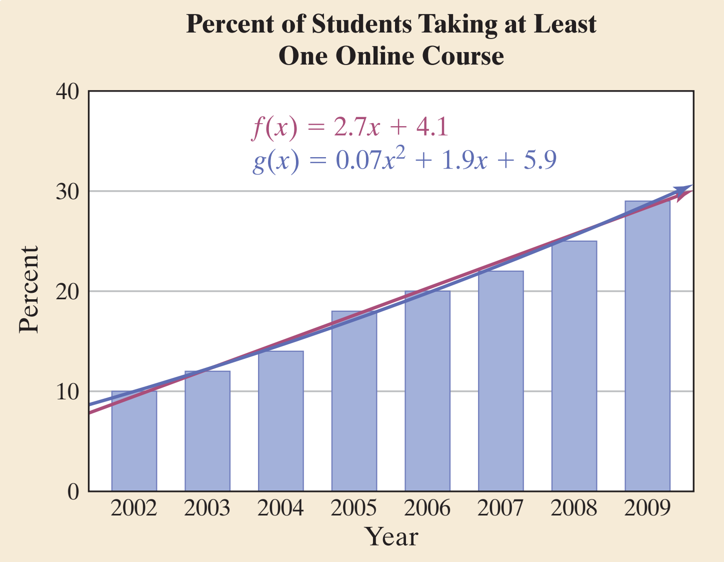
CHAPTER 3

Graphs and Functions

The linear equations and inequalities we explored in Chapter 2 are statements about a single variable. This chapter examines statements about two variables: linear equations and inequalities in two variables. We focus particularly on graphs of those equations and inequalities that lead to the notion of relation and to the notion of function, perhaps the single most important and useful concept in all of mathematics.

We define online courses as courses in which at least 80% of the content is delivered online. Although there are many types of course delivery used by instructors, the bar graph below shows the increase in percent of students taking at least one online course. Notice that the two functions, $f(x)$ and $g(x)$, both approximate the percent of students taking at least one online course. Also, for both functions, x is the number of years since 2000. In Section 3.2, Exercises 81–86, we use these functions to predict the growth of online courses.



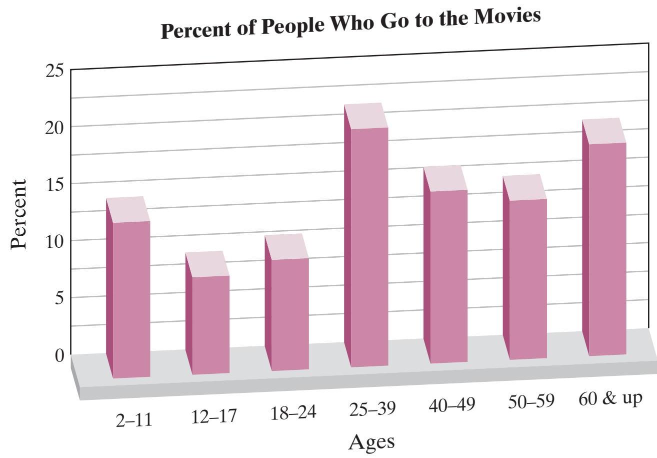
Source: The College Board: Trends in Higher Education Series

3.1 Graphing Equations

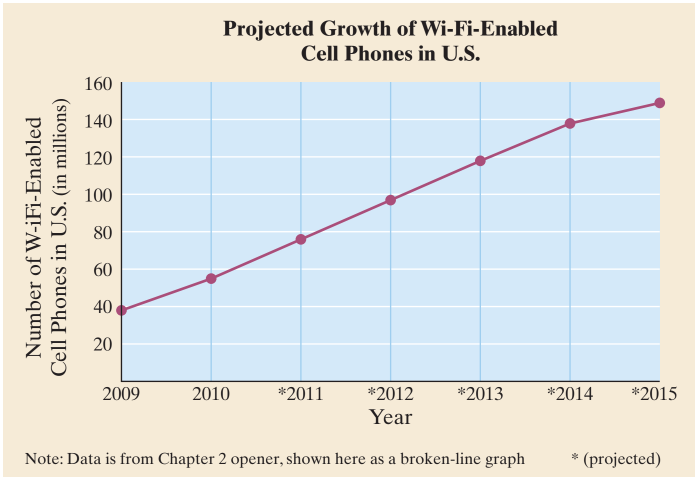
OBJECTIVES

1. Plot Ordered Pairs.
2. Determine Whether an Ordered Pair of Numbers Is a Solution to an Equation in Two Variables.
3. Graph Linear Equations.
4. Graph Nonlinear Equations.

Graphs are widely used today in newspapers, magazines, and all forms of newsletters. A few examples of graphs are shown here.



Source: Motion Picture Association of America



To help us understand how to read these graphs, we will review their basis—the rectangular coordinate system.

OBJECTIVE

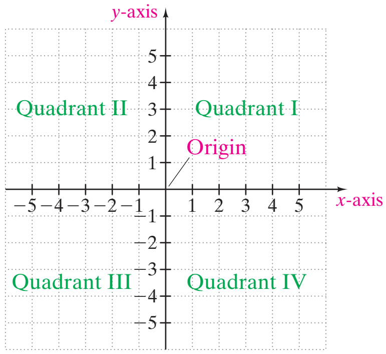
1 Plotting Ordered Pairs on a Rectangular Coordinate System

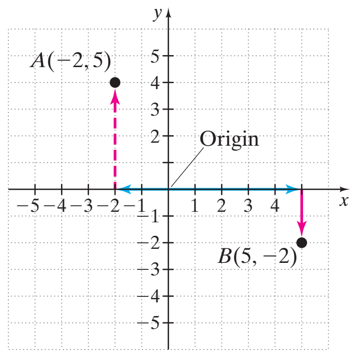
One way to locate points on a plane is by using a **rectangular coordinate system,** which is also called a **Cartesian coordinate system** after its inventor, René Descartes (1596–1650).

A rectangular coordinate system consists of two number lines that intersect at right angles at their 0 coordinates. We position these axes on paper such that one number line is horizontal and the other number line is then vertical. The horizontal number line is called the ***x*-axis** (or the axis of the **abscissa**), and the vertical number line is called the ***y*-axis** (or the axis of the **ordinate**). The point of intersection of these axes is named the **origin.**

Notice in the left figure on the next page that the axes divide the plane into four regions. These regions are called **quadrants.** The top-right region is quadrant I. Quadrants II, III, and IV are numbered counterclockwise from the first quadrant as shown. The x-axis and the y-axis are not in any quadrant.

Each point in the plane can be located, or **plotted,** or graphed by describing its position in terms of distances along each axis from the origin. An **ordered pair,** represented by the notation (x, y), records these distances.





For example, the location of point A in the above figure on the right is described as 2 units to the left of the origin along the x-axis and 5 units upward parallel to the y-axis. Thus, we identify point A with the ordered pair $(-2, 5)$. Notice that the order of these numbers is *critical*. The x-value -2 is called the **x-coordinate** and is associated with the x-axis. The y-value 5 is called the **y-coordinate** and is associated with the y-axis.

Compare the location of point A with the location of point B, which corresponds to the ordered pair $(5, -2)$. Can you see that the order of the coordinates of an ordered pair matters? Also, two ordered pairs are considered equal and correspond to the same point if and only if their x-coordinates are equal and their y-coordinates are equal.

Keep in mind that **each ordered pair corresponds to exactly one point in the real plane and that each point in the plane corresponds to exactly one ordered pair.** Thus, we may refer to the ordered pair (x, y) as the point (x, y).

EXAMPLE 1 Plot each ordered pair on a Cartesian coordinate system and name the quadrant or axis in which the point is located.

a. $(2, -1)$ **b.** $(0, 5)$ **c.** $(-3, 5)$ **d.** $(-2, 0)$ **e.** $\left(-\frac{1}{2}, -4\right)$ **f.** $(1.5, 1.5)$

Solution The six points are graphed as shown.

a. $(2, -1)$ lies in quadrant IV.

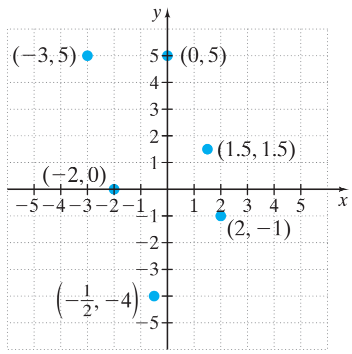
b. $(0, 5)$ is on the y-axis.

c. $(-3, 5)$ lies in quadrant II.

d. $(-2, 0)$ is on the x-axis.

e. $\left(-\frac{1}{2}, -4\right)$ is in quadrant III.

f. $(1.5, 1.5)$ is in quadrant I.



□

PRACTICE

1 Plot each ordered pair on a Cartesian coordinate system and name the quadrant or axis in which the point is located.

a. $(3, -4)$ **b.** $(0, -2)$ **c.** $(-2, 4)$ **d.** $(4, 0)$ **e.** $\left(-1\frac{1}{2}, -2\right)$ **f.** $(2.5, 3.5)$

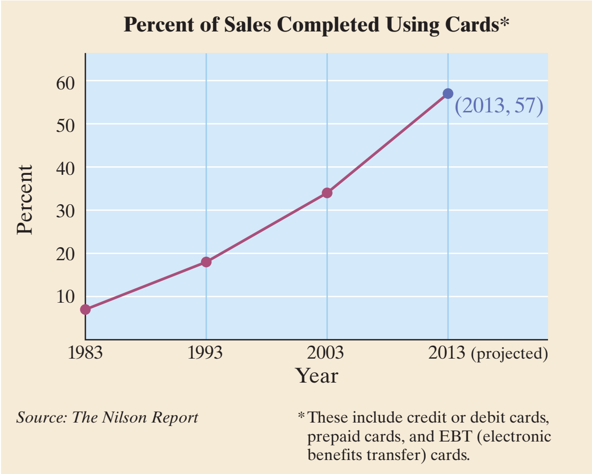
Notice that the y-coordinate of any point on the x-axis is 0. For example, the point with coordinates $(-2, 0)$ lies on the x-axis. Also, the x-coordinate of any point on the y-axis is 0. For example, the point with coordinates $(0, 5)$ lies on the y-axis. These points that lie on the axes do not lie in any quadrants.

✓CONCEPT CHECK

Which of the following correctly describes the location of the point $(3, -6)$ in a rectangular coordinate system?

a. 3 units to the left of the y-axis and 6 units above the x-axis
b. 3 units above the x-axis and 6 units to the left of the y-axis
c. 3 units to the right of the y-axis and 6 units below the x-axis
d. 3 units below the x-axis and 6 units to the right of the y-axis

Many types of real-world data occur in pairs. Study the graph below and notice the paired data $(2013, 57)$ and the corresponding plotted point, both in blue.



This paired data point, $(2013, 57)$, means that in the year 2013, it is predicted that 57% of sales will be completed using some type of card (credit, debit, etc.).

OBJECTIVE

2 Determining Whether an Ordered Pair Is a Solution

Solutions of equations in two variables consist of two numbers that form a true statement when substituted into the equation. A convenient notation for writing these numbers is as ordered pairs. A solution of an equation containing the variables x and y is written as a pair of numbers in the order (x, y). If the equation contains other variables, we will write ordered pair solutions in alphabetical order.

EXAMPLE 2 Determine whether $(0, -12)$, $(1, 9)$, and $(2, -6)$ are solutions of the equation $3x - y = 12$.

Solution To check each ordered pair, replace x with the x-coordinate and y with the y-coordinate and see whether a true statement results.

Let $x = 0$ and $y = -12$.
$$3x - y = 12$$
$$3(0) - (-12) \stackrel{?}{=} 12$$
$$0 + 12 \stackrel{?}{=} 12$$
$$12 = 12 \quad \text{True}$$

Let $x = 1$ and $y = 9$.
$$3x - y = 12$$
$$3(1) - 9 \stackrel{?}{=} 12$$
$$3 - 9 \stackrel{?}{=} 12$$
$$-6 = 12 \quad \text{False}$$

Let $x = 2$ and $y = -6$.
$$3x - y = 12$$
$$3(2) - (-6) \stackrel{?}{=} 12$$
$$6 + 6 \stackrel{?}{=} 12$$
$$12 = 12 \quad \text{True}$$

Thus, $(1, 9)$ is not a solution of $3x - y = 12$, but both $(0, -12)$ and $(2, -6)$ are solutions. □

PRACTICE

2 Determine whether $(1, 4)$, $(0, 6)$, and $(3, -4)$ are solutions of the equation $4x + y = 8$.

Answer to Concept Check: c

OBJECTIVE 3

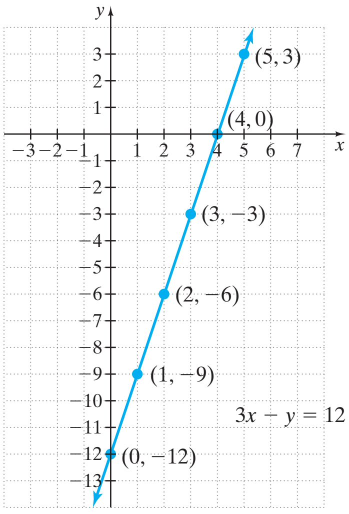
Graphing Linear Equations

The equation $3x - y = 12$, from Example 2, actually has an infinite number of ordered pair solutions. Since it is impossible to list all solutions, we visualize them by graphing.

A few more ordered pairs that satisfy $3x - y = 12$ are $(4, 0)$, $(3, -3)$, $(5, 3)$, and $(1, -9)$. These ordered pair solutions along with the ordered pair solutions from Example 2 are plotted on the following graph.

The graph of $3x - y = 12$ is the single line containing these points. Every ordered pair solution of the equation corresponds to a point on this line, and every point on this line corresponds to an ordered pair solution.

x	y	$3x - y = 12$
5	3	$3 \cdot 5 - 3 = 12$
4	0	$3 \cdot 4 - 0 = 12$
3	−3	$3 \cdot 3 - (-3) = 12$
2	−6	$3 \cdot 2 - (-6) = 12$
1	−9	$3 \cdot 1 - (-9) = 12$
0	−12	$3 \cdot 0 - (-12) = 12$



The equation $3x - y = 12$ is called a linear equation in two variables, and **the graph of every linear equation in two variables is a line.**

Linear Equation in Two Variables

A **linear equation in two variables** is an equation that can be written in the form

$$Ax + By = C$$

where A and B are not both 0. This form is called **standard form.**

Some examples of equations in standard form:

$$3x - y = 12$$
$$-2.1x + 5.6y = 0$$

Helpful Hint

Remember: A linear equation is written in standard form when all of the variable terms are on one side of the equation and the constant is on the other side.

Many real-life applications are modeled by linear equations. Suppose you have a part-time job at a store that sells office products.

Your pay is \$3000 plus 20% or $\frac{1}{5}$ of the price of the products you sell. If we let x represent products sold and y represent monthly salary, the linear equation that models your salary is

$$y = 3000 + \frac{1}{5}x$$

(Although this equation is not written in standard form, it is a linear equation. To see this, subtract $\frac{1}{5}x$ from both sides.)

Some ordered pair solutions of this equation are below.

Products Sold	x	0	1000	2000	3000	4000	10,000
Monthly Salary	y	3000	3200	3400	3600	3800	5000

For example, we say that the ordered pair (1000, 3200) is a solution of the equation $y = 3000 + \frac{1}{5}x$ because when x is replaced with 1000 and y is replaced with 3200, a true statement results.

$$y = 3000 + \frac{1}{5}x$$

$$3200 \stackrel{?}{=} 3000 + \frac{1}{5}(1000) \quad \text{Let } x = 1000 \text{ and } y = 3200.$$

$$3200 \stackrel{?}{=} 3000 + 200$$

$$3200 = 3200 \quad \text{True}$$

A portion of the graph of $y = 3000 + \frac{1}{5}x$ is shown in the next example.

Since we assume that the smallest amount of product sold is none, or 0, then x must be greater than or equal to 0. Therefore, only the part of the graph that lies in quadrant I is shown. Notice that the graph gives a visual picture of the correspondence between products sold and salary.

Helpful Hint

A line contains an infinite number of points and each point corresponds to an ordered pair that is a solution of its corresponding equation.

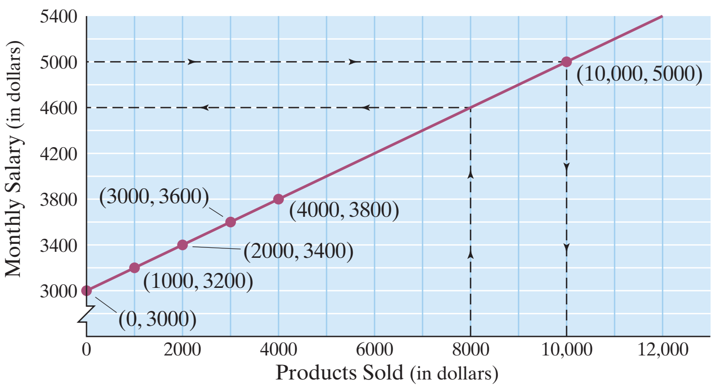
EXAMPLE 3 Use the graph of $y = 3000 + \frac{1}{5}x$ to answer the following questions.

a. If the salesperson sells \$8000 of products in a particular month, what is the salary for that month?

b. If the salesperson wants to make more than \$5000 per month, what must be the total amount of products sold?

Solution

a. Since x is products sold, find 8000 along the x-axis and move vertically up until you reach a point on the line. From this point on the line, move horizontally to the left until you reach the y-axis. Its value on the y-axis is 4600, which means if \$8000 worth of products is sold, the salary for the month is \$4600.



b. Since y is monthly salary, find 5000 along the y-axis and move horizontally to the right until you reach a point on the line. Either read the corresponding x-value from the labeled ordered pair or move vertically downward until you reach the x-axis. The corresponding x-value is 10,000. This means that \$10,000 worth of products sold gives a salary of \$5000 for the month. For the salary to be greater than \$5000, products sold must be greater that \$10,000. □

PRACTICE 3 Use the graph in Example 3 to answer the following questions.

a. If the salesperson sells \$6000 of products in a particular month, what is the salary for that month?

b. If the salesperson wants to make more than \$4800 per month, what must be the total amount of products sold?

Recall from geometry that a line is determined by two points. This means that to graph a linear equation in two variables, just two solutions are needed. We will find a third solution, just to check our work. To find ordered pair solutions of linear equations in two variables, we can choose an x-value and find its corresponding y-value, or we can choose a y-value and find its corresponding x-value. The number 0 is often a convenient value to choose for x and for y.

EXAMPLE 4 Graph the equation $y = -2x + 3$.

Solution This is a linear equation. (In standard form it is $2x + y = 3$.) Find three ordered pair solutions, and plot the ordered pairs. The line through the plotted points is the graph. Since the equation is solved for y, let's choose three x-values. We'll choose 0, 2, and then -1 for x to find our three ordered pair solutions.

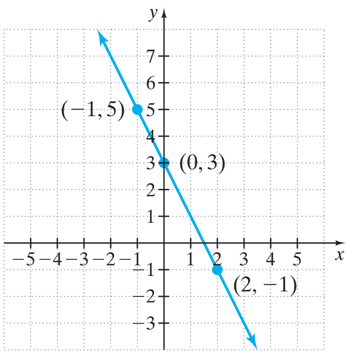
Let $x = 0$
$y = -2x + 3$
$y = -2 \cdot 0 + 3$
$y = 3$ Simplify.

Let $x = 2$
$y = -2x + 3$
$y = -2 \cdot 2 + 3$
$y = -1$ Simplify.

Let $x = -1$
$y = -2x + 3$
$y = -2(-1) + 3$
$y = 5$ Simplify.

The three ordered pairs $(0, 3)$, $(2, -1)$, and $(-1, 5)$ are listed in the table, and the graph is shown.

x	y
0	3
2	−1
−1	5



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PRACTICE 4 Graph the equation $y = -3x - 2$.

Notice that the graph crosses the y-axis at the point (0, 3). This point is called the ***y*-intercept.** (You may sometimes see just the number 3 called the y-intercept.) This graph also crosses the x-axis at the point $\left(\frac{3}{2}, 0\right)$. This point is called the ***x*-intercept.** (You may also see just the number $\frac{3}{2}$ called the x-intercept.)

Since every point on the y-axis has an x-value of 0, we can find the y-intercept of a graph by letting $x = 0$ and solving for y. Also, every point on the x-axis has a y-value of 0. To find the x-intercept, we let $y = 0$ and solve for x.

Finding *x*- and *y*-Intercepts

To find an x-intercept, let $y = 0$ and solve for x.
To find a y-intercept, let $x = 0$ and solve for y.

We will study intercepts further in Section 3.3.

EXAMPLE 5 Graph the linear equation $y = \frac{1}{3}x$.

Solution To graph, we find ordered pair solutions, plot the ordered pairs, and draw a line through the plotted points. We will choose x-values and substitute in the equation. To avoid fractions, we choose x-values that are multiples of 3. To find the y-intercept, we let $x = 0$.

> **Helpful Hint**
> Notice that by using multiples of 3 for x, we avoid fractions.

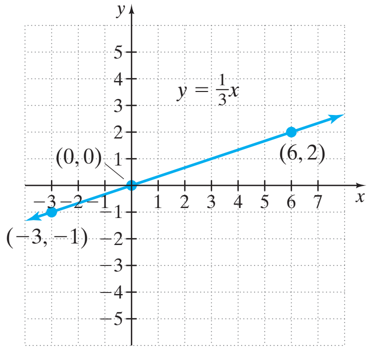
If $x = 0$, then $y = \frac{1}{3}(0)$, or 0.

If $x = 6$, then $y = \frac{1}{3}(6)$, or 2.

If $x = -3$, then $y = \frac{1}{3}(-3)$, or -1.

$y = \frac{1}{3}x$

x	y
0	0
6	2
−3	−1



This graph crosses the x-axis at (0, 0) and the y-axis at (0, 0). This means that the x-intercept is (0, 0) and that the y-intercept is (0, 0). □

> **Helpful Hint**
> Since the equation $y = \frac{1}{3}x$ is solved for y, we choose x-values for finding points. This way, we simply need to evaluate an expression to find the x-value, as shown.

PRACTICE 5 Graph the linear equation $y = -\frac{1}{2}x$.

OBJECTIVE 4

Graphing Nonlinear Equations

Not all equations in two variables are linear equations, and not all graphs of equations in two variables are lines.

EXAMPLE 6 Graph $y = x^2$.

Solution This equation is not linear because the x^2 term does not allow us to write it in the form $Ax + By = C$. Its graph is not a line. We begin by finding ordered pair solutions. Because this graph is solved for y, we choose x-values and find corresponding y-values.

If $x = -3$, then $y = (-3)^2$, or 9.

If $x = -2$, then $y = (-2)^2$, or 4.

If $x = -1$, then $y = (-1)^2$, or 1.

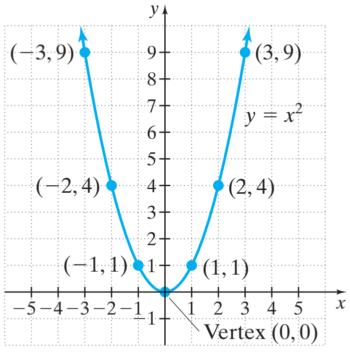
If $x = 0$, then $y = 0^2$, or 0.

If $x = 1$, then $y = 1^2$, or 1.

If $x = 2$, then $y = 2^2$, or 4.

If $x = 3$, then $y = 3^2$, or 9.

x	y
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9



Study the table a moment and look for patterns. Notice that the ordered pair solution (0, 0) contains the smallest y-value because any other x-value squared will give a positive result. This means that the point (0, 0) will be the lowest point on the graph. Also notice that all other y-values correspond to two different x-values. For example, $3^2 = 9$, and also $(-3)^2 = 9$. This means that the graph will be a mirror image of itself across the y-axis. Connect the plotted points with a smooth curve to sketch the graph.

This curve is given a special name, **a parabola.** We will study more about parabolas in later chapters. □

PRACTICE 6 Graph $y = 2x^2$.

EXAMPLE 7 Graph the equation $y = |x|$.

Solution This is not a linear equation since it cannot be written in the form $Ax + By = C$. Its graph is not a line. Because we do not know the shape of this graph, we find many ordered pair solutions. We will choose x-values and substitute to find corresponding y-values.

If $x = -3$, then $y = |-3|$, or 3.

If $x = -2$, then $y = |-2|$, or 2.

If $x = -1$, then $y = |-1|$, or 1.

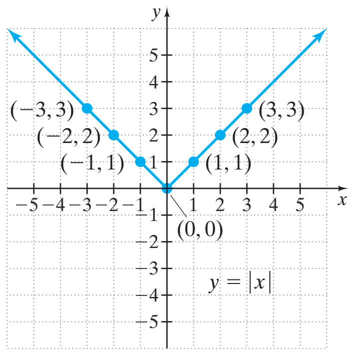
If $x = 0$, then $y = |0|$, or 0.

If $x = 1$, then $y = |1|$, or 1.

If $x = 2$, then $y = |2|$, or 2.

If $x = 3$, then $y = |3|$, or 3.

x	y
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3



Again, study the table of values for a moment and notice any patterns.

From the plotted ordered pairs, we see that the graph of this absolute value equation is V-shaped. □

PRACTICE 7 Graph $y = -|x|$.

Graphing Calculator Explorations

In this section, we begin a study of graphing calculators and graphing software packages for computers. These graphers use the same point plotting technique that we introduced in this section. The advantage of this graphing technology is, of course, that graphing calculators and computers can find and plot ordered pair solutions much faster than we can. Note, however, that the features described in these boxes may not be available on all graphing calculators.

The rectangular screen where a portion of the rectangular coordinate system is displayed is called a **window.** We call it a **standard window** for graphing when both the x- and y-axes display coordinates between −10 and 10. This information is often displayed in the window menu on a graphing calculator as

Xmin = −10
Xmax = 10
Xscl = 1 The scale on the x-axis is one unit per tick mark.
Ymin = −10
Ymax = 10
Yscl = 1 The scale on the y-axis is one unit per tick mark.

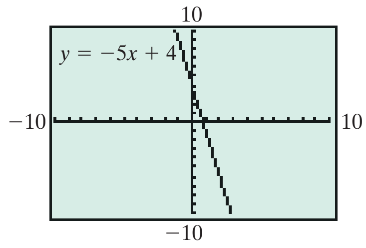
To use a graphing calculator to graph the equation $y = -5x + 4$, press the [Y =] key and enter the keystrokes

[(−)] [5] [x] [+] [4].

↑

(Check your owner's manual to make sure the "negative" key is pressed here and not the "subtraction" key.)

The top row should now read $Y_1 = -5x + 4$. Next, press the [GRAPH] key, and the display should look like this:



Use a standard window and graph the following equations. (Unless otherwise stated, we will use a standard window when graphing.)

1. $y = -3.2x + 7.9$

2. $y = -x + 5.85$

3. $y = \frac{1}{4}x - \frac{2}{3}$

4. $y = \frac{2}{3}x - \frac{1}{5}$

5. $y = |x - 3| + 2$

6. $y = |x + 1| - 1$

7. $y = x^2 + 3$

8. $y = (x + 3)^2$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Some choices may not be used.

line	parabola	1	3
origin	V-shaped	2	4

1. The intersection of the x-axis and y-axis is a point called the __________.

2. The rectangular coordinate system has __________ quadrants and __________ axes.

3. The graph of a single ordered pair of numbers is how many points? __________

4. The graph of $Ax + By = C$, where A and B are not both 0, is a(n)__________.

5. The graph of $y = |x|$ looks __________.

6. The graph of $y = x^2$ is a __________.

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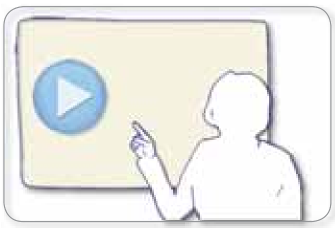

See Video 3.1

Watch the section lecture video and answer the following questions.

OBJECTIVE 1

7. Several points are plotted in Examples 1–3. Where do you start when using this method to plot a point? How does the 1st coordinate tell you to move? How does the 2nd coordinate tell you to move? Include the role of signs in your answer.

OBJECTIVE 2

8. Based on Examples 4 and 5, complete the following statement. An ordered pair is a solution of an equation in ______ variables if, when the variables are replaced with their ordered pair values, a ______ statement results.

OBJECTIVE 3

9. From Example 6 and the lecture before, what is the graph of an equation? If you need only two points to determine a line, why are three ordered pair solutions or points found for a linear equation?

OBJECTIVE 4

10. Based on Examples 7 and 8, complete the following statements. When graphing a nonlinear equation, first recognize it as a nonlinear equation and know that the graph is ______ a line. If you don't know the ______ of the graph, plot enough points until you see a pattern.

3.1 Exercise Set

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Plot each point and name the quadrant or axis in which the point lies. See Example 1.

1. $(3, 2)$ **2.** $(2, -1)$

3. $(-5, 3)$ **4.** $(-3, -1)$

5. $\left(5\frac{1}{2}, -4\right)$ **6.** $\left(-2, 6\frac{1}{3}\right)$

7. $(0, 3.5)$ **8.** $(-5.2, 0)$

9. $(-2, -4)$ **10.** $(-4.2, 0)$

Determine the coordinates of each point on the graph. See Example 1.

11. Point C

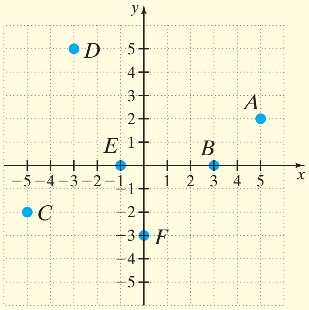
12. Point D

13. Point E

14. Point F

15. Point B

16. Point A



Determine whether each ordered pair is a solution of the given equation. See Example 2.

17. $y = 3x - 5$; $(0, 5)$, $(-1, -8)$

18. $y = -2x + 7$; $(1, 5)$, $(-2, 3)$

19. $-6x + 5y = -6$; $(1, 0)$, $\left(2, \frac{6}{5}\right)$

20. $5x - 3y = 9$; $(0, 3)$, $\left(\frac{12}{5}, -1\right)$

21. $y = 2x^2$; $(1, 2)$, $(3, 18)$

22. $y = 2|x|$; $(-1, 2)$, $(0, 2)$

23. $y = x^3$; $(2, 8)$, $(3, 9)$

24. $y = x^4$; $(-1, 1)$, $(2, 16)$

25. $y = \sqrt{x} + 2$; $(1, 3)$, $(4, 4)$

26. $y = \sqrt[3]{x} - 4$; $(1, -3)$, $(8, 6)$

MIXED PRACTICE

Determine whether each equation is linear or not. Then graph the equation by finding and plotting ordered pair solutions. See Examples 3 through 7.

27. $x + y = 3$ **28.** $y - x = 8$

29. $y = 4x$ **30.** $y = 6x$

31. $y = 4x - 2$ **32.** $y = 6x - 5$

33. $y = |x| + 3$ **34.** $y = |x| + 2$

35. $2x - y = 5$ **36.** $4x - y = 7$

37. $y = 2x^2$ **38.** $y = 3x^2$

39. $y = x^2 - 3$ **40.** $y = x^2 + 3$

41. $y = -2x$ **42.** $y = -3x$

43. $y = -2x + 3$ **44.** $y = -3x + 2$

45. $y = |x + 2|$ **46.** $y = |x - 1|$

47. $y = x^3$
(*Hint:* Let $x = -3, -2, -1, 0, 1, 2$.)

48. $y = x^3 - 2$
(*Hint*: Let $x = -3, -2, -1, 0, 1, 2$.)

49. $y = -|x|$ **50.** $y = -x^2$

51. $y = \frac{1}{3}x - 1$ **52.** $y = \frac{1}{2}x - 3$

53. $y = -\frac{3}{2}x + 1$ **54.** $y = -\frac{2}{3}x + 1$

REVIEW AND PREVIEW

Solve the following equations. See Section 2.1.

55. $3(x - 2) + 5x = 6x - 16$

56. $5 + 7(x + 1) = 12 + 10x$

57. $3x + \frac{2}{5} = \frac{1}{10}$ **58.** $\frac{1}{6} + 2x = \frac{2}{3}$

Solve the following inequalities. See Section 2.4.

59. $3x \le -15$

60. $-3x > 18$

61. $2x - 5 > 4x + 3$

62. $9x + 8 \le 6x - 4$

CONCEPT EXTENSIONS

Without graphing, visualize the location of each point. Then give its location by quadrant or x- or y-axis.

63. $(4, -2)$ **64.** $(-42, 17)$

65. $(0, -100)$ **66.** $(-87, 0)$

67. $(-10, -30)$ **68.** $(0, 0)$

Given that x is a positive number and that y is a positive number, determine the quadrant or axis in which each point lies.

69. $(x, -y)$ **70.** $(-x, y)$

71. $(x, 0)$ **72.** $(0, -y)$

73. $(-x, -y)$ **74.** $(0, 0)$

Solve. See the Concept Check in this section.

75. Which correctly describes the location of the point $(-1, 5.3)$ in a rectangular coordinate system?

a. 1 unit to the right of the y-axis and 5.3 units above the x-axis
b. 1 unit to the left of the y-axis and 5.3 units above the x-axis
c. 1 unit to the left of the y-axis and 5.3 units below the x-axis
d. 1 unit to the right of the y-axis and 5.3 units below the x-axis

76. Which correctly describes the location of the point $\left(0, -\frac{3}{4}\right)$ in a rectangular coordinate system?

a. on the x-axis and $\frac{3}{4}$ unit to the left of the y-axis
b. on the x-axis and $\frac{3}{4}$ unit to the right of the y-axis
c. on the y-axis and $\frac{3}{4}$ unit above the x-axis
d. on the y-axis and $\frac{3}{4}$ unit below the x-axis

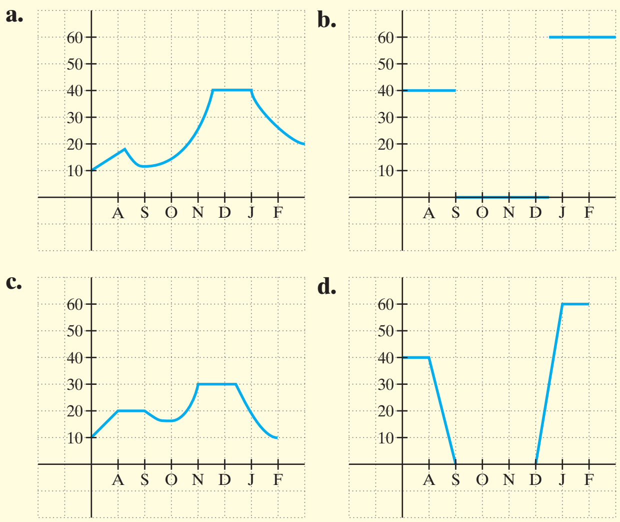
For Exercises 77 through 80, match each description with the graph that best illustrates it.

77. Moe worked 40 hours per week until the fall semester started. He quit and didn't work again until he worked 60 hours a week during the holiday season starting mid-December.

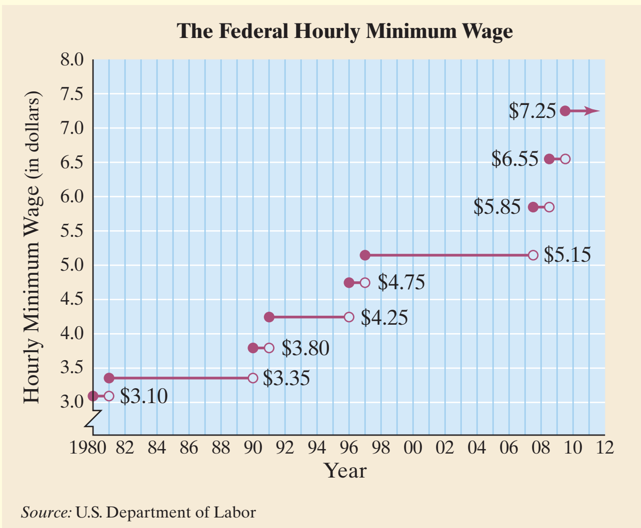
78. Kawana worked 40 hours a week for her father during the summer. She slowly cut back her hours to not working at all during the fall semester. During the holiday season in December, she started working again and increased her hours to 60 hours per week.

79. Wendy worked from July through February, never quitting. She worked between 10 and 30 hours per week.

80. Bartholomew worked from July through February. During the holiday season between mid-November and the beginning of January, he worked 40 hours per week. The rest of the time, he worked between 10 and 40 hours per week.



This broken-line graph shows the hourly minimum wage and the years it increased. Use this graph for Exercises 81 through 84.



81. What was the first year that the minimum hourly wage rose above $5.00?

82. What was the first year that the minimum hourly wage rose above $6.00?

83. Why do you think that this graph is shaped the way it is?

84. The federal hourly minimum wage started in 1938 at $0.25. How much will it have increased by 2011?

85. Graph $y = x^2 - 4x + 7$. Let $x = 0, 1, 2, 3, 4$ to generate ordered pair solutions.

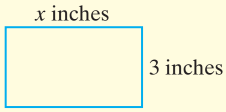
86. Graph $y = x^2 + 2x + 3$. Let $x = -3, -2, -1, 0, 1$ to generate ordered pair solutions.

△ **87.** The perimeter y of a rectangle whose width is a constant 3 inches and whose length is x inches is given by the equation

$$y = 2x + 6$$

a. Draw a graph of this equation.

b. Read from the graph the perimeter y of a rectangle whose length x is 4 inches.



88. The distance y traveled in a train moving at a constant speed of 50 miles per hour is given by the equation

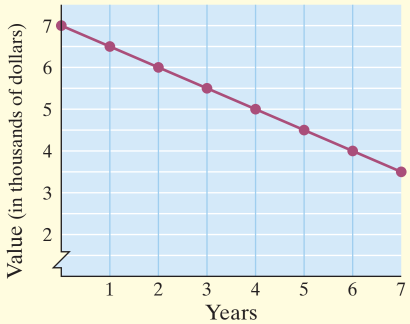
$$y = 50x$$

where x is the time in hours traveled.

a. Draw a graph of this equation.

b. Read from the graph the distance y traveled after 6 hours.

For income tax purposes, the owner of Copy Services uses a method called ***straight-line depreciation*** *to show the loss in value of a copy machine he recently purchased. He assumes that he can use the machine for 7 years. The following graph shows the value of the machine over the years. Use this graph to answer Exercises 89 through 94.*



89. What was the purchase price of the copy machine?

90. What is the depreciated value of the machine in 7 years?

91. What loss in value occurred during the first year?

92. What loss in value occurred during the second year?

93. Why do you think that this method of depreciating is called straight-line depreciation?

94. Why is the line tilted downward?

95. On the same set of axes, graph $y = 2x$, $y = 2x - 5$, and $y = 2x + 5$. What patterns do you see in these graphs?

96. On the same set of axes, graph $y = 2x$, $y = x$, and $y = -2x$. Describe the differences and similarities in these graphs.

Write each statement as an equation in two variables. Then graph each equation.

97. The y-value is 5 more than three times the x-value.

98. The y-value is -3 decreased by twice the x-value.

99. The y-value is 2 more than the square of the x-value.

100. The y-value is 5 decreased by the square of the x-value.

Use a graphing calculator to verify the graphs of the following exercises.

101. Exercise 39

102. Exercise 40

103. Exercise 47

104. Exercise 48

3.2 Introduction to Functions

OBJECTIVES

1. Define Relation, Domain, and Range.
2. Identify Functions.
3. Use the Vertical Line Test for Functions.
4. Find the Domain and Range of a Function.
5. Use Function Notation.

OBJECTIVE

1 Defining Relation, Domain, and Range

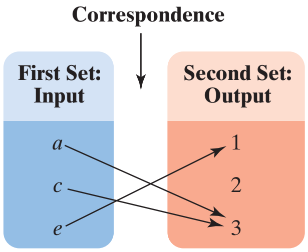
Recall our example from the last section about products sold and monthly salary. We modeled the data given by the equation $y = 3000 + \frac{1}{5}x$. This equation describes a relationship between x-values and y-values. For example, if $x = 1000$, then this equation describes how to find the y-value related to $x = 1000$. In words, the equation $y = 3000 + \frac{1}{5}x$ says that 3000 plus $\frac{1}{5}$ of the x-value gives the corresponding y-value. The x-value of 1000 corresponds to the y-value of $3000 + \frac{1}{5} \cdot 1000 = 3200$ for this equation, and we have the ordered pair (1000, 3200).

There are other ways of describing relations or correspondences between two numbers or, in general, a first set (sometimes called the set of *inputs*) and a second set (sometimes called the set of *outputs*). For example,

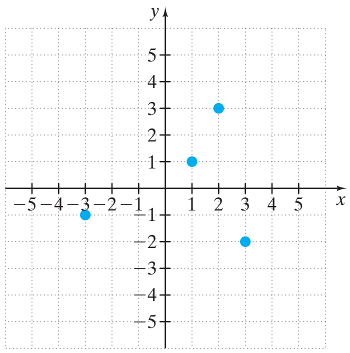
First Set: Input →	***Correspondence*** →	***Second Set: Output***
People in a certain city	Each person's age, to the nearest year	The set of nonnegative integers

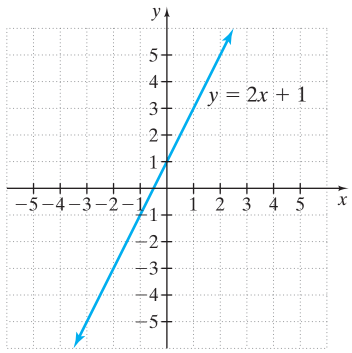
A few examples of ordered pairs from this relation might be (Ana, 4), (Bob, 36), (Trey, 21), and so on.

Below are just a few other ways of describing relations between two sets and the ordered pairs that they generate.



Ordered Pairs
$(a, 3), (c, 3), (e, 1)$


Ordered Pairs
$(-3, -1), (1, 1), (2, 3), (3, -2)$



Some Ordered Pairs
(1, 3), (0, 1), and so on

Relation, Domain, and Range

A **relation** is a set of ordered pairs.
The **domain** of the relation is the set of all first components of the ordered pairs.
The **range** of the relation is the set of all second components of the ordered pairs.

For example, the domain for our relation on the left above is $\{a, c, e\}$ and the range is $\{1, 3\}$. Notice that the range does not include the element 2 of the second set.