1 qt = 4 c

Multiply 9 by 4.   9 × 4 = 36

So, 9 qt = 36 c.

Juan can serve 36 classmates.

Solve.

1. Bill bought $8\frac{1}{4}$ yd of rope. How many inches of rope did Bill buy?

2. Rosa has 8 gal of juice. How many quarts of juice does she have?

3. Terri has $10\frac{1}{2}$ yd of fabric. How many inches of fabric does she have?

4. Jim's backpack weighs 88 oz. How many pounds does the backpack weigh?

5. Gary has 44 ft of hedges along his fence. How many yards of hedges does he have?

6. Monica has a water pitcher that holds 48 oz of water. How many quarts of water does the pitcher hold?

7. Lana poured 24 cups of lemonade. How many pints did she pour?

8. Which is heavier, $2\frac{1}{2}$ tons or 5,000 pounds?

Algebra: Metric Measurements

Mel made 1.45 L of soup. How many milliliters of soup does he have?

You can write a proportion to change metric units of measurement.

Step 1

Use the relationship of liter to milliliters to write a proportion.

$$\frac{1 \text{ L}}{1{,}000 \text{ mL}} = \frac{1.45 \text{ L}}{x \text{ mL}}$$

Step 2

Find the cross products. $x = 1{,}000 \times 1.45$

Step 3

Solve for x. $x = 1{,}450$

So, Mel has 1,450 mL of soup.

Use a proportion to solve.

1. Barbara has 2.7 km of ribbon. How many meters of ribbon does she have?

2. A box weighs 8.7 kg. How many grams does it weigh?

3. Rico has 536 m of rope. How many kilometers of rope does he have?

4. The chef made 2,300 mL of gravy. How many liters of gravy did he make?

5. Vince has 19.5 L of paint. How many mL of paint does he have?

6. Beth's suitcase weighs 1.9 kg. How many grams does it weigh?

7. Rachel has made 3,500 mL of punch. How many liters of punch does she have?

8. Akeem has 15.3 kg of books. How many grams of books does he have?

9. A package weighs 12.3 kg. How many grams does it weigh?

10. Mary's new car is 3 m long. How many kilometers long is her new car?

Relate Customary and Metric

Two systems of measurement are used in the United States, customary and metric. A length, weight, or capacity might be expressed in one of these systems when you need to know the equivalent measure in the other system. You need to know the relationships between the two systems to find equivalent measures.

A recipe for biscuits uses 250 g of flour. You would like to make the biscuits but need to know an equivalent customary measure, such as ounces. How many ounces of flour does the recipe require?

You must find the number of ounces that are equivalent to 250 g.

Step 1: Find the relationship between ounces and grams.
1 oz ≈ 28.35 ← The symbol ≈ is read "is approximately equal to."

Step 2: Think: If 1 oz ≈ 28.35 g, then 2 oz ≈ 2 × 28.35, or 56.7 g.
Continue in this way until you reach a value close to 250 g.

Ounces	1	2	3	4	5	6	7	8	9
Grams	28.35	56.7	85.05	113.4	141.75	170.1	198.45	226.8	255.15

From the table, you can see that 250 g is approximately equal to 9 oz.

Another method uses the relationship between ounces and grams to write a proportion.

$$\frac{1 \text{ oz}}{28.35 \text{ g}} = \frac{x \text{ oz}}{250 \text{ g}}$$

You can then solve the proportion for x to see how many ounces are approximately equivalent to 250 g.

$$28.35x = 250 \quad \text{Find the cross products.}$$
$$x = \frac{250}{28.35}$$
$$x \approx 8.82$$

So, you will need about 9 oz of flour for the biscuit recipe.

Use a proportion to convert to the given unit.

1. 1 kg ≈ 2.2 lb
7 kg ≈ __?__ lb

2. 1 m ≈ 3.28 ft
3 m ≈ __?__ ft

3. 1 qt ≈ 0.95 L
9 L ≈ __?__ qt

_____

_____

_____

4. 1 in. ≈ 2.54 cm
65 in. ≈ __?__ cm

5. 1 mi ≈ 1.61 km
9 km ≈ __?__ mi

6. 1 yd ≈ 0.91 m
250 yd ≈ __?__ m

_____

_____

_____

Name _____

Appropriate Tools and Units

You can measure the length of the same line segment using different scales on a ruler. The smaller the unit you use, the more precise the measurement will be.

inches
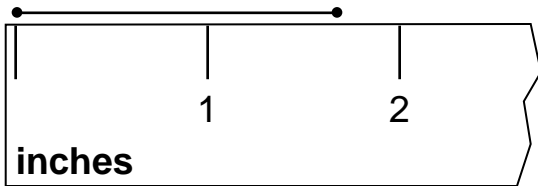
Measured to the nearest inch, the line segment is 2 in. long.

inches
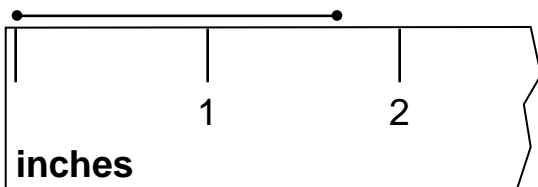
Measured to the nearest half inch, the line segment is $1\frac{1}{2}$ in. long.

inches
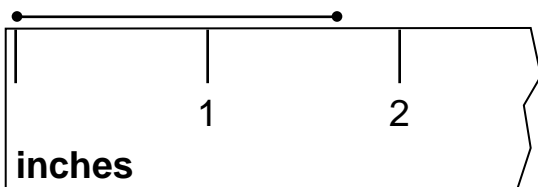
Measured to the nearest quarter inch, the line segment is $1\frac{3}{4}$ in. long.

Notice in the example that there was a different result each time the segment was measured. The result found by measuring to the nearest quarter inch, $1\frac{3}{4}$ in., is the most precise measurement of the segment.

Measure the line segment to the given length.

1. nearest inch; nearest half inch

inches
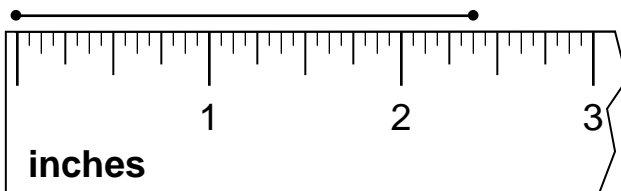
2. nearest centimeter; nearest millimeter
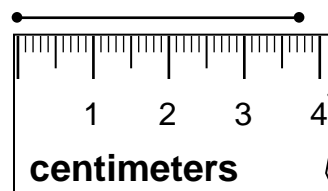
centimeters

3. nearest half inch; nearest quarter inch

inches
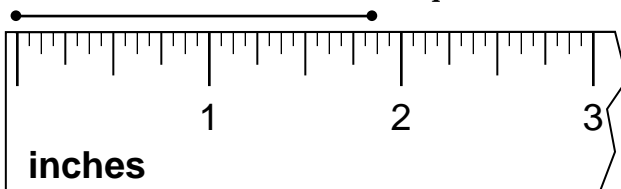
4. nearest centimeter; nearest millimeter
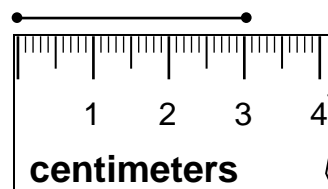
centimeters

5. nearest centimeter; nearest millimeter

6. nearest half inch; nearest quarter inch

Problem Solving Skill: Estimate or Find Exact Answer

When you read a problem, you should find an exact answer only if you have decided that an estimate is not sufficient. However, many problems that you will encounter can be solved by estimating.

Questions you can ask to help you decide on the type of answer include:

- Is it possible to find an exact answer?
  You may only be able to estimate when numbers such as large populations are involved.

- Will finding an exact answer help me in any way?
  You may need, for example, to know an exact amount of money or an exact count of people.

- Is an estimate enough to solve the problem?
  Many problems ask you only to decide whether a certain minimum amount has been reached.

A mail-order company has these 8 orders to ship. The shipping company charges $3.95 per package if the average weight of all the packages is less than 3 lb. Does the mail-order company qualify for the $3.95 rate?

shirts: 4 lb	ties: 1 lb	shoes: 4 lb	shirts: 2 lb
shirts: 2 lb	jeans: 3 lb	shirts: 2 lb	gloves: 1 lb

Step 1: Think about what you know and what you are asked to find.

- You know the weight of each package.
- You are asked to find whether the average weight of the 8 packages is less than 3 lb.

Step 2: Decide on a plan to solve.

- Since you need to find only whether the average weight of the packages is less than 3 lb, you do not need an exact answer. Estimate the average weight.

Step 3: Solve.

- Carry out the plan. Note that there are only 2 packages that weigh more than 3 lb. Both packages weigh just 1 lb more than the 3 lb average. There are 5 packages that each weigh less than 3 lb. So, it is safe to say that the average weight of the packages is less than 3 lb.

The mail-order company qualifies for the $3.95 shipping rate.

Tell whether an estimate or an exact answer is needed. Solve.

1. How much will the mail-order company have to pay the shipper to ship the 8 packages?

2. On a day when 200 packages are to be shipped at the $3.95 rate, will the shipping cost be greater than $1,000?

Perimeter

The **perimeter** of a polygon is the distance around it. To find the perimeter, add the lengths of the sides of the polygon.

Rick wants to find the perimeter of this polygon.

Find the unknown length. Then find the perimeter.

Look at the base of the figure. It has a length of 24 m. The length of the top is equal to 24m. The figure shows that two of the pieces that make up this length are each 6 m. So, $x = 24 - 12$, or 12 m.

Add the lengths of all the sides.

$12 + 6 + 6 + 12 + 24 + 12 + 6 + 6 = 84$

So, the perimeter of the figure is 84 m.
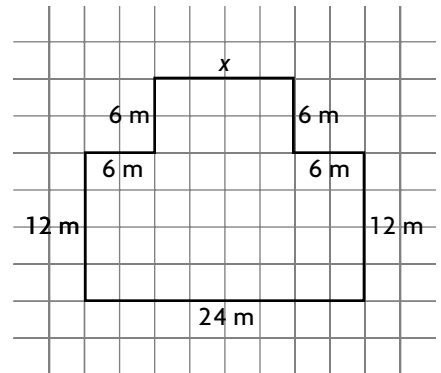
Find the unknown length. Then find the perimeter.

1.

5 ft   5 ft

x   4 ft

10 ft

2.

8 cm

6 cm   6 cm

8 cm   8 cm

6 cm   x   6 cm

3.

12 mi

6 mi

6 mi   10.5 mi

4.5 mi   x

4.

9 yd   9 yd

x   14 yd

16 yd

Reteach   RW103
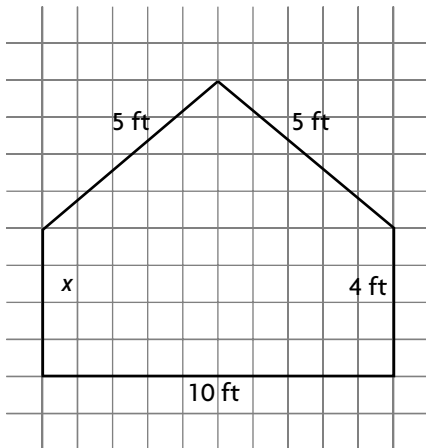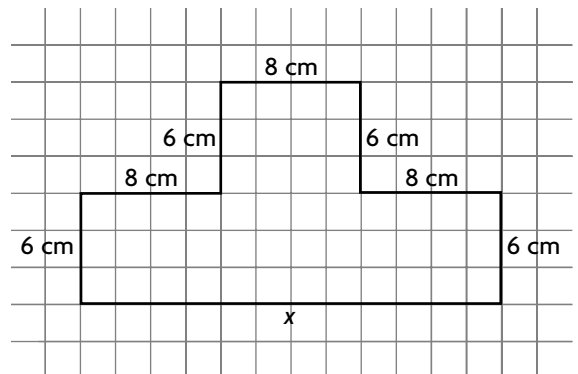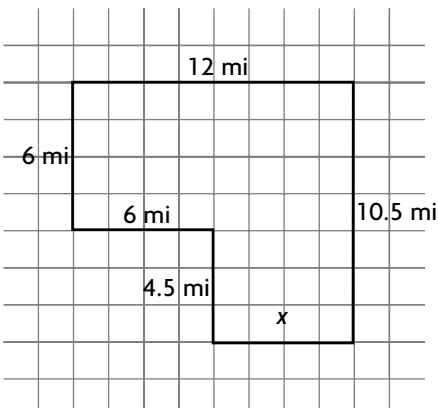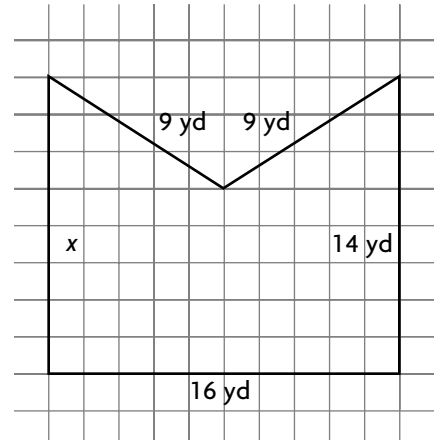
Problem Solving Strategy: Draw a Diagram

Kerry made a rectangular picture frame from 4 wood strips. The perimeter was 72 in. The length was 8 in. greater than the width. The next frame she made was as wide as the first but was 8 in. longer. It had a perimeter of 88 in. Find the dimensions of each frame.

Step 1: Think about what you know and what you are asked to find.

- You know the following about the first frame:
  It was rectangular in shape.
  The perimeter was 72 in.
  The length was 8 in. greater than the width.

- You know the following about the second frame:
  The width was the same as the first frame.
  The length was 8 in. longer than the length of the first frame.
  The perimeter was 88 in.

- You are asked to find the dimensions (length and width) of each frame.

Step 2: Plan a strategy to solve.

- Use the strategy *draw a diagram.*

- Draw the first frame using information in the problem. Experiment until you find a rectangle that meets the requirements.

- Draw the second frame from what you have learned from the first diagram.

14 in. | **A** |

22 in.

14 in. | **B** |
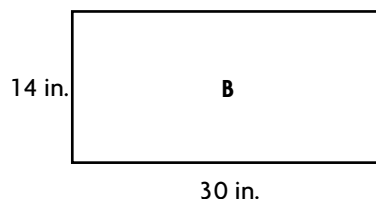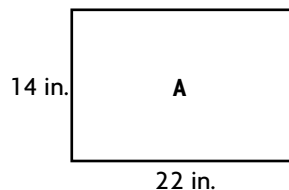
30 in.

Step 3: Solve.

- Carry out the strategy.
  From the problem, you learn that the only rectangle that meets the requirements has length 22 in. and width 14 in.
  From the first diagram, you learn that the dimensions of the second frame must be length 30 in. and width 14 in.

Use the strategy *draw a diagram* to help you solve.

1. Kerry made a triangular frame. Two sides were the same length. The third was 6 in. longer than the other two. The perimeter of the frame was 33 in. How long was each side?

2. Two square frames required a total of 144 in. of wood. The perimeter of one frame was 16 in. greater than the perimeter of the other. What were the perimeters of each frame?

Name _____

Circumference

If you understand the relationship between the diameter of a circle
and its circumference, choosing the correct formula will be easier.
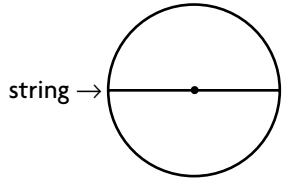
string →

- The string is stretched across the circle so that it touches the
  center point. The length of the string is equal to the diameter
  of the circle.
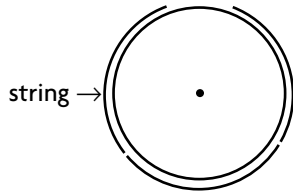
string →

- The string can be used to show the distance around, or
  circumference of the circle.

- It takes 3 string lengths plus a little more to equal the
  circumference. The extra that is needed is about $\frac{14}{100}$ or
  0.14 of the string length.

The ratio of 3.14 diameters to 1 circumference is called *pi*. The symbol used to show this relationship is the Greek letter π. The relationship applies to all circles. If the length of a diameter or radius is given as a mixed number, it may be more convenient to use $\frac{22}{7}$ rather than 3.14.

So, the formula for the circumference of a circle is $C = \pi d$. Read the formula as "Circumference equals pi times the length of a diameter."

Since a diameter is equal to two radii (the plural of radius), the formula can also be written $C = 2\pi r$ or "Circumference equals two times pi times the length of a radius."

Find the circumference of the circle. Use 3.14 or $\frac{22}{7}$ for π.

1. 9 cm

2. 16 m

3. $3\frac{1}{2}$ in.

4. 24 cm

5. 17 mm

6. $6\frac{1}{2}$ in.
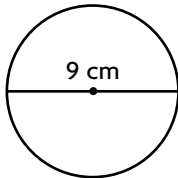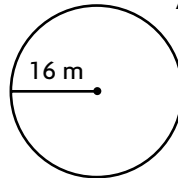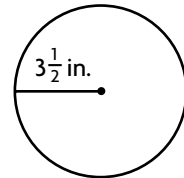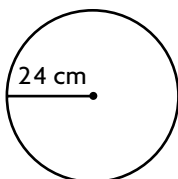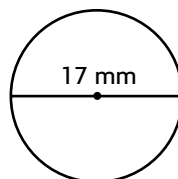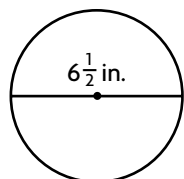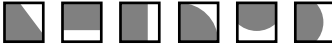
Estimate and Find Area

You can find the area of a figure on a grid by counting the square units inside the figure.

- This is a full square.
- These squares are half full.
- These squares are more than half full. They are almost-full squares.
- These squares are less than half full. They are almost-empty squares.

Here is how to estimate the area of the figure at the right. Each small square on the grid represents 1 ft^2.

- full squares: Count 9. Include all 9.
- almost-full squares: Count 4. Include all 4.
- half-full squares: Count 4. Include $\frac{1}{2} \times 4$, or 2.
- almost-empty squares: Count 3. Do not include them.

Find the sum of the squares counted. $9 + 4 + 2 = 15$

The area of the figure is about 15 ft^2.
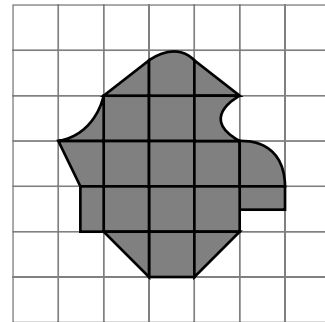
Complete to estimate the area of each figure. Each small square of the grid represents 1 yd^2.

1. There are _____ full squares.

There are _____ almost-full squares.

There are _____ half-full squares.

There are _____ almost-empty squares.
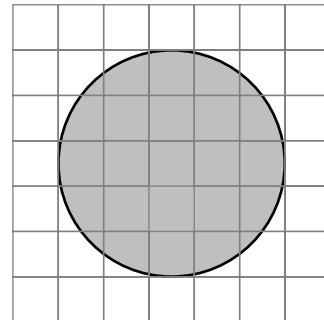
_____ + _____ + ($\frac{1}{2} \times$ _____) = _____ The area is about _____ yd^2.

Estimate the area of each figure. Each small square on the grid represents 1 ft^2.

2.

3.
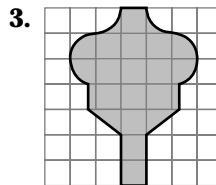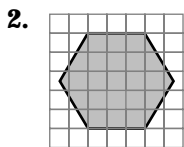
Algebra: Areas of Parallelograms and Trapezoids

This trapezoid is formed by a right triangle and a rectangle. Its area is the sum of the areas of the triangle and the rectangle.

area of the triangle   area of rectangle

$A = \frac{1}{2} bh = 4\text{cm}^2$ $A = bh = 12\text{cm}^2$

Area of trapezoid $= 4 \text{ cm}^2 + 12 \text{ cm}^2$ or 16 cm^2

Check by using the formula: $A = \frac{1}{2} h (b_1 + b_2)$

$A = \frac{1}{2} \times 4 (3 + 5)$

$A = \frac{1}{2} \times 4 \times 8 = 16$
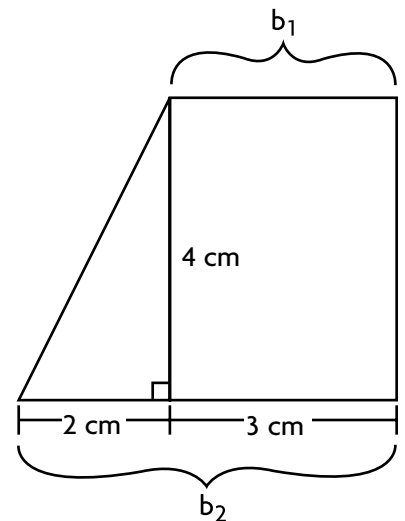
A right triangle of the same size but upside down is added to the opposite side of the rectangle to form this parallelogram. Its area is the sum of the areas of both triangles and the area of the rectangle.

triangle   triangle   rectangle   parallelogram

4cm^2 + 4cm^2 + 12cm^2 = 20cm^2

The area of the parallelogram is 20 cm^2.

Check by using the formula: $A = bh$.

$A = 5 \times 4 = 20$
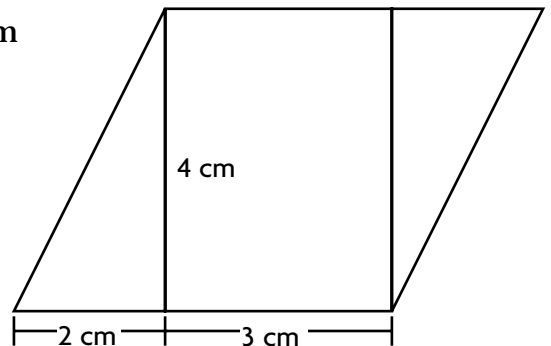
Find the area of each parallelogram. Use the formula $A = bh$.

1. $b = 15$ cm; $h = 8$ cm   **2.** $b = 7$ yd; $h = 5$ yd   **3.** $b = 9$ ft; $h = 7$ ft

$A = $ _____ cm^2 $A = $ _____ yd^2 $A = $ _____ ft^2

Find the area of each trapezoid.. Use the formula $A = \frac{1}{2} h(b_1 + b_2)$.

4. $b_1 = 5$ cm; $b_2 = 6$ cm   **5.** $b_1 = 12$ ft; $b_2 = 5$ ft   **6.** $b_1 = 3$ yd; $b_2 = 7$ yd
$h = 8$ cm $h = 9$ ft $h = 5$ yd

$A = $ _____ cm^2 $A = $ _____ ft^2 $A = $ _____ yd^2

Algebra: Areas of Circles

A circle has a radius (*r*) and a diameter (*d*). Here is how
they are related.

$d = 8$ cm $r = 4$ cm $2 \times r = d$ $\dfrac{d}{2} = r$
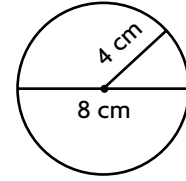
You can use the formula $A = \pi r^2$ to find the area of the circle.

$\pi \approx 3.14$

The symbol $\approx$ means "is approximately equal to."

$A \approx 3.14 \times 4^2$

Multiply. $3.14 \times 4 \times 4 = 50.24$

The area of the circle is about 50 cm^2.

Write the letter of the correct formula from Column 2.

Column 1   **Column 2**

1. rectangle _____   **a.** $A = \pi r^2$

2. parallelogram _____   **b.** $A = lw$

3. circle _____   **c.** $A = \dfrac{1}{2} bh$

4. triangle _____   **d.** $A = bh$

Use the formula $A = \pi r^2$ to find the area of each circle. Round to the
nearest whole number.

5.   **6.**   **7.**

8 in.   17 cm   22 m

_____   _____   _____
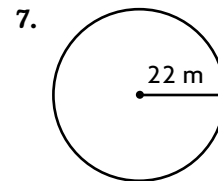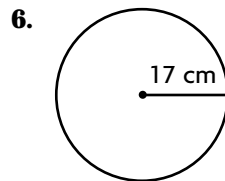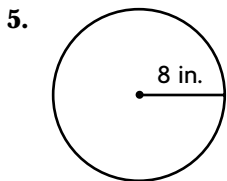
8. $r = 9$ yd _____   **9.** $r = 5$ ft _____   **10.** $r = 7$ in. _____

11. $r = 10$ m _____   **12.** $d = 12$ cm _____   **13.** $d = 30$ in. _____

14. $d = 100$ mm _____ **15.** $r = 25$ yd _____   **16.** $d = 9$ m _____

Algebra: Surface Areas of Prisms and Pyramids

Tony wants to wrap a present. He needs to know how much wrapping paper to buy. The box is shaped like a rectangular prism. How can you find the number of square inches of wrapping paper are needed to wrap the present?

You need to find the area of each of the 6 faces of the box, or the surface area of the box. Each face is a rectangle, so use the formula for the area of a rectangle.

3 in. — front — 8 in. — top — side — 14 in.

The opposite faces have the same area.

Find the area of the front.
$A = lw$
$= 8 \times 3 = 24$

Find the area of the top.
$A = lw$
$= 14 \times 8 = 112$

Find the area of one side.
$A = lw$
$= 14 \times 3 = 42$

Then multiply each area by 2 to include the opposite faces.

front and back:
$24 \times 2 = 48$

top and bottom:
$112 \times 2 = 224$

left and right sides:
$42 \times 2 = 84$

Then add the areas. $48 + 224 + 84 = 356$

So, 356 in.^{2} of wrapping paper is needed to wrap the present.
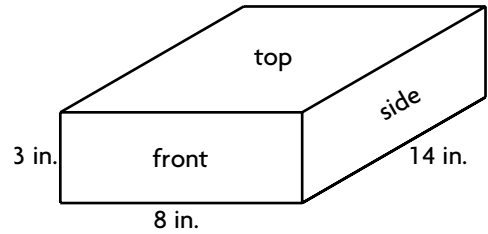
Find the surface area for each rectangular prism.

1. 8 cm, 6 cm, 4 cm

2. 10 ft, 9 ft, 3 ft

3. 11 m, 5 m, 9 m

4. 20 cm, 16 cm, 13 cm

5. 30 in., 18 in., 20 in.

6. 22 m, 22 m, 22 m

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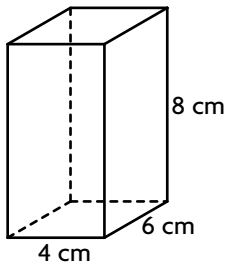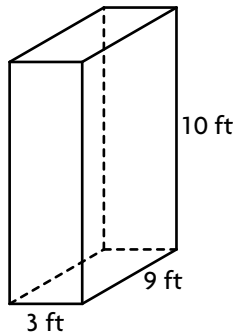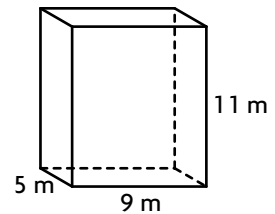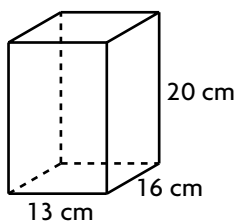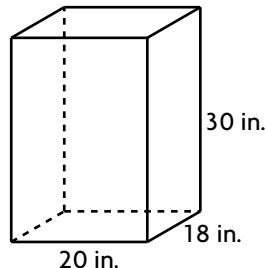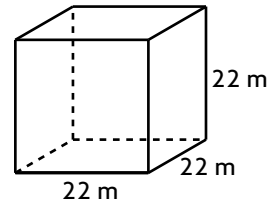
Name _____

Estimate and Find Volume

Hilary has two boxes—a rectangular box that measures 3 ft by 4 ft by 5 ft and a triangular box that measures 3 ft by 4 ft in length and width and 5 ft in height. What is the volume of each box?
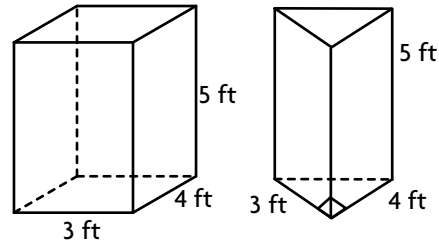
You can find the volume by using the formula $V = Bh$.

V stands for volume, B stands for the area of the base, and h stands for the height.

For a rectangular prism, any side can be the base, since they are all rectangles. For a triangular prism, the base is the triangular side.

Find the volume of the rectangular box.

Step 1 Find the area of the base.

The base is 3 ft by 4 ft.

$A = l \times w$

$A = 3 \text{ ft} \times 4 \text{ ft} = 12 \text{ ft}^2$

Step 2 Multiply the area of the base by the height.

$V = Bh$

$12 \text{ ft}^2 \times 5 \text{ ft} = 60 \text{ ft}^3$

So, the volume of the rectangular box is 60 ft^3.

Find the volume of the triangular box.

Step 1 Find the area of the base.

The base is a right triangle.

$A = \frac{1}{2} bh$

$A = \frac{1}{2} \times 3 \text{ ft} \times 4 \text{ ft} = 6 \text{ ft}^2$

Step 2 Multiply the area of the base by the height.

$V = Bh$

$6 \text{ ft}^2 \times 5 \text{ ft} = 30 \text{ ft}^3$

So, the volume of the triangular box is 30 ft^3.

Find the volume of each figure.

1. 7 in. 7 in. 6 in.

2. 6 cm 8 cm 22 cm

3. 20 m 6 m 10 m

_____   _____   _____
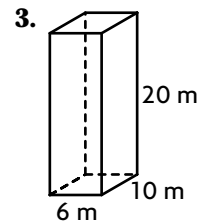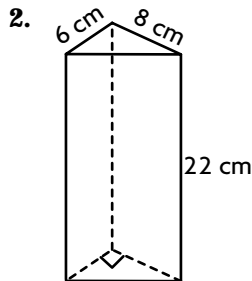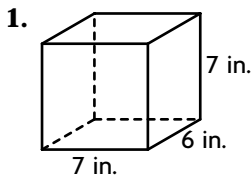
Problem Solving Strategy: Make a Model

How does doubling the dimensions of a rectangular prism affect the volume?

You can answer this question by comparing the models of two rectangular prisms.

Rectangular Prism 1

The rectangular prism below has dimensions of 2 cm × 3 cm × 4 cm.
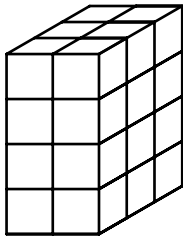
Rectangular Prism 2

The rectangular prism below has dimensions of 4 cm × 6 cm × 8 cm.
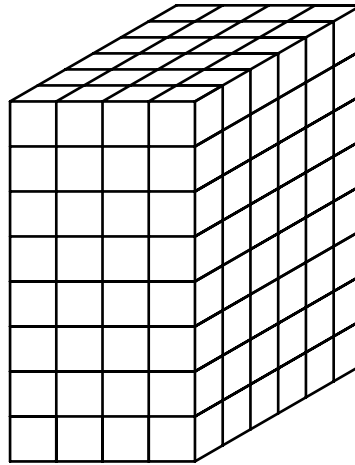
The volume is
2 cm × 3 cm × 4 cm = 24 cm^3.

The volume is
4 cm × 6 cm × 8 cm = 192 cm^3.

Compare the volume of Prism 1 with the volume of Prism 2.

192 ÷ 24 = 8

So, the volume of Prism 2 is 8 times as great as the volume of Prism 1.

Find the volume of each prism. Then double the dimensions and find the volume.

	Length	Width	Height	Volume	Volume (Doubled Dimensions)
1.	5 m	2 m	6 m		
2.	3 ft	2 ft	8 ft		
3.	5 in.	6 in.	7 in.		
4.	10 cm	10 cm	10 cm		
5.	8 m	15 m	2 m		
6.	12 cm	14 cm	15 cm		

Algebra: Volumes of Pyramids

To find the volume of a pyramid use the formula below.

Words	*Volume*	=	$\frac{1}{3}$	times	area of base	times	height of figure
⇓	↓		↓	↓	↓	↓	↓
To	*Volume* (*V*)	=	$\frac{1}{3}$	×	Base (*B*)	×	height (*h*)

⇓

Formula $V = \frac{1}{3}Bh$

Step 1 Find the area of the base. This pyramid has a rectangular base. Use $l \times w$ = area.

$l \times w$ = area of base

$7 \times 8 = 56$ in.^{2}
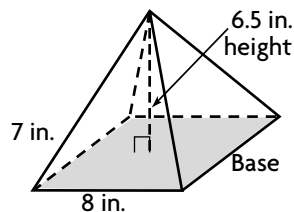
Step 2 Use the area of the base to find the volume. Multiply base times height.

$V = \frac{1}{3}Bh$

$V = \frac{1}{3} \times 56 \times 6.5 = \frac{1}{3} \times 364$

Step 3 Then multiply the product of *B* and *h* by $\frac{1}{3}$.

$V = \frac{1}{3} \times 364$

$V = \frac{364}{3}$

$V = 121\frac{1}{3}$

The volume of the pyramid is $121\frac{1}{3}$ in.^{3}

Find the volume.

1.

11 yd, 9 yd, 4 yd

_____

2.

7.5 m, 8 m, 8 m

_____

3.

5 ft, 5 ft, 8 ft

_____
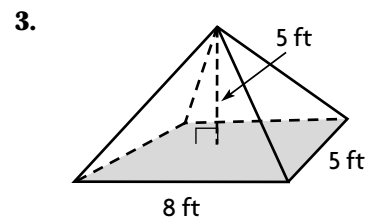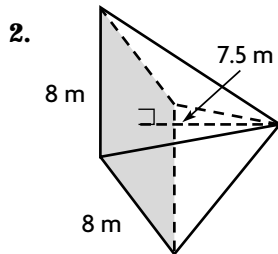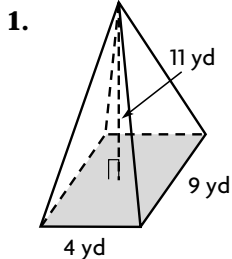
4. rectangular pyramid:
   l = 7 in., *w* = 4.5 in., *h* = 8 in.

_____

5. square pyramid:
   l = 13 ft, *w* = 13 ft, *h* = 15 ft

_____

Name _____

Volumes of Cylinders

The cylinder at the right has a height of
8 in. and a radius of 2 in. A cylinder
has a circular base.

2 in.

8 in.
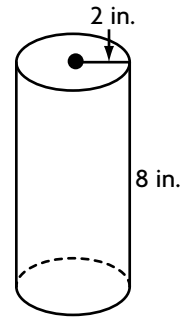
You can find the volume of a cylinder by using the formula $V = Bh$.
The B stands for the area of the base, which is a circle.

Step 1 Find the area of the base.

The base is a circle.

2 in.
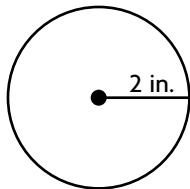
Use the formula $A = \pi r^2$.
Use 3.14 for π.

$A \approx 3.14 \times 2 \text{ in.} \times 2 \text{ in.} \approx 12.56 \text{ in.}^2$

Step 2 Multiply the area of the base, B,
times the height.

8 in.
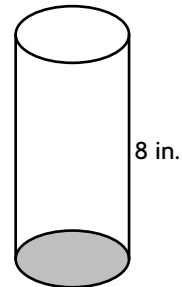
$12.56 \text{ in.}^2 \times 8 \text{ in.} \approx 100.48 \text{ in.}^3$

So, the volume of the cylinder is about 100 in.^{3} to the nearest whole number.

Find the volume of each cylinder. Round to the nearest whole number.

1. 6 cm
12 cm

_____

2. 4 cm
9 cm

_____

3. 10 ft
4 ft

_____

4. 15 m
3 m

_____

5. 8 in.
1 in.

_____

6. 3 ft
5 ft

_____
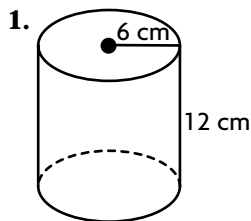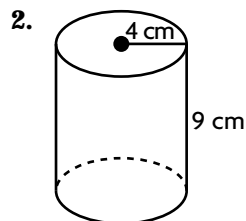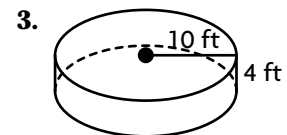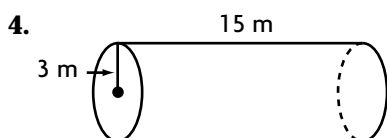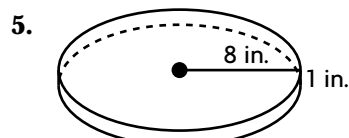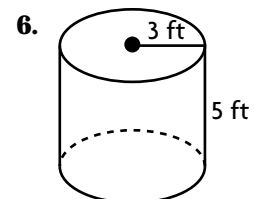
Problem Solving Strategy: Find a Pattern

A problem solving strategy that often works well when events repeat is *find a pattern.*

Ms. Jacobs owns a bookstore. She gets a discount on the number of books she orders. If she orders 4 copies of a book, she will get a $2 discount. For 8 copies, she will get a $5 discount; for 12 copies, an $8 discount; for 16 copies, an $11 discount; and so on until she reaches the maximum discount of $20. How many books does Ms. Jacobs need to order to get the maximum discount?

Step 1 Think about what you want to find and what facts are given.

- You want to find how many books Ms. Jacobs needs to order to get the maximum discount of $20.
- You are given the amount of the discount for the number of books that are ordered.

Step 2 Decide on a strategy for solving.

- Since the number of books increases by the same amount, and the discount increases by the same amount you can use the strategy *Find a Pattern.*

Step 3 Solve.

- Find the patterns:

    books ordered: 4, 8, 12, 16 → increases by 4 each time

    discount: $2, $5, $8, $11 → increases by $3 each time

- Show the patterns in a table to find the number of books that must be ordered to get the maximum discount of $20.

Number of Books	4	8	12	16	20	24	28
Amount of Discount	$2	$5	$8	$11	$14	$17	$20

Ms. Jacobs must order 28 books to get the maximum discount of $20.

Solve the problem by finding a pattern.

1. Alberto and Toshi are on a 7-day hike. They hike 10 mi the first day; 12 mi the second day; 14 mi the third day; and 16 mi the fourth day. If this pattern continues, how many miles will they hike on the seventh day?

2. Janell has decided to jog 1 mi each day this week. She plans to jog $1\frac{1}{2}$ mi each day next week, 2 mi each day the next week, and so on until she jogs 5 mi each day. If this pattern continues, when will that happen?

Patterns in Sequences

John rides the bus to work every day. The bus fare each day is
$3.25. On payday John budgets $20.00 for transportation. How
much of this money will John have after 5 workdays?

You can use a pattern to solve this problem.

Start: $20.00

After 1 day: $20.00 − $3.25 = $16.75

After 2 days: $16.75 − $3.25 = $13.50

After 3 days: $13.50 − $3.25 = $10.25

After 4 days: $10.25 − $3.25 = $7.00

After 5 days: $7.00 − $3.25 = $3.75

Subtract $3.25 from each
term to find the next term.

If you write the terms as a sequence, you have
$20.00, $16.75, $13.50, $10.25, $7.00, $3.75, ...

So, after 5 days John will have $3.75.

___

Write a sequence and solve.

1. Jeff is starting an exercise program. The first week he will jog 5
   mi. If he increases the number of miles by 2 mi each week, in
   how many weeks will he be jogging more than 14 mi?

   _____

2. Pat spends $2.45 for lunch each day. She has $20.00 budgeted for
   lunch. On which day will she need additional lunch money?

   _____

3. Danielle is reading a book. The first day she reads 3 pages. If she
   increases the number of pages she reads by 3 pages each day, on
   which day will she be reading more than 20 pages?

   _____

4. During a five-week reading program, Brad read 2 books the first
   week. If he increases the number of books he reads by 2 each
   week, how many books will he read the fifth week?

   _____

Number Patterns and Functions

Very often, one quantity depends on another quantity. You can write a function to show that relationship.

Eric wants to mail a letter that weighs 6 ounces. The cost of mailing the letter is $.033 for each ounce or part of an ounce. How much will it cost Eric to mail his letter?

Step 1 Look for a pattern:

weight	cost
1 ounce	$0.33
2 ounces	$0.66
3 ounces	$0.99
4 ounces	$1.32

The pattern shows that as the number of ounces increases by one, the cost increases by $0.33.

Step 2 Use the pattern to write an equation.

Let w = number of ounces and c = total cost.

$c = 0.33 \times w$

Step 3 Solve the equation.

$c = 0.33 \times 6$

$c = \$1.98$

It will cost Eric $1.98 to mail his letter.

Solve the problem by finding a pattern.

1. Carolyn wants to mail a package that weights 5 lb. The cost of mailing the package is $0.75 for each pound or part of a pound. Write an equation and solve to find out how much it will cost Carolyn to mail her package.

3. Jack wants to buy a video player for $150.00. He earns $6.25 per hr doing yard work. Write an equation and solve to find out how many hours Jack will have to work to earn enough money to buy the video player.

3. Nina is making a picture frame that is 24 in. long. She wants the length to be three times the width. Write an equation and solve to find out how wide she should make the frame.

4. All 32 members of Sam's computer club are going on a field trip. Each van can hold 6 passengers besides the driver. Write an equation and solve to find out how many vans they will need.

Geometric Patterns

There are many types of patterns based on geometric figures. Study these patterns.

← 1 square, 3 squares, 6 squares, 10 squares, . . . The next figure will have 15 squares.
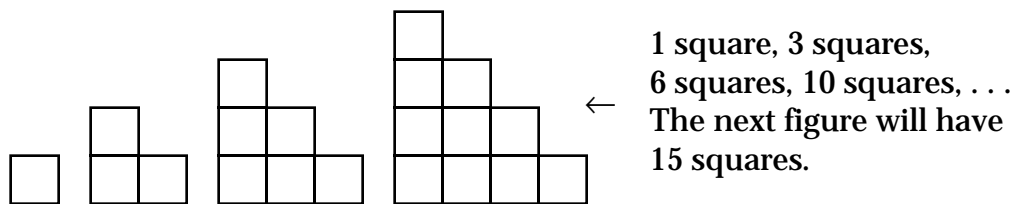
The pattern above is based on the arrangement of the squares. The number of squares on the left side and on the bottom row increases by 1. So the next figure will be 5 squares high and 5 squares wide.

This pattern is based on the type of shape.

←3 sides, 4 sides, 5 sides, . . .
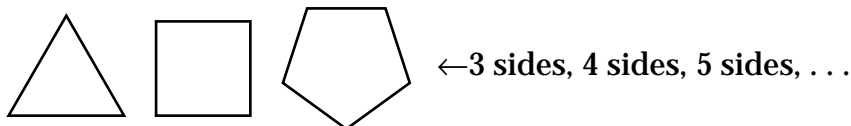
So, the next figure in the pattern will have 6 sides.

A pattern can be based on shading, such as the following pattern. The figure doesn't change, except for the shading pattern.

← $\frac{1}{6}$ is shaded, then $\frac{2}{6}$, and then $\frac{3}{6}$.
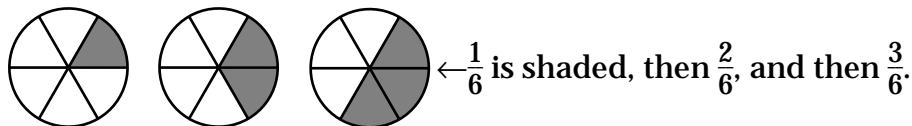
So, the next figure in the pattern will have $\frac{4}{6}$ shaded.

Draw the next two figures in the geometric pattern.

1. _____

2. _____

Transformations of Plane Figures

A **transformation** is a movement that doesn't change the size or shape of a figure.

Three kinds of transformations are:

| Translation | Rotation | Reflection |
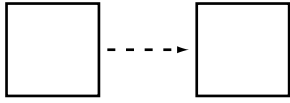
Slide the figure along a straight line.
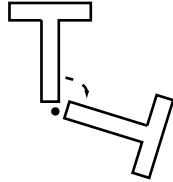
Turn the figure around a point.
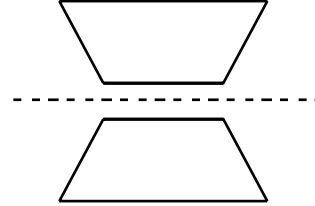
Flip the figure over a line.

Tell whether the drawing shows a translation, rotation, or reflection.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____
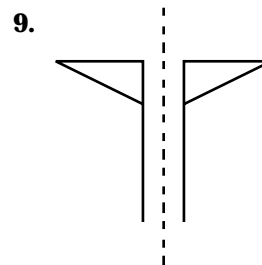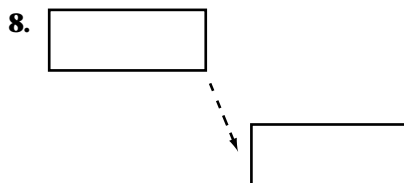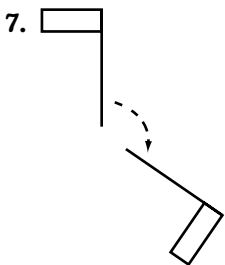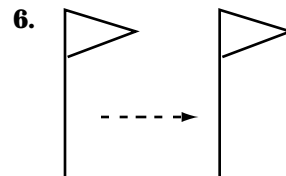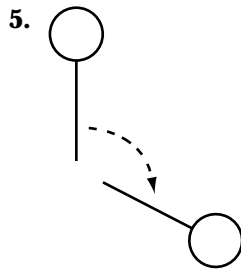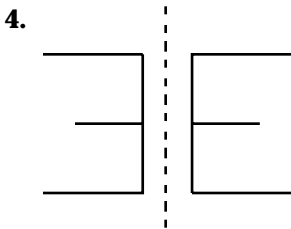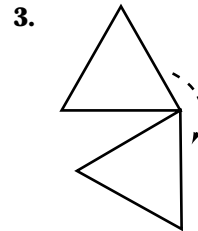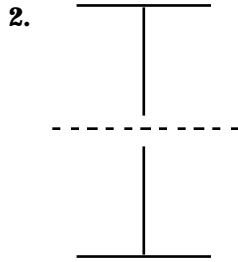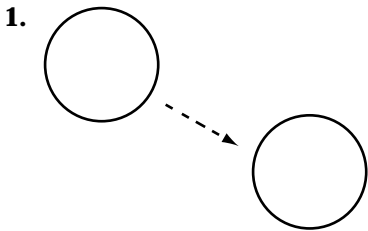
Tessellations

A tessellation is an arrangement of shapes that completely covers a flat surface.

The simplest tessellations involve squares or rectangles. You have seen examples on kitchen floors or on walls, such as the following.
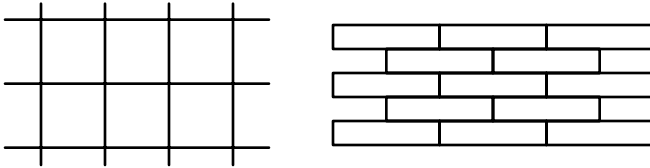
To make a more interesting pattern, start with any square or rectangle.
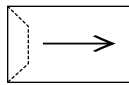
←Cut out a piece of any shape from one side.
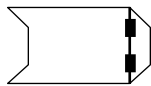
←Translate the piece to the opposite side. Then tape it.

Now tessellate the new shape. Here are two ways to do so.
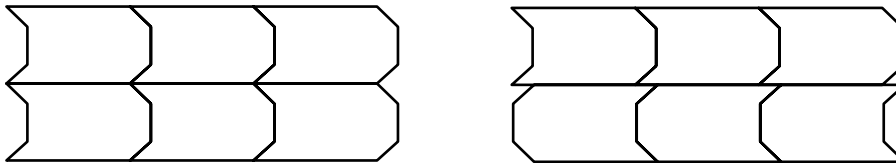
Draw the tessellation shape described by each pattern. Then form two rows of a tessellation.

1.

2.
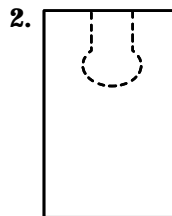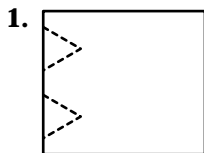
Problem Solving Strategy: Make a Model

Larry is designing a tile mosaic for a floor.
The shape he uses must tessellate a plane.
He wants to use the shape at the right. Can
he use this shape for his design?
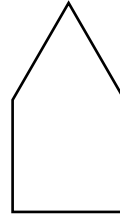
To solve this problem, you can make a model.

Trace and cut out several of the shapes
Larry wants to use. Use the cutouts to
see if the shape will tessellate a plane.

Move the shapes around to see if they will
fit together without gaps or overlaps.

Check that the sum of the angle measures
around a point is 360°.

The shape Larry wants to use will
tessellate the plane.

150°   150°
60°
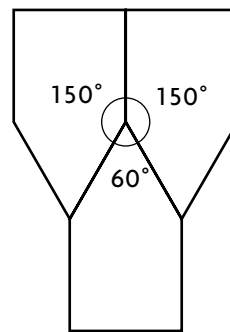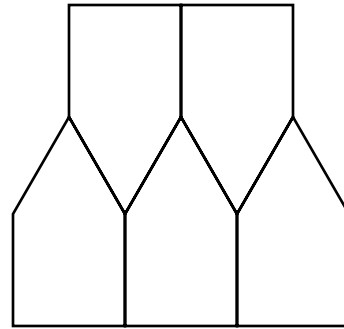
Make a model to solve.

1. Erika is making a design from the
   shape below. She wants the design to
   tessellate. Can she use this shape?
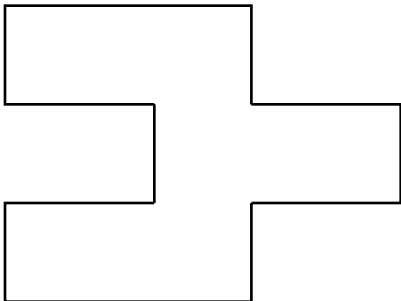
2. For a tabletop design, Shawn uses
   octagons and squares. Will his design
   tessellate a plane?

© Harcourt
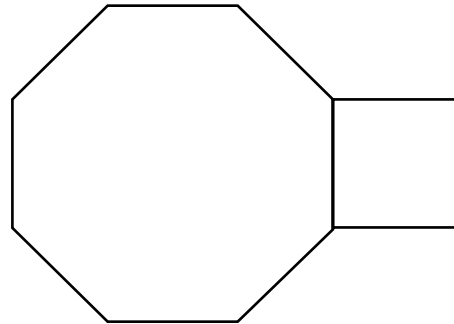
Transformations of Solid Figures

Find the number of unique ways you can
place this figure on top of the gray plane
figure. Like a puzzle, the prism must fit over
the gray square exactly.
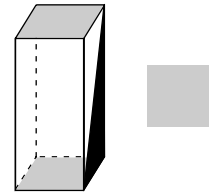
Step 1 Look and think about what you are asked to do.
The figure is a prism. It has two parallel faces
that are squares. It also has two sets of parallel
faces that are rectangles. The gray plane figure is
a two-dimensional square.
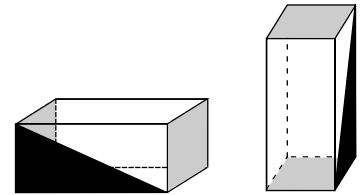
Step 2 Find the faces of the prism that will cover the
gray square. The rectangular faces do not
fit. So find every way each square face will
fit over the gray square.

Step 3 The prism in the position of the original
figure counts as the first way. Rotate
around an imaginary vertical axis to find
the first 4 ways.
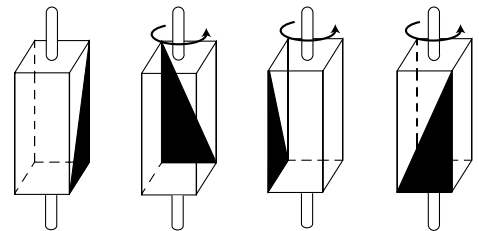
Step 4 Next, rotate the prism around a horizontal
axis so that you find each way the other
square face can be placed.
Again, rotate the prism around a vertical
axis to find the final 4 ways it will fit. There
are a total of 8 ways that the prism fits.
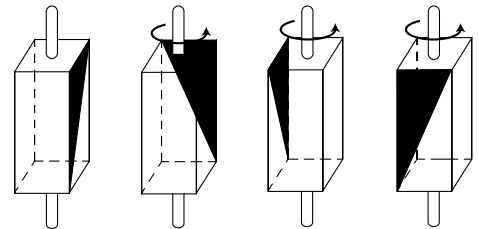
Write the number of ways you can place the solid figure on the plane
figure in the second column and on the plane figure in the third column.

1.

2.

3 cm   3 cm

3 cm
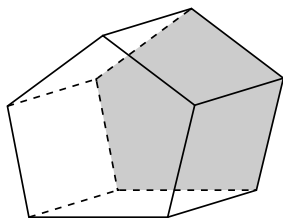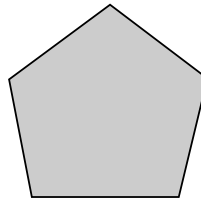
3.
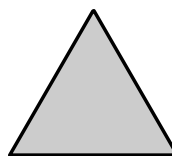
4.
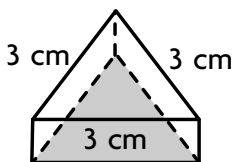
Symmetry

A figure has **line symmetry** if it can be folded so that the two parts of the figure match.

Look at the top figure.
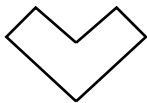
The dashed line is a line of symmetry.

If you fold the figure on the dashed line, the two parts match.

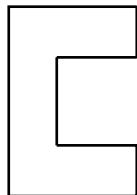The bottom figure shows another line of symmetry.
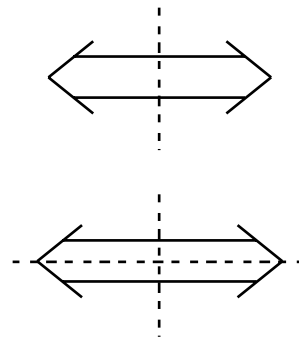
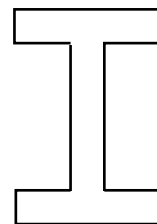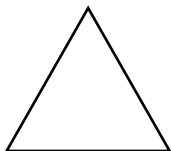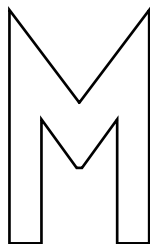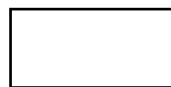Draw the line or lines of symmetry for each figure.
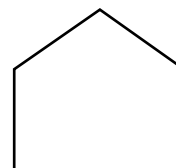
1.

2.

3.

4.

5.

6.

7.

8.

Draw a figure that is not shown above that has at least two lines of symmetry.
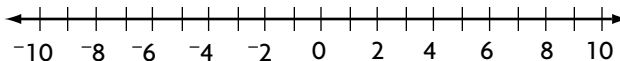
Name _____

Inequalities on a Number Line

An inequality uses the symbols $<$, $>$, $\leq$, or $\geq$. You can use a number line to show the solution of an algebraic inequality.

Graph all of the solutions to $b > 7$.

Step 1

Draw a number line that has the number 7 on it and at least two numbers to the left and right of 7.

-10 -8 -6 -4 -2 0 2 4 6 8 10

Step 2
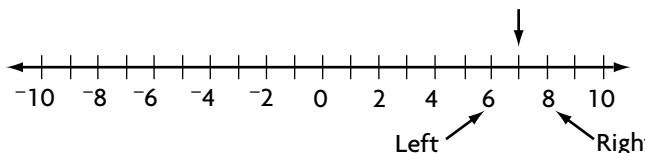
Test the number 7 by substituting it into the inequality.

$b > 7$; $7 > 7$ False

So, the graph does not include the number 7.

Step 3

Decide which side of 7 is the solution. Do this by picking a number to the left of 7 and a number to the right of 7.

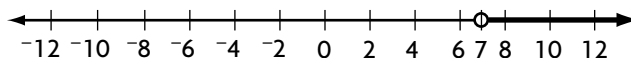To the left of 7 is 6, and to the right of 7 is 8.

-10 -8 -6 -4 -2 0 2 4 6 8 10

Left   Right

Step 4

Test each number. Graph points for the solutions to the side that is true.

Left: $b > 7$; $6 > 7$ False

Right: $b > 7$; $8 > 7$ True

-12 -10 -8 -6 -4 -2 0 2 4 6 7 8 10 12

All values greater than 7 are included in the solution. The circle around 7 means that 7 is not included in the solution.

For each inequality, test three numbers: the number itself, the whole number to the left, and the whole number to the right. Write *true* or *false*.

1. $x > 13$ **2.** $x \leq 9$ **3.** $x \geq {}^-2$

_____   _____   _____

_____   _____   _____

Graph each inequality on a number line.

4. $x < 5$ **5.** $x \leq {}^-6$

◄   ◄

Name _____

Graph on the Coordinate Plane

Finding coordinates or graphing points on a coordinate plane can be done using these steps.

## Finding Coordinates

To find the coordinates for point A:

- From the point, move straight up to the x-axis.
- Read the number. 5 is the x-coordinate.
- From the point, move to the left to the y-axis.
- Read the number. ⁻3 is the y-coordinate. The ordered pair is (5,⁻3).
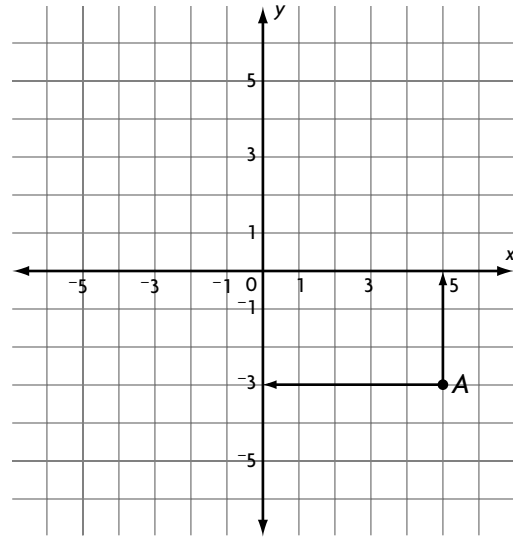
Graphing Points

To graph point B (⁻2,⁻4):

- The first number indicates direction left or right. Starting at the origin, move 2 units to the left on the x-axis. Draw a vertical line through this point.
- The second number indicates direction up or down. Starting at the origin, move 4 units down. Draw a horizontal line through this point.
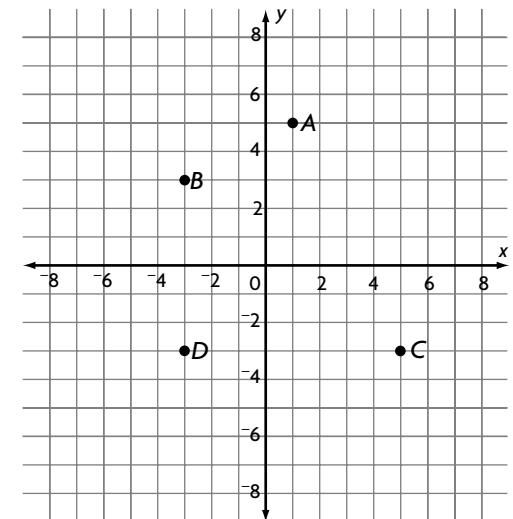- The point where the vertical line and horizontal line cross is the point B (⁻2,⁻4).
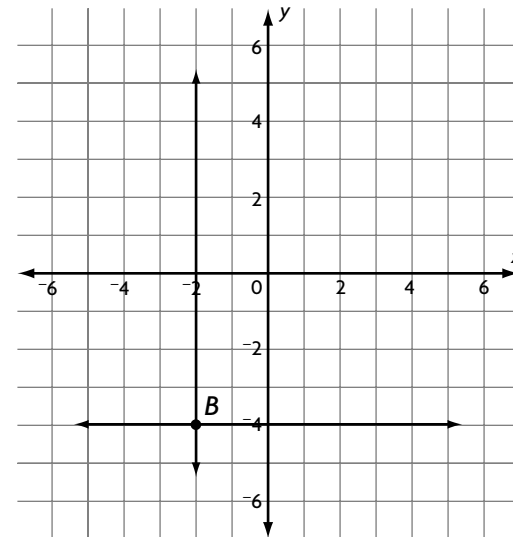
Write the ordered pair for each point on the coordinate plane.

1. point A **2.** point B

_____   _____

3. point C **4.** point D

_____   _____

Graph each point on the coordinate plane.

5. G (0,3)   **6.** H (0,⁻5)   **7.** I (2,⁻3)

8. J (4,2)   **9.** K (⁻6,4)

RW124   **Reteach**

Graph Functions

This is a function table.
The first row represents the
values of *x*, and the second
row represents the values
of *y*. To graph the function,
you need to

x	⁻3	⁻2	⁻1	0	1	2	3
y	3	4	5	6	7	8	9

- write ordered pairs like (⁻3, 3).
- locate the ordered pairs on a coordinate plane.

You can also write an algebraic equation
by looking at the pattern in the table.

- The value of *y* is 6 more than the value of *x*.
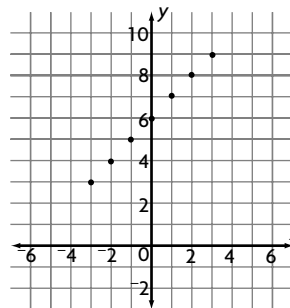
$$y = x + 6$$

Ordered Pairs

(⁻3,3); (⁻2,4); (⁻1,5); (0,6); (1,7); (2,8); (3,9)

Graph Ordered Pairs

A

x	⁻3	⁻2	⁻1	0	1
y	1	2	3	4	5

1. List the ordered pairs for Table A.

_____

3. Locate the points for the ordered
pairs of the function from Exercise 1.

5. Write an equation relating *y* to *x* using
the data in Table A.

_____

B

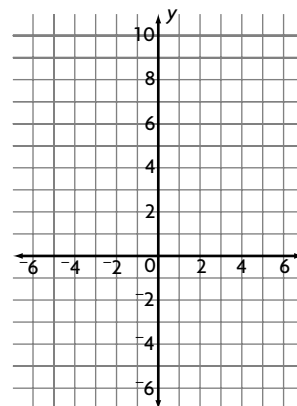| *x*	⁻3	⁻2	⁻1	0	1
y	9	6	3	0	⁻3

2. List the ordered pairs for Table B.

_____
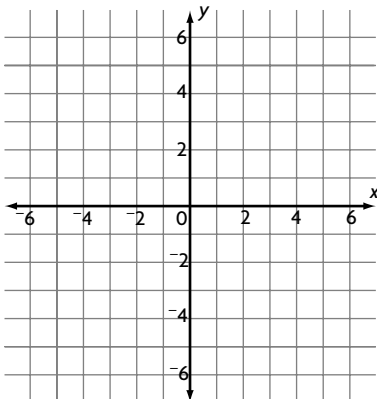
4. Locate the points for the ordered
pairs of the function from Exercise 2.

6. Write an equation relating *y* to *x* using
the data in Table B.

_____

Problem Solving Skill: Make Generalizations

One problem solving strategy that often works well when one quantity depends on another is to make a generalization. You can show a generalization by writing an equation that describes the relationship between the two quantities.

Each of the tables in the school cafeteria can seat 8 students. How many tables does the cafeteria need to seat 128 students?

Step 1   Use the information you are given to make a table. Let x stand for one quantity and y for the other quantity. Then look for a pattern.

Number of Tables (x)	1	2	3	4	5
Number of Students (y)	8	16	24	32	40

The table shows that each x-value is $\frac{1}{8}$ the corresponding y-value.

Step 2   Use the pattern to write an equation.

$$\text{Number of Tables} \longrightarrow x = \frac{1}{8}y \longleftarrow \text{Number of Students}$$

Note that the equation can be used to find the number of tables (x) for any number of students (y). That's why the equation is called a generalization.

Step 3   Solve the equation.
To solve the equation, replace y with the number of students.

$$y = 128 \qquad x = \frac{1}{8} \times 128 \qquad x = \frac{128}{8} \qquad x = 16$$

The cafeteria needs 16 tables to seat 128 students.

Write an equation to show the generalization. Then solve.

1. Each of the tables in the school cafeteria can seat 12 students. How many tables (x) will the cafeteria need to seat 252 students (y)?

    _____

2. Mark is riding on a train pulled by an old steam engine. The train averages a speed of 45 mi per hr. How many miles (y) will Mark travel in 8 hr (x)?
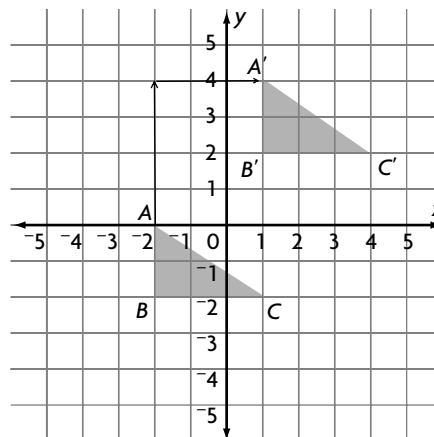
    _____

Name _____

Graph Transformations

The three basic transformations for figures in a coordinate plane are translations, reflections, and rotations.

- A translation moves the figure right or left, or up or down, or both.

  Figure *ABC* is translated up and right. Follow point *A* as it moves 4 units up and 3 units right. The coordinates of *A'* are (1,4). The new figure is triangle *A'B'C'*.

- A reflection is a flip across a line. Rectangle *DEFG* is reflected across the *y*-axis.

  After a point is reflected, the distance from the line stays the same. Both *E* and *E'* are 1 unit from the *y*-axis.
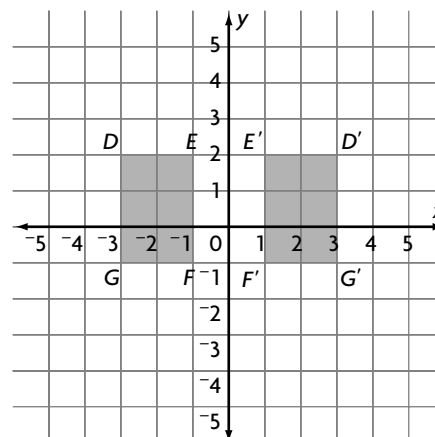
- A rotation about the origin can be clockwise or counterclockwise. It can be any number of degrees.

  Think of tracing paper placed over the graph and attached at the origin. You can draw the figure on the tracing paper and then rotate it.

  Triangle *HJK* is rotated 90° clockwise about the origin.

The coordinates of triangle *PQR* are (0,0), (2,0), and (0,4). Find the new coordinates after the transformation of triangle *PQR*.

1. a translation 4 units to the right

   _____

2. a reflection across the *x*-axis

   _____

3. a 90° counterclockwise rotation about the origin

   _____