Manhattan Review®

2nd Edition

New SAT 2016

SAT® Quantitative Question Bank

Joern Meissner

TURBOCHARGE YOUR PREP

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Turbocharge Your SAT: Quantitative Question Bank

part of the 2nd Edition Series

April 20th, 2016

- ☐ *Designed as per the Revised SAT*
- ☐ *Classified into four sub-sections:*
    - *Heart of Algebra*
    - *Problem Solving & Data Analysis*
    - *Passport to Advanced Math*
    - *Advanced topics in Math*
- ☐ *Complete & Challenging Training Sets: 300 questions*
- ☐ *Ample questions on new additions:*
    - *Trigonometry & Complex Numbers*
    - *Linear graphic & Quadratic graphic equations*
    - *The concept of Higher order Thinking*
    - *Application of Functions*
- ☐ *Each question tagged with 'With calculator' or 'Without calculator'*

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About the Turbocharge your SAT Series

The Turbocharge Your SAT Series was created to provide students with comprehensive and highly effective SAT preparation for maximum SAT performance. Thousands of students around the world have received substantial score improvements by using Manhattan Review's SAT prep books. Now in its updated 2nd edition for the new SAT in 2016, the full series of 12 guides is designed to provide SAT students with rigorous, thorough, and accessible SAT instruction for top SAT scores. Manhattan Review's SAT prep books precisely target each testing area and deconstruct the different test sections in a manner that is both student-centered and results-driven, teaching test-takers everything they need to know in order to significantly boost their scores. Covering all of the necessary material in mathematics and verbal skills from the most basic through the most advanced levels, the Turbocharge Your SAT Series is the top study resource for all stages of SAT preparation. Students who work through the complete series develop all of the skills, knowledge, and strategies needed for their best possible SAT scores.

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About the Company

Manhattan Review's origin can be traced directly back to an Ivy League MBA classroom in 1999. While teaching advanced quantitative subjects to MBAs at Columbia Business School in New York City, Professor Dr. Joern Meissner developed a reputation for explaining complicated concepts in an understandable way. Prof. Meissner's students challenged him to assist their friends, who were frustrated with conventional test preparation options. In response, Prof. Meissner created original lectures that focused on presenting standardized test content in a simplified and intelligible manner, a method vastly different from the voluminous memorization and so-called tricks commonly offered by others. The new methodology immediately proved highly popular with students, inspiring the birth of Manhattan Review.

Since its founding, Manhattan Review has grown into a multi-national educational services firm, focusing on preparation for the major undergraduate and graduate admissions tests, college admissions consulting, and application advisory services, with thousands of highly satisfied students all over the world. Our SAT instruction is continuously expanded and updated by the Manhattan Review team, an enthusiastic group of master SAT professionals and senior academics. Our team ensures that Manhattan Review offers the most time-efficient and cost-effective preparation available for the SAT. Please visit www.ManhattanReview.com for further details.

About the Founder

Professor Dr. Joern Meissner has more than 25 years of teaching experience at the graduate and undergraduate levels. He is the founder of Manhattan Review, a worldwide leader in test prep services, and he created the original lectures for its first test preparation classes. Prof. Meissner is a graduate of Columbia Business School in New York City, where he received a PhD in Management Science. He has since served on the faculties of prestigious business schools in the United Kingdom and Germany. He is a recognized authority in the areas of supply chain management, logistics, and pricing strategy. Prof. Meissner thoroughly enjoys his research, but he believes that grasping an idea is only half of the fun. Conveying knowledge to others is even more fulfilling. This philosophy was crucial to the establishment of Manhattan Review, and remains its most cherished principle.

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Chapter 1

Introduction

Dear Students,

Here at Manhattan Review, we constantly strive to provide you the best educational content for standardized test preparation. We make a tremendous effort to keep making things better and better for you. This is especially important with respect to an examination such as the SAT. As you know that from Spring'16, SAT goes for a major change. The revised SAT is challenging now. A typical SAT aspirant is confused with so many test-prep options available. Your challenge is to choose a book or a tutor that prepares you for attaining your goal. We cannot say that we are one of the best, it is you who has to be the judge.

The revised SAT will focus more on Algebra and Data Analysis. The book is classified into four sub-sections: Heart of Algebra, Problem Solving & Data Analysis, Passport to Advanced Math, and Advanced topics in Math. Each of the four sub-sections are classified topic-wise. We have incorporated more questions on real-life situational charts and graphs in the book. The new additions to the revised SAT: Trigonometry & Complex Numbers have been dealt with ample number of practice questions. The book includes quite a many questions on linear graphic equations and quadratic graphic equations. Questions on Application of Functions have been given enough weightage. There is section dedicated to questions related to the concept of Higher order Thinking. As you know that there would be a No-Calculator math section, so we have tagged each of the 300 questions with either 'With calculator' or 'Without calculator'.

In a nut shell, Manhattan Review's SAT-Question Bank is holistic and comprehensive for the practice; it is created so because we listen to what students need. Should you have any query, please feel free to write to us at info@manhattanreview.com.

Happy Learning!

Professor Dr. Joern Meissner
& The Manhattan Review Team

Chapter 2

Introduction to Revised SAT

The SAT has changed and the Revised SAT will take effect in the Spring of 2016. The revised SAT will comprise two major sections: one, Evidence-based Reading & Writing and two, Math. The essay, which now is optional is excluded from being a compulsory part of SAT Writing section. Evidence-based Reading & Writing has two sections: one, Reading (only Reading, no Critical word prefixed to it, but that does not mean that the new Reading Test will not test critical aspects of reading) and two, Writing & Language Test. This section has gone for a major change in its format. Questions testing your skills at writing, grammar, & language aspects will be taken up from a passage. With both Reading & Writing & Language sections being passage-based, they may also include info-graphics within the passages, and there would be one or two questions based on a graph or a chart. You may have a flavor of some math in the Reading passages & the Writing passages.

While the format of the Math test remains unchanged, there are new additions in Math section. It will focus more on Algebra and Data Analysis. You will see more questions on real-life situational charts and graphs in the test. There is an addition of two new topics: Trigonometry & Complex Numbers. There would be one or two questions testing your higher order thinking. Those questions may be in a set of two questions and would have a lengthy narration. Another special category of questions would be one in which you would be asked to interpret a situation described mathematically in word; there would be four options, each being at least two lines, and only one of the options is correct. Another change to the math section is that there would be a section of No-Calculator.

Two noticeable changes in the Revised SAT are: one, there is no negative marking and two, there would be only four options in MCQs.

Following table does a comparative analysis of The old SAT vs. The revised SAT.

2.1 The Old SAT vs. The Revised SAT

	Old SAT	Revised SAT

Sections	• Math
	• Critical Reading
	• Writing (incl. Essay)
		• Math
		• Evidence-based Reading & Writing
		  • Reading
		  • Writing & Language Test
		• Essay (optional; exclusive of Writing & Language test)

Content	• Reasoning Skills • Contextual vocabulary • Applied mathematical problems	• Reasoning Skills & knowledge of real-world situations • Evidence-based Reading, Writing, & Math problems • Introduction of graphs & charts in passages, thereby testing associated questions (even calculation-based questions) • Contextual vocabulary in broader contexts • Introduction of Trigonometry & Complex Numbers in math • Higher Order thinking questions in math

Question types	• Multiple Choice Questions (MCQ) • Student-produced response Questions (Grid-In) in math	• Multiple Choice Questions (MCQ) • Student-produced response Questions (Grid-In) in math
Number of options for MCQs	5 (A through E)	**4 (A through D)**
Negative marking	$-\frac{1}{4}$ for wrong answer	**No negative marking**
Scoring	• Total score: 600–2400; incl. scores from Critical Reading, Writing, & Math (each score from 200–800) • Writing score includes Essay score	• Total score: 400–1600; incl. scores from Evidence-based Reading and Writing, & Math (each scored from 200–800) • Essays are scored separately (1–4) • Sub-scores & Cross-scores (contribution from selected areas)
Timing	• 3 hours 45 minutes	• 3 hours (excluding essay) • 3 hours 50 minutes (including essay)
Calculator access	Throughout the math section	There would be a No-Calculator section in the math section

2.2 Revised SAT Math content

Content	Topics	Number of questions	
		Calculator	No-Calculator
Heart of Algebra	Fundamental concepts used in Algebra; arranging formulae, linear equations, inequalities, etc.	11	8
Problem Solving & Data Analysis	Understanding qualitative & quantitative data, analyzing relationship, incl. graphical; Ratio & Proportion, Percents, and units of measurements	17	None (All questions with calculator acess)
Passport to Advanced Math	Advanced concepts in Algebra, incl. quadratic & higher order equations, polynomials	7	9
Advanced topics in Math	Geometry (2D, area, volume; & 3D), Trigonometry, Complex numbers	3	3
	Total number of questions (58) **Total time (80 minutes)**	**38 (55 minutes)**	**20 (25 minutes)**

Chapter 3

Practice Questions

3.1 Heart of Algebra

3.1.1 Algebraic Expressions

1. What is the value of $(m - 3)^3$ if $m = \left(\frac{5}{3} + \frac{5}{6} + \frac{1}{2}\right)$? [Without calculator]

(A) -1

(B) $-\dfrac{1}{8}$

(C) 0

(D) $\dfrac{1}{27}$

2. The value of Q is given by the relation $Q = 17m$. If Q and m are integers with Q greater than 150 and less than 160, then which of the following is the value of m? [With calculator]

(A) 9

(B) 1

(C) 12

(D) 14

3. The value of Q is given by the relation $Q = 17m$. If m is an integer greater than 15 and less than 23, then which of the following is the value of Q? [With calculator]

(A) 255

(B) 290

(C) 306

(D) 391

4. If $N = \dfrac{S}{\frac{p}{q} - 1}$, then express q in terms of N, S, and p. [Without calculator]

(A) $q = \dfrac{Np}{N + S}$

(B) $q = \dfrac{Np}{N - S}$

(C) $q = \dfrac{N}{N + S}$

(D) $q = \dfrac{N}{Np + S}$

5. Calculate the value of $\frac{3}{\frac{3}{4}-\frac{5}{12}}$. [Without calculator]

(A) $\frac{3}{4}$

(B) $\frac{9}{4}$

(C) 9

(D) 12

6. **Grid-In:**

If $p = \frac{1+\frac{x}{y}}{1+\frac{xy}{z^2}}$, $x = \frac{z}{2}$ and $y = 3z$, find the value of p. [With calculator]

7. **Grid-In:**
If $a = 216$, find the value of $a^{\frac{1}{3}} - 1$. [With calculator]

8. **Grid-In:**
Find the value of $\left(\frac{1}{x-9}\right)^n + \left(\frac{1}{x}\right)^{1-n}$; where $x = 25, n = \frac{1}{2}$. [With calculator]

9. **Grid-In:**
Find the value of $\frac{a^2+b^2}{(a+b)^2+(a-b)^2}$, when $a = 2.3,\ b = 1.7$. [Without calculator]

10. Find the value of the expression $\frac{x^2-12x+32}{x-4}$, when $x = 6.5$. [Without calculator]

(A) -1.5

(B) 2.5

(C) 45.16

(D) 58.83

11. Find the value of the expression: $\frac{x^2-12x+32}{x+4}$, when $x = -6.5$. [With calculator]

(A) -1.5

(B) 2.5

(C) 45.16

(D) 60.9

12. Grid-In:

Let # be an operation on x and y defined as $x\#y = \frac{x^2+y^2}{x+y}$. Find the value of 1#1#5 given that the operations are performed from left to right? [With calculator]

13. Grid-In:

If a mixed fraction is denoted by $Z\frac{Z}{3} = \frac{8}{3}$, then what is the value of $\frac{1}{2Z-3}$? [Without calculator]

3.1.2 Numbers

14. Which of the following must be odd? [Without calculator]

I. The sum of 5 consecutive integers

II. The sum of 14 consecutive integers

III. The product of 11 consecutive integers

(A) Only I

(B) Only II

(C) Only III

(D) Both I and II

15. Three consecutive numbers are selected from the set of integers from 1 to 30. Suppose P is the product of the numbers drawn. Which of the following must be true? [Without calculator]

I. P is an integer multiple of 3.

II. P is an integer multiple of 4.

III. P is an integer multiple of 6.

(A) Only I

(B) Only II

(C) Both I and III

(D) Both II and III

16. If $a, b, \&\ c$ are single digit integers and $N = 11000 \times a + 10 \times b + c$, find the digit in the hundred's place of number N. [Without calculator]

(A) 0

(B) 1

(C) a

(D) b

17. Grid-In:
How many two digit numbers exist such that the numbers are 54 more than their reversed numbers? [Without calculator]

18. Grid-In:
Let # be a digit from 0 to 9 such that $\frac{1\#}{3\#} + \frac{4}{7} = 1$.

(Note: Here 4# and 9# represent two digit numbers). Find the value of the digit #. [With calculator]

19. Bob writes down all consecutive integers starting with 1 to 100 and adds them to get a number P. Alfred writes down all consecutive even numbers starting with 2 till 200 and adds them to get a number Q. Choose the correct relation between P and Q. [Without calculator]

(A) $P = Q$

(B) $P = 2Q$

(C) $P = \frac{Q}{2}$

(D) $P = Q^2$

20. If $2a = 5b = 3c = 6d$ for four positive integers $a, b, c, \&\ d$, which of the following definitely does NOT represent an integer? [Without calculator]

(A) $\dfrac{ad}{3}$

(B) $\dfrac{ac}{d}$

(C) $\dfrac{ac}{bd}$

(D) $\dfrac{ab}{cd}$

21. If $ab = cd$ for four positive integers $a, b, c, \&\ d$, which of the following must NOT be true? [Without calculator]

(A) $\dfrac{ad}{c} = \dfrac{d^2}{b}$

(B) $\dfrac{a^2 + c^2}{c^2} = \dfrac{d^2 + b^2}{b^2}$

(C) $\dfrac{\frac{1}{b} + a}{a} = \dfrac{\frac{1}{c} + d}{d}$

(D) $\dfrac{ab^2}{c} = d$

22. If $2a = 5b = 3c = 6d$ for four positive integers $a, b, c$and d, which of the following definitely does NOT represent an integer? [Without calculator]

(A) $\dfrac{ad}{3}$

(B) $\dfrac{ac}{d}$

(C) $\dfrac{ac}{bd}$

(D) $\dfrac{ab}{cd}$

23. **Grid-In:**

The symbol @ is defined such that $a@b = a^2 - b^2$. If $x@4 = \frac{2x+8}{5}$, find the value of x given that $x \neq -4$. [Without calculator]

24. A three-digit number yields a remainder 1 when it is divided either by 100 or by 60. How many three-digit numbers are there with this property? [Without calculator]

(A) One

(B) Two

(C) Three

(D) Four

25. If u and v are integers and $uv = -7$, then $(u + v)^2 =$

(A) -36

(B) 4

(C) 36

(D) 49

3.1.3 Factors, Multiples, LCM, & HCF

26. How many factors of 48 are greater than $\sqrt{48}$? [Without calculator]

(A) 4

(B) 5

(C) 6

(D) 8

27. **Grid-In:**
A rectangular floor of dimensions 48 feet by 60 feet is to be tiled, using identical square tiles. What is the minimum number of square tiles required? [With calculator]

28. Amy decides to write down every fourth number starting with one till 50. While at the same time, Mary decides to write down every third number starting with one till 50. How many numbers would be common to both of them? [With calculator]

(A) 3

(B) 4

(C) 5

(D) 6

29. After multiplying the following numbers by 5, all the numbers have the same number of distinct prime factors EXCEPT [Without calculator]

(A) 15

(B) 28

(C) 30%

(D) 22%

30. Which of the following numbers has the highest exponent for any of its prime factors? [Without calculator]

(A) 48

(B) 72

(C) 250

(D) 484

31. If $75^n * 15 = 3^6 * 5^m$, find the value of $(m + n)$ if m and n are positive integers. [Without calculator]

(A) 5

(B) 8

(C) 11

(D) 16

32. If $1500 = 30^m * 50^n * 5$, find the value of $(m - n)$ if m and n are positive integers. [Without calculator]

(A) 0

(B) 1

(C) 2

(D) 3

3.1.4 Linear equations

33. If $x + y = 21$ and $2x = 5y$, find the value of $\sqrt{x-y} + \left(\frac{x}{y}\right)^2$. [With calculator]

(A) 3.16

(B) 9.16

(C) 9.25

(D) 15.25

34. The cost of 75 apples and 45 oranges is \$12. At the same rate for each fruit, what would be the price of 50 apples and 30 oranges? [Without calculator]

(A) \$4

(B) \$12

(C) \$18

(D) \$20

35. A man bought some apples and some mangoes. The number of apples he bought was 4 less than twice the number of mangoes. Had he bought 6 mangoes more, the number of apples and mangoes would have been equal. How many mangoes did the man buy? [With calculator]

36. In the city Xuangho, fine for speeding on the highway is calculated in a strange manner. There is a flat fine of \$60 for anyone found driving above the speed limit of 100 miles/hour and an additional fine of \$3 for each mile/hour if found driving above the limit of 120 miles/hour. Lucy was fined \$90. At what speed (in miles/hour) was Lucy driving? [With calculator]

(A) 90

(B) 110

(C) 150

(D) 190

37. Seven thieves steal x gold coins from a treasury, x being an even number. On trying to distribute the coins equally among themselves, they find that there are 6 coins left over. How many coins, when added to x, may result in a number of coins divisible among 14 thieves? [Without calculator]

(A) 13

(B) 23

(C) 36

(D) 42

38. Presently, Amy is six years younger to Jane. Five years ago, Jane was thrice as old as Amy. What will be Jane's age ten years from now? [Without calculator]

(A) 8

(B) 14

(C) 18

(D) 24

3.1.5 Absolute numbers

39. If $|5x| + 30 = 40$, what is x? [Without calculator]

(A) -2

(B) 2

(C) ± 2

(D) $\sqrt{2}$

40. What is the value of $\left[\dfrac{-|-25| \times -|2|}{(-|-5|)^2}\right]$? [Without calculator]

(A) -2

(B) 2

(C) ± 2

(D) $\sqrt{2}$

41. If $\dfrac{2x}{3y} = 0$, which of the following MUST be true? [Without calculator]

(A) $y \neq 0$

(B) $y = 0$

(C) $x \neq 0$

(D) $x < y$

42. **Grid-In:**

If $|3x - 4| = 11$, what is x? [Without calculator]

43. **Grid-In:**

If $\dfrac{|13x - 4|}{5} + 3 = 10$, what is x? [Without calculator]

3.1.6 Inequalities

44. If $|2x + 1| < 7$, what is the range of value of x? [Without calculator]

(A) $4 > x > -3$

(B) $-4 > x > 3$

(C) $x < 3$

(D) $-4 < x < 3$

45. If $\frac{1}{x-1} > 1$, what is the range of value of x? [Without calculator]

(A) $0 > x > -2$

(B) $0 < x < 1$

(C) $0 < x < 2$

(D) $1 < x < 2$

46. If $\frac{-2|x|}{3y} > -1$, which of the following MUST be true? [Without calculator]

(A) $x > 0$ & $y > 0$

(B) $|x| < 3$

(C) $y > 0$

(D) $x < y$

47. If $-3 < x < -1$, then which of the following could be true? [Without calculator]

I. $x^2 < 2x$

II. $x^2 = 2x$

III. $x^2 > 2x$

(A) I only

(B) II only

(C) III only

(D) All three

48. If $a^3bc^2 < 0$, which of the following must be correct? [Without calculator]

(A) $a < 0$ & $c < 0$

(B) $b < 0$ & $c < 0$

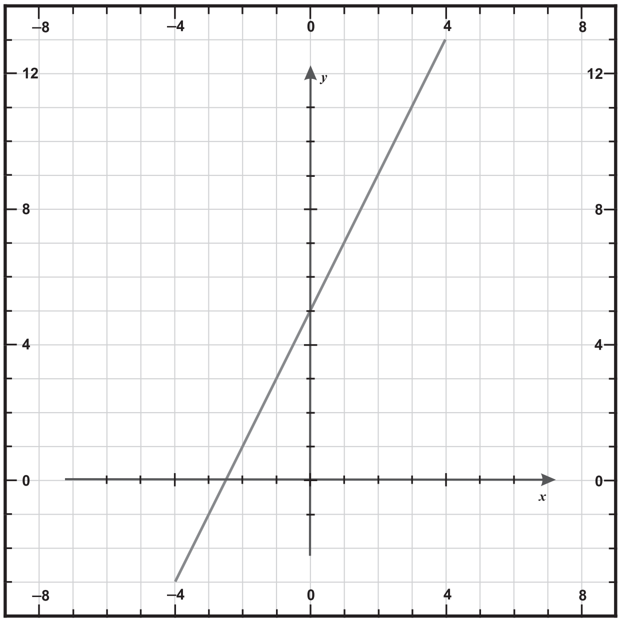
(C) $a < 0$ & $b > 0$

(D) $a < 0$ & $b > 0$ or $a > 0$ & $b < 0$

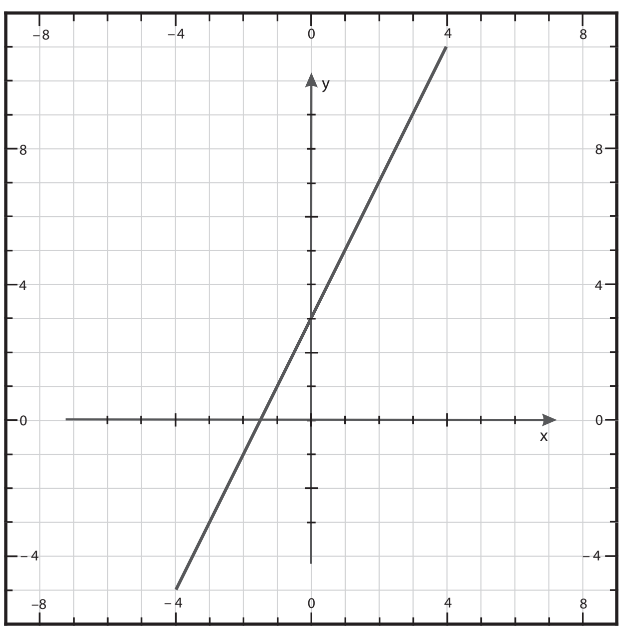
49. Grid-In:

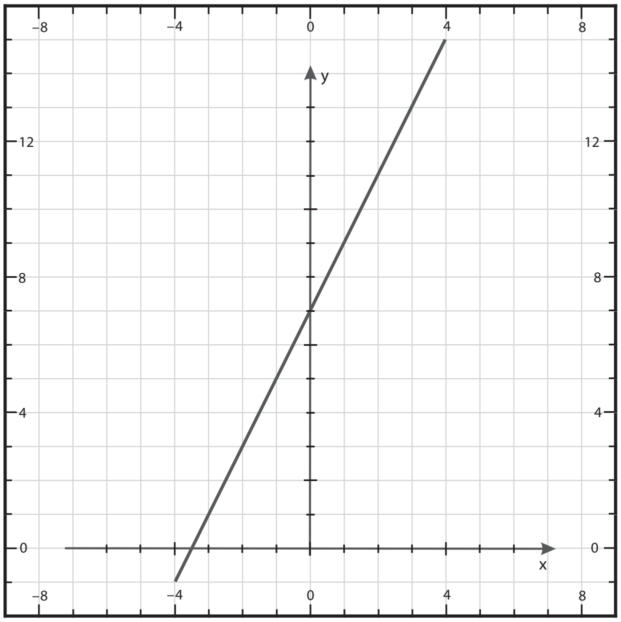
If x & y are integers and $6 > x > 3$ and $-4 < -y < 7$, how many number of values can $(3x + 2y)$ have? [Without calculator]

3.1.7 Graphic equations & Inequalities

50. 

The graph of $f(x) = 2x + 5$ is shown above. Choose from the options below the correct graph of $g(x) = 2(x - 1) + 5$. [Without calculator]

(A) 



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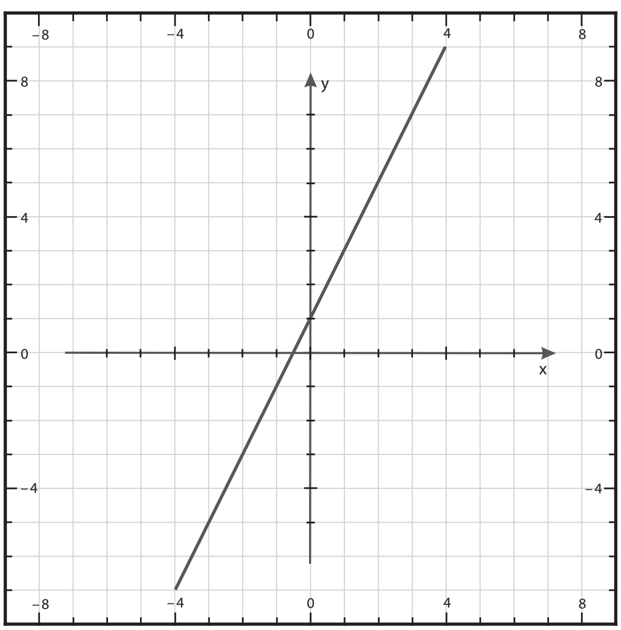
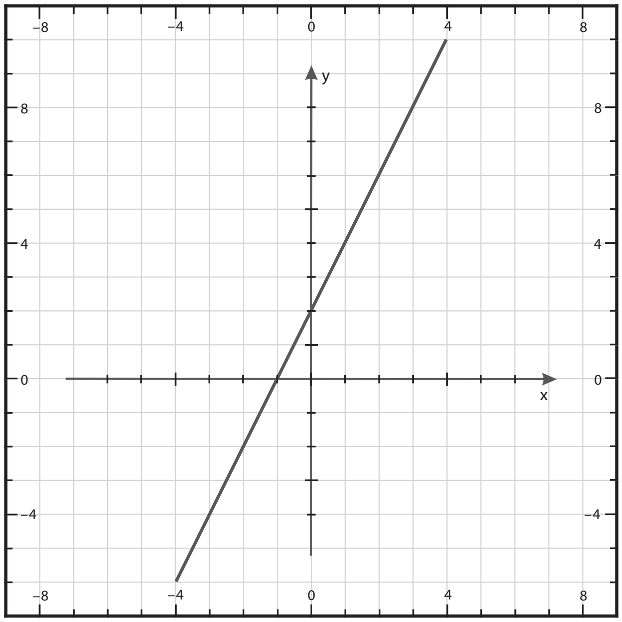
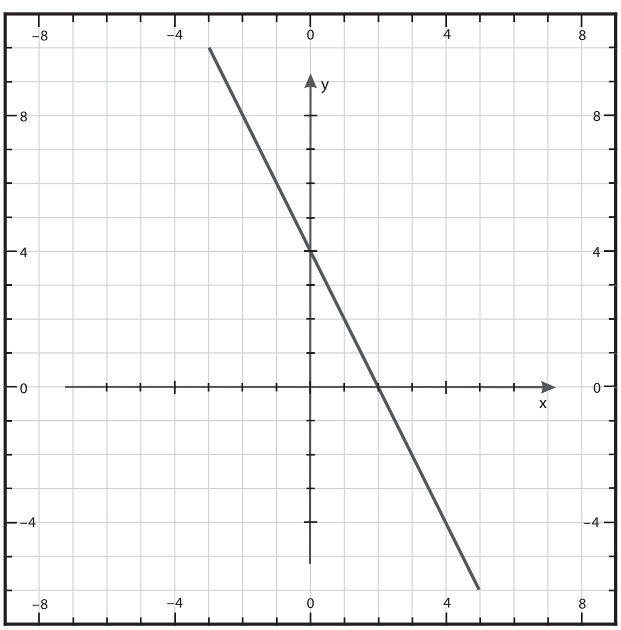
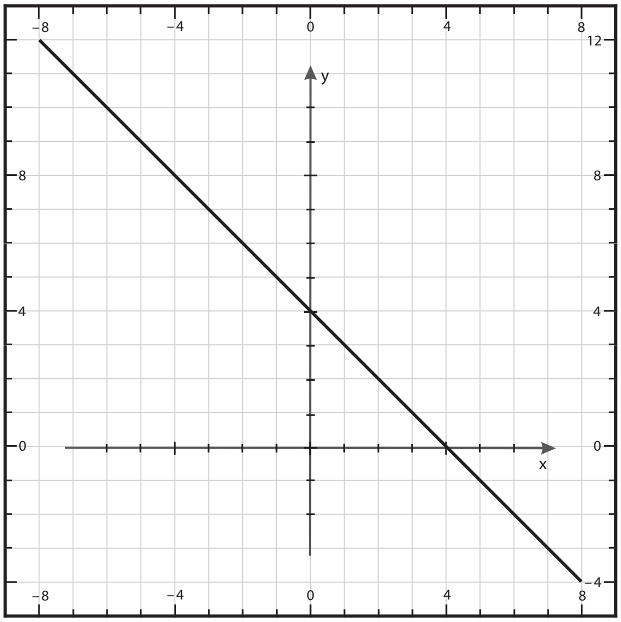


(C)

(D) 

51.  $y = f(x)$



$y = g(x)$

The graphs of $f(x)$ and $g(x)$ are shown above. Choose from the options below the correct relation between $g(x)$ and $f(x)$. [Without calculator]

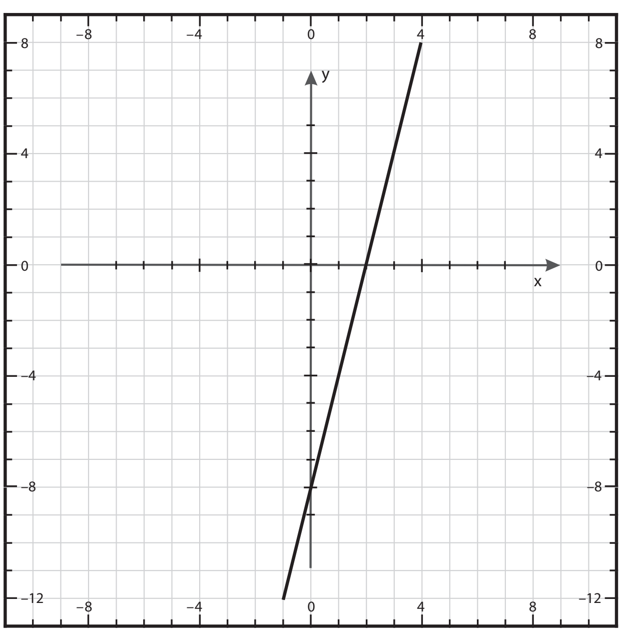
(A) $g(x) = 2f(x)$

(B) $g(x) = \dfrac{f(x)}{2}$

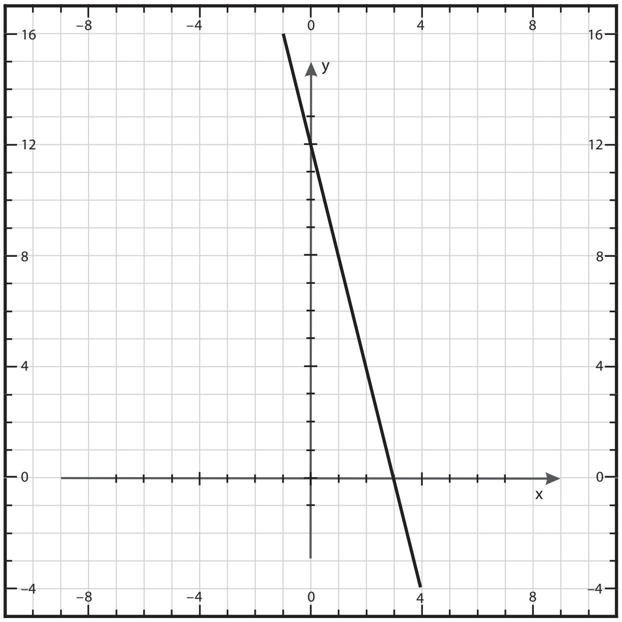
(C) $g(x) = f(2x)$

(D) $g(x) = f\left(\dfrac{x}{2}\right)$

52.



$y = f(x)$



$y = g(x)$

The graphs of $f(x)$ and $g(x)$ are shown above. Choose from the options below the correct relation between $g(x)$ and $f(x)$. [Without calculator]

(A) $g(x) = f(x) - 1$

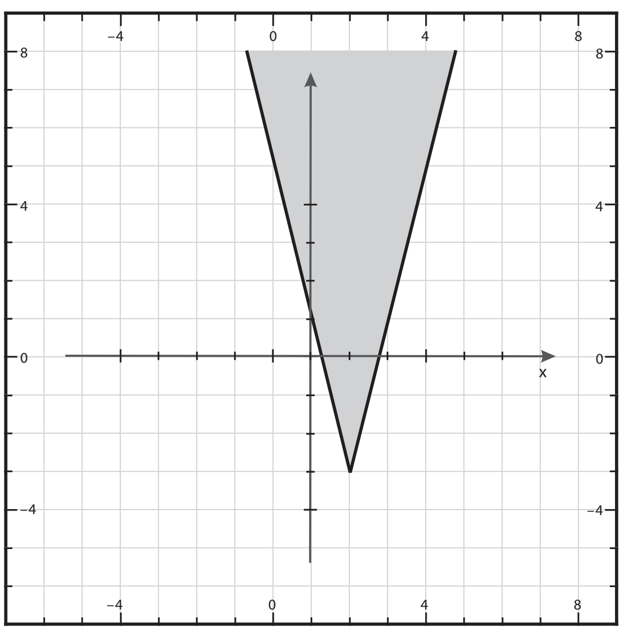
(B) $g(x) = -f(x-1)$

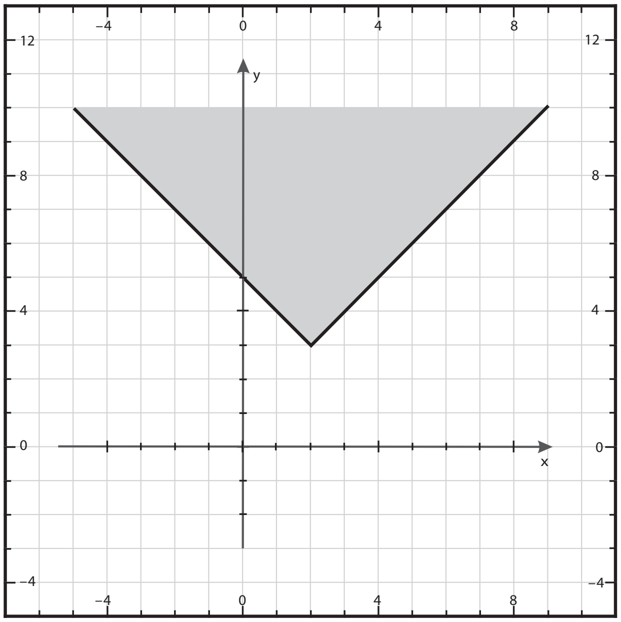
(C) $g(x) = -f(x) + 1$

(D) $g(x) = -f(2x) - 1$

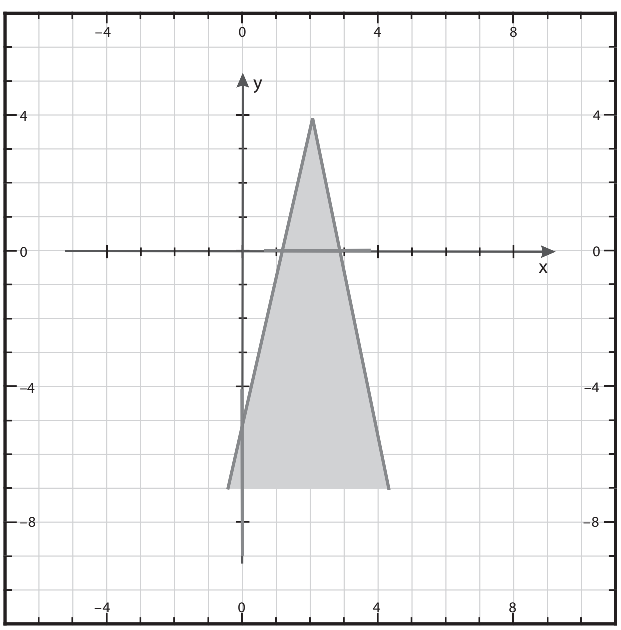
53. Choose the graph from the options below that correctly depicts the region denote by $f(x) \geq 4|x-2| - 3$ (the region has been shaded in the options). [Without calculator]

(A)



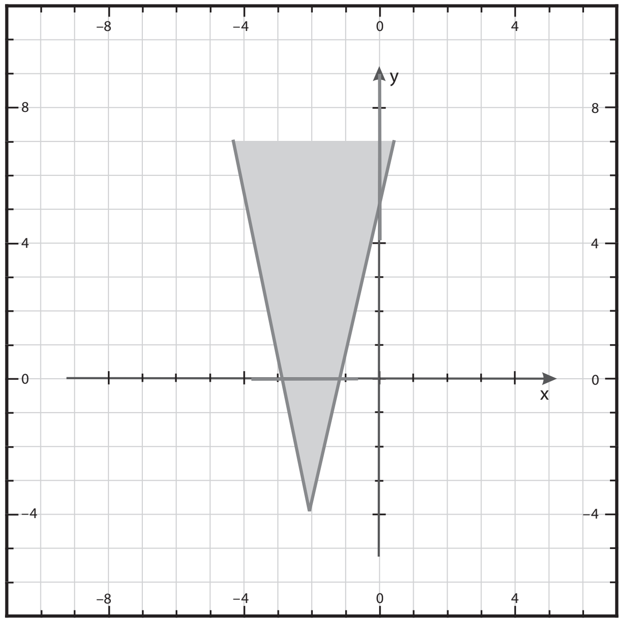


(B)



(C)

(D)



3.1.8 Functions

54. If $f(x) = 3x^3 - 2x^2 + 2x + 2$, what is the value of $f(-2)$? [Without calculator]

(A) -34

(B) -14

(C) 14

(D) 25

55. If $g(t) = 2t^3 - 3t^2 + \frac{3}{t^2} + \frac{2}{t^3}$, what is the value of $g(-\frac{1}{t})$. [Without calculator]

(A) $-g(t)$

(B) $g(t)$

(C) $-2g(t)$

(D) $2g(t)$

56.

$f(x)$	3	2	10	23
x	0	1	-1	-2

Based on the table above, which of the following could represent $f(x)$? [Without calculator]

(A) $f(x) = -4x^2 + 3x + 3$

(B) $f(x) = -5x^2 + 4x + 3$

(C) $f(x) = 3x^2 + 4x + 3$

(D) $f(x) = -4x^2 - 3x + 3$

57. **Grid-In:**

Given $f(x) = 3x - 4$, for what value of x, does $5f(x) - 3 = f(3x - 4)$? [Without calculator]

58. **Grid-In:**

If for all positive integers $m, [m] = -|\sqrt{m}|$ when m is odd & $[m] = -\sqrt{m}$ when m is even; for all negative integers $m, [m] = m^2$ when m is odd & $[m] = -|m^2|$ when m is even, what is the value of $\frac{[49]^4.[-4]}{[-49].[4]}$? [Without calculator]

59. If $[X]$ is the smallest integer greater than or equal to X, what is the value of $[-2.2] + [3] + [5.2]$? [Without calculator]

(A) 5

(B) 6

(C) 7

(D) 8

60. An operation @ is defined by the equation $a@b = \dfrac{(a+b)}{(a-b)}$, for all numbers a and b such that $a \neq b$. If $a@c = 2$, then what is the value of c in terms of a? [Without calculator]

(A) $-a/3$

(B) $-3a$

(C) $a/3$

(D) $3a$

3.2 Problem Solving & Data Analysis

Problem Solving & Data Analysis section always has calculator access or no question will be asked in No-Calculator section.

3.2.1 Time & Work

61. Amy and Bob decide to paint one wall of a building. Working alone, Amy takes 12 hours to paint the entire wall while Bob takes 18 hours for the same. Amy painted the wall for 4 hours and then Bob took over and completed the wall. How long did it take for them to paint the entire wall? [With calculator]

(A) 12

(B) 16

(C) 27

(D) 31

62. Grid-In:
John can do a piece of work in 20 days. He worked initially for 12 days after which he asked his friend Matt to complete the remaining work for him. The work required to be done by Matt is what percentage less than the work already done by John? [With calculator]

63. Joe takes 6 hours to water the plants in his garden. When Joe works with his brother Jack, he takes 4 hours to water the plants in the garden. How long would it take Jack alone to water the plants in the garden? [With calculator]

(A) \$1.2

(B) \$2.0

(C) \$10

(D) \$12

3.2.2 Mixtures

64. A 50 ml. mixture of wine and water has only 15 percent wine. If a person requires 27 ml. of wine, what volume of the mixture must he have access to? [With calculator]

(A) 81

(B) 108

(C) 150

(D) 180

65. 50 grams of a certain metal makes only 15 percent of the total yield from an ore. If a technician observes 27 grams of this metal, what percent of the total yield does it make? [With calculator]

(A) 4.5%

(B) 5.4%

(C) 8.1%

(D) 27.78%

66. The ratio of quantity of milk to water in a mixture was 1: 3. After sometime, 60 liters of milk was added to the mixture. Then, the ratio of quantity of milk to water in the mixture became 3: 5. What was the total quantity of the initial mixture? [With calculator]

(A) 120

(B) 135

(C) 225

(D) 300

67. An energy drink concentrate widely available in the market contains 10% carbohydrates, 40% protein, 30% glucose and 20% minerals. John buys this drink and mixes it with water so that water forms 60% of the mixture. What percentage of the new mixture thus formed is constituted of minerals? [With calculator]

(A) 4%

(B) 8%

(C) 12%

(D) 16%

68. Alfred, a tour operator wanted to increase his fleet of cars. For that, he wanted to know the preference of car types among his customers. A survey revealed that 40% of the customers preferred sedans while the remaining preferred SUVs. Among the 40% who preferred sedans, 45% went for Volkswagen while 55% went for Honda. What percentage of the cars should Alfred keep as Honda sedans so that it is commensurate with the findings of the survey? [With calculator]

(A) 18%

(B) 22%

(C) 40%

(D) 55%

3.2.3 Word Problems

69. Each new instance of good behavior in a school provides increasing award points to students on his or her 'Behavior Reflection sheet'. A student gets one point for the first such instance and the incremental value in award points for each new instance is 3 points. On which numbered instance will a student achieve 16 points, a qualification to get outstanding behavior reward? [With calculator]

(A) 4

(B) 5

(C) 6

(D) 7

70. John and Mary have some marbles. The total number of marbles with John and Mary is 75 times the ratio of the number of marbles with John to that with Mary. If Mary has 30 marbles, find the number of marbles with John. [With calculator]

(A) 20

(B) 30

(C) 45

(D) 50

71. John has twice the amount of money as Peter has. If John gives \$50 to Peter, the ratio of the amount of money with them is reversed. Find the total amount of money with both of them. [With calculator]

(A) 50

(B) 100

(C) 150

(D) 200

72. In the addition shown below, each letter represents a unique digit from 1 to 9. Find the value corresponding to the letter 'R'. [With calculator]

$$\begin{array}{ccccc} & & P & Q & R \\ + & S & P & S & P \\ \hline & 1 & 4 & 4 & P \\ \hline \end{array}$$

(A) 1

(B) 2

(C) 4

(D) 6

73. Jack, Tom, Beth, and Ruth each have a number of marbles. Jack has 10 marbles, the least, while Tom has 27 marbles, the highest. Which of the following could be the average of the number of marbles present with all of them, given that no two of them has the same number of marbles? [With calculator]

(A) 13

(B) 17

(C) 23

(D) 26

74. In an entrance examination, $\frac{3}{5}^{\text{th}}$ of all candidates were male. $\frac{1}{3}^{\text{rd}}$ of all candidates failed to clear the entrance examination. $\frac{1}{4}^{\text{th}}$ of all male candidates failed to clear the examination. Among the candidates who cleared the examination, what fraction is female? [With calculator]

(A) 32.5%

(B) 54.2%

(C) 67.5%

(D) 75.0%

75. A man distributed his entire wealth between his two sons John and Jack such that John got three times as much wealth as Jack. If John got \$ 5000 more than Jack, how much wealth did the father have? [With calculator]

(A) \$2500

(B) \$7500

(C) \$10000

(D) \$12500

76. A boy took an examination that had $+3$ marks for a correct response and -1 for a wrong one. If there were a total of 60 questions and he attempted all of them to get a score of 100, how many questions did he get wrong? [With calculator]

(A) 5

(B) 20

(C) 40

(D) 45

77. In an election between two candidates Joseph and Judith, 60% of the total number of voters did not vote for Joseph and 50% of the total number of voters did not vote for Judith. If it is known that each voter can vote for only one candidate, what percentage of voters did not cast their votes? [With calculator]

(A) 5

(B) 10

(C) 20

(D) 30

78. In an award ceremony, books were distributed to students of different standards. $\frac{2}{5}^{\text{th}}$ of the books were distributed to students from lower-middle standard. Of the remaining, $\frac{1}{3}^{\text{rd}}$ were distributed to students from high school. The remaining books were kept in the school library. What percentage of the total books was stocked in the school library? [With calculator]

(A) 6.67%

(B) 20%

(C) 26.7%

(D) 40%

79. **Grid-In:**
John, Mary, and Joe played a game of marbles. At the end of the game, John had 15 marbles more than Mary, but only one-third of what Joe had. If each player initially had 20 marbles, what was the total number of marbles with John after the game? [With calculator]

80. **Grid-In:**
A certain dealer replenishes his stock when it drops to 15 items. Every time he replenishes his stock to 40 items. On average, he needs to replenish his stock once every two weeks. The cost of each item is $5. What is the dealer's total expense (in dollars) on replenishing his stock in a month? [With calculator]

81. **Grid-In:**
A man purchased bulbs at 4 for every $6. However, he observed that 2 out of every 5 bulbs were defective. What was the price (in dollars) per usable bulb that the man paid? [With calculator]

82. Grid-In:

A man refueled his car one day. However, the very next day, the price of fuel fell by $2.5 per gallon. The man calculated that he could have saved $75 had he fueled one day later. What quantity (in gallon) of fuel had the man purchased? [With calculator]

83. Grid-In:

Find the value of x, if 10% of 20% of $x = 4$. [With calculator]

84. Grid-In:

An airline has a policy of charging for baggage carried by its customers. It charges $\$x$ for luggage up to 5 lbs. For every lb of luggage above 5 lbs, it charges $\$y$ extra. A customer paid $\$a$ for his luggage. If it is known that $a > x$, and the weight of luggage carried by the customer in an integer, then find the expression that gives the weight of luggage carried by the customer. [With calculator]

(A) $5 + (a - x)$

(B) $5 + \frac{a - x}{y}$

(C) $5 + \frac{a}{y}$

(D) $5 + \frac{x}{y}$

85. Grid-In:

A man's salary increases from $\$x$ per month to $400 per month as a result of a 25% hike. Find the value of $(x - 100)$. [With calculator]

86. Grid-In:

A manufacturing firm manufactures screws, rivets and nuts. The average time taken to manufacture a screw, rivet and nut is 10 seconds, 20 seconds and 80 seconds, respectively. If the firm manufactures screws from 10 AM to 11 AM, rivets from 11 AM to 12 noon, and nuts from 12 noon to 2 PM, what percentage of the total items manufactured are nuts? [With calculator]

87. In a promotional campaign, three products P, Q, & R manufactured by a particular company are distributed equally among three dealers; each dealer gets at least 2 items of each product. There are 22 items of P, 37 items of Q, and 13 items of R. What could be the maximum number of items of R stocked with any dealer? [With calculator]

(A) 2

(B) 9

(C) 19

(D) 20

88. The operating cost of a bus consists of a fixed maintenance cost calculated on a per day basis and fuel cost calculated at the rate of $\$n$ per mile travelled. The bus operates 6 days a week, making 3 trips per day covering a distance of d miles per trip. The total operating cost of the bus for an entire week is $\$c$. How much is the daily maintenance cost ($) of the bus? [With calculator]

(A) $(c - 3dn)$

(B) $\frac{(c - 3dn)}{7}$

(C) $\frac{(c - 18dn)}{6}$

(D) $\frac{(c + 6dn)}{2}$

89. Of 4500 items on an assembly line, 30% turn out to be defective and are sent for rework. Of these, 80% are rectified and sent for processing and the rest are scrapped. How many more items are rectified after rework than those which are scrapped? [With calculator]

(A) 270

(B) 810

(C) 1080

(D) 3420

90. A freezer has a powerful ice forming system; it can form 2 inches of ice in every 40 minutes. The freezer was kept operational for 4 hours and after that it was switched off for 2 hours. Once switched off, the ice melts at the rate of 0.5 inches per hour. The freezer was again switched on for 2 hours. What is the total thickness of ice now? [With calculator]

(A) 8

(B) 11

(C) 17

(D) 18

91. In a charity organized by a school, books were given away to children. $\frac{3}{11}^{\text{th}}$ of the books were given away to children less than 8 year old. Of the remaining books, $\frac{3}{4}^{\text{th}}$ were given away to children aged between 8 and 12 years. The rest were given away to children older than 12 years. What fraction of the total books was given away to children older than 12 years? [With calculator]

92. The number of student enrollments for a mathematics course in a college reduced from 235 in the year 2014 to 200 in the year 2015. What is the percentage reduction in the enrollment of students from 2014 to 2015? [With calculator]

93. To manufacture an item, process A requires assembling of 540 components. A machine can manufacture 60 such components in 25 minutes. To manufacture the same item, process B requires assembling of 300 components. A machine can manufacture 50 such components in 15 minutes. How much manufacturing time in percent can be saved if process A is replaced by process B (assume that assembling the components happens instantly)? [With calculator]

94. In Singapore, in the year 2011, the number of voters increased by 2.4 million and the number of people eligible to vote increased by 10.8 million. Find the percentage of voters among people eligible to vote in 2011 if the number of people eligible to vote and the number of people who actually voted in 2010 were 192 million and 54 million, respectively. [With calculator]

(A) 22.22%

(B) 27.81%

(C) 28.12%

(D) 29.38%

95. To gain entry for the Bachelor's programs in different colleges, students need to appear for the AST entrance test. Of all students who register for the test, only 80% appear for the test. Of these students, only 75% clear the test and gain entry into a college of his/her liking. By what percentage is the number of students who do not appear for the test after registering less than the number of students who clear the test? [With calculator]

(A) 33.33%

(B) 40%

(C) 60%

(D) 66.66%

96. To gain entry for the Bachelor's programs in different colleges, students need to appear for the AST entrance test. A total of 50,000 students register for the test in any given year. Of all students who register for the test, only 80% appear for the test. Of these students, only 75% clear the test and gain entry into a college of his/her liking. What is the difference between the number of students who do not appear for the test after registering and the number of students who clear the test? [With calculator]

(A) $10,000$

(B) $20,000$

(C) $30,000$

(D) $40,000$

97. To gain entry for the Bachelor's programs in different colleges, students need to appear for the AST entrance test. A total of 50,000 students register for the test in any given year. Of all students who register for the test, only 80% appear for the test. Of these students, only 75% clear the test and gain entry into a college of his/her liking. What is the difference between the number of students who do not appear for the test after registering and the number of students who clear the test? [With calculator]

(A) 16.66%

(B) 20%

(C) 30.55%

(D) 44%

98. Running advertisements on prime-time television costs $12,000 per 10 seconds. Running the same advertisement early morning reduces the cost by 30%. What is the total daily expenditure (in thousands of dollars) if a 30-second advertisement is to be run once during prime-time and twice early morning? [With calculator]

(A) 28.80

(B) 51.00

(C) 86.40

(D) 97.20

99. A sprinter covers d feet up a steep slope at a speed of s feet/second. While running down the slope, she followed the exact same route and found that she was able to run at a much faster pace of $(2s + 1)$ feet/s. What was the average speed (in feet/second) of the sprinter for the entire round trip? [With calculator]

(A) $\dfrac{\sqrt{s\,(2s+1)}}{2}$

(B) $\dfrac{(3s+1)d}{2s(2s+1)}$

(C) $\dfrac{4s+1}{2}$

(D) $\dfrac{2s\,(2s+1)}{3s+1}$

100. A sprinter covers some distance up a slope at a speed of 3.6 feet/second. While running down the slope, she followed the exact same route and found that she was able to run much faster at 6 feet/second. She took a total of 6 minutes for her round trip. What was the one way distance covered by the sprinter? [With calculator]

(A) 750 feet

(B) 810 feet

(C) 864 feet

(D) 1620 feet

101. A car travelling from California to Los Angeles travelled the first d miles at an average speed of 60 miles/hr and the remaining 60 miles it covered in 6 hours. What is the average speed (in miles/hr) of the car for the entire journey? [With calculator]

(A) $\dfrac{60\,(\mathrm{d}+60)}{\mathrm{d}+360}$

(B) $\dfrac{60\,(\mathrm{d}+60)}{\mathrm{d}+6}$

(C) $\dfrac{d}{60}+10$

(D) 35

102. In a certain class, 40 students play football and 30 students play rugby. If 10 students play both games and 20% of the students in the class do not play any of the two games, how many students are there in the class? [With calculator]

103. At a gas station, cars arrive for fueling every p minutes during the peak hours from 9:00 am to 1:00 pm and every q minutes during the non-peak hours from 3:00 pm to 6:00pm. If cars do not arrive at the gas station at any time other than those mentioned above, what is the average number of cars that arrive at the station every minute? [With calculator]

(A) $\dfrac{4q+3p}{12pq}$

(B) $\dfrac{4p+3q}{7pq}$

(C) $\dfrac{4q + 3p}{7pq}$

(D) $\dfrac{7pq}{4q + 3p}$

104. A fruit seller stocks apples, oranges, mangoes and guavas. The number of apples with him is one-third the number of mangoes and the number of guavas with him is one-fourth the number of oranges. If the number of mangoes is half the number of guavas, what fraction of the total fruits are mangoes? [With calculator]

105. A shopkeeper sells CDs, Pen-drives and DVDs. The number of CDs with him is one-third the number of Pen-drives and the number of DVDs with him is one-fourth the number of Pen-drives. What fraction of the total items are DVDs? [With calculator]

106. Grid-In:

A container used in large container ships has dimensions of $4 \times 6 \times 20$ meters. It is usually filled with smaller boxes which have dimensions of $4 \times 3 \times 2$ meters. What is the maximum number of smaller boxes that can fit in a container? [With calculator]

107. A bicycle rider calibrated his distance measuring device at a shop at the entrance to the bridge. On the bridge, due to vibration of the bridge, the device malfunctioned and underestimated the distance by 10 centimeters for every 15 meters traveled. What was the distance in meters shown by the device at the end of the bridge, if the length of the bridge is known to be 1800 meters? [With calculator]

(A) 1704

(B) 1710

(C) 1779

(D) 1788

108. Grid-In:

A Wall Street firm hired 140 students with an MBA degree and 40 students with an MA degree. The cost of hiring a student with an MA degree is half of that for the student with an MBA degree. What percent of the total cost of hiring is the cost of hiring all of the MA students? [With calculator]

109. During t_1 hours in the morning, customers arrive at a bank at the rate of r_1 per hour, and during t_2 hours in the afternoon, customers arrive at a bank at the rate of r_2 per hour. What is the average rate of arrival in customers per hour to the bank during the whole day? [With calculator]

(A) $\dfrac{r_1 + r_2}{\frac{r_1}{t_1} + \frac{r_2}{t_2}}$

(B) $\dfrac{r_1 + r_2}{t_1 + t_2}$

(C) $\dfrac{r_1 t_1 + r_2 t_2}{t_1 + t_2}$

(D) $\dfrac{t_1 + t_2}{r_1 t_1 + r_2 t_2}$

3.2.4 Profit & Loss

110. A dealer bought a bicycle for $120 and later sold it to a customer for $90. What was the percent loss in the transaction? [With calculator]

(A) 10.0

(B) 25.0

(C) 30.0

(D) 33.3

111. To attract more customers, a distributor provides a 20% discount on the listed price, while maintains a 10% profit in the transaction. If the price at which the distributor procured the product is $120, what should be the listed price of the product? [With calculator]

(A) $100

(B) $125

(C) $130

(D) $165

112. By selling 16 toffees for $4, a man makes neither profit nor loss. How many toffees should he sell for $10 to make 25% profit? [With calculator]

(A) 32

(B) 36

(C) 40

(D) 50

113. A shopkeeper sells balloons at 10 cents apiece or at $2 per box if purchased in boxes containing 25 balloons each. How much does a customer save if he purchases 500 balloons by purchasing boxes than by purchasing in single pieces? [With calculator]

(A) $5

(B) $10

(C) $40

(D) $50

114. A shopkeeper sells balloons either in single pieces priced at 10 cents apiece or in boxes containing 25 each at \$2 per box. What is the minimum amount that a customer has to spend if he wants to purchase exactly 310 balloons? [With calculator]

(A) \$12.8

(B) \$25.6

(C) \$25.0

(D) \$26.0

115. **Grid-In:**
A pen costs thrice as much as a pencil. A boy purchased 10 pens and 10 pencils. His total cost is \$20. What is the cost (in cents) of 5 pencils? [With calculator]

116. **Grid-In:**
A pencil costs 50 cents. A pen costs thrice as much as a pencil. With \$20, a boy wants to purchase some pens and pencils. What is the maximum number of items he can purchase if he has to purchase at least 3 pens? [With calculator]

117. **Grid-In:**
A shopkeeper sets the selling price of a DVD such that the selling price of 7 DVDs equals the cost price of 9 DVDs. Given that a customer buys 9 DVDs, what percentage profit does the shopkeeper make? [With calculator]

118. **Grid-In:**
A shopkeeper sets the selling price of a DVD such that the selling price of 7 DVDs equals the cost price of 9 DVDs. Given that a customer buys 10 DVDs, and saves \$30. What is the cost price (in \$) of each DVD? [With calculator]

119. **Grid-In:**
A restauranteur promotes his restaurant by offering a scheme: “Dine 5, and Pay for 4”. If a group of 5 dines in the restaurant, what percent of discount will they get? [With calculator]

120. **Grid-In:**
A shopkeeper promotes his shop by offering a scheme: “Buy 5, and get 3 free”. If a man wants 8 shirts, what percent of discount will he get? [With calculator]

121. When Brian bought a boat, he found out that the municipality charged him 7 percent tax for marina services. If Brian paid \$6,885.45 for the boat, what was the price of the boat before marina services tax? [With calculator]

(A) $6,012.34$

(B) 6535.00

(C) $6,435.00$

(D) $7,367.43$

122. Initially, the price of an item had been expressed in terms of whole dollars. A store raised the price of an item by exactly 10 percent. Which of the following could NOT be the resulting price of the item? [With calculator]

(A) \$5.50

(B) \$6.60

(C) \$11.70

(D) \$75.90

123. **Grid-In:**

Michael runs a book shop. For every customer who buys four books, he sells the first book at the listed price, but gives 10% discount on each of the other three books. Assuming each book is priced at \$60, what is the net amount (in dollars) paidby a customer who buys four books? [With calculator]

3.2.5 Interest & Compounding

124. **Grid-In:**
In a laboratory experiment on bacteria, it was observed that the count of bacteria doubles in number in every 60minutes. In order to control their growth, scientist introduced a chemical agent, which resulted in the bacteria multiplying to thrice their count in two hours. By what percentage was the growth of the bacteria reduced? [With calculator]

125. An investor puts in \$1000 for a year with a financial institute. Which of the following terms of payment of interest would be most lucrative to the investor? [With calculator]

(A) interest calculated at the end of the year at the rate of 10% per annum

(B) interest calculated and compounded at the end of every 6 months at the rate of 9% per annum

(C) interest calculated and compounded at the end of every 3 months at the rate of 8% per annum

(D) All the above terms of payment of interest would yield the same interests

126. An investment pays at the rate of $6\frac{1}{4}\%$ on the annual basis. If the same investment pays at an annual interest rate of 6% on a semi-annual basis without compounding, instead of $6\frac{1}{4}\%$ on a annual basis, what is the gain or the loss in payments as per the new terms at the end of the year to the investor who invests $20,000$ dollars? [With calculator]

(A) Loss of \$32

(B) Loss of \$50

(C) Gain of \$18

(D) Gain of \$50

127. During a festive season, an e-commerce website offers two schemes on buying iphones. The first offer demands an outright payment of 20% of the phone price and 5 equal monthly installments, with each being 20% of the phone price, while the second offer demands an outright payment of 10% of the phone price and 12 equal monthly installments, with each being 10% of the phone price. By what percent is the first offer cheaper than the second? [With calculator]

(A) 7.69

(B) 8.33

(C) 11.11

(D) 20

128. If the bacteria count of a culture increases by 10% per minute, during what interval will the bacteria count be more than 1.5 times the initial bacteria count? [With calculator]

(A) 2-3 minutes

(B) 3-4 minutes

(C) 4-5 minutes

(D) 5-6 minutes

3.2.6 Ratio & Proportion

129. To make a dye, one needs to have red and yellow paints in the ratio 3 : 5. How many liters of the dye can be made if there are 13.5 liters of red paint and 18 liters of yellow paint? [With calculator]

(A) 10.8

(B) 22.5

(C) 28.8

(D) 36.0

130. To make a perfume, a manufacturer needs to mix oil, fragrance and water in the ratio 1 : 8 : 3 by volume. What maximum quantity of perfume can the manufacturer make using 60 ml. fragrance? [With calculator]

(A) 15

(B) 40

(C) 75

(D) 90

131. Two cities A and B are connected by three routes R, S, and P whose lengths are in the ratio 3 : 4 : 3. Three people travel from A to B in a SUV, a sedan, and a bike along the routes R, S, & P respectively. The amount of fuel spent by the SUV, sedan and bike is in the ratio 6 : 2 : 3. What is the ratio of their fuel efficiencies i.e. fuel consumed per unit distance traveled by the SUV, sedan and bike? [With calculator]

(A) 4 : 1 : 2

(B) 1 : 4 : 2

(C) 18 : 8 : 9

(D) 3 : 2 : 2

132. A 40 liters mixture of milk and water had 80% concentration of milk. A part of the mixture was replaced with an equal quantity of water to achieve 50% concentration of milk. What volume of the mixture (in liters) was replaced to achieve the desired concentration? [With calculator]

(A) 12

(B) 15

(C) 20

(D) 32

133. A 40 liters mixture of milk and water had 80% concentration of milk. Some water was added to the mixture in order to achieve 50% concentration of milk. What is the volume of water (in liters) that was added to achieve the desired concentration? [With calculator]

(A) 12

(B) 24

(C) 32

(D) 64

134. In a laboratory experiment, a concentrated mixture containing 80% of a chemical is processed in a special container. 40% of this mixture is separated and stored for future use. The container is then filled to its full capacity by water. The chemical forms what percentage of the final mixture? [With calculator]

(A) 8%

(B) 12%

(C) 32%

(D) 48%

3.2.7 Combinatorics

135. 3-digit numbers are formed using the digits 2, 5, 8, and 9. Repetition of digits is allowed. How many of these numbers would be even? [With calculator]

(A) 8

(B) 16

(C) 32

(D) 64

136. How many different license numbers can be allotted using 3 letters and 3 digits? The letters and the digits may appear in any order. Repetition of the letters and the digits is allowed. [With calculator]

(A) $C_3^{26}.C_3^{10}$

(B) $2.C_3^{26}.C_3^{10}$

(C) $2.26^3.10^3$

(D) 36^3

137. How many 5-digit numbers can be formed containing exactly two '3s', appearing together? Other digits can be repeated. [With calculator]

(A) $3.C_3^9$

(B) $4.C_3^9$

(C) $9^4 + 32.9^3$

(D) $9^3 + 24.9^2$

138. In a polygon of 7 sides, how many different triangles can be drawn using the vertices of the polygon as the vertices of the triangles? [With calculator]

(A) 32

(B) 34

(C) 35

(D) 56

139. In how many 3-member teams can be formed out of 6 individuals provided that one specific individual must be in the team? [With calculator]

(A) 5

(B) 10

(C) 15

(D) 20

140. In how many ways can 2-members: either only boys or only girls be chosen from a class comprising 8 boys and 10 girls? [With calculator]

(A) 28

(B) 73

(C) 153

(D) 1260

141. How many distinct words (meaningful or meaningless) can be formed using all the letters of the word — “Banana” ? [With calculator]

(A) 9

(B) 10

(C) 120

(D) 720

142. **Grid-In:**
In how many ways can 2 girls or 3 boys be chosen from a class comprising 10 boys and 12 girls? [With calculator]

143. **Grid-In:**
How many distinct words (meaningful or meaningless) can be formed using all the letters of the word — “Banana” such that no two “Ns" come together? [With calculator]

144. **Grid-In:**
How many spy-codes are possible taking all the symbols together with the following shapes, if $\square$ must NOT be at the beginning and $\lozenge$ must NOT be at the end? [With calculator]

$\square \trianglerighteq \otimes \oslash \lozenge$

3.2.8 Probability

145. A box contains 30 balls, of which 8 are black, 10 are green, and 12 are yellow. If two balls are drawn from the box at random, what is the probability that both the balls will be NOT be green? [With calculator]

(A) 8/9

(B) 3/29

(C) 24/29

(D) 26/29

146. A fair coin is tossed four times. What is the probability of getting at least one 'Tail'? [With calculator]

(A) 1/16

(B) 1/4

(C) 3/4

(D) 15/16

147. Jack, Parker, and Herman are given an assignment to collectively solve a problem. The probabilities of Jack, Parker, and Herman to solve the problem individually are 4/5, 2/3, and 3/4 respectively. What is the probability that the problem will be solved? [With calculator]

(A) 1/60

(B) 13/60

(C) 59/60

(D) 2/13

148. 3 actors are to be chosen out of 5 — Jack, Steve, Elad, Suzy, and Ali. What is the probability that if Jack is selected then Steve would be left out, and if Steve is selected then Jack would be left out? [With calculator]

(A) 1/5

(B) 3/5

(C) 4/5

(D) 5/6

149. A box contains p number of silver coins and q number of gold coins. One coin is randomly drawn from the box but not replaced; thereafter a second coin is randomly drawn. What is the probability that both the coins drawn are silver? [With calculator]

(A) $\dfrac{pq}{(p+q)^2}$

(B) $\dfrac{p(p-1)}{(p+q)(p+q-1)}$

(C) $\dfrac{(p-1)(q-1)}{(p+q)^2}$

(D) $\dfrac{pq}{(p+q)(p+q-1)}$

150. A super mall issues coupons to its customers at the entrance of the mall. The coupons are numbered from 100 to 400, inclusive. Customers who have coupons with the numbers divisible either by '3' or by '5' win gifts. What is the probability that a random customer wins a gift? [With calculator]

(A) 141/300

(B) 141/301

(C) 161/301

(D) 161/300

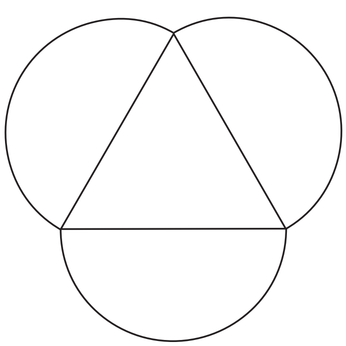
151. A box contains 5 black and some green balls. If two balls are drawn from the box at random, and the probability that both the balls are green is 1/6, how many green balls are in the box? [With calculator]

(A) 3

(B) 4

(C) 5

(D) 6



152.

Above figure shows a designer dartboard. It has a triangle at the center with side 4 cm. Each side of the triangle is the diameter of respective semi-circle. If Brian throws a dart, and it hits the dartboard, what is the probability that it hits triangular part of the dartboard? [With calculator]

(A) 1/2

(B) 1/4

(C) $\dfrac{2\sqrt{3}}{2\sqrt{3}+6\pi}$

(D) $\dfrac{4\sqrt{3}}{4\sqrt{3}+6\pi}$

153. Grid-In:

A pair of die is rolled. What is the probability of getting the sum of numbers greater than 10 on their upper faces? [With calculator]

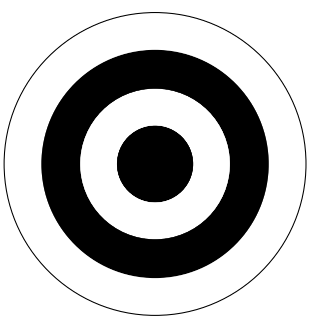
154. Grid-In:

A coin is tossed three times. What is the probability of getting 'Tails' on the first two tosses and a 'Head' on the third toss? [With calculator]

155. Grid-In:

A die is rolled four times. What is the probability of getting a number greater than '2' in the first time, greater than '3' in the second time, greater than '4' in the third time, and greater than '5' in the fourth time? [With calculator]

156. Grid-In:



Above figure shows a dartboard. Radius of the the outermost circle is 4 cm., the radius of the innermost black circle is 1 cm., and the width of the black ring is 1 cm. If Brian throws a dart, and it hits the dartboard, what is the probability that it hits any of the black parts of the dartboard? [With calculator]

3.2.9 Statistics

Questions on Average (Arithmetic Mean)

157. Average (arithmetic mean) of x and y is 80, and the average of x and z is 200; what is the value of $\frac{(z-y)}{8}$? [With calculator]

(A) –30

(B) 0

(C) 30

(D) 80

158. Three numbers have their average (arithmetic mean) equals to 6. The middle number is 7. If the average of two larger numbers is 8, what is the average of the two smaller numbers? [With calculator]

(A) 2

(B) 4

(C) 4.5

(D) 6

159. The average (arithmetic mean) of six numbers is 7.5. If one of the six numbers is multiplied by 4, the average of the numbers decreases by 0.5, which of the six numbers is multiplied by 4? [With calculator]

(A) –3

(B) –2

(C) 0

(D) 2

160. A student's average (arithmetic mean) score on 5 tests is 38. What must be the student's score on the 6^{th} test for the average score on all the 6 tests to be 41? [With calculator]

(A) 40

(B) 44

(C) 50

(D) 56

161. For the first x days, the average (arithmetic mean) rainfall was 32 millimeter per day. If today's rainfall of 56 millimeter increased the average rainfall to 36 millimeter per day, what is the value of x? [With calculator]

(A) 2

(B) 3

(C) 5

(D) 6

162. If all the elements of a set are multiplied by a constant, the average (arithmetic mean) remains unchanged, which of the following statements must true? [With calculator]

(I) Average (arithmetic mean) = 0

(II) Sum of the elements of the set equals 0.

(III) The set contains at least one positive and one negative number

(A) I only

(B) II only

(C) III only

(D) I, and II only

Questions on Median

163. If two sets, set X: $\{-2, 23, 4, -8, 6\}$, and set Y: $\{0, -2, -8, 8, 12, 13, y\}$ have equal median values, what is the value of y? [With calculator]

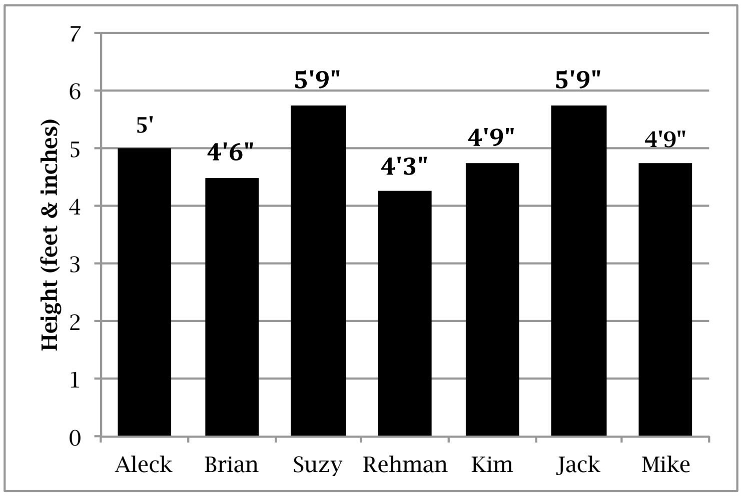
(A) 0

(B) 2

(C) 3

(D) 4

164. According to the chart given below, which of the following student represent the median height of seven students? [With calculator]



(A) Aleck

(B) Suzy

(C) Kim

(D) Rehman

165. **Grid-In:**
As per the data in the above chart, what is the average (Mean) height of all the seven students in the nearest integer in centimeters? [With calculator]

166. **Grid-In:**
As per the data in the above chart, what is the Range of heights of all the seven students in inches? [With calculator]

167. If the median of X consecutive integers is 50.5, which is the smallest integer among the following? [With calculator]

(A) $51 - X/2$

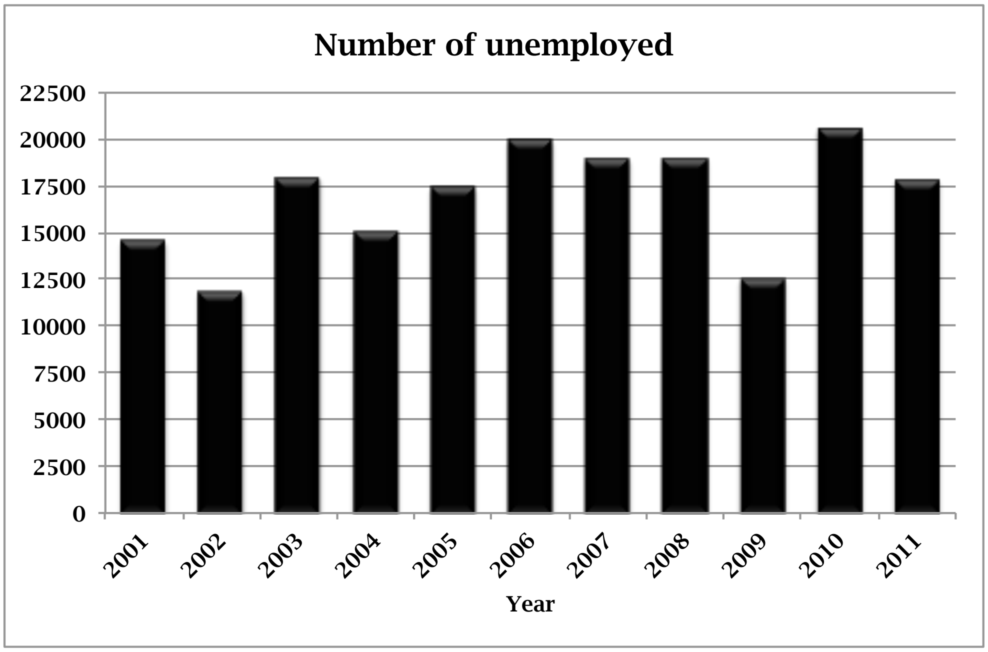
(B) $50.5 - X/2$

(C) $50 - X/2$

(D) $49.5 - X/2$

(E) $49 - X/2$

168. According to the chart given below, which of the following is closest to the median number of unemployed for the years 2001 to 2011, inclusive? [With calculator]



(A) 11500

(B) 12500

(C) 17000

(D) 18000

Questions on Average and Median

169. Number of defects in eight of the nine batches of batteries are: 2, 4, 6, 5, 3, 3, 7, 8, respectively. If the median defect per batch for all the nine batches equals the average (arithmetic mean) defect per batch for all the nine batches, which of the following would be the number of defects in the ninth batch? [With calculator]

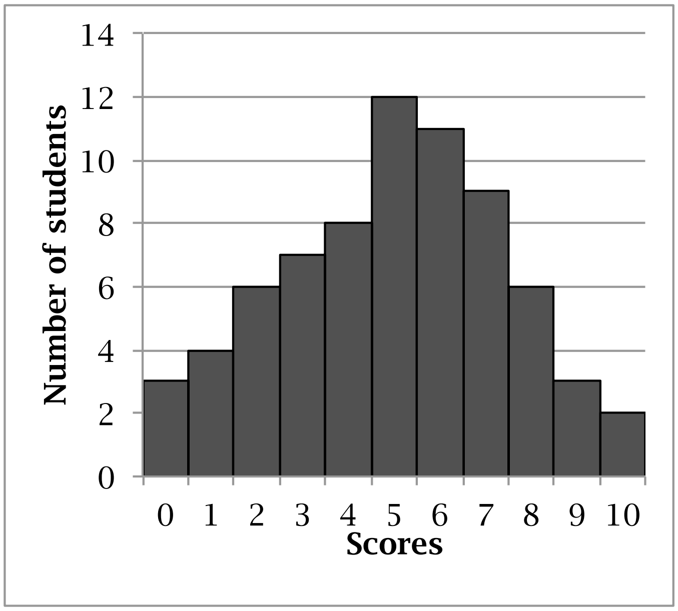
(A) 2

(B) 3

(C) 4

(D) 7

170. According to the histogram given below, what is the modal score (Mode) for 71 students? [With calculator]



(A) 4

(B) 5

(C) 6

(D) 12

171. **Grid-In:**
As per the data in the above histogram, what is the average (Mean) score of all 71 students? [With calculator]

Questions on Range

172. Which of the following could be the range of set {– 43, 76, a, b, c}? [With calculator]

(I) –129

(II) 112

(III) 154

(A) I only

(B) II only

(C) III only

(D) Only I and II

Questions on Standard deviation

173. Which of the following sets have a standard deviation less than the standard deviation of set X: {10, 20, 30, 40}? [With calculator]

$P = \{2, 12, 22, 32\}$
$Q = \{-3, 7, 17, 27\}$
$R = \{-6, 4, 14, 24\}$

(A) Set P

(B) Set Q

(C) Set R

(D) None

174. If set X is $\{p, q, r\}$, which of the following sets must have the same standard deviation as has set X? [With calculator]

(A) $\{p + 1, q, r\}$

(B) $\{p, q + 1, r\}$

(C) $\{p, q, r + 1\}$

(D) $\{p + qr, q(1 + r), r(q + 1)\}$

175. If one of P and Q is a positive number and the other is a negative number, which of the following could be the standard deviation of set $\{P, -12, -8, -4, 0, 4, 8, Q\}$? [With calculator]

(I) –4

(II) –2

(III) 0

(A) I only

(B) II only

(C) I and II only

(D) None of these

176. A group of five friends collected WWE stickers. They found out that they had an average of 15 stickers. If the maximum number of stickers with any of them was 18 and each friend had a distinct number of stickers, what could be the minimum number of stickers with any of them? [With calculator]

(A) 3

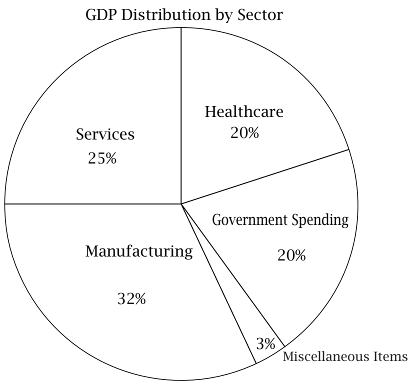
(B) 6

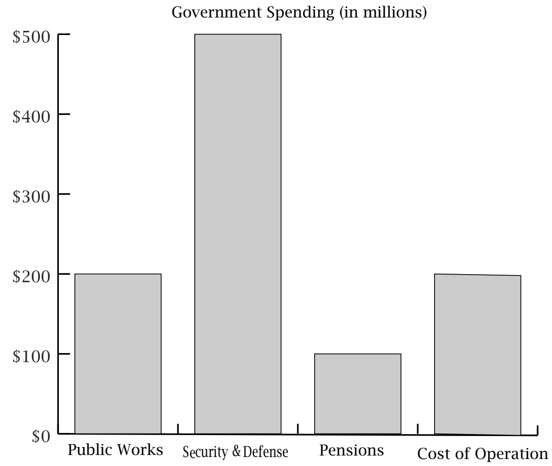
(C) 9

(D) 12

3.2.10 Data Analysis

177. Percentage distribution of GDP & Government spending ($M) for Country U during year N:





During Year N, the government's cost of operation was what percent of the total GDP of Country U? [With calculator]

(A) 4%

(B) 5%

(C) 8%

(D) 20%

178. Making use of the information in the previous problem, if Management Consulting Services, a part of Services, in the economy of Country N have the same dollar value as Miscellaneous Items, what percent of Services is accounted for by Management Consulting Services? [With calculator]

(A) 60%

(B) 15%

(C) 12%

(D) 6%

179.

Week	Number of data requests
1	20,000
2	20,000
3	15,000
4	5,000

The table above shows the approximate number of data requests at the internet site of a new reality show for the first 4 weeks after it went on air for the first time. The cost of advertisement per week on the show's website was computed to be 10% of the product of the number of data request per week and the average length of a data request per week. What was the total advertising cost for the first 4 weeks from the website, if the average length of a data request was estimated to be 4 minutes? [With calculator]

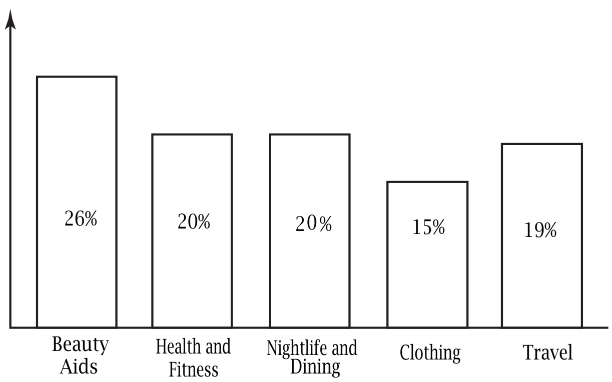
(A) $\$8,000$

(B) $\$18,000$

(C) $\$20,000$

(D) $\$24,000$

180.



During a certain year Michelle spent her annual trust fund allowance according to the chart above. How much did Michelle spend on clothing, if she spent $\$38,000$ traveling? [With calculator]

(A) $\$8,000$

(B) $\$18,000$

(C) $\$20,000$

(D) $\$30,000$

The following table presents quarter-wise sales and inventory for Apex Corporation for 10 steel products, in metric tons (MT). The following two questions are based on it.

S - Sales (MT); I - Inventory (MT)

Products	**Quarter I**		**Quarter II**		**Quarter III**		**Quarter IV**		**Yearly Total**	
	S	**I**	**S**	**I**	**S**	**I**	**S**	**I**	**S**	**I**
CR pipe	24	12	15	8	4	8	13	4	56	32
CR sheet	16	8	12	12	16	4	15	21	59	45
CR tube	15	15	16	8	6	15	8	4	45	42
CR wire	15	0	24	8	18	13	13	12	70	33
HR pipe	24	12	21	8	5	21	12	15	62	56
HR sheet	18	15	15	16	0	0	14	3	47	34
HR tube	8	9	8	21	8	0	9	2	33	32
HR wire	24	26	12	16	15	15	8	4	59	61
HS Billet	21	24	21	0	18	5	15	12	75	41
MS Billet	10	8	15	0	18	2	15	16	58	26
Total	**175**	**129**	**159**	**97**	**108**	**83**	**122**	**93**	**564**	**402**

181. Which of the following quarters has the highest average (arithmetic mean) sales in metric tons (MT)? [With calculator]

(A) Quarter I

(B) Quarter II

(C) Quarter III

(D) Quarter IV

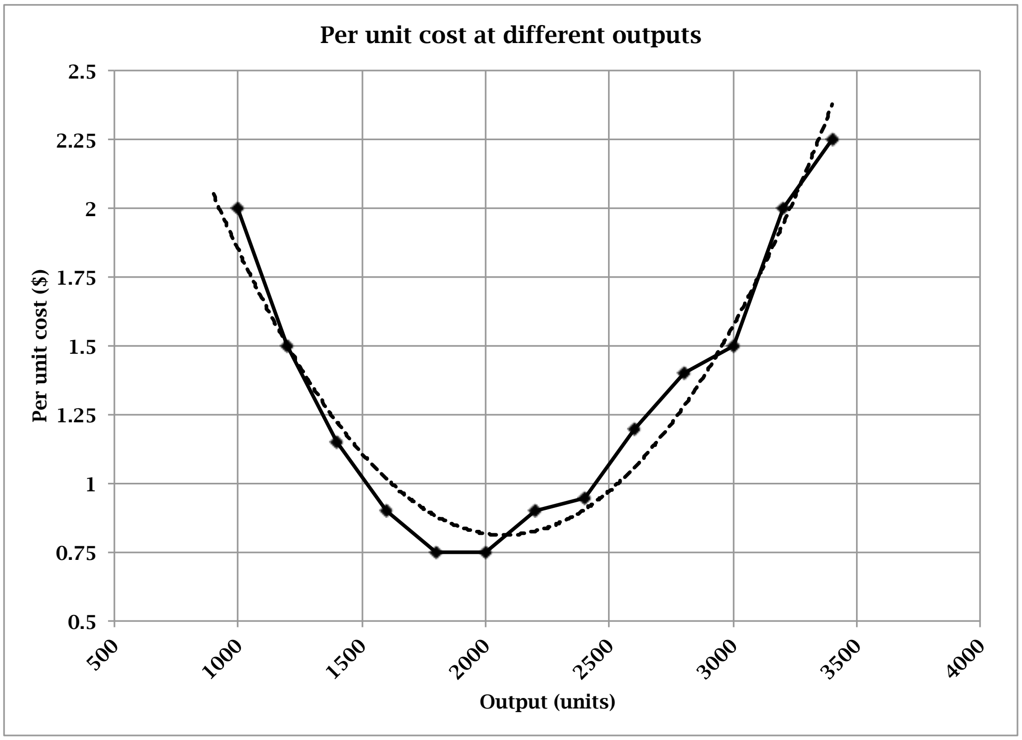
182. Which of the following statements is correct? [With calculator]

(A) In quarter I, there is only one product that has the highest sale.

(B) In quarter II, for 'HS billet' and 'MS billet', the ratio of 'sale to inventory' is 0.

(C) In quarter III, the product that has the highest inventory also has the least sale.

(D) In quarter IV, the product that has the highest inventory also has the highest sale.



Above graph shows distribution of per unit unit cost ($) of an automotive component at different outputs of a machine. The best estimation curve is also shown with dashed line. The following two questions are based on it.

183. What is the best estimate for the output for the machine such that the per unit cost of the component is least? [With calculator]

(A) 800

(B) 1000

(C) 2100

(D) 3400

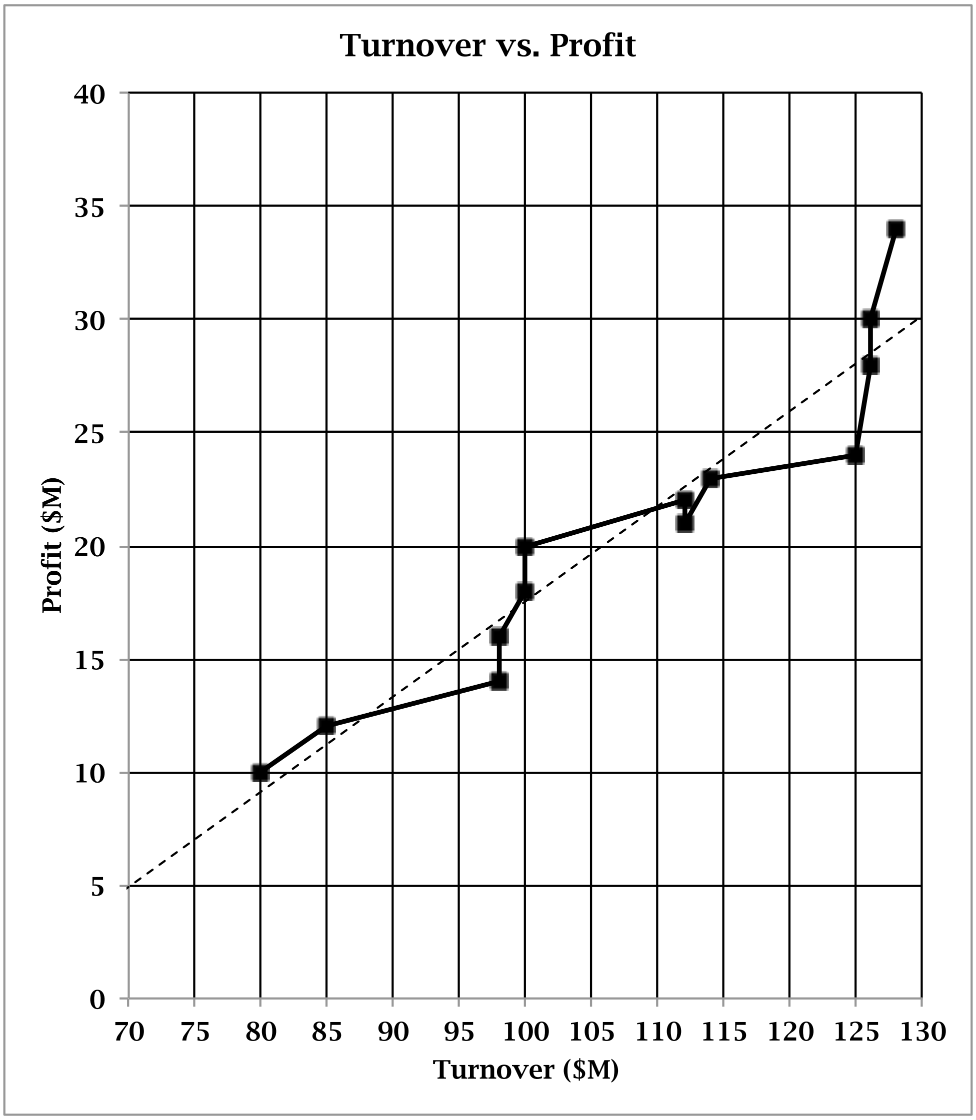
184. According to the above graph, at what output would the per unit cost be the same as it is at 1200 units of output? [With calculator]

(A) 1000

(B) 2000

(C) 3000

(D) 3400



Above graph shows a scatterplot for the distribution of turnovers ($M) and their corresponding profits ($M). The data is modeled as a linear relationship (a dashed trend-line) where p is profit and t is turnover. The following two questions are based on it.

185. What is the estimated profit ($M) for the turnover of $125M? [With calculator]

(A) 25

(B) 28

(C) 30

(D) 125

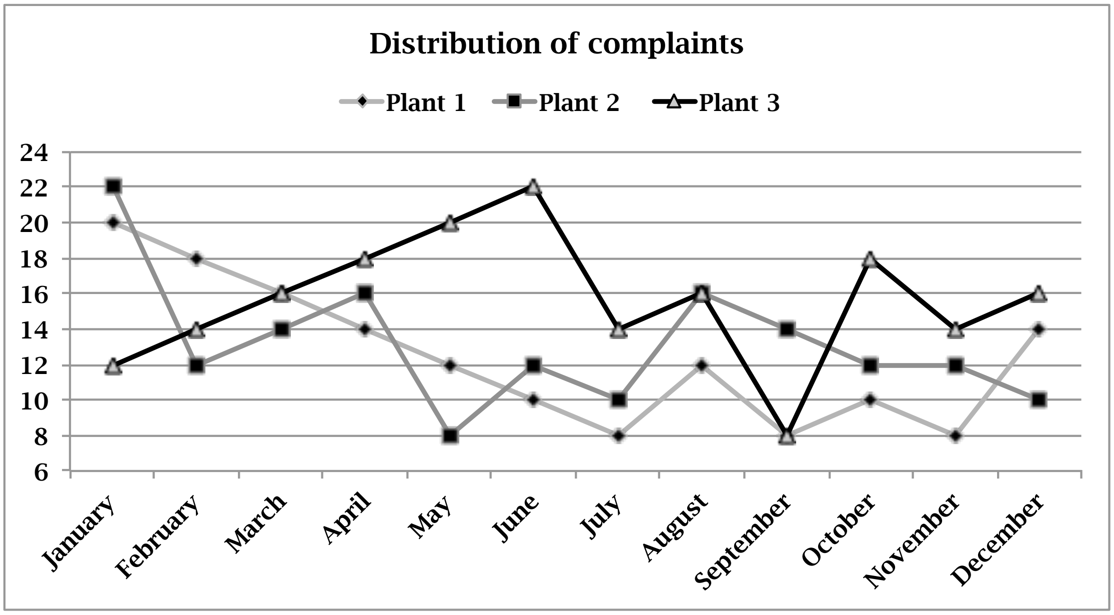
186. Deduce a linear equation that estimates the trend-line, using p and t? [With calculator]

(A) $p = 2.4t - 29.17$

(B) $p = 0.42t - 29.17$

(C) $p = 0.42t - 0.34$

(D) $p = 0.42t + 29.17$



Above line graph shows numbers of complaints received for a product over the year for three plants. The following two questions are based on it.

187. How many total number of complaints in three plants in the month of January is more than that in the month of June? [With calculator]

(A) 10

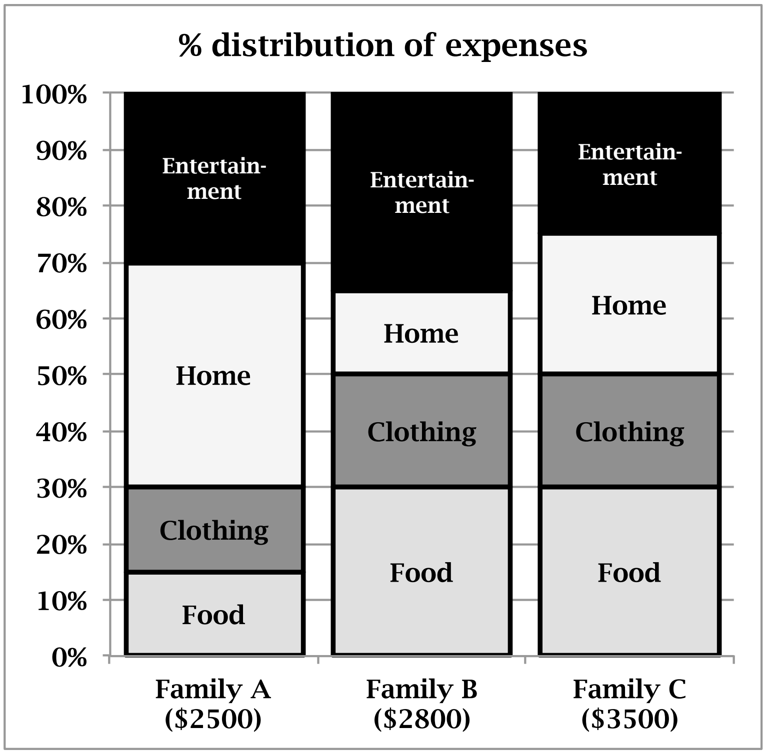
(B) 12

(C) 14

(D) 16

188. Grid-In:

For how many months is the number of complaints for plant 3 more than the number of complaints in corresponding months for plant 1 and for plant 2? [With calculator]



Above bar chart shows the percentage distribution of expenses for following heads: Food, Clothing, Home, and Entertainment for three families. Total expenses incurred by each family are also mentioned in the chart. The following two questions are based on it.

189. Which of the following families does the highest expenses on 'Entertainment' head? [With calculator]

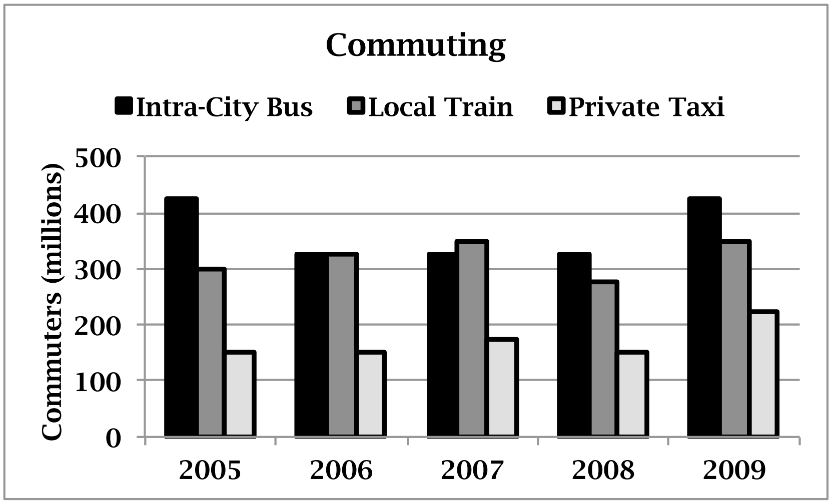
(A) Family A

(B) Family B

(C) Family C

(D) Family A & B

190. Grid-In:
How much dollars does the family that spends least on 'Clothing' head among all the families spend on 'Clothing'? [With calculator]



Above multiple bar chart shows distribution of commuters for 5 years on the basis of mode of transport used by them, namely, Intra-City Bus, Local Train, and Private taxi.

191. In which of the following years, most number of commuters traveled, taking all three modes of transport together? [With calculator]

(A) 2006

(B) 2007

(C) 2008

(D) 2009

192. **Grid-In:**

Considering all the years from 2005 to 2009, what is the least number of commuters traveled (in millions), taking all three modes of transport together? [With calculator]

3.3 Passport to Advanced Math

3.3.1 Polynomials

193. Which of the following expressions represent

$(2x^5 - 3x^4 - 4x^2 + 3) - (2x^4 - 3x^2 + 4x + 13) - (2x^3 - 3x^2 - 4x)$? [Without calculator]

(A) $2x^5 - 3x^4 + 8x + 16$

(B) $2x^5 - 5x^4 - x^2 - 10$

(C) $2x^5 - x^4 + 2x^2 + 8x - 10$

(D) $2x^5 - 3x^4 + 2x^2 + 16$

194. Which of the following expressions is the equivalent to

$13x^5y^{-3}z^2 \times 4x^{-2}y^3z^{-3}$ [Without calculator]

(A) $52x^{-10}y^{-9}z^{-6}$

(B) $52x^3z^{-1}$

(C) $52x^3yz^{-6}$

(D) $52x^{-10}y^{-1}z^6$

195. Which of the following expressions represent

$(-3x^4 + 3) \times (2x^4 + 4x)$? [Without calculator]

(A) $-6x^8 + 6x^4 + 3x^5 + 12x$

(B) $-6x^8 - 6x^4 - 3x^5 + 12x$

(C) $-6x^8 + 6x^4 - 3x^4 + 12x$

(D) $-6x^{16} + 6x^4 - 3x^5 + 12x$

196. Which of the following expressions represent

$\frac{6x^5 - 33x^4 + 21x^3 - 6x^2 - 30x}{3x}$? [Without calculator]

(A) $2x^4 - 11x^3 + 7x^2 - 2x - 10x$

(B) $2x^4 - 11x^3 + 7x^2 - 2x - 10$

(C) $2x^4 - 11x^4 + 7x^2 - 2x - 10$

(D) $2x^4 - 11x^3 + 7x^2 - 2x$

197. Which of the following expressions represent $\dfrac{3x^3 - 24x^2 - 4x + 32}{x - 8}$? [Without calculator]

(A) $x - 8$

(B) $3x^2 - 2x - 4$

(C) $3x^2 - 4$

(D) $(x - 8)(3x^2 - 4)$

198. Which of the following expressions represent $\dfrac{8x^3 + 125}{2x + 5}$? [Without calculator]

(A) $4x^2 - 10x - 25$

(B) $4x^2 - 20x + 25$

(C) $4x^2 - 10x + 25$

(D) $4x^2 + 10x + 25$

199. Which of the following expressions represent $\dfrac{8 - 243x^6}{3x - \sqrt{2}}$? [Without calculator]

(A) $(2 + 3x)(4 + 18x^2 + 81x^4)$

(B) $(\sqrt{2} - 3x)(4 + 18x^2 + 81x^4)$

(C) $4 + 18x^2 + 81x^4$

(D) $(\sqrt{2} + 3x)(4 + 18x^2 + 81x^4)$

3.3.2 Quadratic equations

200. **Grid-In:**
What is the integer value of x that satisfies the equation: $\frac{3}{x-1} - \frac{2}{2x-3} = 1$? [With calculator]

201. What is the sum of values of x that satisfies the equation: $\frac{3}{x} + \frac{x}{6} = \frac{3}{2}$? [Without calculator]

(A) 2

(B) 3

(C) 6

(D) 9

202. What is the integer value of x that satisfies the equation: $\frac{12}{x+4} + \frac{6}{x-6} = 4$? [Without calculator]

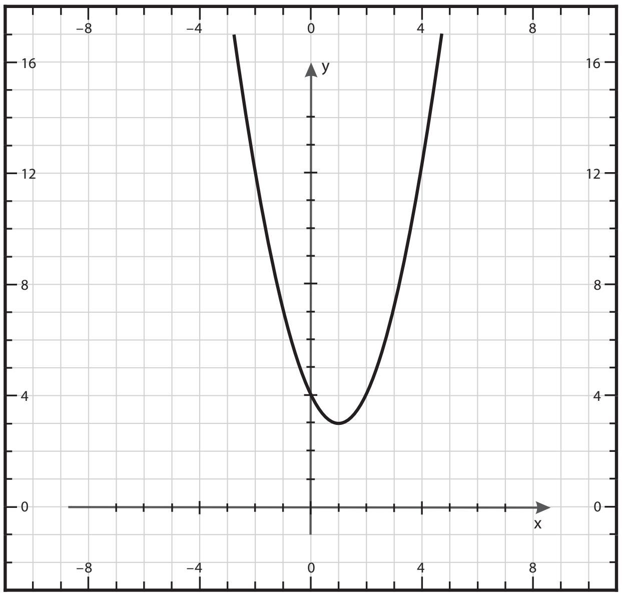
(A) – 3

(B) – 8

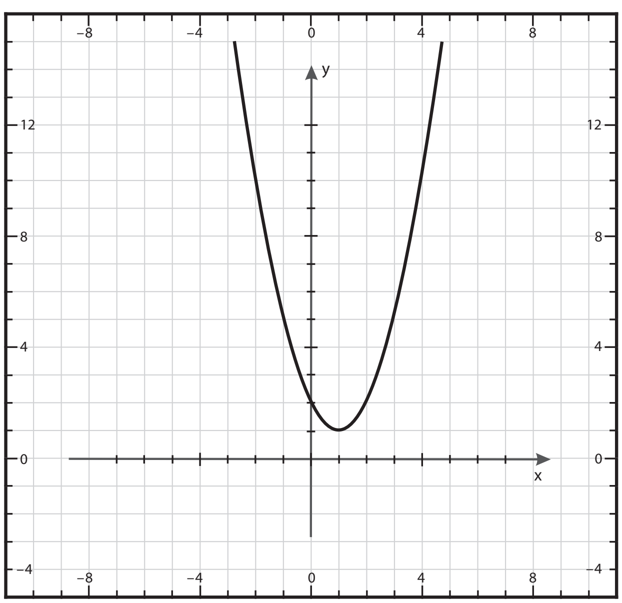
(C) 8

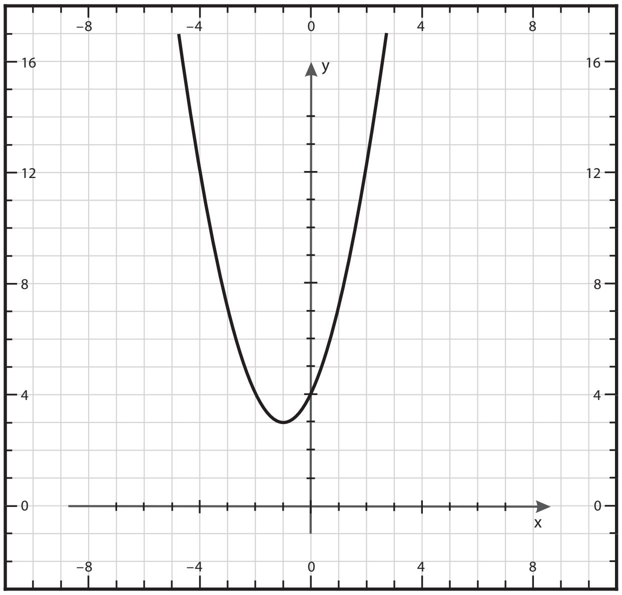
(D) 12

Graphic quadratic equations

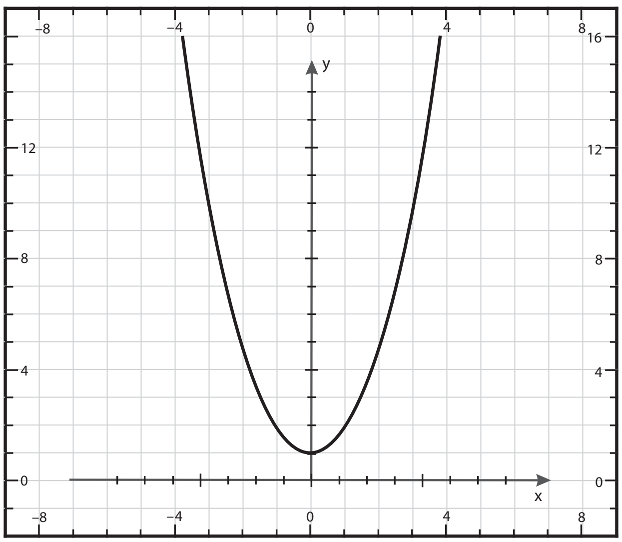
203. 

The graph of $f(x) = (x-1)^2 + 3$ is shown above. Choose from the options below the correct graph of $g(x) = (x+1)^2 + 1$. [Without calculator]

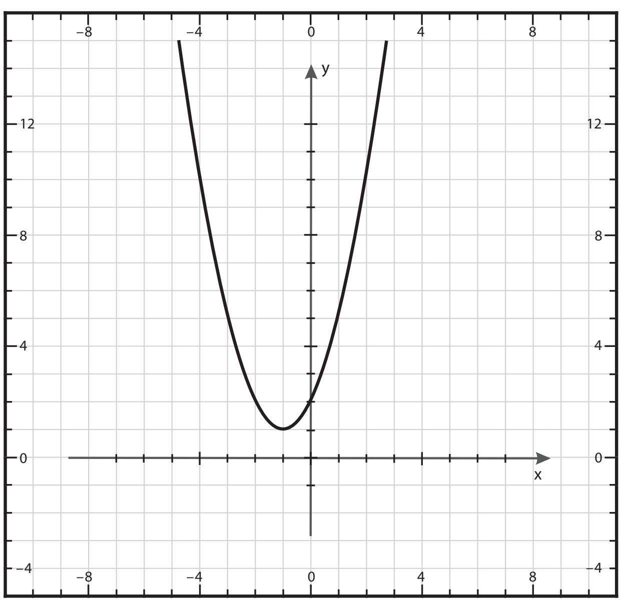
(A) 



(B)

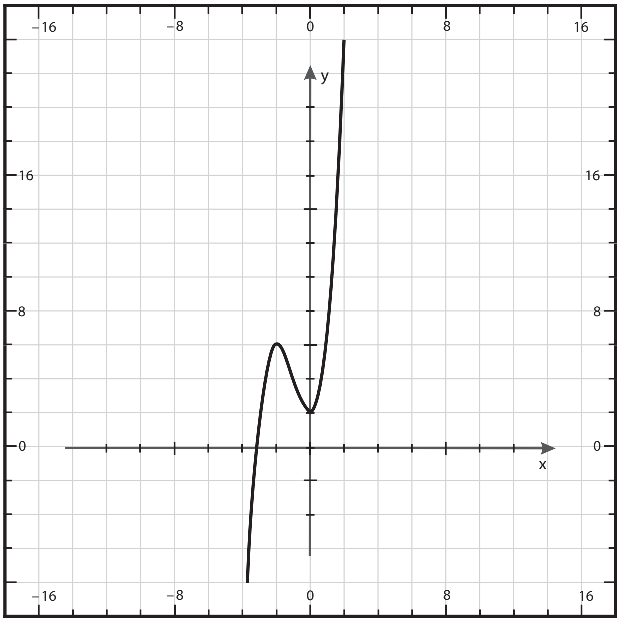


(C)

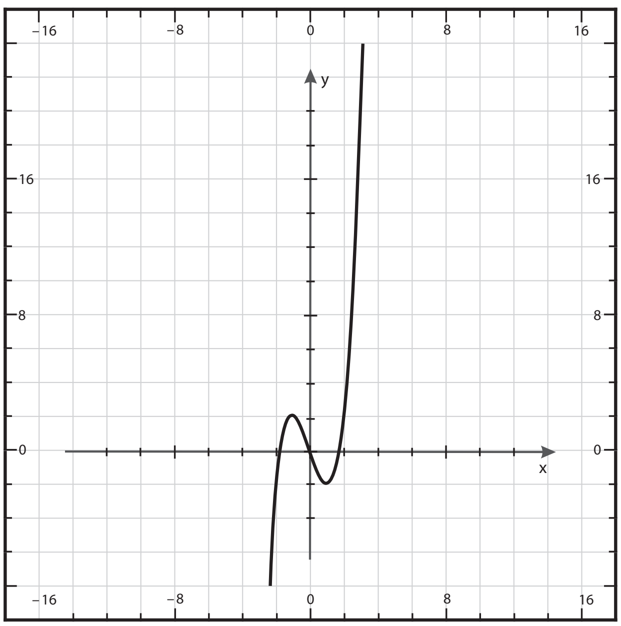


(D)

204.



$y = f(x)$



$y = g(x)$

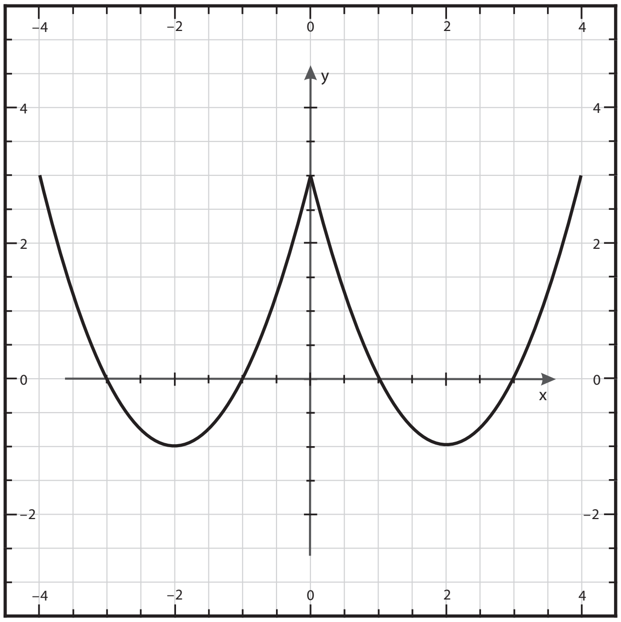
The graphs of $f(x)$ and $g(x)$ are shown above. Choose from the options below the correct relation between $g(x)$ and $f(x)$. [Without calculator]

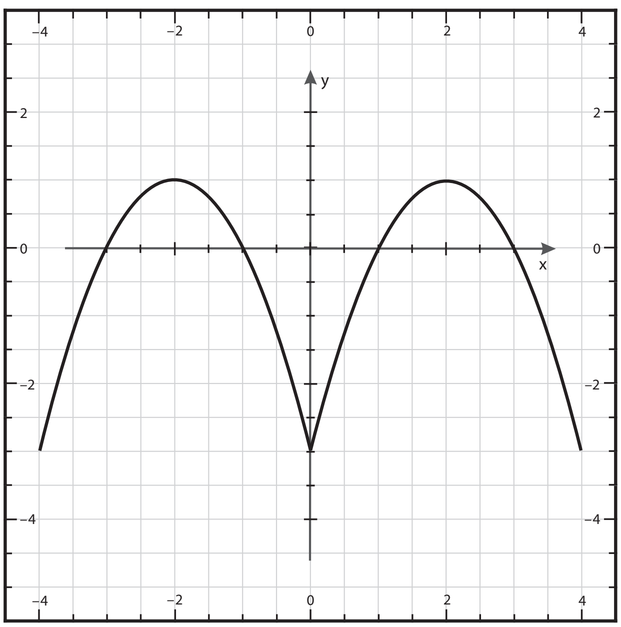
(A) $g(x) = f(x+1)$

(B) $g(x) = f(x+1) - 2$

(C) $g(x) = f(x-1) + 4$

(D) $g(x) = f(x-1) - 4$

205.  $y = f(x)$

 $y = g(x)$

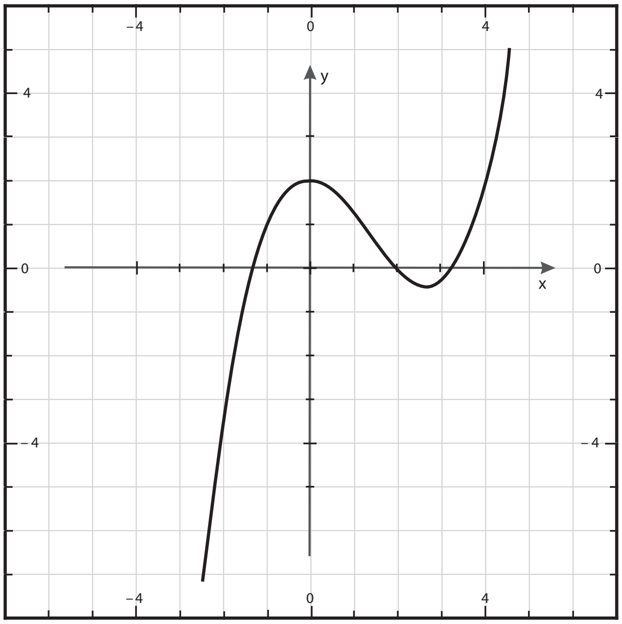
The graphs of $f(x)$ and $g(x)$ are shown above. Choose from the options below the correct relation between $g(x)$ and $f(x)$. [Without calculator]

(A) $g(x) = f(x-1)$

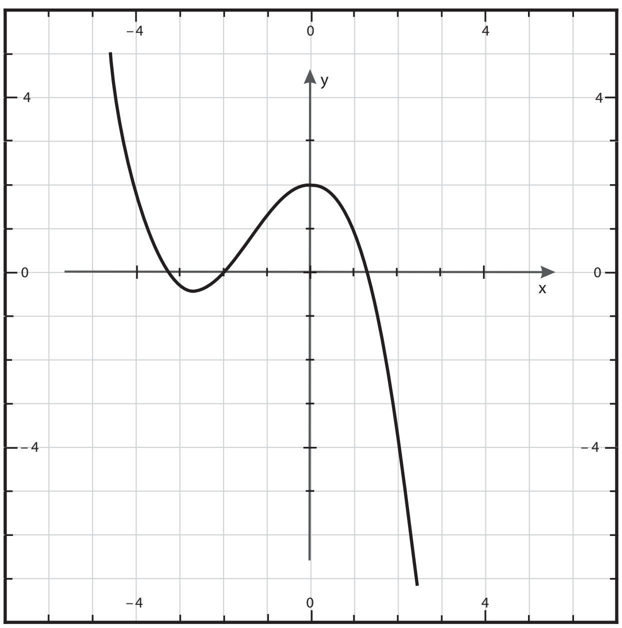
(B) $g(x) = f(-x)$

(C) $g(x) = -f(x)$

(D) $g(x) = f(x) - 12$

206. 

$y = f(x)$



$y = g(x)$

The graphs of $f(x)$ and $g(x)$ are shown above. Choose from the options below the correct relation between $g(x)$ and $f(x)$. [Without calculator]

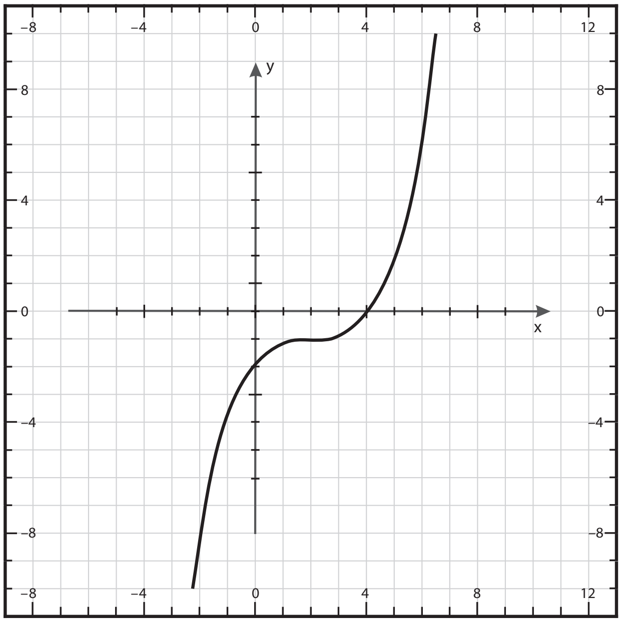
(A) $g(x) = f(x) + 4$

(B) $g(x) = f(-x)$

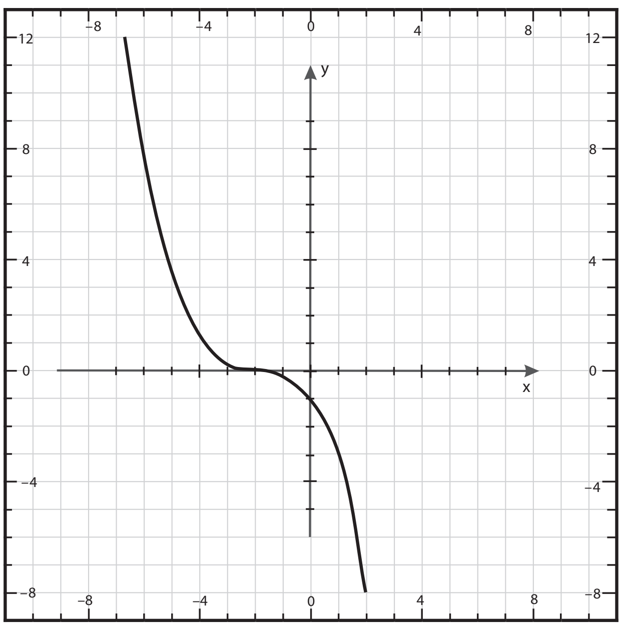
(C) $g(x) = -f(x)$

(D) $g(x) = f(x + 4)$

207.



$y = f(x)$



$y = g(x)$

The graphs of $f(x)$ and $g(x)$ are shown above. Choose from the options below the correct relation between $g(x)$ and $f(x)$. [Without calculator]

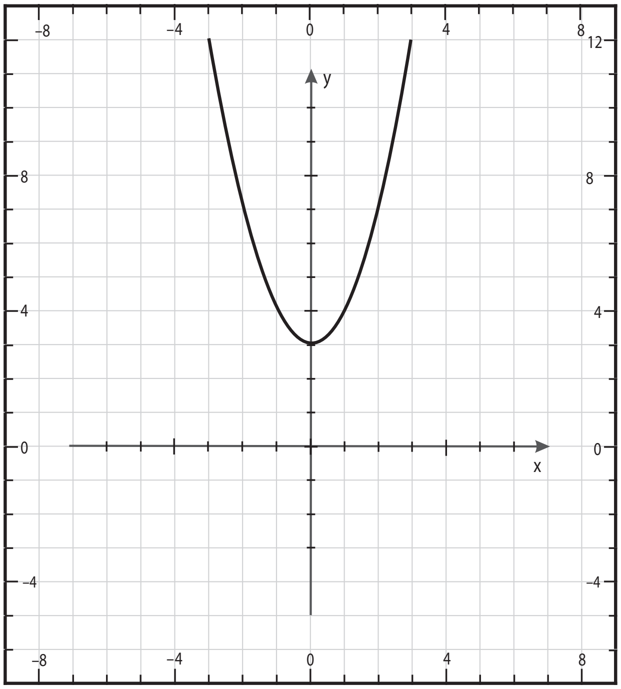
(A) $g(x) = -f(x) + 1$

(B) $g(x) = f(-x) + 1$

(C) $g(x) = -f(x) - 1$

(D) $g(x) = f(-x) - 1$

208.



$f(x) = x^2 + 3$

The graph of $f(x) = x^2 + 3$ is shown above. If $g(x) = \frac{x^2}{3} + 3$, then what changes, if made to the graph of $f(x)$ would make it look similar to the graph of $g(x)$? [Without calculator]

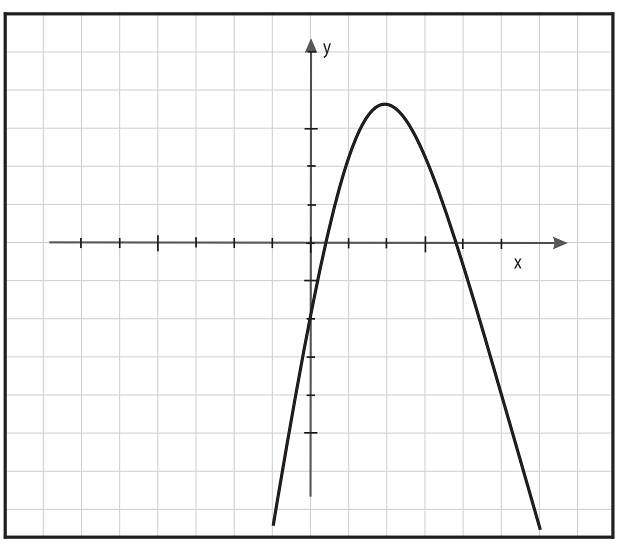
(A) Shift the graph of $f(x)$ up by 2 units

(B) Shift the graph of $f(x)$ down by 2 units

(C) Make the graph of $f(x)$ narrow but do not shift the graph

(D) Make the graph of $f(x)$ wide but do not shift the graph

209.



The graph of the quadratic function $f(x) = ax^2 + bx + c$ is shown above. Choose the correct statement from the options below. [Without calculator]

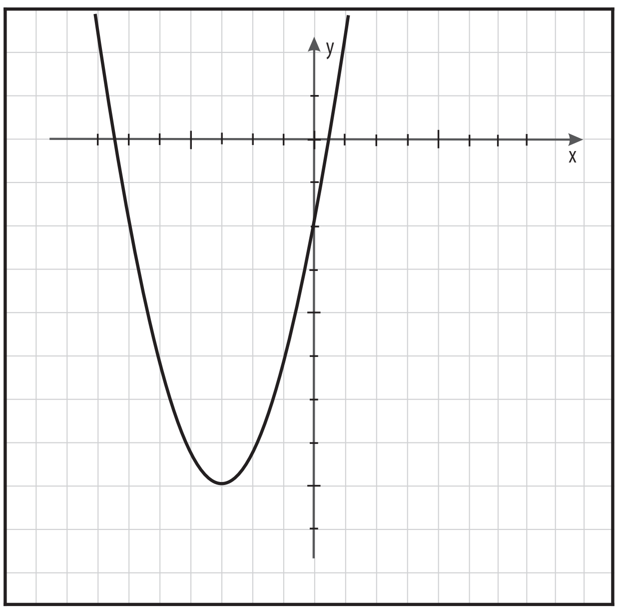
(A) $a > 0, b < 0, c > 0$

(B) $a > 0, b > 0, c > 0$

(C) $a > 0, b > 0, c < 0$

(D) $a < 0, b > 0, c < 0$

210.



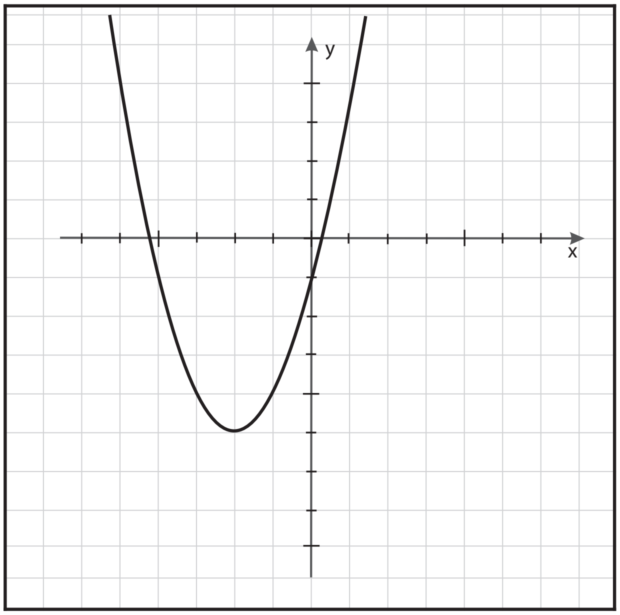
The graph of the quadratic function $f(x) = ax^2 + bx + c$ is shown above. Choose the correct statement from the options below. [Without calculator]

(A) $a > 0, b < 0, c > 0$

(B) $a > 0, b > 0, c > 0$

(C) $a > 0, b > 0, c < 0$

(D) $a < 0, b > 0, c < 0$

211. 

The graph of the quadratic function $f(x) = ax^2 + bx + c$ is shown above. Choose the correct statement from the options below. [Without calculator]

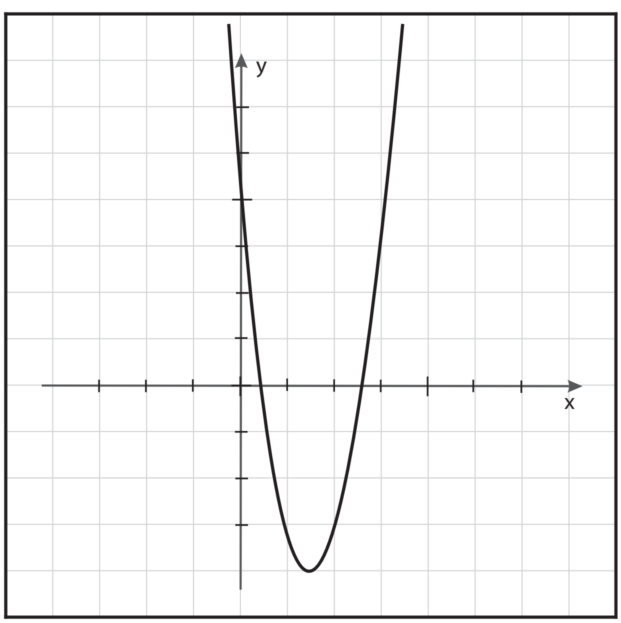
(A) The intercept on the y axis is equal to a

(B) The intercept on the y axis is equal to $-|b|$

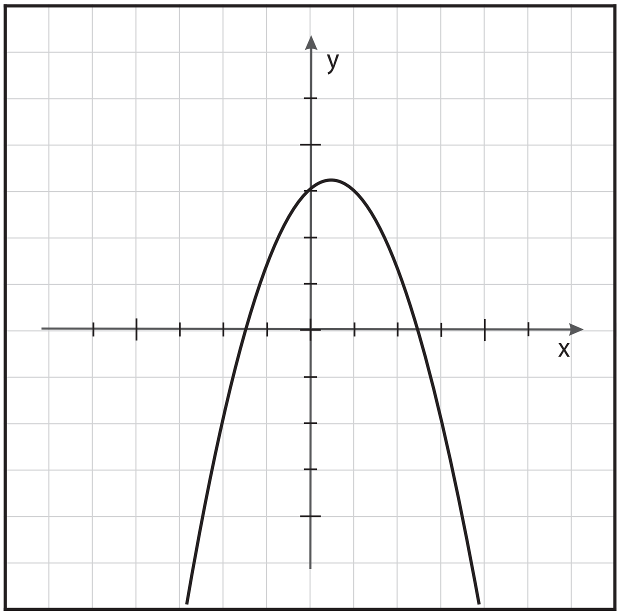
(C) The intercept on the y axis is equal to $-c$

(D) The intercept on the y axis is equal to $-|c|$

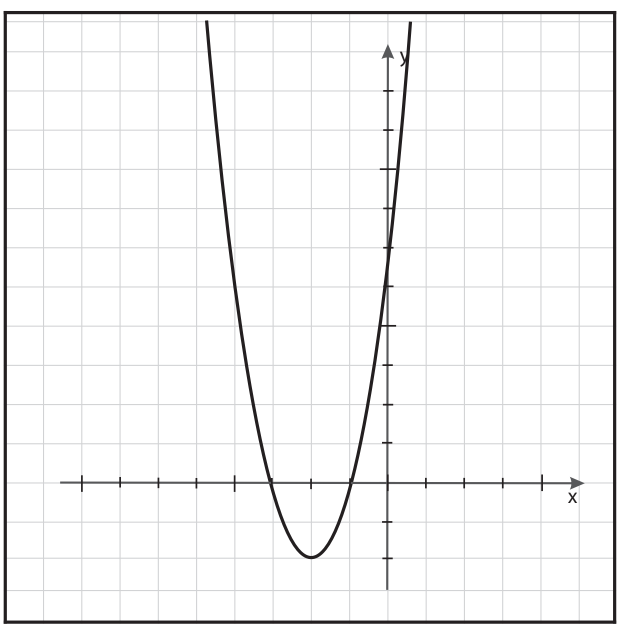
212. Which of the following could be the correct graph of $f(x) = x^2 - 12x + 20$? [Without calculator]

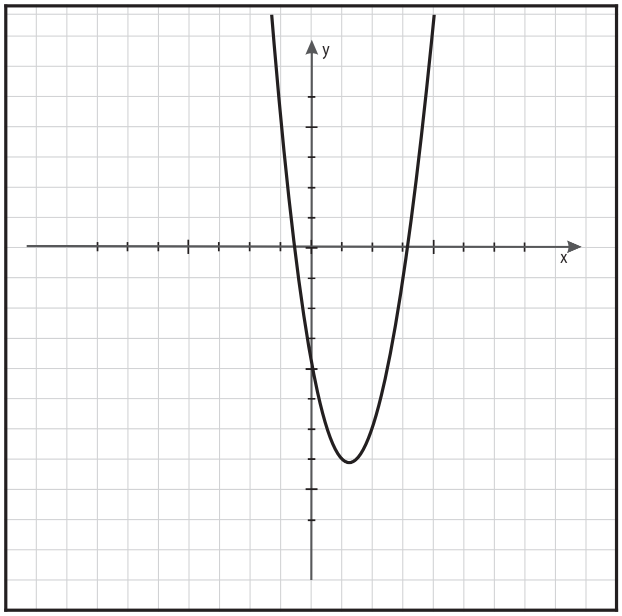
(A) 

(B)

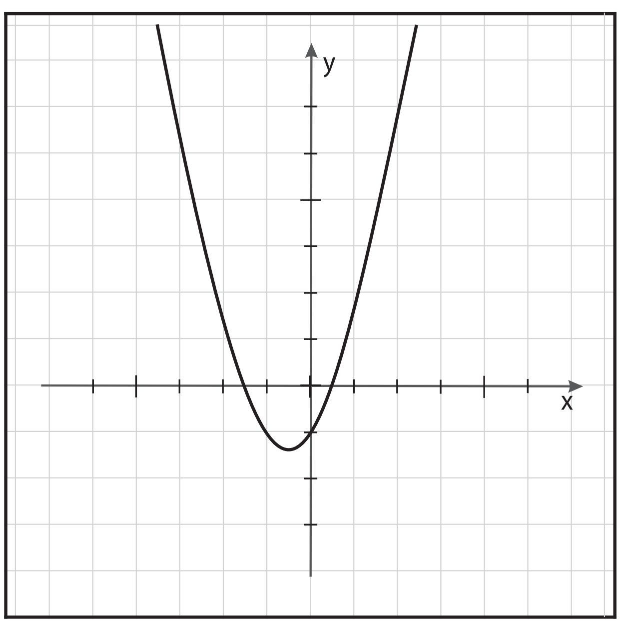


(C)

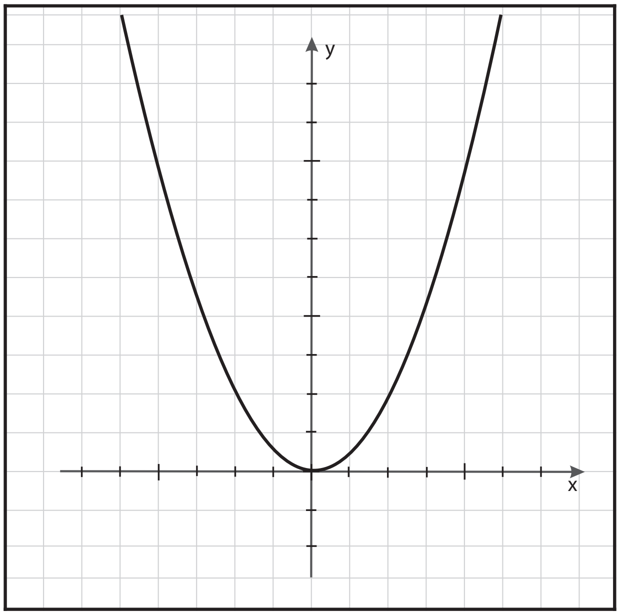


(D)


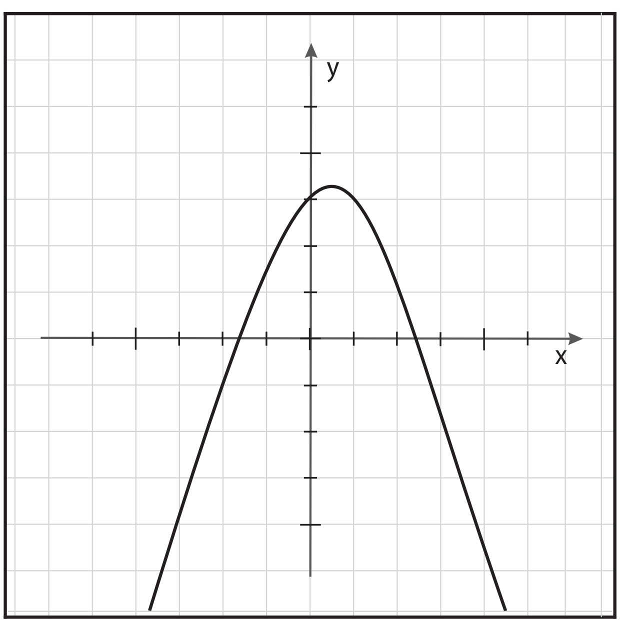
213. Which of the following could be the correct graph of $f(x) = x^2 + 6x + 9$? [Without calculator]

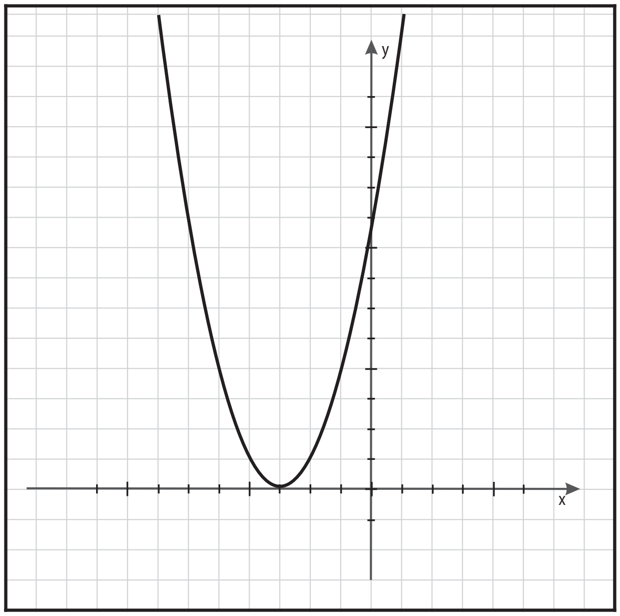
(A)


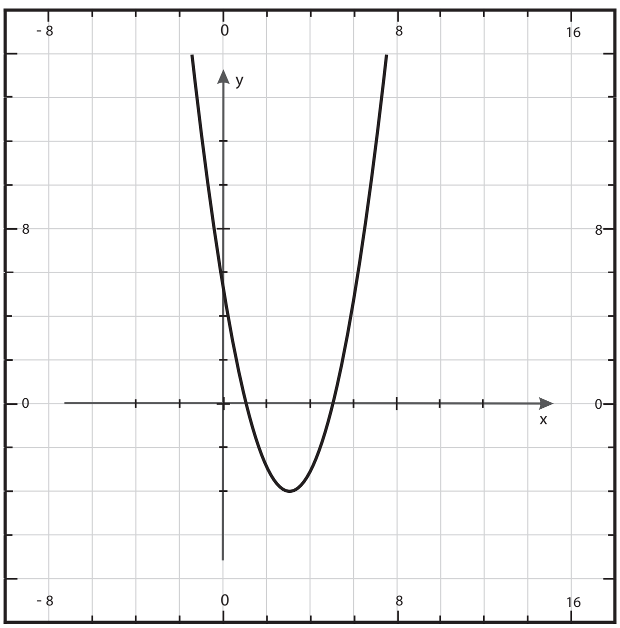
(B)



(C)



(D) 

214. 

$y = f(x)$

Which one is the correct expression that corresponds to the graph of $f(x)$ shown above? [Without calculator]

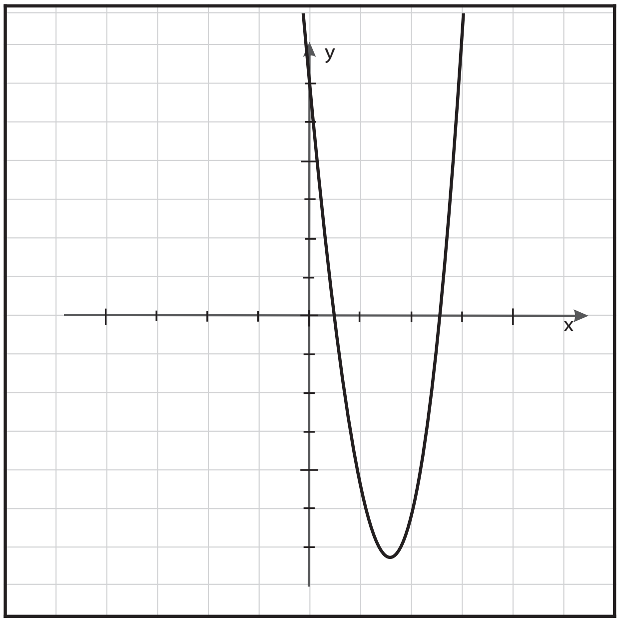
(A) $f(x) = x^2 + 8$

(B) $f(x) = x^2 + 4x + 6$

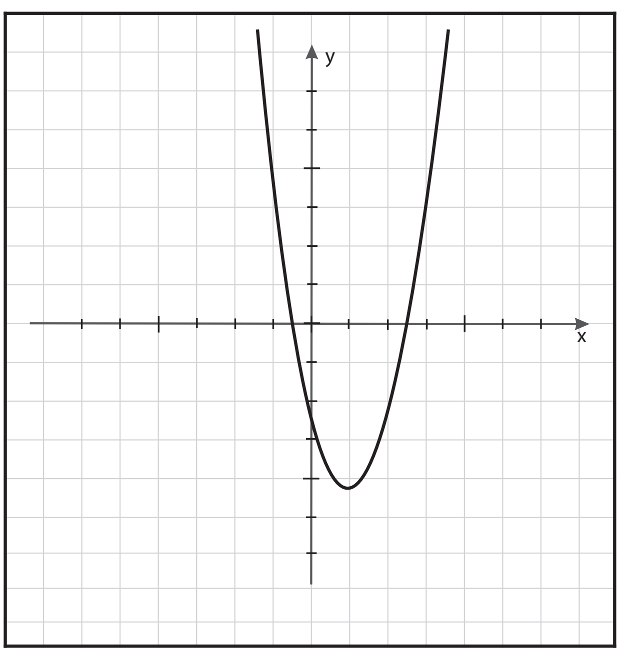
(C) $f(x) = (x - 3)^2 - 4$

(D) $f(x) = -x^2 + 4x - 8$

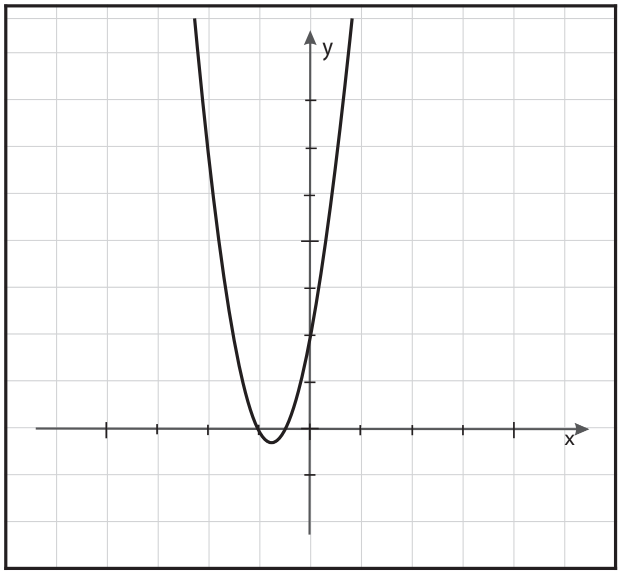
215. Which of the following could be the correct graph of $f(x) = x^2 + (a^2 - b)x + b^2 - a^2$, given that a and b are positive with $a > 1$ and $0 < b < 1$? [Without calculator]

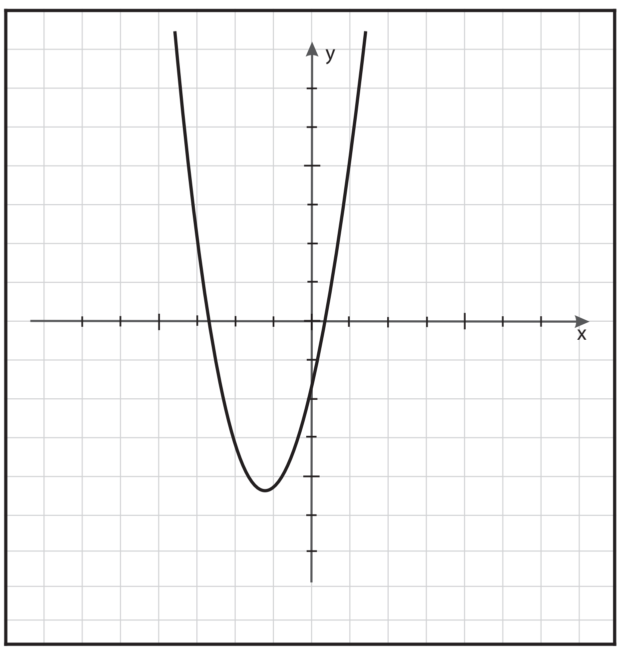


(A)

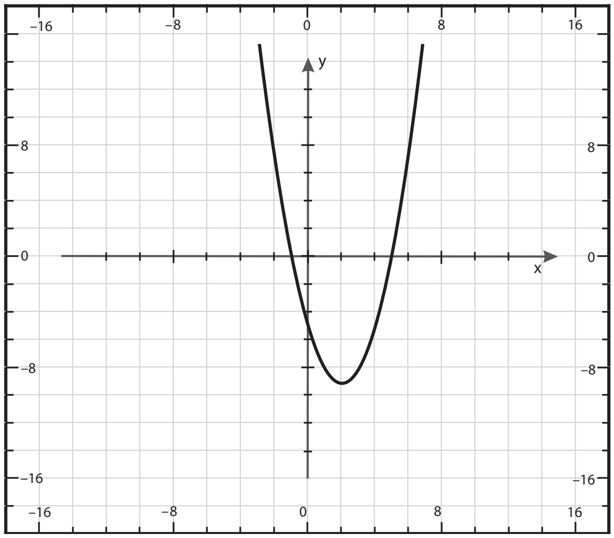


(B)

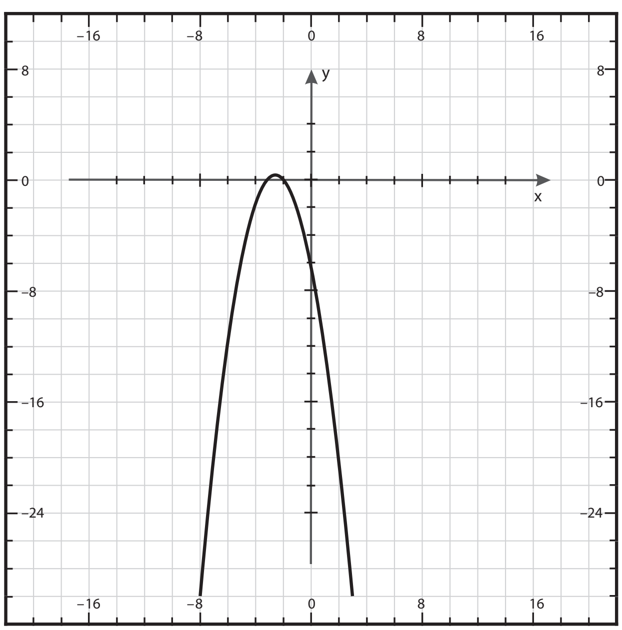
(C) 

(D) 

216. Which of the following is the correct graph of $f(x) = -x^2 + (a-b)x + b^2$, given that a and b are positive with $0 < a < 1$ and $3 < b < 5$? [Without calculator]

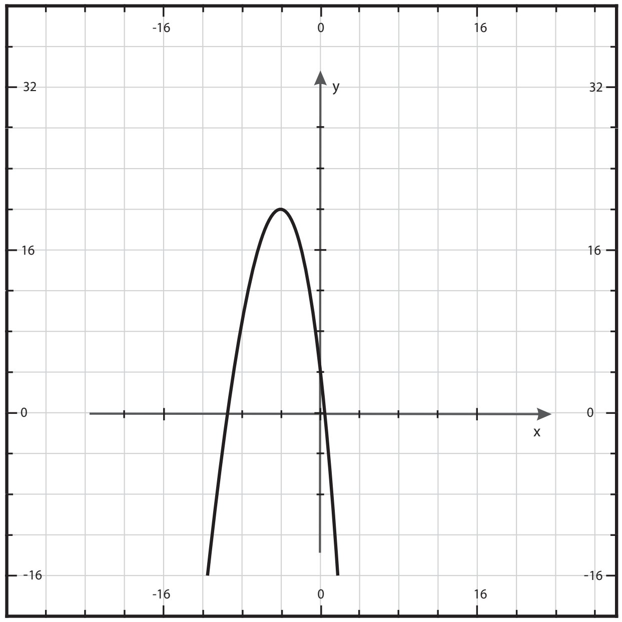


(A)

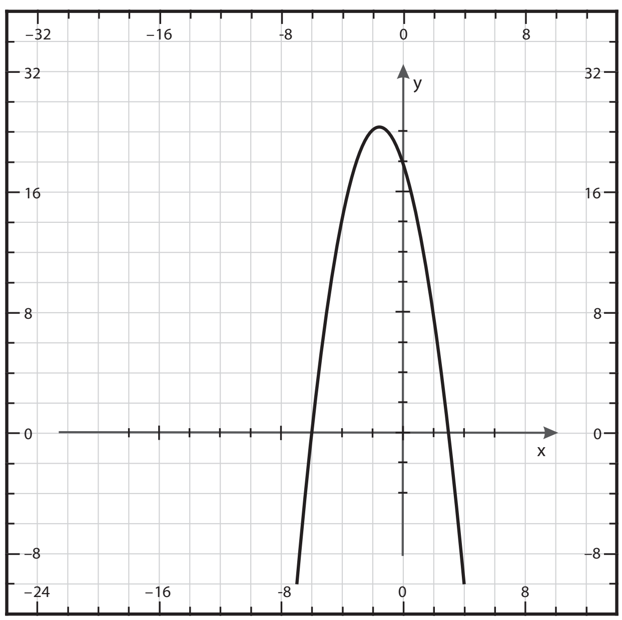


(B)

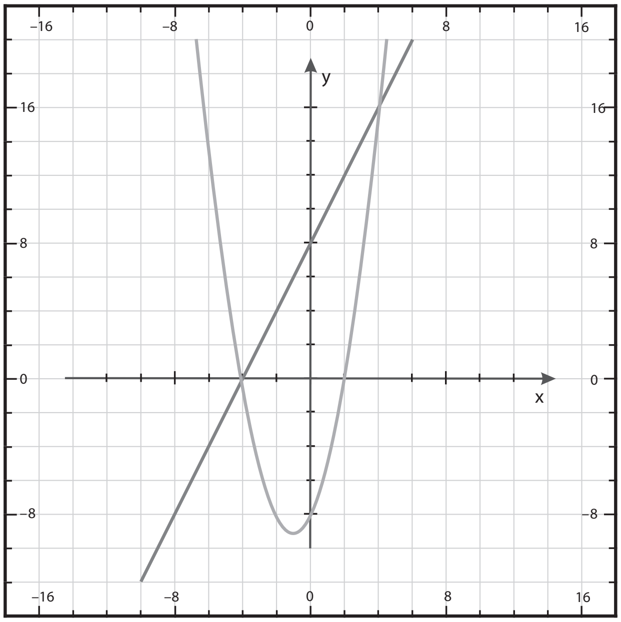
(C)



(D)



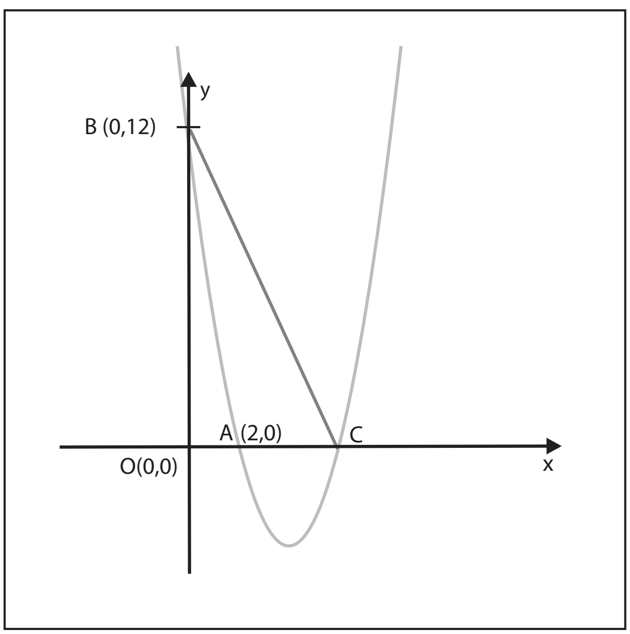
217.



Grid-In:

The graphs of $f(x) = x^2 + 2x - 8$ and $g(x) = 2x + 8$ are shown above. For how many integer values of x is $g(x) > f(x)$? [Without calculator]

218.



Grid-In:

The graph of $f(x) = x^2 + ax + b$ is shown above. What is the area of the $\triangle$BOC? [Without calculator]

3.3.3 Advanced equations

219. What is the value of x that satisfies the equation: $\sqrt{2x-8} = \sqrt{x+5} - 3$? [Without calculator]

(A) 4

(B) 12

(C) 36

(D) 76

220. What is the non-zero value of x that satisfies the equation: $\left(2^{x+2}\right)^{x+6} = 4^6$? [Without calculator]

(A) – 8

(B) – 2

(C) 2

(D) 8

221. **Grid-In:**
What is the value of x that satisfies the equation: $3^{2x-1} = 243$? [Without calculator]

222. What is the absolute difference between the values of x that satisfy the equation: $\sqrt{3x-8} - x + 2 = 0$? [Without calculator]

(A) – 1

(B) 1

(C) 3

(D) 4

223. How many values of x satisfy the equation: $\left(x^2 - 17\right)^2 = 64$? [Without calculator]

(A) 1

(B) 2

(C) 3

(D) 4

224. **Grid-In:**

What is the value of x that satisfies the equation: $\sqrt{x-8} = \sqrt{x+8} - 2$? [Without calculator]

225. What is the integer value of x that satisfies the equation: $\sqrt{(-2)^{x-5}} = 2^{(x-6)}$? [With calculator]

(A) 4

(B) 7

(C) 8

(D) 10

3.3.4 Application of functions

Following three questions are based on the following scenario:

Pete fires a pistol pointing it upwards. The height attained by the bullet above the ground t seconds after the pistol was fired is given by $h(t) = -t^2 + 12t + 30$.

226. What is the maximum height above the ground level reached by the bullet? [With calculator]

(A) 30

(B) 36

(C) 60

(D) 66

227. What is the earliest time when the height of the bullet above the ground is 57 feet? [With calculator]

(A) 2 seconds

(B) 3 seconds

(C) 6 seconds

(D) 9 seconds

Solution:

Since the height above the ground is 57 feet, we have:

Given that $h(t) = -t^2 + 12t + 30 = 57$

$$=> -t^2 + 12t - 27 = 0$$

$$=> t^2 - 12t + 27 = 0 => t^2 - 9t - 3t + 27 = 0$$

$$=> t(t-9) - 3(t-9) = 0$$

$=> (t-3)(t-9) = 0 => t = 3$ or 9.

Since we want the earliest time, we have $t = 3$ i.e. after three seconds.

Hence, the correct option is (B).

Alternatively, you can also get the answer by plugging in the option values in the function $h(t)$ and get the answer.

228. By what time does the bullet reach the same level above the ground as the level from which the bullet was fired? [With calculator]

(A) 6 seconds

(B) 9 seconds

(C) 12 seconds

(D) 18 seconds

Following two questions are based on the following scenario:

The total cost of manufacturing flower vases by the Potter House follows the function $c(x) = x^2 - 17x + 80$, where x denotes the number of vases manufactured. The vases are sold at a constant price of \$25 per vase.

229. For how many of the following vases manufactured by the company will there be neither profit nor loss? [With calculator]

(A) 3

(B) 40

(C) 72

(D) 123

230. For how many vases manufactured by the company is the profit made the maximum? [With calculator]

(A) 10

(B) 13

(C) 18

(D) 21

Following three questions are based on the following scenario:

In a laboratory experiment, it was found that the number of bacteria triples every hour. The number of bacteria at the end of six hours was found to be 72900.

231. If the number of bacteria initially during the start of the experiment be N, which of the following functions best describes the number of bacteria t hours after the start of the experiment? [With calculator]

(A) $N^3 * t$

(B) $3^N * t$

(C) N^{3t}

(D) $3^t * N$

232. How many hours after the experiment started was the number of bacteria one-third of the final number of bacteria, 72900? [With calculator]

(A) 2

(B) 3

(C) 4

(D) 5

233. What is the initial number of bacteria with which the experiment was started? [Without calculator]

(A) 100

(B) 500

(C) 1000

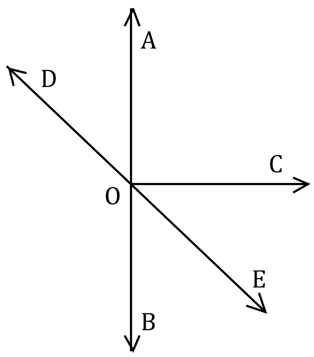
(D) 1500

234. Grid-In:
After how many hours from midnight is the number of vehicles on the street the maximum? [Without calculator]

235. Grid-In:
After how many hours from midnight is the number of vehicles on the street the same as that at midnight?

3.4 Advanced Topics in Math

3.4.1 Geometry



236.

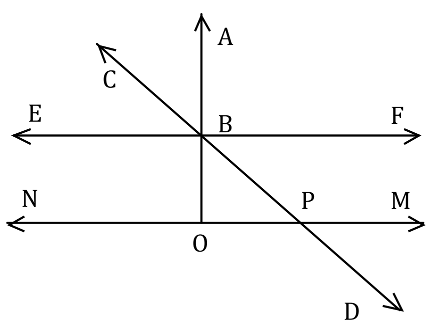
In the figure above, lines AB, DE intersect at O. Ray OC is perpendicular to AB. If angle AOD is twice of angle COE, what is the measure of angle EOA? [Without calculator]

(A) 30^0

(B) 60^0

(C) 120^0

(D) 150^0



237.

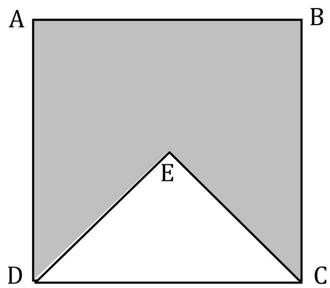
In the figure above, lines MN and CD intersect at P. Ray OA is perpendicular to MN. Lines EF, CD and ray OA intersect at B. If lines MN and EF are parallel and $\angle$MPD is 20^0 more than $\angle$ABC, what is the measure of $\angle$CBE? [Without calculator]

(A) 35^0

(B) 45^0

(C) 55^0

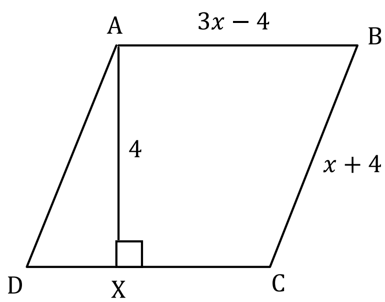
(D) 65^0



238.

Grid-In:

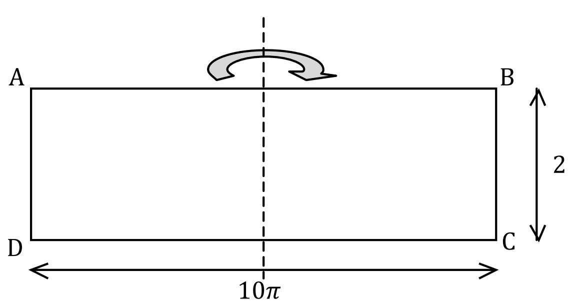
ABCD is a square of side 2 units. The height of the triangle DEC inside the square is 1 unit. What is the area (in square units) of the shaded region? [Without calculator]



239.

Grid-In:

In the above parallelogram ABCD of perimeter 24 units, the sides have been mentioned in terms of x.If AX, the perpendicular from A on CD is 4 units, what is the area (in square units) of ABCD? [Without calculator]



240.

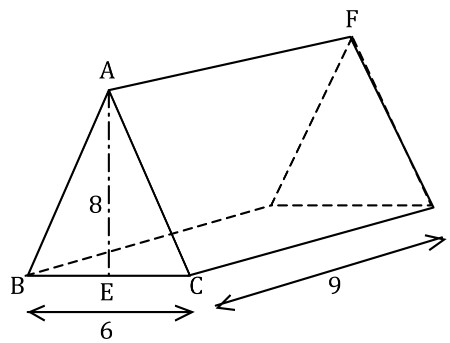
A rectangular sheet of paper ABCD is folded as shown in the figure above to form an open cylinder so that the edges BC and AD overlap. If AB = 10π units and BC = 2 units, what is the volume (in cubic units) enclosed by the above cylinder? [Without calculator]

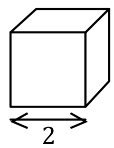
(A) 50

(B) 25π

(C) 50π

(D) 100π





241.

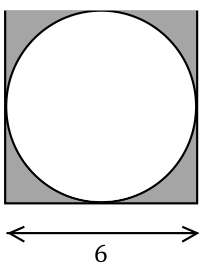
The triangular prism shown above is made of iron. The triangle ABC has base BC of 6 units and height AE as 8 units. The length of the prism, AF is 9 units. It is melted and recast into small identical cubes (shown above) having sides of 2 units each. How many such cubes can be made if there is no loss of metal during recasting? [With calculator]

(A) 8

(B) 27

(C) 36

(D) 54



242.

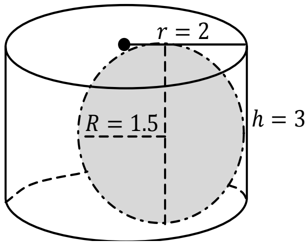
The square above has sides of 6 units. A circle in inscribed in the square touching the four sides. What is the area of the shaded region? [Without calculator]

(A) $36(\pi - 1)$

(B) $24 - 9\pi$

(C) $18\,(4 - \pi)$

(D) $9\,(4 - \pi)$



243.

What is the volume (in cubic units) of the largest sphere that can be placed completely inside a cylinder of radius 2 units and height 3 units? [With calculator]

(A) $\frac{4\pi}{3}$

(B) $\frac{9\pi}{2}$

(C) $\frac{32\pi}{3}$

(D) 36π

244. The total surface area of a cube is $96k$. What is the volume of the cube? [Without calculator]

(A) $64k$

(B) $64k\sqrt{k}$

(C) $64k^2$

(D) $384k\sqrt{6k}$

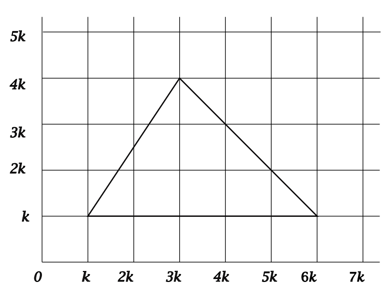
245. The radius of a cylinder is three times the radius of a sphere. If the volume of the sphere is double that of the cylinder, what is the ratio of the height to the radius of the cylinder? [Without calculator]

(A) 2 : 81

(B) 2 : 9

(C) 4 : 81

(D) 4 : 9



246.

If the area of the triangle above is 35, what is the value of k? [Without calculator]

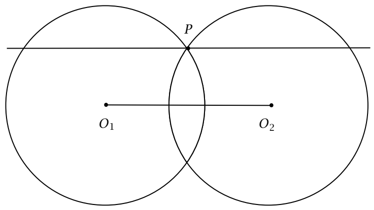
(A) $\sqrt{2}$

(B) $\frac{\sqrt{21}}{3}$

(C) $\sqrt{3}$

(D) $\frac{\sqrt{42}}{3}$

247.



In the figure above, two circles with radii equal to 10 units of length overlap such that their centers, O_1 and O_2, are 16 units apart. What is the distance between the line joining the centers of the circles and the line drawn through point P parallel to the line connecting the centers? [Without calculator]

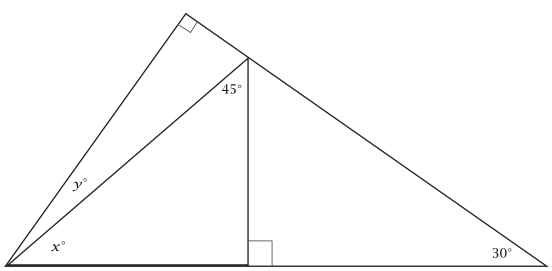
(A) 3

(B) 5

(C) 6

(D) 8

248.



Note: Figure not drawn to scale

In the figure above, what is the value of $(2x - y)$? x and y are denoted as the degree of corresponding angles. [Without calculator]

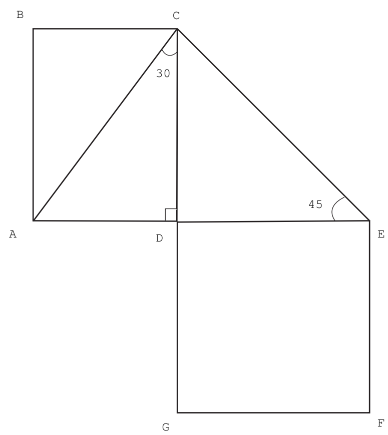
(A) 30

(B) 45

(C) 60

(D) 75

249.



If the area of rectangle $ABCD$ is $4\sqrt{3}$, then what is the area of the square $DEFG$? [Without calculator]

(A) $2\sqrt{3}$

(B) 4

(C) $4\sqrt{3}$

(D) 12

250. The total distance between points A and C is 150 kilometers. Point B lies between these two points. If the stretch between A and B is five times longer than that between B and C, what is the distance between points B and C?

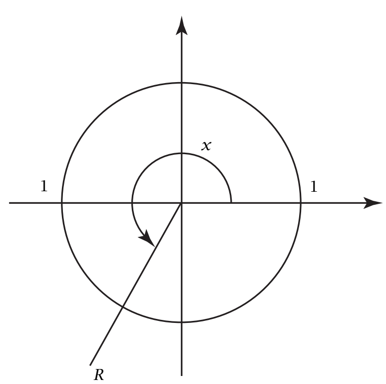
(A) 18

(B) 20

(C) 22

(D) 24

251.



Ray R starts its counter-clockwise revolution around the origin from the point $(1,0)$ in a coordinate plane. After it travels for one full revolution and an additional angle of $x°$, as shown in the diagram, it stops at the above-illustrated position in the third quadrant. Let D denote the total distance traveled by the original point of $(1, 0)$ on ray R.

Which statement(s) about D are true? [Without calculator]

I. $\frac{5\pi}{4} < D < \frac{3\pi}{2}$

II. $-\pi < -D < -\frac{\pi}{2}$

III. $3\pi < D < \frac{7\pi}{2}$

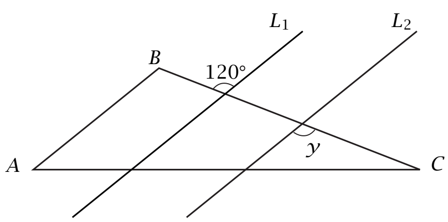
(A) I only

(B) II only

(C) III only

(D) I and III only

252.



In $\triangle ABC$ above, $AB \| L_1$ & $L_1 \| L_2$. What is the measure of angle y?

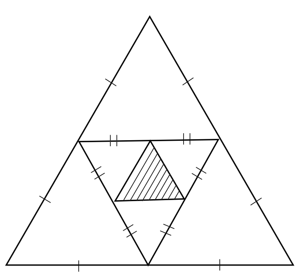
(A) 80

(B) 90

(C) 100

(D) 120

253.



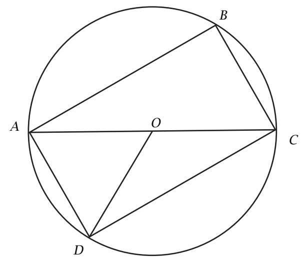
In the figure above each triangle is equilateral, what fraction of the largest triangular region is shaded? [Without calculator]

(A) $\dfrac{1}{16}$

(B) $\dfrac{1}{8}$

(C) $\frac{3}{16}$

(D) $\frac{1}{4}$

254. 

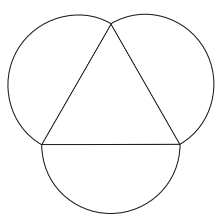
A rectangle is inscribed in the circle in the figure drawn above. The circle has a center at point O and a radius of length 6. The area of $\triangle ABC$ is $18\sqrt{3}$. What is the area of $\triangle ODC$ if points B, D, and O lie on the same line?

(A) 12

(B) $9\sqrt{3}$

(C) $6\sqrt{3}$

(D) 10

255. 

Above is the base of a solid which is 5 centimeters in height. The base consists of an equilateral triangle and three semi-circles on each side of the triangle. Each side of the triangle is 40 centimeters. What is the volume in cubic centimeters of the solid? [Without calculator]

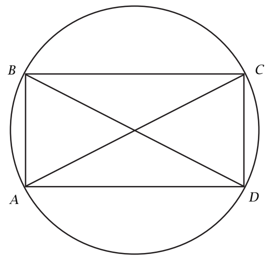
(A) $2,400\pi + 400\sqrt{3}$

(B) $3,000\pi + 2,000\sqrt{3}$

(C) $12,000\pi + 2,000\sqrt{3}$

(D) $14,400\pi$

256.



Rectangle $ABCD$ with a perimeter of 60 is inscribed in a circle with a radius of $7.5\sqrt{2}$. What is the area of $ABCD$? (Note: Figure not drawn to scale.)

(A) 150

(B) 225

(C) 450

(D) 750

257. The number of diagonals in a polygon having n sides is given be the formula $\frac{n(n-3)}{2}$. What is the value of n if it is known that the number of diagonals is double the number of sides of the polygon? [Without calculator]

(A) 4

(B) 5

(C) 7

(D) 8

258. For a rectangle, the difference between the length and breadth is 4 inches. If the length of the rectangle was increased by 6 inches keeping the breadth constant, the area increased by 120 square inches. Find the breadth of the rectangle. [Without calculator]

(A) 10

(B) 20

(C) 24

(D) 30

259. When the side of a square was doubled, the area of the square increased by 300 square inches. What was the initial length (in inches) of a side of the square? [With calculator]

(A) 10

(B) $10\sqrt{2}$

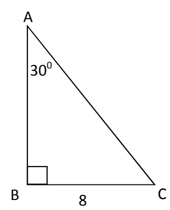
(C) $10\sqrt{3}$

(D) 20

3.4.2 Trigonometry

260. In the figure shown below, ABC is a triangle, right angled at B. What is the length of AC?

[Without calculator]



(A) 4

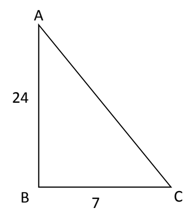
(B) $\frac{16}{\sqrt{3}}$

(C) 16

(D) $16\sqrt{3}$

261. In the figure shown below, ABC is a triangle right angled at B. What is the length of AC?

[Without calculator]



(A) 17

(B) $\sqrt{527}$

(C) 25

(D) 31

262. In a right angled isosceles triangle, the length of the hypotenuse is 8. What is the sum of the lengths of the other two sides? [Without calculator]

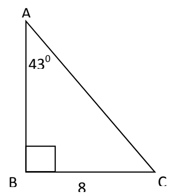
(A) 4

(B) $4\sqrt{2}$

(C) 8

(D) $8\sqrt{2}$

263. In the figure shown below, ABC is a triangle right angled at B. What is the difference between AC and AB? [Without calculator]



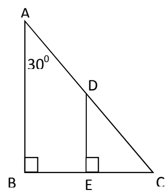
(A) $\dfrac{8}{\text{Sin } 43^0} - \dfrac{8}{\tan 43^0}$

(B) $\dfrac{8}{\text{Sin } 43^0} - \dfrac{8}{\text{Cos } 43^0}$

(C) $\dfrac{8}{\text{Cos } 43^0} - \dfrac{8}{\tan 43^0}$

(D) $\dfrac{8}{\tan 43^0} - \dfrac{8}{\text{Sin } 43^0}$

264. In the figure shown below, ABC is a triangle right angled at B. Through D, a perpendicular is dropped on BC so that BE = EC. If it is known that BC = 8, find the length of DC. [Without calculator]



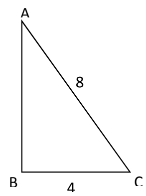
(A) $4\sqrt{2}$

(B) 8

(C) $8\sqrt{2}$

(D) $8\sqrt{3}$

265. In the figure shown below, ABC is a triangle right angled at B. What is the value of angle A? [Without calculator]



(A) 30^0

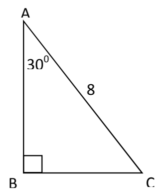
(B) 45^0

(C) 60^0

(D) 75^0

266. Matt has a garden in the shape of a right angled triangle with one of the acute angles as 30^0. If the longest side of the triangle is 8 m long, what is the perimeter of the garden?

[Without calculator]



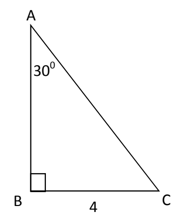
(A) $4\sqrt{3}$

(B) $12 + \dfrac{4}{\sqrt{3}}$

(C) $8 + 4\sqrt{3}$

(D) $12 + 4\sqrt{3}$

267. Andrew drew a right angled triangle with one angle as 30^0. If the length of the smallest side is 4, what is the area of the triangle? [Without calculator]



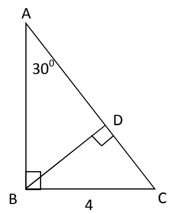
(A) $\frac{8}{\sqrt{3}}$

(B) $8\sqrt{3}$

(C) $12\sqrt{3}$

(D) $16\sqrt{3}$

268. In the figure shown beside, ABC is a triangle, right angled at B. Through B, a line is drawn perpendicular to AC which meets AC in D. What is the length of BD? [Without calculator]



(A) $\dfrac{2}{\sqrt{3}}$

(B) $\dfrac{4}{\sqrt{3}}$

(C) $2\sqrt{3}$

(D) $4\sqrt{3}$

269. A company decided to launch a promotional campaign using a hot air balloon. The balloon was tied to a point on the ground using a rope 30 meter long. One day, because of a strong wind, the rope made an angle of 60^0 with the ground. How high (in meters) was the balloon above the ground level that day? [Without calculator]

(A) $\dfrac{15}{\sqrt{3}}$

(B) 15

(C) $\dfrac{30}{\sqrt{3}}$

(D) $15\sqrt{3}$

270. What is the value of $\left(\frac{3\pi}{4}^c + \frac{11\pi}{5}^c - \frac{7\pi}{10}^c\right)$, when converted to degrees? [Without calculator]

(A) 135^0

(B) 245^0

(C) 405^0

(D) 810^0

271. Which of the following is the correct value of $\left(\frac{7\pi}{4}^c + \frac{4\pi}{3}^c - \frac{5\pi}{12}^c - 180^0\right)$? [Without calculator]

(A) $\dfrac{5\pi}{4}^c$

(B) $\dfrac{5\pi}{3}^c$

(C) 240^0

(D) 450^0

272. Which of the following statement(s) is/are correct about the angle $\frac{11\pi}{3}$ radians? [With calculator]

I. It is equivalent to 660^0.

II. The angle falls in the third quadrant.

III. Cos$\frac{11\pi}{3}$ is positive.

(A) Only I

(B) Only II

(C) Both I and II

(D) Both I and III

273. Which of the following statement(s) is/are correct about the angle $\frac{13\pi}{4}$ radians? [Without calculator]

I. It is equivalent to 585^0.

II. The value of tan$\frac{13\pi}{4}$ is -1.

III. The value of Cos$\frac{13\pi}{4}$ is $\frac{1}{\sqrt{2}}$.

(A) Only I

(B) Only II

(C) Both I and II

(D) Both I and III

274. Which of the following statement(s) is/are correct about the angle θ (in radians) if it is known that Cos$\theta = \frac{\sqrt{3}}{2}$? [Without calculator]

I. The angle falls in the first quadrant or the second quadrant.

II. The value of tanθ could be negative.

III. The value of Sinθ must be positive.

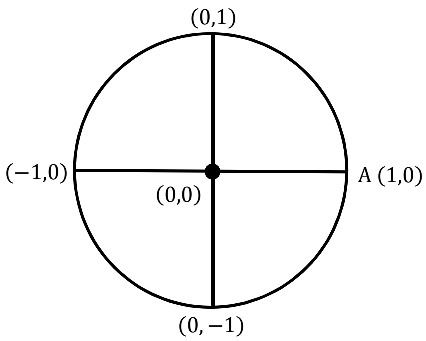
(A) Only I

(B) Only II

(C) Both I and III

(D) Both II and III

275. An ant started moving in the anti-clockwise direction around the unit circle from the point A as shown below. At a point of time, it stopped at a point after covering a total angle of $\frac{7\pi}{6}^{c}$. What could be the possible co-ordinates of the point on the circle where the ant stopped? [Without calculator]



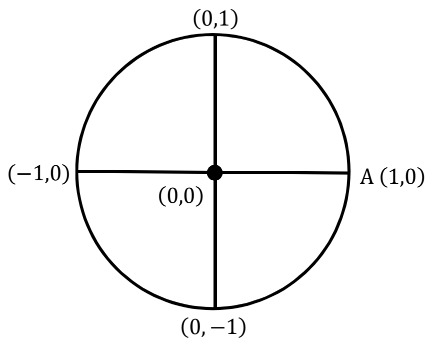
(A) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(B) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

(C) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(D) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

276. Andrew started tracing out the circumference of the unit circle from the point A as shown below in the clockwise direction. He stopped after tracing out an angle of $\frac{5\pi}{4}^{c}$. What could be the possible co-ordinates of the point on the circle where he stopped? [Without calculator]



(A) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(D) $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

277. If the angle θ satisfies the condition that $\frac{4\pi}{5} \leq \theta \leq \frac{5\pi}{6}$, then which of the following statement(s) is/are correct? [Without calculator]

I. The value of Cosθ is positive.

II. The value of Cos (2θ) is positive.

III. The value of Sin (2θ) is negative.

(A) Only II

(B) Both I and II

(C) Both II and III

(D) Both I and III

278. If the angle θ satisfies the condition that $\frac{3\pi}{4} < \theta < \frac{4\pi}{5}$, then which of the following statement(s) is/are correct? [Without calculator]

I. The value of tanθ is negative.

II. The value of tan (2θ) is negative.

III. The value of $\tan(3\theta)$ is positive.

(A) Only I

(B) Only III

(C) Both II and III

(D) I, II and III

279. If $\theta = 150^0$, then which of the following statement(s) is/are correct? [Without calculator]

I. $\text{Sin}(2\theta)$ and $\text{Cos}(4\theta)$ have the same sign.

II. $\tan(2\theta)$ and $\text{Cos}(5\theta)$ have opposite signs.

III. $\text{Sin}(4\theta)$ and $\tan(5\theta)$ have the same sign.

(A) Only I

(B) Only III

(C) Both I and II

(D) None of the statements

3.4.3 Complex Numbers

280. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $\sqrt{-25}$? [Without calculator]

(A) $5i$

(B) $5i^2$

(C) $25i$

(D) $25i^2$

281. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $\frac{4\sqrt{-9}}{3i}$? [With calculator]

(A) $4i$

(B) $12i^2$

(C) 4

(D) 12

282. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $(3 - i) \times (2 + 5i)$? [Without calculator]

(A) $1 + 17i$

(B) $1 + 13i$

(C) $11 + 13i$

(D) $11 + 17i$

283. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $(2 - 3i) + (1 + 8i)$? [With calculator]

(A) $3 + 11i$

(B) $3 + 5i$

(C) $1 + 5i$

(D) $9 - i$

284. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $(2 - 3i) \times (1 + 8i) + i \times (2 + 5i)$? [Without calculator]

(A) $29 + 15i$

(B) $21 - 22i$

(C) $31 + 15i$

(D) $21 + 15i$

285. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $(i^2 + i^3 + i^4 + i^5)$? [Without calculator]

(A) 0

(B) $1 + i$

(C) $1 - i$

(D) $-i$

286. The imaginary number i is defined such that $i^2 = -1$.

Calculate the value of $\sqrt{-3} \times \sqrt{-6} \times \sqrt{-8} \times i$ [Without calculator]

(A) $12i$

(B) $-12i$

(C) 12

(D) -12

287. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $(i \times i^2 \times i^3 \times i^4)$? [With calculator]

(A) -1

(B) 1

(C) i

(D) $-i$

288. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $\{(1 - i) + (1 + i)\} \times \{(1 + i) \times (1 - i)\}$? [With calculator]

(A) $2 + 2i$

(B) $4i$

(C) -4

(D) 4

289. The imaginary number i is defined such that $i^2 = -1$. Which of the following expressions is equivalent to $\left(\dfrac{1-i}{1+i}\right)$? [Without calculator]

(A) $-i$

(B) i

(C) $2i$

(D) 2

3.5 Higher Order Thinking Questions

290. Alex, Betty, and Cherry working together have to complete a job Working alone, Alex, Betty, and Cherry can do a job in 8, 24, and 48 hours, respectively; however they decided that each will work on the job successively for an hour, and anyone can take a start, and others will follow.

Part 1

Among Alex, Betty, and Cherry, who must start first, and others to follow in order so that they take minimum hours to complete the job? [With calculator]

Part 2

Among Alex, Betty, and Cherry, who must start first, and others to follow in order so that they take maximum hours to complete the job? [With calculator]

291. Which of the following description represents the expression $\frac{yh}{x}$? [With calculator]

(A) The time, in hours, that it takes x persons working at the same rate to complete a task that can be completed by y persons in h hours.

(B) The time, in hours, that it takes x persons working at the same rate to complete a task that can be completed by h persons in y hours.

(C) The time, in hours, that it takes $2h$ persons working at the same rate to complete a task that can be completed by x persons in $2y$ hours.

(D) The time, in hours, that it takes y persons working at the same rate to complete a task that can be completed by $2x$ persons in $2h$ hours.

292. Which of the following situations can be mathematically represented by the equation $\frac{1}{x} - \frac{1}{y} = \frac{1}{10}$? [With calculator]

(A) An empty tank can be filled in 10 hours with the help of two pipes: one, an inlet pipe, running alone can fill the tank in y hours and two, an outlet pipe, running alone can empty the tank in x hours.

(B) An empty tank can be filled in 10 hours with the help of two pipes: one, an inlet pipe, running alone can fill the tank in x hours and two, an outlet pipe, running alone can empty the tank in y hours.

(C) A top-up tank can be emptied in 10 hours with the help of two pipes: one, an outlet pipe, running alone can empty the tank in y hours and two, an inlet pipe, running alone can fill the tank in x hours.

(D) A top-up tank can be emptied in 5 hours with the help of two pipes: one, an outlet pipe, running alone can empty the tank in $2x$ hours and two, an inlet pipe, running alone can fill the tank in $2y$ hours.

293. $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$

Above expression represents that an empty tank can be filled in z hours with the help of two pipes: one, an inlet pipe, running alone can fill the tank in x hours and two, an outlet pipe, running alone can empty the tank in y hours.

Which of the following must be correct? [With calculator]

(A) $x < y < z$

(B) $x < z < y$

(C) $x < y$

(D) $x > y > z$

294. $\frac{10}{x} + \frac{5}{y} = \frac{1}{6}$

Roger needs to complete a job of assembling a carton using two types of machines; 5 machines work faster than other 10 machines. Using 15 machines, Roger completes the job in 6 hours.

Which of the following describes what the expression $\frac{5}{y}$ represents in the above equation? [With calculator]

(A) The time, in hours, that it takes the slower machines to complete the job alone.

(B) The time, in hours, that it takes the slower machines to complete $\frac{1}{6}$ of the job alone.

(C) The portion of the job that the slower machines would take to complete in one hour.

(D) The portion of the job that the faster machines would take to complete in one hour.

295. Roger is a globe-trotter. He frequently travels to the US, Latin America, Gulf and India for his computer peripheral business. He has to deal in few currencies: Brazilian Real, Indian Rupee, US Dollar, and Kuwaiti Dinar. His international bank converts the given currencies per Roger's request. The bank charges a currency conversion fee of 5% of the converted amount.

The table given below represents the ratios of currencies- Brazilian Real to Dollar, Rupees to Dollar, and Dollar to Kuwaiti Dinar for the period of 5 months.

Months & Currency ratios	Brazilian Real to Dollar	Rupees to Dollar	Dollar to Kuwaiti Dinar
July	2.60	56.50	3.50
August	2.50	48.90	3.40
September	2.40	61.50	3.60
October	2.70	65.21	3.40
November	2.50	70	3.50

Part I

On July 1, Roger wants to buy in India, few motherboards, each costing Rupees 5500/-. How much Dinar must he have in his bank account to buy a motherboard? [With calculator]

Part II

A Mac machine costs Real 2400 in September. Roger has with him in cash: $1100, Dinar 250, and Rupees 50,000. Using which of the following currency (ies) would be able to buy Mac? (Assume that he can use only one type of currencies for the transaction. [With calculator]

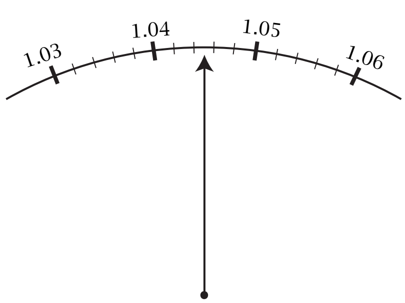
(A) Using only Dollar

(B) Using only Rupee

(C) Using only Dinar

(D) Using only Rupee and using only Dinar

296.



An ammeter is calibrated to show the strength of electrical current. What is the strength of the electrical current shown? [With calculator]

(A) 1.042

(B) 1.044

(C) 1.045

(D) 1.046

297. $CI = P[1 - (1 + 5\%)^4]$

Above formula for calculating compound interest, CI, has standard notations; P represents principal or the sum lent, 5% is the rate at which the sum P is lent, and 4 is time period for which CI is calculated.

Which of the following would NOT mathematically represent above equation? [With calculator]

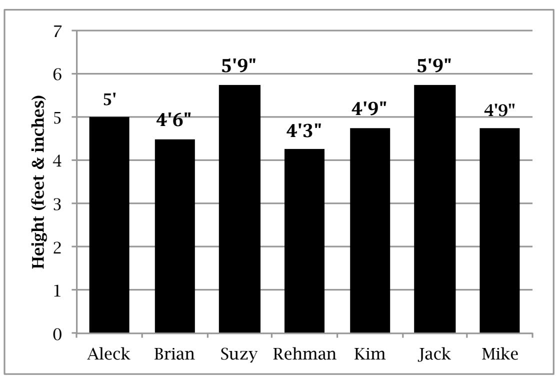
(A) Compound interest for the principal P, lent at a rate of 5% per annum for 4 years.

(B) Compound interest for the principal P, lent at a rate of 10% per annum for 2 years, compounded half yearly.

(C) Compound interest for the principal P, lent at a rate of 20% per annum for 1 years, compounded quarterly.

(D) Compound interest for the principal P, lent at a rate of 1.25% per annum for 16 years, compounded quarterly.

298. The Pie chart given below represents the percentage distribution of undergraduate degrees earned by Post-Graduate B-School students in a select few faculties: Art, Science, Engineering, Commerce, Management, Humanities, and others.



Based on the given information, solve the following two parts of the question.

Part 1:

If 126 students earned their undergraduate degrees from the Art faculty, then what is the number of students who earned their undergraduate degrees from the Engineering faculty exceeds that from the Management faculty? [With calculator]

Part 2:

How many undergraduate students from the Engineering faculty are more than that from the Science faculty? [With calculator]

299. ${}^5_3C.3!$

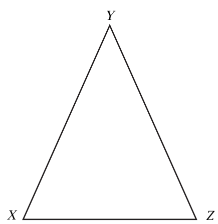
Which of the following description represents the above expression? [With calculator]

(A) Number of ways a team of three can be formed out of five players.

(B) Number of ways two teams of three can be formed out of five players.

(C) Number of ways unique photographs of three persons can be taken out of five persons.

(D) Number of ways three persons can be selected out of five players to be seated in a round table arrangement.

300. 

$\triangle XYZ$ above is isosceles triangle and has a perimeter 60. Which of the following information, if provided, can help calculate its unique area? (Note: Figure is not drawn to scale.) [Without calculator]

(A) $XZ = 24$

(B) $Y^\circ = 30^\circ$

(C) $XY : YZ : XZ :: 3 : 3 : 4$

(D) Sum of two sides

Chapter 4

Answer-key

Answer Key

(1) C

(2) A

(3) C

(4) A

(5) D

(6) 7/15 or 0.46 or 0.47

(7) 5

(8) 9/20

(9) 1/2

(10) A

(11) D

(12) 13/3 or 4.33.

(13) 1

(14) B

(15) C

(16) A

(17) 4

(18) 5

(19) C

(20) D

(21) D

(22) D

(23) 22/5 or 4.4

(24) C

(25) C

(26) B

(27) 20

(28) C

(29) A

(30) A

(31) D

(32) A

(33) C

(34) B

(35) 10

(36) C

(37) C

(38) D

(39) C

(40) B

(41) A

(42) $x = 5$ or $-7/3$

(43) $x = 3$ or $-31/3$

(44) D

(45) C

(46) C

(47) D

(48) D

(49) 30

(50) A

(51) D

(52) B

(53) A

(54) A

(55) A

(56) C

(57) $x = 7/6$

(58) 8

(59) C

(60) C

(61) B

(62) 33.3

(63) D

(64) D

(65) C

(66) D

(67) B

(68) B

(69) C

(70) A

(71) C

(72) D

(73) B

(74) A

(75) C

(76) B

(77) B

(78) D

(79) 15

(80) 250

(81) \$2.5 or 2.50

(82) 30

(83) 200

(84) B

(85) 220

(86) 14.3

(87) B

(88) C

(89) B

(90) C

(91) 2/11 or 0.18

(92) 14.9

(93) 60

(94) B

(95) D

(96) B

(97) D

(98) C

(99) D

(100) B

(101) A

(102) 75

(103) C

(104) 3/34 or 0.09

(105) 3/19 or 0.16

(106) 20

(107) D

(108) 12.5

(109) C

(110) B

(111) D

(112) A

(113) B

(114) C

(115) 250

(116) 34

(117) 28.6

(118) 13.5

(119) 20

(120) 37.5

(121) C

(122) C

(123) 222

(124) 25

(125) A

(126) B

(127) B

(128) C

(129) C

(130) D

(131) A

(132) B

(133) B

(134) D

(135) C

(136) D

(137) D

(138) C

(139) B

(140) B

(141) B

(142) 111

(143) 20

(144) 114

(145) D

(146) D

(147) C

(148) B

(149) B

(150) B

(151) B

(152) D

(153) 1/12

(154) 1/8

(155) 1/54

(156) 3/8

(157) C

(158) C

(159) A

(160) D

(161) C

(162) D

(163) D

(164) C

(165) 151

(166) 18

(167) A

(168) D

(169) D

(170) B

(171) 4.97

(172) C

(173) D

(174) D

(175) D

(176) C

(177) A

(178) C

(179) D

(180) D

(181) A

(182) D

(183) C

(184) C

(185) B

(186) B

(187) A

(188) 7

(189) B

(190) 375

(191) D

(192) 750

(193) B

(194) B

(195) D

(196) B

(197) C

(198) C

(199) D

(200) 2

(201) D

(202) C

(203) D

(204) D

(205) C

(206) B

(207) B

(208) D

(209) D

(210) C

(211) D

(212) A

(213) D

(214) C

(215) D

(216) D

(217) 7

(218) 36

(219) A

(220) A

(221) 3

(222) B

(223) D

(224) 17

(225) B

(226) D

(227) E

(228) C

(229) B

(230) D

(231) D

(232) D

(233) C

(234) 10

(235) 20

(236) C

(237) C

(238) 3

(239) 20

(240) C

(241) B

(242) D

(243) B

(244) B

(245) A

(246) D

(247) C

(248) D

(249) D

(250) D

(251) C

(252) D

(253) A

(254) B

(255) B

(256) B

(257) C

(258) B

(259) A

(260) C

(261) C

(262) D

(263) A

(264) B

(265) B

(266) D

(267) B

(268) C

(269) D

(270) C

(271) B

(272) D

(273) A

(274) B

(275) D

(276) B

(277) C

(278) D

(279) C

(280) A

(281) C

(282) C

(283) B

(284) D

(285) A

(286) C

(287) A

(288) D

(289) A

(290) Part I-15.5; Part II-17

(291) D

(292) B

(293) C

(294) D

(295) Part I-29.2; Part II-A

(296) C

(297) D

(298) Part I-126; Part II-50%

(299) C

(300) C

Chapter 5

Solutions

5.1 Heart of Algebra

5.1.1 Algebraic Expressions

1. We note that the denominators of all the fractions are multiples of 2 and 3 and have an LCM as 6.

We can write the value of m as:

$m = \left(\frac{5}{3} + \frac{5}{6} + \frac{1}{2}\right) = \frac{5\times2}{3\times2} + \frac{5}{6} + \frac{1\times3}{2\times3} = \frac{10}{6} + \frac{5}{6} + \frac{3}{6} = \frac{18}{6} = 3.$

Thus, $(m-3)^3 = (3-3)^3 = 0$.

Hence, the correct option is (C).

2. We know that: $150 < Q < 160$

Or, $150 < 17m < 160$

Dividing throughout by 17: $\frac{150}{17} < m < \frac{160}{17}$ => $8.82 < m < 9.41$

Since m is an integer, the only value qualifies for $m = 9$.

Hence, the correct option is (A).

3. We know that: $15 < m < 23$ => $16 \le m \le 22$ (since m is an integer).

Also, $Q = 17m$

So, the minimum value of $Q = 16 \times 17 = 272$ and the maximum value of $Q = 22 \times 17 = 374$.

So, $272 \le Q \le 374$.

Hence, options (A), (D) and (E) are eliminated. The correct option is between (B) and (C).

The correct option will be that number which is divisible by 17.

Option (C) is divisible by 17 while (B) is not.

[Note: We do not need to check both (B) and (C). We check any one option. If that option is divisible by 17, it must be the answer since there cannot be more than one option correct. If, however that option turns out be not divisible by 17, the other option must be correct.]

Hence, the correct option is (C).

4. We have:

$$N = \frac{S}{\frac{p}{q} - 1} \Rightarrow N = \frac{S}{\frac{p-q}{q}} \Rightarrow N = \frac{Sq}{p-q}$$

$$=> Np - Nq = Sq => q\,(N + S) = Np$$

$$=> q = \frac{Np}{N + S}.$$

Hence, the correct answer is (A).

5.

$$\frac{3}{\frac{3}{4} - \frac{5}{12}} = \frac{3}{\frac{3\times3}{4\times3} - \frac{5}{12}} = \frac{3}{\frac{9}{12} - \frac{5}{12}}$$

$$= \frac{3}{\frac{9-5}{12}} = \frac{3}{\frac{4}{12}} = 3 \times \frac{12}{4}$$

$$= 3 \times 3 = 9.$$

Hence, the correct answer is (D).

6. We know that: $p = \dfrac{1 + \frac{x}{y}}{1 + \frac{xy}{z^2}}$.

We see that x and y are expressed in terms of z. So, if we substitute the value of x and y in the ratio $\frac{x}{y}$, we will get a numerical value (since z will get eliminated). Similarly, for the ratio $\frac{xy}{z^2}$, if we substitute the values of x and y, then z^2 will be eliminated and we will get a numerical value.

Let us calculate the value of $\frac{x}{y}$ and plug in the above expression.

Now, $\frac{x}{y} = \frac{\frac{z}{2}}{3z} = \frac{z}{2} \times \frac{1}{3z} = \frac{1}{6}$. By replacing the given values of x and y.

Calculating the value of $\frac{xy}{z^2}$, a term of the denominator of the expression for p.

Also, $\dfrac{xy}{z^2} = \dfrac{\left[\left(\frac{z}{2}\right) \times 3z\right]}{z^2} = \dfrac{\left(\frac{3z^2}{2}\right)}{z^2} = \dfrac{3}{2}$.

Thus, we have: $p = \dfrac{\left(1 + \frac{1}{6}\right)}{\left(1 + \frac{3}{2}\right)} = \dfrac{7}{6} \times \dfrac{2}{5} = \dfrac{7}{15}$.

Hence, the correct answer is 7/15 or 0.46 or 0.47 (rounded off).

7. $a = 216 = 6^3$; representing in exponential form

$$=> a^{\frac{1}{3}} = (6^3)^{\frac{1}{3}} = 6^{3\times\left(\frac{1}{3}\right)} = 6.$$

Thus, $a^{\frac{1}{3}} - 1 = 6 - 1 = 5$.

Hence, the correct answer is 5.

8.

$$\left(\frac{1}{x-9}\right)^n + \left(\frac{1}{x}\right)^{1-n} = \left(\frac{1}{25-9}\right)^{\frac{1}{2}} + \left(\frac{1}{25}\right)^{1-\frac{1}{2}} = \left(\frac{1}{16}\right)^{\frac{1}{2}} + \left(\frac{1}{25}\right)^{\frac{1}{2}} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}.$$

Thus, the value of the given expression is $\frac{9}{20}$.

Hence, the correct answer is 9/20.

9. We have: $\frac{a^2+b^2}{(a+b)^2+(a-b)^2} = \frac{a^2+b^2}{(a^2+2ab+b^2)+(a^2-2ab+b^2)} = \frac{a^2+b^2}{2(a^2+b^2)} = \frac{1}{2}$. [By expanding $(a+b)^2$ & $(a-b)^2$]

Thus, the value of the expression is $\frac{1}{2}$ irrespective of the values of a, b.

Hence, the answer is $\frac{1}{2}$.

A traditional approach to solve this question would have been time consuming.

Let us see how.

By replacing the values of a & B in the expression, we get,

$$\frac{a^2+b^2}{(a+b)^2+(a-b)^2} = \frac{2.3^2+1.7^2}{(2.3+1.7)^2+(2.3-1.7)^2} = \frac{5.29+2.89}{4^2+0.6^2} = \frac{8.18}{16+0.36} = \frac{1}{2}.$$

Hence, the correct answer is 1/2.

10.

$$\frac{x^2-12x+32}{x-4} = \frac{x^2-4x-8x+32}{x-4} = \frac{x\,(x-4)-8\,(x-4)}{x-4} = \frac{(x-4)\,(x-8)}{x-4} = (x-8).$$

Hence, the value of the expression $= x - 8 = 6.5 - 8 = -1.5$.

[Note: One could also have simply plugged in the value of $x = 6.5$ in the expression and calculated the value. However, the calculation would have been tedious.]

Hence, the correct option is (A).

11.

$$\frac{x^2-12x+32}{x+4} = \frac{x^2-4x-8x+32}{x+4} = \frac{x\,(x-4)-8\,(x-4)}{x+4} = \frac{(x-4)\,(x-8)}{x+4}$$

$$= \frac{(-6.5-4)(-6.5-8)}{(-6.5+4)} = \frac{-10.5\times-14.5}{-2.5} = 60.9.$$

Hence, the correct option is (D).

12. We need to calculate the value of 1#1#5 performing the operations from left to right.

So, we have: $(1\#1)\#5 = \left(\frac{1^2+1^2}{2}\right)\#5 = \frac{2}{2}\#5 = 1\#5 = \frac{1^2+5^2}{1+5} = \frac{26}{6} = \frac{13}{3} = 4.33$.

Hence, the correct answer is 13/3 or 4.33.

13. The mixed fraction $Z\frac{Z}{3}$ can be manipulated as a proper fraction, and equal $= \frac{3Z+Z}{3} = \frac{4Z}{3}$.

Thus, we have: $\frac{4Z}{3} = \frac{8}{3} => Z = \frac{8}{3} * \frac{3}{4} = 2$.

Hence, the value of $\frac{1}{2Z-3} = \frac{1}{2*2-3} = \frac{1}{4-3} = 1$.

Hence, the correct answer is 1.

5.1.2 Numbers

14. We know the following results for summation:

Even + Even = Even

Odd + Odd = Even

Even + Odd = Odd

In general, the following are valid:

The sum of any number of even terms is even

The sum of even number of odd terms is even

The sum of odd number of odd terms is odd

For product, we have the following results:

Even x Even = Even

Odd x Odd = Odd

Even x Odd = Even

In general, an even number multiplied with any integer (even or odd) always results in an even number.

Statement I: In the case of 5 consecutive integers, there are 2 cases possible:

a) There are 3 odd numbers and 2 even numbers: Here, sum of the 3 odd numbers is odd and the sum of the even numbers is even. So, the total is the sum of one odd number and one even number which is odd (example: 3, 4, 5, 6, 7).

b) There are 2 odd numbers and 3 even numbers:

Here, sum of the 2 odd numbers is even and the sum of the even numbers is even. So, the total is the sum of two even numbers which is even (example: 2, 3, 4, 5, 6).

So, statement I is false (since we need the result to be definitely odd).

Statement II: In the case of 14 consecutive integers, there will always be 7 odd numbers and 7 even numbers. Here, sum of the 7 odd numbers is odd and the sum of the even numbers is even. So, the total is the sum of one odd number and one even number which is odd (example: 3, 4, 5, 6, 7 . . . 15, 16).

So, statement II is always true.

Statement III: When we consider 11 consecutive integers, there would be either 5 odd and 6 even numbers or 6 odd and 5 even numbers. In either case, there would be at least one even number. Hence, the product will always be even.

So, statement III is false.

Hence, the correct answer is (B).

15. We need to understand the following points:

a) When we select any 2 consecutive integers, we always have one number even and one number odd.

To validate this, take any 2 consecutive integers: (2, 3), (11, 12), (17, 18), etc.

b) When we select any 3 consecutive integers, we always have a number, a multiple of 3.

To validate this, take any 3 consecutive integers: (1, 2, 3), (11, 12, 13), (22, 23, 24), etc.

Thus, we can conclude that the product of any three consecutive integers must be a multiple of 3. Also, since three consecutive integers include the case of two consecutive integers as well, the product is also a multiple of 2. Hence, the product is a multiple of 6.

Thus, both statements I and III are correct.

Statement II may or may not be correct.

For example, the product of the numbers in the set (11, 12) is a multiple of 4, however, the product of the numbers in the set (17, 18) is not a multiple of 4.

[Note: In general, we can say: The product of N consecutive positive integers is always divisible by N!]

Hence, the correct answer is (C).

16. $11000a = aa000$ (for example, $11000 \times 2 = 22000$, etc.)

Similarly, $10b = b0$

Hence, if we add the numbers, we get:

$$\begin{array}{ccccc} a & a & 0 & 0 & 0 \\ & + & & b & 0 \\ & & & + & c \\ \hline a & a & 0 & b & c \\ \hline \end{array}$$

Thus, we get a 5 digit number: $a\ a\ 0\ b\ c$.

The hundred's digit is thus 0.

Hence, the correct option is (A).

17. Let us assume that the number N has the digits a and b in the ten's and unit's places respectively.

Thus, the number can be expressed as $10a + b$.

The reverse of this number is $10b + a$.

Hence, we have: $(10a + b) - (10b + a) = 54 \Rightarrow 9(a - b) = 54$

$$\Rightarrow a - b = 6$$

Hence, possible values of a and b can be enumerated as:

a	b	$N = 10a + b$
6	0 (minimum value)	60
7	1	71
8	2	82
9 (maximum value)	3	93

Thus, there are 4 possible such numbers.

Hence, the correct answer is 4.

18. We have: $\frac{1\#}{3\#} + \frac{4}{7} = 1 \Rightarrow \frac{1\#}{3\#} = 1 - \frac{4}{7} = \frac{7-4}{7} = \frac{3}{7}$.

Thus, we have: $\frac{1\#}{3\#} = \frac{3}{7}$.

Now, we need to find two numbers 1# and 3#, having the same unit digits and are in the ratio 3 : 7.

Thus, we need to find a multiple of 3 and a multiple of 7, each having the same unit digit and starts with '1' and '3', respectively. After an observation, we find that the common factor to the multiples must be '5' (since $3 * 5 = 15$ and $7 * 5 = 35$).

Hence, the value of # is 5.

Hence, the correct answer is 5.

19. The sum of the numbers written down by Bob is: $P = 1 + 2 + 3 \cdots + 100$.

The sum of the numbers written down by Alfred is: $Q = 2 + 4 + 6 \cdots + 200 = 2(1 + 2 + 3 \cdots + 100)$.

Thus, we can clearly see that $Q = 2P \Rightarrow P = \frac{Q}{2}$.

Hence, the correct option is (C).

20. Here, the relation between four integers, a, b, c, d are given. We do not know the actual values of any of the integers. However, from the relation, we can find out the ratio of the integers.

Since $2a = 5b = 3c = 6d$, let us assume that all of these are equal to a multiple of the LCM of 2, 5, 3 and 6. The LCM of 2, 5, 3 and 6 is 30 (Since this is a ratio questions, the choice of the initial number we assume will not affect the final answer. We need to choose a convenient value which would make our calculations easy).

So, let us assume that $2a = 5b = 3c = 6d = 30k$ where k is some constant whose value is unknown.

Thus, we have:

$$a = \frac{30k}{2} = 15k.$$

$$b = \frac{30k}{5} = 6k.$$

$$c = \frac{30k}{3} = 10k.$$

$$d = \frac{30k}{6} = 5k.$$

Now, we need to find which of the options does not represent an integer.

For this, we substitute the values of a, b, c, d in the options one-by-one.

Starting with option (A):

$$\frac{ad}{3} = \frac{15k * 5k}{3} = 25k^2.$$

Now, if the value of k be assumed as an integer, then option (A) would result in an integer value.

Option (B):

$$\frac{ac}{d} = \frac{15k * 10k}{5k} = 30k.$$

Now, if the value of k be assumed as an integer, then option (B) would result in an integer value.

Option (C):

$$\frac{ac}{bd} = \frac{15k * 10k}{6k * 5k} = 5$$

Thus, option (C) obviously results in an integer value.

Since one of the options must not result in an integer value, the answer must be option (D). We do not need to verify option (D) as well.

However, for your curiosity, let us check option (D):

$$\frac{ab}{cd} = \frac{15k * 6k}{10k * 5k} = \frac{9}{5}$$

Thus, option (D) obviously does not result in an integer value.

Hence, the correct option is (D).

21. Here, the relation between four integers, a, b, c, d are given. We do not know the actual values of any of the integers.

We need to find which of the options is incorrect. For this, we need to check the options one-by-one.

Starting with option (A):

$$ab = cd => \frac{a}{c} = \frac{d}{b}.$$

Multiplying d to both sides of the equation: $\frac{ad}{c} = \frac{d^2}{b}$

Hence, option (A) is true.

Option (B):

$$ab = cd => \frac{a}{c} = \frac{d}{b}.$$

Squaring both sides: $\frac{a^2}{c^2} = \frac{d^2}{b^2}$

Adding 1 to both sides: $1 + \frac{a^2}{c^2} = 1 + \frac{d^2}{b^2} => \frac{a^2+c^2}{c^2} = \frac{d^2+b^2}{b^2}$

Hence, option (B) is true.

Option (C):

$ab = cd \ldots$(i)

Adding 1 to both sides: $1 + ab = 1 + cd \ldots$(ii)

Dividing (ii) by (i):

$$\frac{1+ab}{ab} = \frac{1+cd}{cd} => \frac{b\left(\frac{1}{b}+a\right)}{ab} = \frac{c\left(\frac{1}{c}+d\right)}{cd} => \frac{\frac{1}{b}+a}{a} = \frac{\frac{1}{c}+d}{d}$$

Hence, option (C) is true.

Hence, we can conclude that option (D) must be incorrect since there must be one option which is not true. We do not need to verify option (D).

However, for your curiosity, let us verify option (D):

$$ab = cd => \frac{ab}{c} = d$$

Multiplying b to both sides:

$$\frac{ab^2}{c} = bd$$

But , option (D) says: $\frac{ab^2}{c} = d$

Hence, option (D) is not true.

Alternate approach:

Let us assume some values of a, b, c, d which satisfy the given relation $ab = cd$.

A set of possible values could be $a = 2, b = 12, c = 3, d = 8$ (we should try not to use the easy values like $a = b = c = d = 1$ as these tend to satisfy all the options given in such questions).Now, we substitute these values in each option and one-by-one check which of the options is not satisfied.

Option (A):

$\frac{ad}{c} = \frac{2*8}{3} = \frac{16}{3}$ and $\frac{d^2}{b} = \frac{8^2}{12} = \frac{64}{12} = \frac{16}{3}$

Thus, both the Left hand side (LHS) and the Right hand side (RHS) give the same value. Hence, option (A) is true.

Option (B):

$\frac{a^2+c^2}{c^2} = \frac{2^2+3^2}{3^2} = \frac{13}{9}$ and $\frac{d^2+b^2}{b^2} = \frac{8^2+12^2}{12^2} = \frac{208}{144} = \frac{13}{9}$

Thus, both the Left hand side (LHS) and the Right hand side (RHS) give the same value. Hence, option (B) is true.

Option (C):

$\frac{\frac{1}{b}+a}{a} = \frac{\frac{1}{12}+2}{2} = \frac{25}{24}$ and $\frac{\frac{1}{c}+d}{d} = \frac{\frac{1}{3}+8}{8} = \frac{25}{24}$

Thus, both the Left hand side (LHS) and the Right hand side (RHS) give the same value. Hence, option (C) is true.

Option (D):

$\frac{ab^2}{c} = \frac{2*12^2}{3} = \frac{288}{3} = 96 \neq d$

Thus, both the Left hand side (LHS) and the Right hand side (RHS) do not give the same value. Hence, option (D) is not true.

Hence, the correct option is (D).

22. Here, the relation between four integers, a, b, c, d are given. We do not know the actual values of any of the integers. However, from the relation, we can find out the ratio of the integers.

Since $2a = 5b = 3c = 6d$, let us assume that all of these are equal to a multiple of the LCM of 2, 5, 3 and 6. The LCM of 2, 5, 3 and 6 is 30 (Since this is a ratio questions, the choice of the initial number we assume will not affect the final answer. We need to choose a convenient value which would make our calculations easy).

So, let us assume that $2a = 5b = 3c = 6d = 30k$ where k is some constant whose value is unknown.

Thus, we have:

$$a = \frac{30k}{2} = 15k.$$

$$b = \frac{30k}{5} = 6k.$$

$$c = \frac{30k}{3} = 10k.$$

$$d = \frac{30k}{6} = 5k.$$

Now, we need to find which of the options does not represent an integer.

For this, we substitute the values of a, b, c, d in the options one-by-one.

Starting with option (A):

$$\frac{ad}{3} = \frac{15k * 5k}{3} = 25k^2.$$

Now, if the value of k be assumed as an integer, then option (A) would result in an integer value.

Option (B):

$$\frac{ac}{d} = \frac{15k * 10k}{5k} = 30k.$$

Now, if the value of k be assumed as an integer, then option (B) would result in an integer value.

Option (C):

$$\frac{ac}{bd} = \frac{15k * 10k}{6k * 5k} = 5.$$

Thus, option (C) obviously results in an integer value.

Since one of the options must not result in an integer value, the answer must be option (D). We do not need to verify option (D) as well.

However, for your curiosity, let us check option (D):

$$\frac{ab}{cd} = \frac{15k * 6k}{10k * 5k} = \frac{9}{5}.$$

Thus, option (D) obviously does not result in an integer value.

Hence, the correct option is (D).

23. This question is based on the identity: $a^2 - b^2 = (a + b)(a - b)$.

Since $a@b = a^2 - b^2$, we have: $x@4 = x^2 - 4^2 = (x + 4)(x - 4)$.

Thus, we have: $(x + 4)(x - 4) = \frac{2x+8}{5} => (x + 4)(x - 4) = \frac{2}{5} * (x + 4)$

$$=> (x + 4)(x - 4) - \frac{2}{5} * (x + 4) = 0$$

Taking $(x + 4)$ common: $(x + 4)\left\{(x - 4) - \frac{2}{5}\right\} = 0 => (x + 4)\left(x - 4 - \frac{2}{5}\right) = 0$

$$=> (x + 4)\left(x - \frac{22}{5}\right) = 0.$$

Since $x \neq -4$, we can say that $x = \frac{22}{5} = 4.4$.

Hence, the correct answer is $22/5$ or 4.4.

24. Three-digit numbers yielding 1 in the remainder when divided by 100 are $101, 201, 301, 401, 501, 601, 701, 801, \& \ 901$ – 9 numbers. If we subtract 1 from these numbers, we will get following 9 numbers: $100, 200, 300, 400, 500, 600, 700, 800, \& \ 900$.

Among these numbers, we have to select the numbers that are completely divisible by 60. and these numbers are $300, 600$, and 900 – 3 numbers.

Hence, the correct option is (C).

25. Given that $uv = -7$, if product of two integers is negative, one of the integer would be negative and the other would be positive, so u and v are of opposite signs.

If we factorize '7', we get '1×7', thus u and v can take a numerical value of 1 or 7. So, their sum is either 6 or -6, and $6^2 = 36$, or $(-6)^2 = 36$.

Hence, the correct option is (C).

5.1.3 Factors, Multiples, LCM, & HCF

26. The key to this question is to decompose 48 into its prime factors and use these prime factors to construct the factors of 48 that are greater than $\sqrt{48}$.

Since $7^2 = 49$, $\sqrt{48} < 7$.

We understand that a factor is necessarily an integer, and we are looking for factors which are greater than 7. Prime number decomposition of 48 is $2^4 \times 3$.

To determine a complete set of factors: $2, 2, 2, 2, 3$, we need to work systematically:

$2 \times 2 = 4$
$2 \times 3 = 6$
$2 \times 2 \times 2 = 8$
$2 \times 2 \times 3 = 12$
$2 \times 2 \times 2 \times 2 = 16$
$2 \times 2 \times 2 \times 3 = 24$
$2 \times 2 \times 2 \times 2 \times 3 = 48$

Thus, the factors of 48 which are more than $\sqrt{48}$ are: $8, 12, 16, 24, 48$ i.e. 5 in total.

Hence, the correct answer is (B).

27. The minimum number of squared identical tiles will be used when we use square tiles of the largest possible sides. The key word is 'identical' i.e. all the tiles must be of the same dimensions.

The maximum side of a square tile will be the HCF (Highest Common Factor) of the lengths of the sides of the rectangular floor i.e. the HCF of 48 and 60.

$$48 = 2^4 * 3$$

$$60 = 2^2 * 3 * 5$$

Hence, the HCF of 48 and 60 = $2^2 * 3 = 12$ feet.

So, square tiles of size 12 feet by 12 feet should be used.

Thus, number of square tiles required = $\frac{\text{total area}}{\text{area of each square}} = \frac{48*60}{12*12} = 20$.

Thus, 20 square tiles need to be used.

Hence the correct answer is 20.

28. Amy writes down every fourth number starting with one till 50, hence she writes down the numbers: 1, 5, 9, 13 and so on.

Mary writes down every third number starting with one till 50, hence she writes down the numbers: 1, 4, 7, 10, 13 and so on.

Thus, we see that the first number common to both of them after 1 is 13.

We can get the starting number (i.e. 13) in another way:

Since both Amy and Mary start with the same number (i.e. 1), with Amy counting every fourth number and Mary counting every third number, they would reach a common number after a gap equal to the LCM of 4 and 3 i.e. 12.

Thus, the next number that both reach is: $1 + 12 = 13$.

Thus, in general, they would keep reaching the same numbers always at gaps of 12.

If one observes carefully, the number after 13 which they both reach is: $13 + 12 = 25$.

Thus, the numbers common to both are: 1, 13, 25, 37 and 49. Hence, they have 5 numbers in common.

Alternately, we can also calculate the # of common numbers in the following way:

We know that the starting number is 1 and the gap between any two consecutive common numbers is 12. Thus, from 1 to 50, i.e. a total gap of $50 - 1 = 49$, we try to find out how many gaps of 12 can be accommodated.

This can be calculated by dividing 49 by 12 and finding the quotient, which, in this case is 4.Thus, there are 4 such gaps possible.

Hence, the # of common numbers will be one more than the number of gaps = $1 + 4 = 5$. Hence, the total # of numbers common = 5.

Hence, the correct option is (C).

29. We need to find the distinct number of prime factors for each of the numbers when it is multiplied by 5. Then, we need to select the number which has a different number of such distinct prime factors. Here, we need to work with each option.

Starting with option (A):

$$15 = 5^1 * 3^1.$$

Thus, $15 * 5 = 5^2 * 3^1$(Multiplying it with '5').

Hence, the number of distinct prime factors are **two**, viz. 3 and 5.

Option (B):

$$28 = 2^2 * 7^1.$$

Thus, $28 * 5 = 2^2 * 7^1 * 5^1$.

Hence, the number of distinct prime factors are **three**, viz. 2, 5 and 7.

So, we have already got two numbers, 15 & 28, having different number of distinct prime factors. According to the question, only one number will have different number of distinct prime factors. Hence, in the next option, the number will have either **two** or **three** distinct prime factors. Whichever of two or three gets repeated, the other one will be the exception and hence, our answer. So, we would not have to check the fourth number.

Option (C):

$$30 = 2^1 * 3^1 * 5^1.$$

Thus, $30 * 5 = 2^1 * 3^1 * 5^2$.

Hence, the number of distinct prime factors are **three**, viz. 2, 3 and 5.

Hence, we know that option (A) must be the exception since it has **two** distinct prime factors while options (A) and (C) have **three** each. It implies that option D: 22 must have **three** distinct prime factors as there would be only one option as an exception!

For your curiosity, let us work out option (D):

$$22 = 2^1 * 11^1.$$

Thus, $22 * 5 = 2^1 * 11^1 * 5^1$.

Hence, the number of distinct prime are **three**, viz. 2, 11 and 5.

Thus, option (D) is not the exception.

Hence, the correct option is (A).

30. We need to check each number for the highest exponent of any of its prime factors. In order to do that, we need to break each number into its prime form.

Here, we need to work with each option.

Option (A):

$$48 = 2 * 2 * 2 * 2 * 3 = 2^4 * 3^1.$$

Thus, the highest exponent for its prime factors is 4 since we have 2^4 as a factor.

Option (B):

$$72 = 2 * 2 * 2 * 3 * 3 = 2^3 * 3^2.$$

Thus, the highest exponent for its prime factors is 3 since we have 2^3 as a factor.

Option (C):

$$250 = 2 * 5 * 5 * 5 = 2^1 * 5^3.$$

Thus, the highest exponent for its prime factors is 3 since we have 5^3 as a factor.

Option (D):

$$484 = 2 * 2 * 11 * 11 = 2^2 * 11^2.$$

Thus, the highest exponent for its prime factors is 2 since we have 2^2 as well as 11^2 as factors.

Hence, among the given numbers, the one with the highest exponent for any of its prime factors is 48 and the corresponding exponent is 4 (exponent for 2).

Hence, the correct option is (A).

31. The left hand side of the equation:

$$75^n * 15 = \left(5^2 * 3\right)^n * 3^1 * 5^1 = 5^{2n+1} * 3^{n+1}.$$

Comparing this to the right hand side of the equation and equating the exponents, we get:

From the exponents of 3: $n + 1 = 6 => n = 5$.

From the exponents of 5: $2n + 1 = m => m = 2 * 5 + 1 = 11$.

Thus, the value of $(m + n) = 11 + 5 = 16$.

Hence, the correct option is (D).

32. Breaking 1500 into its prime factors:

$$1500 = 2 * 2 * 3 * 5 * 5 * 5 = 2^2 * 3^1 * 5^3$$

The right hand side of the equation:

$$6^m * 50^n * 5 = (2 * 3)^m * \left(5^2 * 2\right)^n * 5 = 2^m * 3^m * 5^{2n} * 2^n * 5 = 2^{m+n} * 3^m * 5^{2n+1}$$

Comparing 1500 to the right hand side of the equation and equating the exponents, we get:

From the exponents of 2: $m + n = 2$.

From the exponents of 3: $m = 1$.

Thus, $n = 2 - m = 2 - 1 = 1$.

We can verify that the exponent of 5 would match for $n = 1$: $5^{2n+1} = 5^{2*1+1} = 5^3$.

Thus, the value of $(m - n) = 1 - 1 = 0$.

Hence, the correct option is (A).

5.1.4 Linear equations

33. We know that $2x = 5y => x = \frac{5y}{2}$.

Hence, substituting the value of x in the equation $x + y = 21$, we get:

$$\frac{5y}{2} + y = 21 => \frac{7y}{2} = 21 => y = \frac{2}{7} * 21 = 6.$$

Thus, $x = \frac{5}{2} * 6 = 15$.

Alternatively, we could say that $2x = 5y => \frac{x}{y} = \frac{5}{2} => x = 5 * k,\ y = 2 * k$, where k is a constant of proportionality.

Now, $x + y = 21 => 5k + 2k = 21 => 7k = 21 => k = 3$.

Thus, $x = 5 * k = 5 * 3 = 15$ and $y = 2 * k = 2 * 3 = 6$.

Thus, $\sqrt{x - y} + \left(\frac{x}{y}\right)^2 = \sqrt{15 - 6} + \left(\frac{5}{2}\right)^2 = \sqrt{9} + (2.5)^2 = 3 + 6.25 = 9.25$.

Hence, the correct option is (C).

34. The total cost of 75 apples and 45 oranges is given as \$12.

Now, that the cost of any combination of apples and oranges cannot be calculated from the information given as we do not know the per piece price of an apple and an orange.

However, if we look carefully, we find that 50 apples $= \frac{2}{3} * 75$ apples and 30 oranges $= \frac{2}{3} * 45$ oranges. The factor of $\frac{2}{3}$ is common to both the fruits.

Thus, we need to calculate the cost of $\frac{2}{3}^{\text{rd}}$ of the quantity of the given combination of fruits.

Thus, the required cost of 50 apples and 30 oranges would be $\frac{2}{3}$ of the cost of 75 apples and 45 oranges $= \frac{2}{3} * \$12 = \$8$.

Hence, the correct option is (B).

Alternate approach:

Let the cost of an apple be $\$x$ and that of an orange be $\$y$.

Thus, the cost of 75 apples and 45 oranges would be $\$\,(75x + 45y)$.

So, we have: $75x + 45y = 12 => 3 * (25x + 15y) = 12 => 25x + 15y = 4 \ldots$(i)

We now need to calculate the cost of 50 apples and 30 oranges i.e. the value of $(50x + 30y)$.

$(50x + 30y)$ can be factored as given below.

$50x + 30y = 2 * (25x + 15y) = 2 * 4 = 8$ (Plugging in the value of $25x + 15y = 4$ from equation (i)).

Hence, the correct option is (B).

35. Let the number of apples be x and the number of mangoes be y.

Since the number of apples was 4 less than twice the number of mangoes, we have:

$x = 2y - 4 \ldots$(i)

Also, we know that if the man had bought 6 more mangoes, the number of apples and mangoes would have been equal. Thus, we have:

$y + 6 = x \ldots$(ii)

Substituting x from (ii) in (i), we get:

$$y + 6 = 2y - 4 => y = 10.$$

Thus, the man had bought 10 mangoes.

Hence, the correct answer is 10.

36. Let us assume that Lucy was traveling at x miles/hour; $x >$120 miles/hour. Here, x must be greater than 120 as the fine \$90 is greater than \$60, which implies that she must have sped over 120 miles/hr.

Thus, her fine would be calculated as: $\$[60 + 3\,(x - 120)]$.

Hence, we have: $60 + 3x - 360 = 90 => 3x = 450 => x = 150$.

Thus, Lucy was driving at 150 miles/hour.

Hence, the correct option is (C).

37. Let y be the number of coins each thief gets.

Then, $x = 7y + 6$.

Since x & 6 are even, it implies that $7y$ must be even and hence, y is even (Odd$\times$ Even = Even).

Let the number that is to be added to x be denoted by a.

Thus, we see that $(x + a)$ is divisible by 14, implying that $(x + a)$ is even (An odd number is not divisible by an even number.).

We know that x is even, hence, we deduce that a is also even.

Hence, we can say that the answer is either option (C) or (D) as only those are even numbers.

We need to verify which one of the two options is correct.

To verify with $a = 36$:

$$x + 36 = 7y + 6 + 36 = 7y + 42.$$

Since y is even, $7y$ is a multiple of 14. Also, 42 is a multiple of 14.

Hence, $7y + 42$ is divisible by 14.

Hence, $a = 36$ satisfies the condition given in the problem. Thus, $a = 42$ is obviously not correct.

Hence, the correct answer is (C).

Alternate approach:

We know that when the even number x is divided by 7, the remainder comes to 6.

Obviously, when the same even number x is divided by 2×7 i.e. 14, the remainder would still come to 6.

Thus, we need to add such a number to x, which would leave a remainder of $14 - 6 = 8$ when divided by 14 (only then the remainders would add up to $6 + 8 = 14$ and the resulting number would thus be divisible by 14).

We see that 36 is the only number which when divided by 14 leaves a remainder of 8. Hence, 36 must be the correct answer.

Hence, the correct answer is (C).

38. Let the age of Jane and Amy at present be x and y years respectively.

Thus, we know: $y = x - 6 \Rightarrow x - y = 6 \ldots$ (i)

Also, five years ago, the ages of Jane and Amy were be $(x - 5)$ and $(y - 5)$ years, respectively.

Thus, we have: $x - 5 = 3(y - 5) \Rightarrow x - 3y = -10 \ldots$ (ii)

Subtracting (ii) from (i): $2y = 16 \Rightarrow y = 8$.

Thus, $x = y + 6 = 14$.

Thus, Jane's age at present is 14 years.

Thus, after 10 years, Jane's age would be $14 + 10 = 24$ years.

Hence, the correct option is (D).

Alternate approach (using a single variable):

Let, five years ago, the age of Amy was y years.

Thus, at that time, Jane's age = $3y$ years.

Thus, the present ages of Jane and Amy are $(3x + 5)$ and $(x + 5)$ years.

Hence, as per the first condition, we have: $y + 5 = (3y + 5) - 6 => 2y = 6 => y = 3$.

Thus, Jane's age five years ago = $3 \times 3 = 9$ years.

Thus, present age of Jane = $9 + 5 = 14$ years.

Thus, after 10 years, Jane's age would be $14 + 10 = 24$ years.

Hence, the correct option is (D).

5.1.5 Absolute numbers

39. $|5x| + 30 = 40$

=> $|5x| = 10$

=> $|x| = 10/5 = 2$

=> $|x| = 2$

=> $x = \pm 2$.

Hence, the correct option is (C).

40. $\left[\dfrac{-|-25| \times -|2|}{(-|-5|)^2}\right]$

$= \left[\dfrac{-25 \times -2}{(-5)^2}\right]$

$= \dfrac{50}{25} = 2.$

Hence, the correct option is (B).

41. $\dfrac{2x}{3y} = 0$ can be reduced to $\dfrac{x}{y} = 0 \times \dfrac{3}{2} = 0$.

From $\dfrac{x}{y} = 0$, we can deduce two results: one, $x = 0$ & two, $y \neq 0$.

Option D MUST not necessarily be true. Say, for a condition $(x > y)$, for example, $x = 0$ & $y = -3$, $\dfrac{x}{y} = 0$ can be true; while options B & C MUST be wrong.

Hence, the correct option is (A).

42. $|3x - 4| = 11$ means that either $3x - 4 = +11$ or $3x - 4 = -11$

If $3x - 4 = +11$, then $3x = 4 + 11 = 15$ => $x = 5$.

And if $3x - 4 = -11$, then $3x = -11 + 4 = -7$ => $x = -7/3$.

Hence, the corrects answers are $x = 5$ or $-7/3$.

43. $\dfrac{|13x - 4|}{5} + 3 = 10$ => $\dfrac{|13x - 4|}{5} = 7$ => $|13x - 4| = 35$

$|13x - 4| = 35$ means that either $13x - 4 = +35$ or $13x - 4 = -35$

If $13x - 4 = +35$, then $13x = 4 + 35 = 39$ => $x = 3$.

And if $13x - 4 = -35$, then $3x = -35 + 4 = -31$ => $x = -31/3$.

Hence, the corrects answers are $x = 3$ or $-31/3$.

5.1.6 Inequalities

44. $=> |2x+1| < 7 \begin{cases} 2x+1 < 7 => 2x < 6 => x < 3. \\ \\ 2x+1 > -7 => 2x > -8 => x > -4. \end{cases}$

So, the range is $-4 < x < 3$.

Hence, the correct option is (D).

45. Since LHS of the inequity $\frac{1}{x-1} > 1$ is greater than 1, hence $(x-1)$ must be positive.

We can manipulate the inequality as

$\left[\frac{1}{(x-1)} \times (x-1)\right] > \left[1 \times (x-1)\right]$

$=> 1 > (x-1)$

$=> 2 > x$; since we know that $(x-1)$ is a positive number, we can multiply both the sides of inequity with a positive number; had it been not known we cannot do it, so considering $(x-1)$ as positive, the range of x would be $0 < x < 2$.

Hence, the correct option is (C).

46. By multiplying the inequality $\frac{-2|x|}{3y} > -1$ with "–", it will transform to $\frac{2|x|}{3y} < 1$; note the sign change of inequality (from > to <). When we multiply an inequality with "–", the sign changes.

The inequality can be manipulated to $2|x| < 3y$.

Since the LHS, $2|x|$ is always a positive number and the RHS, $3y$ is greater than the LHS, it means that $3y$ or y MUST be positive or $y > 0$.

Hence, the correct option is (C).

Option A MUST not be true as unlike y, x may assume a negative number while keeping $|x|$ a positive number.

Option D MUST not be true as for say, ($x > y$), for an example $x = 3$, & $y = 2.5$, the inequality $2|x| < 3y$ holds true.

47. This is 'Could be true' type of questions. Even if the option is true for only one circumstance, it is the correct answer. 'Could be true' type of questions are different from 'Must be true' type of questions, in which the condition given must be true for all the circumstances.

After multiplying the inequality $-3 < x < -1$ with "–", it can be written as $3 > x > 1$; note the sign change,

I. $x^2 < 2x$: For $1 < x < 2$, the inequality $x^2 < 2x$ is true, else not. Say $x = 1/2$, then $(1/2)^2 < 2.(1/2) => 1/4 < 1$; so it can be true.

Alternatively, in case of inequality, if x is a positive number which is true in the given inequality ($1 < x < 3$), then we can cancel x from both the sides. So, $x^{\cancel{2}} < 2\cancel{x} => x < 2$. $x < 2$ is within the range of $1 < x < 3$, so option (I) is correct.

II. $x^2 = 2x => x = 2; x = 2$ is within the range of $1 < x < 3$, so option (II) is also correct. We must not even consider $x = 0$ as $x = 0$ is out of range of given constraints $1 < x < 3$.

III. $x^2 > 2x => x^{\cancel{2}} > 2\cancel{x} => x > 2$; Similarly, $x > 2$ is within the range of $1 < x < 3$, so option (III) is also correct.

Hence, the correct option is (D).

48. Since in the inequality $a^3bc^2 < 0$, we see that on the LHS side, there is a term c^2, so we cannot conclude whether c is positive/negative as for c being positive/negative, c is always positive.

However, we can sure conclude that $c \neq$ as $a^3bc^2 < 0$. There is no such option of $c \neq$, so we move ahead.

We can deduce that a^3bc^2 is negative (Less than 0) because of either a^3 or b, thus a^3b must be negative. The sign of a^3 would depend on the sign of a, if a is positive, a^3 is positive, and if a is negative, a^3 is negative.

a^3b cab be negative if and only if one of a or b is negative and the other is positive. Option D states this. Option C is incorrect as it is not MUST be correct option.

Hence, the correct option is (D).

49. The direction of both the inequalities must match. Currently, the directions of inequalities: $6 > x > 3$ and $-4 < -y < 7$ are opposite. $-4 < -y < 7$ can be written as $4 > y > -7$.

Since we want $3x$, so by multiplying '3' to $6 > x > 3$, we get $18 > 3x > 9$; similarly, by multiplying '3' to $4 > y > -7$, we get $8 > 2y > -14$.

Adding the inequalities, we get...

$(18 > 3x > 9) + (8 > 2y > -14) => 26 > 3x + 2y > -5$.

The minimum value for $3x + 2y$ is -4 and the maximum is 25, thus the set of values is $\{25, 24, 23, \ldots\ldots, 0, -1, -2, -3, -4\}$. There are a total of 30 values.

Hence, the correct answer is 30.

5.1.7 Graphic equations & Inequalities

50. The given graph is $f(x) = 2x + 5$.

We need to find the graph of $g(x) = 2(x - 1) + 5$.

Here, we see that x has been replaced with $(x - 1)$.

Thus, the graph would shift by one unit right (since x has been reduced by one).

To find the correct graph, let us pick one reference point on the original graph.

Let the point be the one where $f(x)$ intersects the X-axis. The point is $(-2.5, 0)$.

Thus, on shifting, this point would move to $(-2.5, 0 + 1, 0) => (-1.5, 0)$.

Thus, the graph of $g(x)$ should intersect the X-axis at $(-1.5, 0)$.

This is satisfied by the graph shown in option (A).

Hence, the correct option is (A).

51. If we compare the graphs of $f(x)$ and $g(x)$, it seems that $g(x)$ is less steep that $f(x)$ is. Also, we can observe that the point of intersection of $f(x)$ with the Y-axis remains unchanged in $g(x)$ while the point of intersection of $f(x)$ with the X-axis has changed.

To verify this, let us find the coordinates of the point of intersection of $f(x)$ with the X-axis and the Y-axis. The coordinates of the above points are $(2, 0)$ and $(0, 4)$. Thus, the magnitude of the slope of the line $y = f(x)$ is $\frac{4}{2} = 2$ (magnitude of the slope equals the ratio of the magnitude of the y intercept and the magnitude of the x intercept).

The corresponding points of intersection of $g(x)$ with the X-axis and the Y-axis are $(4, 0)$ and $(0, 4)$. Thus, the magnitude of the slope of the line $y = g(x)$ is $\frac{4}{4} = 1$.

Thus, we see that at the point where the graphs intersect with the X-axis, the value of x has doubled and the magnitude of the slope of the line $y = g(x)$ has halved from $y = f(x)$.

Thus, we can say that $g(x) = f\left(\frac{x}{2}\right)$.

Thus, option (D) satisfies the above condition.

Hence, the correct option is (D).

52. On comparing the graphs of $f(x)$ and $g(x)$, it appears that the graph of $f(x)$ has been inverted i.e. reflected about the X-axis to get the graph of $g(x)$.

However, simply reflecting the graph of $f(x)$ does not lead to $g(x)$.

Let us pick some reference point in order to compare the graphs of $f(x)$ and $g(x)$.

Let the reference points be the one where $f(x)$intersects the X-axis, which is $(2, 0)$.

In the graph of $g(x)$, the intersection happens at $(3, 0)$.

Thus, we can say that the graph of $f(x)$ has also been shifted one place to the right i.e. x in $f(x)$ has been replaced by $(x - 1)$.

Thus, we can say that there are two changes from $f(x)$ to $g(x)$.

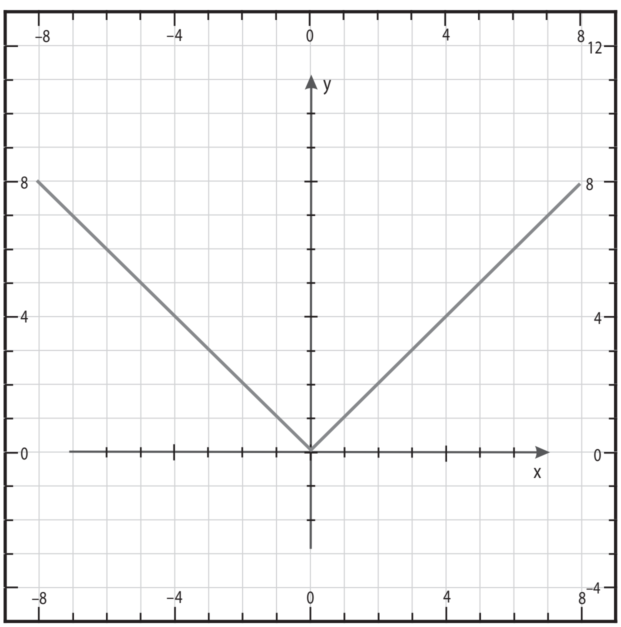
First, we have reflected the graph of $f(x)$ about the X-axis and then shifted it one unit to the right.

Thus, we can say that $g(x) = -f(x - 1)$.

Thus, option (B) satisfies the above condition.

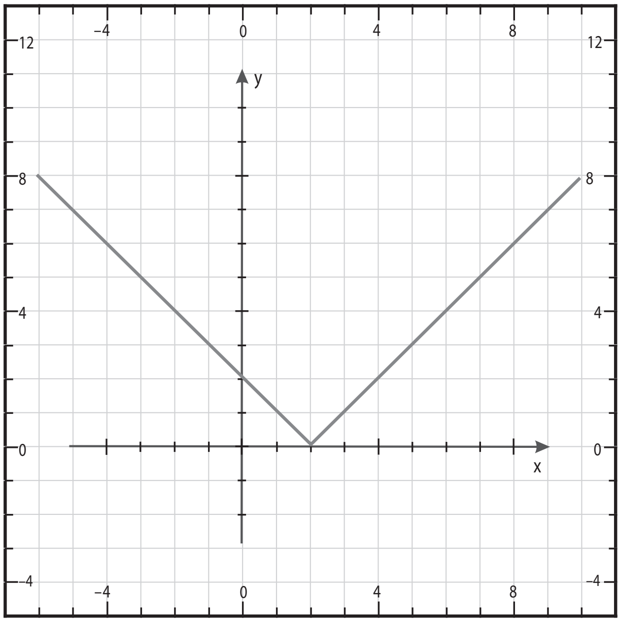
Hence, the correct option is (B).

53. Let us look at the graph of $|x|$:



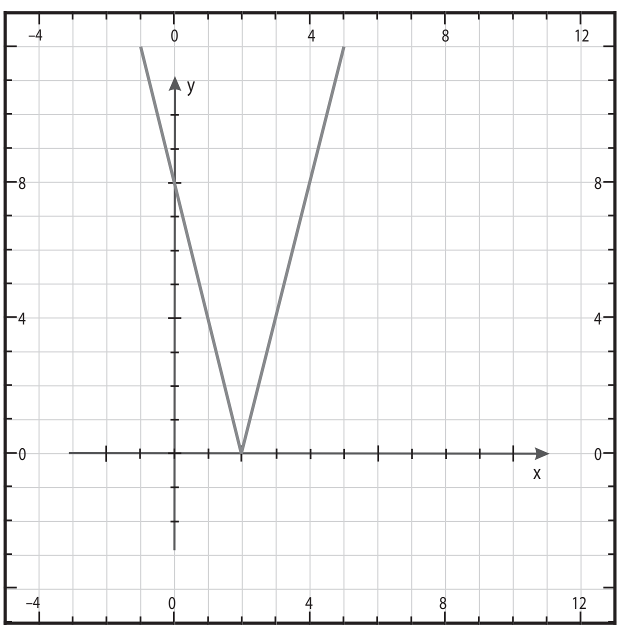
From the above graph of $|x|$, we will try to derive the graph of $4|x - 2| - 3$.

Let us now look at $|x - 2|$.



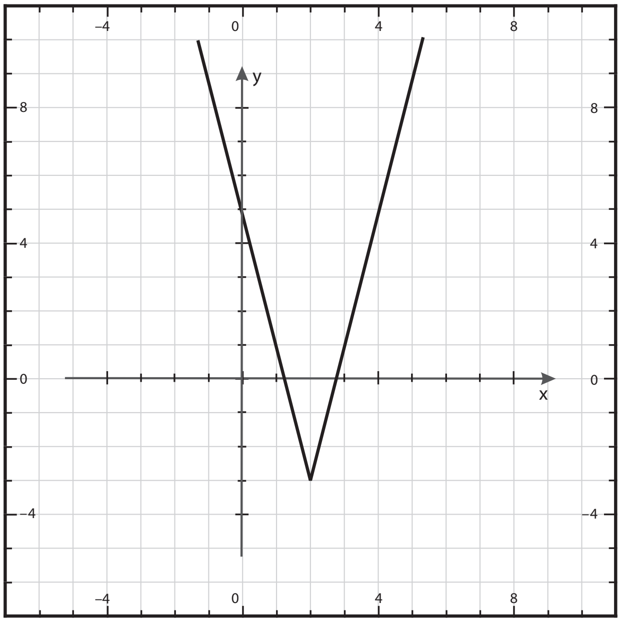
This will be the graph of $|x|$ shifted by 2 units right (since x has been replaced by $(x-2)$).

Let us now look at $4|x-2|$.



This will stretch the graph of $|x-2|$ vertically making it steeper.

Now, finally we look at the graph of $4\,|x-2|-3$.



This will shift the graph of $4|x - 2|$ down by 3 units (since we have subtracted 3 from the function).

Thus, we have arrived at the graph of $4\,|x - 2| - 3$.

However, we need to find the region denoted by $f\,(x) \geq 4\,|x - 2| - 3$.

We can see that all the points which lie inside the V created by the graph have y values which are higher than the values given by $4\,|x - 2| - 3$.

To verify the above statement, we pick any point in the V region, say the point $(2, 0)$.

Let us calculate the value of $4\,|x - 2| - 3$ at the point $x = 2$. It gives us $4\,|2 - 2| - 3 = -3$, i.e. the value of the lowest point of the graph.

It is clear that the y value of the point $(2, 0)$ i.e. $0 > -3$.

Thus, the point $(2, 0)$ satisfies the region $f\,(x) \geq 4\,|x - 2| - 3$.

Thus, the region denoted by $f\,(x) \geq 4\,|x - 2| - 3$ will be the shaded region as shown in option (A).

Hence, the correct option is (A).

5.1.8 Functions

54. Plug in the value of $x = -2$ in the function,

we get $f(2) = 3.(-2)^3 - 2.(-2)^2 + 2.(-2) + 2 = -24 - 8 - 4 + 2 = -34$.

Hence, the correct option is A.

55. $g(-1/t) = 2(-1/t)^3 - 3(-1/t)^2 + \frac{3}{(-1/t)^2} + \frac{2}{(-1/t)^3}$

$=> -\frac{2}{t^3} - \frac{3}{t^2} + 3t^2 - 2t^3 = -(\frac{2}{t^3} + \frac{3}{t^2} - 3t^2 + 2t^3) = -g(t)$.

so $g(-\frac{1}{t}) = -g(t)$.

Hence, the correct option is A.

56. Since the table provides us 4 values of $f(x)$ for x, we must check each option one by one to see which one represents the given values.

Let us plug-in values of x in each option and check the value for $f(x)$; if all the 4 values for x matched with the respective table values of $f(x)$, it is the correct option.

(A) $f(x) = -4x^2 + 3x + 3$

For $x = 0, f(0) = -4.(0)^2 + 3.0 + 3 = 3$; since the value of $f(x)$ matches with the its table value, we must plug-in the next value of x. For $x = 1, f(1) = -4.(1)^2 + 3.1 + 3 = -4 + 3 + 3 = 2$; since the value of $f(x)$ matches with the its table value, we must plug-in the next value of x. For $x = -1, f(-1) = -4.(-1)^2 + 3.(-1) + 3 = -4 - 3 + 3 = -4$; since the value of $f(x)$ does NOT match with the its table value, we will not further as this cannot be an answer. Let's try option (B).

(B) $f(x) = -5x^2 + 4x + 3$ For $x = 0, f(0) = -5.(0)^2 + 4.0 + 3 = 3$; since the value of $f(x)$ matches with the its table value, we must plug-in the next value of x. For $x = 1, f(1) = -5.(1)^2 + 4.1 + 3 = -5 + 4 + 3 = 2$; since the value of $f(x)$ matches with the its table value, we must plug-in the next value of x. For $x = -1, f(-1) = -5.(-1)^2 + 4.(-1) + 3 = 5 - 4 + 3 = 4$; since the value of $f(x)$ does NOT match with the its table value, we will not further as this cannot be an answer. Let's try option (C).

(C) $f(x) = 3x^2 - 4x + 3$ For $x = 0, f(0) = 3.(0)^2 - 4.0 + 3 = 3$; since the value of $f(x)$ matches with the its table value, we must plug-in the next value of x. For $x = 1, f(1) = 3.(1)^2 - 4.1 + 3 = 3 - 4 + 3 = 2$; since the value of $f(x)$ matches with the its table value, we must plug-in the next value of x. For $x = 1, f(-1) = 3.(-1)^2 - 4.(-1) + 3 = 3 + 4 + 3 = 10$; since the value of $f(x)$ matches with the its table value, we must plug-in the next value of x. For $x = -2, f(-2) = 3.(-2)^2 - 4.(-2) + 3 = 12 + 8 + 3 = 23$;

Since the values of $f(x)$ matches with the its table values, option (c) is the correct answer. We must not check option (D) now.

(D) $f(x) = -4x^2 - 3x + 3$

Hence, the correct option is (C).

57. Since $f(x) = 3x - 4$, hence $f(3x-4) = 3.(3x-4) - 4 = 9x - 12 - 4 = 9x - 16$: replacing '$x$' with $3x - 4$.

Now, $5f(x) - 3 = f(3x-4) => 5.(3x-4) - 3 = 9x - 16$; substituting the values of $f(x) = 3x - 4$ and $f(3x-4) = 9x - 16$.

We get, $15x - 20 - 3 = 9x - 16 => 15x - 9x = 23 - 16 => 6x = 7 => x = 7/6$.

Hence, the correct answer is $x = 7/6$.

58. $[49] = -|\sqrt{49}| = -7$; as 49 is a positive and an odd integer.

$[-4] = -|(-4)^2| = -|16| = -16$; as –4 is a negative and an even integer.

$[-49] = (-49)^2 = 49^2$; as –49 is a negative and an odd integer.

$[4] = -\sqrt{4} = -2$; as 4 is a positive and an even integer.

So, $\frac{[49]^4.[-4]}{[-49].[4]} = \frac{(-7)^4\times-16}{49^2\times-2} = \frac{\cancel{49^2}\times\cancel{-16}8}{\cancel{49^2}\times\cancel{-2}}$

Hence, the correct answer is 8.

59. Set of integers greater than or equal to -2.2 is {–2, –1, 0, ...}; among these, the smallest is -2, so, $[-2.2] = -2$.

Similarly, set of integers greater than or equal to 3 is {3, 4, 5, ...}; among these, the smallest is 3, so, $[3] = 3$.

Similarly, set of integers greater than or equal to 5.2 are {6, 7, 8, ...}; among these, the smallest is 6, so, $[5.2] = 6$.

So, $[-2.4] + [3] + [5.2] = -2 + 3 + 6 = 7$.

Hence, the correct option is C.

60. $a@c = \dfrac{(a+c)}{(a-c)} = 2 => a + c = 2 \times (a - c) => a + c = 2a - 2c => c = -a/3$.

Hence, the correct option is C.

5.2 Problem Solving & Data Analysis

Problem Solving & Data Analysis section always has calculator access or no question will be asked in No-Calculator section.

5.2.1 Time & Work

61. Since Amy takes 12 hours to paint the entire wall, in 4 hours she had painted only $\frac{4}{12} = \frac{1}{3}^{\text{rd}}$ of the wall.

Thus, Bob has to paint the remaining $\left(1 - \frac{1}{3}\right) = \frac{2}{3}^{\text{rd}}$ of the wall.

We know that Bob takes 18 hours to paint the entire wall.

Thus, to paint $\frac{2}{3}^{\text{rd}}$ of the wall, he would take $\frac{2}{3} \times 18 = 12$ hours.

Hence, it took $(4 + 12) = 16$ hours for them to paint the entire wall.

Hence, the correct answer is (B).

62. John had already completed 12 days of work.

So, the work left (to be done by Matt) is equivalent to 8 days work of John. Note that it is 8 days of work at John's rate.

Hence, the percentage by which the work required to be done by Matt is less than the work already done by John = $\frac{12-8}{12} = \frac{4}{12} \times 100 = 33.33\% = 33.3\%$ (rounded off)

Hence, the answer is 33.3.

63. Let the total work be one unit.

Thus, Joe takes 6 hours to complete 1 unit work. Hence, in one hour, he does $\frac{1}{6}^{\text{th}}$ part of the work.

Also, Joe and Jack together take 4 hours to complete the work.

Hence, in one hour, they do $\frac{1}{4}^{\text{th}}$ part of the work.

Thus, in one hour, Jack alone would do $\left(\frac{1}{4}-\frac{1}{6}\right) = \left(\frac{1}{12}\right)^{\text{th}}$ part of the work.

Hence, to complete the entire work, time taken by Jack = $\frac{1}{\frac{1}{12}} = 12$ hours.

Hence, the correct option is (D).

Alternate approach:

Joe takes 6 hours and Joe and Jack together take 4 hours to water the plants.

Hence, let us take the total work to be the LCM of 6 and 4 i.e. 12 units.

Thus, in one hour, Joe does $\frac{12}{6} = 2$ units of work.

Also, in one hour, Joe and Jack together do $\frac{12}{4} = 3$ units of work.

Hence, we can conclude that Jack does $3-2 = 1$ unit of work in each hour.

Hence, to complete the total work of 12 units, Jack would take $\frac{12}{1} = 12$ hours.

5.2.2 Mixtures

64. Given that the concentration of wine is 15%.

To get 15 ml. of wine, we need 100 ml. of the mixture.

To get 27 ml. of wine, we need: $\frac{100}{15} \times 27 = 180$ ml. of mixture.

In the question, the data regarding total volume of mixture, 50 ml. is unnecessary (redundant).

Hence, the correct option is (D).

65. We know that 50 grams of the metal makes 15% of the total yield.

Thus, 1 gram of the metal makes $= \frac{15}{50}\%$ of the total yield.

Thus, 27 grams of the metal would make $= \frac{15}{50} \times 27 = 8.1\%$ of the yield.

Hence, the correct option is (C).

66. Let the quantity of milk and water in the mixture be x and $3x$ respectively; given that the ratio is 1:3.

Now, 60 liters milk was added.

So, quantity of milk in the mixture changed to $(x + 60)$ liters.

However, the quantity of water remained unchanged at $3x$ liters.

Thus, we have: $\frac{x+60}{3x} = \frac{3}{5} \Rightarrow 5x + 300 = 9x \Rightarrow 4x = 300$.

Thus, the total volume of the initial mixture $= x + 3x = 4x = 4 \times 75 = 300$ liters.

Hence, the correct answer is (D).

Alternate approach (an approach that can help solve the sum mentally):

Milk	:	Water
1	:	3
3	:	5

Since no water was added, the quantity of water must remain same. However, in the above diagram we see that the scaled quantity of water has changed from 3 to 5.

To make these same, we multiply 5 with the first ratio and 3 with the second ratio. Thus, we have:

Milk	:	Water
$1 \times 5 = 5$	:	$3 \times 5 = 15$
$3 \times 3 = 9$	:	$5 \times 3 = 15$

Now, we see that scaled quantity of milk has changed from 5 to 9 i.e. increased by 4 units. But, we know that these 4 units are equivalent to 60 liters (water added). Thus, 1 such unit is equivalent to 15 liters.

Now, total volume on ratio scale of initial mixture is 5 + 15 = 20 units.

Thus, 20 units are equivalent to $20 \times 15 = 300$ liters.

Hence, the correct answer is (D).

67. Let us assume that the total final mixture be 100 units (since this is a percentage based question, the choice of the initial value would not affect the final answer. We choose 100 as it is a number convenient to work with).

In the new mixture formed with water, water forms 60% of the mixture and hence, the energy drink concentrate forms $(100 - 60) = 40\%$ of the mixture i.e. 40% of 100 units = 40 units.

The original concentrate had 20% minerals, i.e. in 100 units of the energy drink concentrate, minerals constituted 20% of the whole.

Thus, in 40 units of the concentrate, minerals constitute 20% of 40 = 8 units.

Thus, percentage of minerals in the new mixture = $\frac{8}{100} * 100 = 8\%$.

Hence, the correct option is (B).

68. Let us assume that the total number of customers be 100 (since this is a percentage based question, the choice of the initial value would not affect the final answer. We choose 100 as it is a number convenient to work with).

Thus, 40% of 100 = 40 customers prefer sedans.

Among the customers who prefer sedans, 55% preferred Honda.

Hence, number of customer who prefer Honda = 55% of 40 = $\frac{55}{100} * 40 = 22$.

Thus, percentage of total customers who prefer a Honda sedan = $\frac{22}{100} * 100 = 22\%$.

Hence, according to the survey, Alfred should keep 22% of his cars as Honda sedans.

Hence, the correct option is (B).

5.2.3 Word Problems

69. A student gets 1 point for the first instance of good behavior.

After this, a student gets $1 + 3 = 4$ points for the second instance, $4 + 3 = 7$ points for the third instance and so on. The values have been tabulated below:

# Instance	Points awarded for that instance
1	1
2	4
3	7
4	10
5	13
6	16

Hence, on the 6^{th} instance, a student would get 16 points to get outstanding behavior reward.

Hence, the correct option is (C).

70. The key is to translate the problem into an equation.

If the number of marbles with John and Mary be j and m respectively, we have:

$j + m = 75 \times \left(\frac{j}{m}\right) \ldots$(i)

Now, we know that $m = 30$.

Substituting this value in (i): $j + 30 = 75 \times \left(\frac{j}{30}\right)$

$$=> j + 30 = \frac{5j}{2} => \frac{5j}{2} - j = 30 => \frac{3j}{2} = 30 => j = 20.$$

Thus, the number of marbles with John is 20.

Hence, the correct answer is (A).

Alternate approach (using a single variable):

Let John have j marbles.

We know that Mary has 30 marbles.

Thus, we have: $j + 30 = 75 \times \left(\frac{j}{30}\right) => j + 30 = \frac{5j}{2} => \frac{5j}{2} - j = 30$

$$=> \frac{3j}{2} = 30 => j = 30 \times \frac{2}{3} = 20.$$

Thus, the number of marbles with John is 20.

Hence, the correct answer is (A).

71. Let Peter have $\$x$.

Thus, John has $\$2x$.

After John gives \$50 to Peter, the amount of money with John and Peter becomes $\$(2x - 50)$ and $\$(x + 50)$ respectively.

Since the ratio gets reversed, Peter now has twice the amount of money as John has.

Thus: $\frac{x+50}{2x-50} = 2 => x + 50 = 4x - 100 => 3x = 150$.

Thus, they together had \$150 (since the total amount = $x + 2x = 3x$).

[Note: There is no need to calculate the amount of money present with them individually since we have been asked to find the total amount. Since their amounts are x and $2x$, their total is $3x$ which we have already obtained.]

Hence, the correct answer is (C).

72. Here, we see that in the final result, the leftmost position is 1.

This implies that when we added 'P' and 'P' in the hundred's place, there was no carry (since $S \neq 0$; given that each letter represents a unique digit from 1 to 9). Thus, the value of 's' must be 1.

Since the addition of 'P' and 'P' in the hundred's place did not result in a carry and has the total as 4, 'P' must be 2 (if there had been a carry, the value of 'P' would have been 7).

Now, let us focus in the unit's place: 'P' + 'R' gives 8 (may be with carry or without carry).

Since 'P' is 2, 'P' + 'R' cannot result in a carry and give 8.

This implies that there was no carry and hence, 'R' must be 6.

[Note: Following part is not required since we have already answered the question. However, let us find out for the sake of curiosity, the values of 'Q' and 's'.

Finally, we focus in the ten's position: 'Q' + 's' gives 4.

We can get 4 as follows: 1 + 3 = 4 or 2 + 2 = 4 (Note that we cannot assign 0 to them).

However, since 'P' is already 2, neither 'Q' nor 's' may be 2 (given that, each letter represents a **unique** digit from 1 to 9.). Thus, 'Q' and 's' must be 1 or 3 in any order.]

Hence, the correct answer is (D).

73. Let us consider a scenario such that the maximum number of marbles that could be present with all 4 of them together.

Among 4 of them, the maximum number of marbles is 27 (with Tom) and the least number of marbles is 10 (with Jack). To maximize the total, we assume that the other two have 26 and 25 marbles, respectively, with them (since no two can have the same number of marbles, we cannot take them to be 27, each). Thus, the maximum total = 10 + 25 + 26 + 27 = 88 marbles.

Hence, the maximum average = $\frac{88}{4}$ = 22 marbles.

Let us now consider a scenario such that the minimum number of marbles that could be present with all 4 of them together.

To minimize the total, we assume that the other two have 11 and 12 marbles, respectively, with them (since no two can have the same number of marbles, we cannot take them to be 10, each). Thus, the minimum total = 10 + 11 + 12 + 27 = 60 marbles.

Hence, the minimum average = $\frac{60}{4}$ = 15 marbles.

Thus, the average number of marbles with them should be a number between 15 and 22. Among the options, only 17 lies in the range of 15–22.

Hence, the correct answer is (B).

74. We see that the fractions used in the problem are $\frac{3}{5}, \frac{1}{3}$, & $\frac{1}{4}$.

This means that we would need to multiply the number of candidates with the above fractions.

Thus, it is best to assume a number for the total number of candidates which is divisible by $5, 3, 4$ (i.e. the denominators of the fractions).

LCM of $5, 3, 4 = 60$. So, let us assume that there are 60 candidates.

The following table gives the breakup of the candidates:

	Cleared	Not Cleared	Total
Male	$36 - 9 = 27$	$\frac{1}{4} \times 36 = 9$	$\frac{3}{5} \times 60 = 36$
Female	$40 - 27 = 13$	$20 - 9 = 11$	$60 - 36 = 24$
Total	$60 - 20 = 40$	$\frac{1}{3} \times 60 = 20$	60

The working is shown below:

Total male candidates $= \frac{3}{5} \times 60 = 36$.

Total female candidates $= 60 - 36 = 24$.

Total candidates who did not clear the examination $= \frac{1}{3} \times 60 = 20$.

Male candidates who did not clear the examination $= \frac{1}{4} \times 36 = 9$.

Male candidates who cleared the examination $= 36 - 9 = 27$.

Female candidates who cleared the examination = 40 – 27 = 13.

Female candidates who did not clear the examination = 20 – 9 = 11.

Total candidates who cleared the examination = 60 – 20 = 40.

So, among the candidates who cleared the examination, females constitute = $\frac{13}{40} \times 100 =$ 32.5%.

Hence, the correct answer is (A).

75. Let us assume that Jack received $\$x$.

Thus, John received $\$3x$.

Thus, we have: $3x = 5000 + x \Rightarrow x = 2500$.

Thus, total wealth of the father = $x + 3x = 4x = \$(4 \times 2500) = \$10000$.

Hence, the correct answer is (C)

76. Let us assume that the boy got x responses wrong.

Thus, he must have got $(60 - x)$ responses correct (since there are in all 60 questions and he attempted all of them).

Thus, his marks from correct responses = $3 \times (60 - x) = 180 - 3x$.

His marks from the wrong responses = $(-1) \times x = -x$.

Thus, his total marks = $(180 - 3x) + (-x) = 180 - 4x$.

Hence, we have: $180 - 4x = 100 \Rightarrow 4x = 80 \Rightarrow x = 20$.

Thus, he had got 20 responses wrong.

Hence, the correct answer is (B).

77. Since there are only two candidates and no voter voted for both the candidates, we can say that if x% voters did not vote for a particular candidate, then $(100 - x)$% of the voters must have voted for that candidate.

Since 60% voters did not vote for Joseph, it implies that $(100 - 60) = 40$% voters voted for Joseph.

Also, since 50% voters did not vote for Judith, it implies that $(100 - 50) = 50$% voters voted for Judith.

Thus, total percentage of voters who voted = $(40 + 50) = 90$%.

Thus, $(100 - 90) = 10$% of the voters did not cast their vote.

Hence, the correct answer is (B).

78. Let the number of books present initially be B.

Number of books distributed among students of lower-middle school = $\frac{2B}{5}$.

Number of books left = $B - \frac{2B}{5} = \frac{3B}{5}$.

Number of books distributed among students of high school = $\frac{3B}{5} \times \frac{1}{3} = \frac{B}{5}$.

Number of books left = $\frac{3B}{5} - \frac{B}{5} = \frac{2B}{5}$.

These books were stacked in the school library.

Thus, percentage of books stocked in the school library = $\dfrac{\frac{2B}{5}}{B} \times 100 = \dfrac{2B}{5B} \times 100 = 40\%$.

Hence, the correct answer is (D).

Alternate approach:

We can see that the fractions used in the problem are $\dfrac{2}{5}$ & $\dfrac{1}{3}$.

So, let us take the number of books as the LCM of $5, \& \ 3$ i.e. 15.

Thus, the number of books distributed among students of lower-middle school = $\frac{2}{5} \times 15 = 6$.

Number of books left = $15 - 6 = 9$.

Number of books distributed among students of high school = $9 \times \frac{1}{3} = 3$.

Number of books left = $9 - 3 = 6$.

These books were stacked in the school library.

Thus, percentage of books stocked in the school library = $\frac{6}{15} \times 100 = \frac{2}{5} \times 100 = 40\%$.

[Note that the calculations involved are much easier now than in the earlier approach]

Hence, the correct answer is (D).

79. The main idea is that the total number of marbles remained the same; the marbles were only distributed among the players.

Total number of marbles = $3 \times 20 = 60$.

Let x, y, and z represent the final number of marbles with John, Mary and Joe, respectively.

Thus: $x + y + z = 60 \ldots$(i)

Again: $x = y + 15 => y = x - 15 \ldots$(ii)

Also, $x = \frac{z}{3} => z = 3x \ldots$(iii)

Substituting the values of y from (ii) & z from (iii) in (i):

$$x + (x - 15) + 3x = 60$$

$$=> 5x = 75 => x = \frac{75}{5} = 15.$$

Thus, John has 15 marbles after the game.

Hence, the correct answer is 15.

80. The number of items the dealer needs in order to replenish his stock = 40 – 15 = 25.

The cost of each item is \$5.

Hence, his total cost for one such replenishment = $\$5 \times 25 = \$125$.

He needs to replenish once every two weeks i.e. twice each month.

Thus, his cost of replenishing his stock in a month $\$125 \times 2 = \$250$.

Hence, the correct answer is 250.

81. To solve the problem, it is helpful to realize that the number of bulbs can be any. So, we pick one which is suitable for doing the calculations. Here, the convenient number of bulbs in the lot is the one which is divisible by both 5 and 4.

Thus, let the number of bulbs be 20 (LCM of 4 and 5).

The cost of 20 bulbs = $\$\frac{6}{4} \times 20 = \$30$.

Number of defectives = $\frac{2}{5} \times 20 = 8$.

So, number of usable bulbs = 20 – 8 = 12.

So, price paid per usable bulb = $\$\frac{30}{12} = \$2.50$.

Hence, the answer is 2.5 or 2.50.

Alternate Approach:

Cost of a bulb = $\$\frac{6}{4} = \$1.50$.

Since 2 out of 5 bulbs are defective, it means that he had a loss of $\frac{2}{5} \times 100\% = 40\%$ or he paid \$1.50 for 60% of bulbs.

=> Realizable price per bulb = $\frac{1.50}{60\%} = \frac{1.50}{0.60} = \$2.50$.

Hence, the correct answer is \$2.5 or 2.50.

82. The man's savings of \$75 corresponds to the price reduction on the quantity of fuel he purchased.

Let he purchased x gallons.

Thus: $2.5 \times x = 75 => x = \frac{75}{2.5} = 30$ gallons.

Thus, the man had purchased 30 gallons of fuel.

Hence, the correct answer is 30.

83. We have: 10% of 20% of $x = 4$

$$=> \frac{10}{100} \times \frac{20}{100} \times x = 4$$

$$=> x = 4 \times \frac{100}{20} \times \frac{100}{10} = \frac{10000}{50} = 200.$$

Hence, the correct answer is 200.

84. Charge for the first 5 lbs of luggage = $\$x$.

Total charge paid by the customer = $\$a$.

Hence, extra charges paid = $\$(a - x)$.

Charge per lb of extra luggage = $\$y$.

Hence, weight of extra luggage = $\frac{a-x}{y}$ lbs.

Hence, total weight of luggage carried by the customer = $5 + \frac{a-x}{y}$ lbs.

Hence, the correct option is (B).

85. The initial salary was $\$x$.

As a result of a 25% increase, the salary will increase by 25% or $\frac{1}{4}$ times the initial value i.e. it will become $\$\left(x + \frac{1}{4} \times x\right) = \$\frac{5x}{4}$.

Given that the salary after the increase = \$400.

Thus: $\frac{5x}{4} = 400 => x = 400 \times \frac{4}{5} = 320$.

Hence, $x - 100 = 320 - 100 = 220$. Note that the question does not ask for the value of x; it asks for the value of $(x - 100)$. Beware!

Hence, the correct answer is 220.

86. The firm manufactures screws, rivets and nuts for 60 minutes, 60 minutes and 120 minutes, respectively or $60 \times 60 = 3600$ seconds, $60 \times 60 = 3600$ seconds, $120 \times 60 = 7200$ seconds, respectively.

Number of screws manufactured = $\frac{3600}{10} = 360$

Number of rivets manufactured = $\frac{3600}{20} = 180$

Number of nuts manufactured = $\frac{7200}{80} = 90$

Thus, total items manufactured = $360 + 180 + 90 = 630$.

Thus, nuts expressed as a percentage of the total items = $\frac{90}{630} \times 100 = \frac{1}{7} \times 100 = 14.28\% =\sim 14.3\%$.

Hence, the correct answer is 14.3.

Alternate approach:

Ratio of the time taken to manufacture a screw, a rivet & a nut = 10 : 20 : 80 = 1 : 2 : 8.

Thus, ratio of the number of screws, rivets & nuts manufactured in the same duration $= \frac{1}{1} : \frac{1}{2} : \frac{1}{8} = 8 : 4 : 1$.

However, the duration for which screws, rivets & nuts are manufactured = 60 : 60 : 120 = 1 : 1 : 2.

Thus, ratio of the number of screws, rivets & nuts manufactured $= 8 \times 1 : 4 \times 1 : 1 \times 2 = 8 : 4 : 2 = 4 : 2 : 1$.

Thus, nuts expressed as a percentage of the total items $= \frac{1}{4+2+1} \times 100 = \frac{1}{7} \times 100 = 14.28\% =\sim 14.3\%$.

Hence, the correct answer is 14.3.

87. Total number of items = 22 + 37 + 13 = 72.

Since each dealer has the same number of items, each must have $= \frac{72}{3} = 24$ items.

Now, according to the question, there must be at least 2 items of each product with each dealer.

Hence, to maximize the number of items of R with any dealer, we assume that the the dealer has 2 items of P, 2 items of Q, and the rest are all of R. Hence, the maximum number of items of R should be = 24 − 2 − 2 = 20.

However, we can see that there are only 13 items of R. Hence, the above logic is not correct.

Let us analyze this in a different manner:

Let us give 2 items of R to each of two dealers. Thus, we are left with = 13 − 2 − 2 = 9 items of R.

All the above 9 items of R can be given to the third dealer.

Thus, the maximum number of items of R that can be present with any dealer is 9.

Hence, the correct option is (B).

88. The bus makes a total of $6 * 3 = 18$ trips in a week.

Thus, total distance travelled by the bus in a week = $(18 * d)$ miles.

Thus, total fuel cost for the week = \$ $(18 * d) * n$ = \$$(18 * d * n)$

We know that the total operating cost for the week is \$$c$.

c = Total fuel cost + Maintenance cost

Hence, the maintenance cost for the week = \$ $(c - 18dn)$

This is the maintenance cost over 6 days (since the bus operates on 6 days of the week).

Thus, the daily maintenance cost = \$$\frac{(c-18dn)}{6}$.

Hence, the correct option is (C).

89. Total number of items = 4500.

Number of defective = 30% of 4500 = $\frac{30}{100} * 4500 = 1350$.

Number of items rectified = 80% of 1350 = $\frac{80}{100} * 1350 = 1080$.

Number of items scrapped = 1350 – 1080 = 270.

Hence, number of items rectified after rework are more than those which are scrapped by = 1080 – 270 = 810.

Hence, the correct option is (B).

Alternate approach:

Percentage of defective items = 30%.

Thus, as a percentage of the total items, the number of defective items rectified after rework

= 80% of 30% = $\frac{80}{100} * 30\% = 24\%$.

Again, as a percentage of the total items, the number of defective items scrapped

= (100 – 80)% of 30% = $\frac{20}{100} * 30\% = 6\%$.

Hence, defective items rectified after rework is more than that scrapped items by

= (24 – 6) % of the total items = 18% of the total items = $\frac{18}{100} * 4500 = 810$.

Hence, the correct option is (B).

90. Ice forms at the rate of 2 inches in every 40 minutes or $\frac{40}{60} = \frac{2}{3}$ hours.

Thus, Rate of ice formation = $\frac{2}{\frac{2}{3}} = 3$ inch per hour

Thus, in 4 hours, total thickness of ice formed = $3 * 4 = 12$ inches.

The freezer was switched off for 2 hours and the ice melted at the rate of 0.5 inches per hour.

Thus, total amount of ice melted = $2 * 0.5 = 1$ inch.

Thus, ice left = 12 – 1 = 11 inches.

Finally, the freezer was put on for another 2 hours

Ice formed during these 2 hours = $3 * 2 = 6$ inches.

Thus, total thickness of ice now = $11 + 6 = 17$ inches.

Hence, the correct answer is (C).

91. Fraction of the total books given to children aged less than 8 years = $\frac{3}{11}$.

Hence, fraction of books remaining = $1 - \frac{3}{11} = \frac{8}{11}$.

Fraction of the total books given to children aged between 8 and 12 years = $\frac{3}{4}$ part of remaining books $\left(\frac{8}{11}\right) = = \frac{3}{4} * \frac{8}{11} = \frac{6}{11}$.

Hence, fraction of books remaining = $\frac{8}{11} - \frac{6}{11} = \frac{2}{11}$ = books were given to children older than 12 years.

Hence, fraction of books given to children older than 12 years = $\frac{2}{11}$.

Hence, the correct answer is 2/11 or 0.18.

92. The percentage reduction in the enrollment of students from 2014 to 2015 is calculated as:

% Reduction = $\frac{\text{Reduction}}{\text{Value in 2014}} * 100 = = \frac{235-200}{235} * 100 = 14.89\% \sim 14.9\%$

Hence, the correct answer is 14.9.

93. To manufacture the item, process A requires 540 components.

These components can be manufactured @ 60 in 25 minutes.

Thus, time taken to manufacture 540 components = $\frac{540}{60} * 25 = 9 * 25 = 225$ minutes.

To manufacture the same item, process B requires 300 components.

These components can be manufactured @ 50 in 15 minutes.

Thus, time taken to manufacture 540 components = $\frac{300}{50} * 15 = 6 * 15 = 90$ minutes.

Thus, percentage reduction in time by replacing process A with process B

$$= \frac{\text{Reduction in time}}{\text{Time taken by process A}} * 100$$

$$= \frac{225 - 90}{225} * 100 = \frac{135}{225} * 100 = 60\%.$$

Hence, the correct answer is 60.

94. Number of people who were eligible to vote in 2010 = 192 million.

This number increased by 10.8 million.

Hence, the number of people who were eligible to vote in 2011 = 192 + 10.8 = 202.8 million.

Number of people who voted in 2010 = 54 million.

This number increased by 2.4 million.

Hence, the number of people who voted in 2011 = 54 + 2.4 = 56.4 million.

Hence, the percentage of voters among people eligible to vote in 2011

$= \frac{56.4}{202.8} * 100 = 27.81\%$

Hence, the correct option is (B).

95. Let us assume that the total number of students registering for the AST be 100 (since this is a percentage based question, the assumed initial value will have no effect on the answer).

Number of students appearing for the test = 80% of 100 = 80.

Thus, number of students who do not appear after registering = 100 − 80 = 20.

Of the 80 students who appear for the test, only 75% clear the test.

Thus, number of students who clear the test = 75%of 80 = $\frac{75}{100} * 80 = 60$.

Thus, the percentage by which the number of students who do not appear for the test after registering less than the number of students who clear the test is given by:

$$\frac{\text{\# who clear the test} - \text{\# who do not appear after registering}}{\text{\# who clear the test}} * 100 = \frac{60 - 20}{60} * 100 = 66.66\%.$$

Hence, the correct option is (D).

96. Total number of students registering for the AST = 50,000.

Number of students appearing for the test = 80% of 50,000 = 40,000.

Thus, number of students who do not appear after registering = 50,000 − 40,000 = 10,000.

Of the 40,000 students who appear for the test, only 75% clear the test.

Thus, number of students who clear the test = 75%of 40,000 = $\frac{75}{100} * 40{,}000 = 30{,}000$.

Thus, the difference between the number of students who do not appear for the test after registering and the number of students who clear the test

= 30,000 − 10,000 = 20,000

Hence, the correct option is (B).

97. The man purchased goods at the rate of 12 for \$5 and sold them at the rate of 10 for \$6.

Let us assume that the number of goods purchased = LCM of 12 and 10 = 60 (since this is a percentage based question, the assumed initial value will have no effect on the answer. So, we chose a number i.e. 60 which is divisible by both 12 and 10 to make our calculations easier).

Thus, the total cost of 60 goods = $\$\frac{5}{12} * 60 = \$25$.

The total sales proceeds generated by selling all the goods = $\$\frac{6}{10} * 60 = \$36$.

Thus, profit made on selling all the goods = $\$\,(36 - 25) = \$11$.

Hence, percentage profit generated = $\frac{\text{profit}}{\text{total cost}} * 100 = \frac{11}{25} * 100 = 44\%$.

Hence, the correct option is (D).

98. Cost of a 10 second advertisement during prime-time = \$12000.

The cost of the same advertisement early morning is 30% lower than that during prime-time.

Thus, cost of a 10 second advertisement during early morning = $\$(12000 - 30\%$ of $12000) = \$8400$.

Since the advertisement is of 30 seconds duration, we first calculate the cost for the given 10 second duration advertisement and then multiply the answer by $\frac{30}{10} = 3$.

Hence, total cost of running a 10 second advertisement once during prime-time and twice early morning

$$= \$\,(12000 + 8400 * 2) = \$28800.$$

Hence, total cost of running a 30 second advertisement once during prime-time and twice early morning

$=\$28800 * 3 = \$864000 = \$86.4$ thousand dollars.

Hence, the correct option is (C).

99. We know that the average speed is calculated as: $\frac{\text{Total distance}}{\text{Total time}}$.

In the above problem, the sprinter follows the exact same route during both ways and hence, her distance was identical in either case.

Let us assume that the distance travelled be the product $s(2s + 1)$.

Since speed is a ratio of distance and time, the ratio of distance ant time would remain the same irrespective of the value assumed for the distance. Here we assumed the distance to be the product (or the LCM) of the speeds since this would help us in our calculations.

Since the speed up the slope was s, time taken to cover the distance = $\frac{s(2s+1)}{s} = (2s + 1)$.

Again, since the speed down the slope was $(2s+1)$, time taken to cover the distance = $\frac{s(2s+1)}{2s+1} = s$.

Thus, total time taken = $(2s+1) + s = (3s+1)$.

Total distance covered going up the slope and then down = $2s(2s+1)$

Hence, her average speed for the entire journey = $\frac{\text{Total distance}}{\text{Total time}} = \frac{2s(2s+1)}{3s+1}$.

Hence, the correct option is (D).

100. Let us first calculate the average speed of the sprinter for the entire journey.

Let us assume that the time taken to go up the slope be t minutes.

Since the total time is 6 minutes, so time taken to come down the slope = $(6-t)$ minutes.

Speed of the sprinter up the slope = 3.6 feet/second.

Hence, distance covered up the slope = $3.6t * 60 = 216t$ feet (The speed is in feet/second while time was in minutes. So, we multiplied 60 to convert the time from minutes to seconds).

Similarly, speed of the sprinter down the slope = 6 feet/s.

Hence, distance covered up the slope = $6(6-t) * 60 = 2160 - 360t$ feet (The speed is in feet/second while time was in minutes. So, we multiplied 60 to convert the time from minutes to seconds).

Since distance covered both ways is the same, we have:

$$216t = 2160 - 360t => 576t = 2160 => t = \frac{2160}{576} = 3.75.$$

Thus, time taken to go up the slope = 3.75 minutes.

Thus, one-way distance = $3.6t * 60 = 3.6 * 3.75 * 60 = 810$ feet.

Hence, the correct option is (B).

101. We know that the average speed is calculated as: $\frac{\text{Total distance}}{\text{Total time}}$.

For the first part of the journey:

Distance = d miles.

Speed = 60 miles/hr.

Hence, time taken = $\frac{d}{60}$ hours.

For the second part of the journey:

Distance = 60 miles.

Time taken = 6 hours.

Hence, total distance for the entire journey = $(d + 60)$miles.

Total time taken for the journey = $\left(\frac{d}{60} + 6\right)$ hours.

Hence, average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{\text{d}+60}{\frac{\text{d}}{60}+6} = \frac{60(\text{d}+60)}{\text{d}+360}$ miles/hr.

Hence, the correct option is (A).

102. We need to find the total number of students in the class.

The main idea is that students who play both games are already counted among the students who play football as well as among the students who play rugby.

The best idea is to break the students in three categories: those who play only football, those who play only rugby and those who play both games.

The number of students who play only football is the difference between the number of students who play football and the number of students who play both games = $40 - 10 = 30$.

The number of students who play only rugby is the difference between the number of students who play rugby and the number of students who play both games = $30-10 = 20$.

Thus, total number of students who play some game is the sum of the students who play only football, who play only rugby and who play both games = $30 + 20 + 10 = 60$.

Now, we know that of the total number of students, 20% do not play any game. Thus, the number of students who play some game is$(100 - 20) = 80\%$ of the total students.

Thus, we have: 80% of (total students) = 60 => total students $= \frac{100}{80} * 60 = 75$.

Hence, total students in the class = 75.

Hence, the correct answer is 75.

103. Total duration of peak hours (from 9:00 am to 1:00 pm) = 4 hours = $4 * 60 = 240$ minutes.

Since cars arrive every p minutes, number of cars that arrive for fueling during this time $= \frac{240}{p}$ cars.

Total duration of non-peak hours (from 3:00 pm to 6:00 pm) = 3 hours = $3 * 60 = 180$ minutes.

Since cars arrive every q minutes, number of cars that arrive for fueling during this time $= \frac{180}{q}$ cars.

Thus, total number of cars that arrived for fueling = $\left(\frac{240}{p} + \frac{180}{q}\right)$.

Total time duration = $4 + 3 = 7$ hours = $7 * 60 = 420$ minutes.

Thus, average number of cars arriving per minute

$= \frac{\left(\frac{240}{p} + \frac{180}{q}\right)}{420} = \frac{240q+180p}{420pq} = \frac{60(4q+3p)}{420pq} = \frac{4q+3p}{7pq}$.

Hence, the correct option is (C).

104. We have the following information:

#Apples $=\frac{1}{3}*$ #Mangoes = > #Mangoes = 3* #Apples. . . (i)

#Guavas $=\frac{1}{4}*$ #Oranges = > #Oranges = 4* #Guavas . . . (ii)

#Mangoes $=\frac{1}{2}*$ #Guavas = > #Guavas = 2* #Mangoes . . . (iii)

Since this is a ratio questions, the choice of the initial number we assume will not affect the final answer. We need to choose a convenient value for the number of Apples. Then, we can find out the number of fruits of all the other types using the relations above.

Let us assume #Apples = 1.

Thus, from (i): #Mangoes = $3 * 1 = 3$.

From (iii): #Guavas = $2 * 3 = 6$.

From (ii): #Oranges = $4 * 6 = 24$.

Thus, total number of fruits = $1 + 3 + 6 + 24 = 34$.

Thus, fraction of total fruits that are Mangoes = $\frac{3}{34} = 0.088 \sim 0.09$.

Hence, the correct answer is 3/34 or approximately 0.09.

105. We have the following information:

#CDs $=\frac{1}{3}*$#Pen-drives = > #Pen-drives= 3 * #CDs . . . (i)

#DVDs $=\frac{1}{4}*$#Pen-drives = >#Pen-drives= 4*#DVDs . . . (ii)

Since this is a ratio questions, the choice of the initial number we assume will not affect the final answer. We need to choose a convenient value for the number of Pen-drives. Then, we can find out the number of items of all the other types using the relations above.

We can see that number of Pen-drives is thrice the number of CDs as well as four times the number of DVDs. So, we can say that the number of Pen-drives is a multiple of both 3 and 4 and hence, a multiple of their LCM. The LCM of 3 and 4 is 12.

So, let us assume #Pen-drives= 12.

Thus, from (i): #CDs $=\frac{1}{3}*12 = 4$.

From (ii): #DVDs $=\frac{1}{4}*12 = 3$.

Thus, total number of items = $12 + 4 + 3 = 19$.

Thus, fraction of total items that are DVDs = $\frac{3}{19} = 0.157 =\sim 0.16$.

Hence, the correct answer is 3/19 or approximately 0.16.

106. The largest number of boxes that can fit is obtained by placing the lowest dimension of the box, 2 meters, against the largest dimension of a container, 20 meters. So, the length of the container can allow for $\frac{20}{2} = 10$ boxes, and the width of the container allows for $\frac{6}{3} = 2$ boxes to be fit, and the height of the container fits just $\frac{4}{4} = 1$ box. Hence, the total number of boxes that fit in a single container is $10 \times 2 \times 1 = 20$.

Hence, the correct answer is 20.

107. If the device was losing 10 centimeters for every 15 meters, then we need to know how many 15-meter intervals are there in 1800 meters. Conveniently, $1800/15 = 120$. So, at the end of the bridge, the device was off by 1200 centimeters, or 12 meters. Since the device underestimated the distance, the length of the bridge, according to the device, was $1800 - 12 = 1788$ meters.

Hence, the correct option is (D).

108. Suppose it costs a dollar to hire an MA student, so it costs 2 dollars to hire an MBA. Then the total cost of hiring is $2 \cdot 140 + 40 = 320$ dollars, including the cost of hiring of the students with MA degrees is 40 dollars. So, the cost of hiring all MA's is $\frac{40}{320} times 100\% = 12.5\%$.

Hence, the correct answer is 12.5.

109. The average rate of arrival during the day is the ratio of "the total number of customers who arrive at the bank during the day" to "the total number of hours: $t_1 + t_2$ ".

The rate of arrival during the morning is given in customers per unit of time. So, multiplying the rate by hours will yield the number of customers arriving to the bank in the morning. The same is true for the afternoon.

Hence, the total number of customers who arrive to the bank during the day is $r_1 \times t_1 + r_2 \times t_2$, and the average rate of arrival for the day is $\dfrac{r_1t_1 + r_2t_2}{t_1 + t_2}$.

Hence, the correct option is (C).

5.2.4 Profit & Loss

110. The key to this question is to know that, by definition, the percent change in price is the difference of old and new prices divided by the old price, or, mnemonically, it is 'new minus old divided by old'. So, given the numbers of the problem, the percent change is:

$$\frac{90-120}{120} \times 100 = -\frac{30}{120} \times 100 = -25\%$$

Since the selling price is less than the cost price, the dealer incurred a loss which is shown by the "−" sign in the answer. Since the question specifically asks for the loss, we do not need to append the "−" with the answer.

Hence, the correct option is (B).

111. The cost price for the distributor is \$120.

His selling price is 10% higher than the cost i.e. $\$\left(120 + 120 \times \frac{10}{100}\right) = \$132$.

However, he wishes to get this price of \$132 after giving a 20% discount on the listed price of the product.

Thus, if the listed price be $\$x$, then he gave $\$\frac{x}{5}$ off on the price (20% = 1/5).

Thus, his selling price would be $\$\left(x - \frac{x}{5}\right) = \$\frac{4x}{5}$.

Hence, $\frac{4x}{5} = 132 \Rightarrow x = 132 \times \frac{5}{4} = 33 \times 5 = 165$.

Hence, the listed price for the product must have been \$165.

Hence, the correct answer is (D).

Alternate approach:

We can understand from the question that the listed price must be more than the cost price for the distributor.

Thus, option (A) cannot be correct.

Again, the listed price must be more than the selling price of the product.

His selling price is 10% higher than the cost i.e. $\$\left(120 + 120 \times \frac{10}{100}\right) = \$132$.

Thus, options (B) and (C) cannot be correct either.

Hence, the correct answer is (D).

112. We know that the cost of 16 toffees is \$4.

Thus, to make 25% profit, the price at which the man must sell those 16 toffees is $\$\left(4 + 4 \times \frac{25}{100}\right) = \$5$.

So, for \$5, the man sells 16 toffees.

Thus, for \$10, the man would sell $\frac{16}{5} \times 10 = 32$ toffees.

Hence, the correct answer is (A).

113. The cost of 50 balloons if purchased in single pieces = $\$0.10 * 500 = \$50$.

To purchase the balloons in boxes, one would need to buy $\frac{500}{25} = 20$ boxes.

The cost of 20 boxes $\$2 * 20 = \$40$.

Hence, savings in cost = $\$50 - \$40 = \$10$.

Hence, the correct option is (B).

114. The cost of balloons purchased in single pieces = \$0.10.

The effective cost of each balloon if purchased in boxes = $\$\frac{2}{25} = \$0.08$.

Hence, we see that the cost of balloons is lower if purchased in boxes. Hence, we need to purchase the balloons as much as possible in boxes in order to minimize the cost.

Now, we know that each box has 25 balloons.

Hence, in order to buy 310 balloons, the maximum number of boxes that can be purchased is the quotient when we divide 310 by 25, which is 12 (since 12 boxes would give $12 * 25 = 300$ balloons).

The remaining $310 - 300 = 10$ balloons would have to be purchased in single pieces.

The cost of 12 boxes = $\$2*12 = \$24$ and the cost of 10 single balloons = $\$0.10*10 = \$1$.

Hence, the total cost = $\$24 + \$1 = \$25$.

Hence, the correct option is (C).

115. Let the cost of one pencil be $\$x$.

Thus, the cost of one pen = $\$3x$ (since a pen costs thrice as much as a pencil).

Hence, the total cost of 10 pens and 10 pencils = $\$\,(10 * x + 10 * 3x) = \$40x$.

Thus, $40x = 20 \Rightarrow x = 0.50$.

Hence, the cost of 5 pencils = $\$5 * 0.50 = \$2.50$ = 250 cents.

Hence, the correct answer is 250.

116. We know that the cost of one pencil is \$0.50.

Thus, the cost of one pen = $\$3 * 0.50 = \$1.50$ (since a pen costs thrice as much as a pencil).

Since the boy must purchase 3 pens, expense for the pens = $\$3 * 1.50 = \$4.50$.

Amount of money left with the boy = \$ (20 − 4.50) = \$15.50.

Since the cost of a pen is more than that of a pencil, the number of items would be maximized if the boy purchased only pencils with the remaining amount.

Since the cost of each pencil is \$0.50, the number of pencils he can buy = $\$\frac{15.5}{0.50} = 31$.

Hence, the boy can purchase 31 pencils. Hence, the total items he can purchase = $31+3 = 34$.

Hence, the correct answer is 34. Note that '31' as an answer is wrong since the total number of items includes '3 pens' too.

117. We know that the selling price of 7 DVDs equals the cost price of 9 DVDs.

Let us assume that the selling price of 7 DVDs = cost price of 9 DVDs = LCM of 7 and 9 = \$63.

We take the LCM, a convenient number, since we need to take a value divisible by both 7 and 9. We could have taken \$100 as well, but then the calculation would have then been difficult.

Thus, the selling price of 9 DVDs = $\$\frac{63}{7} * 9 = \$81$.

Thus, the profit made on 9 DVDs = \$ (81 − 63) = \$18.

Hence, percentage profit

$= \frac{\text{Total profit made in selling 9 DVDs}}{\text{Cost price of 7 DVDs}} * 100\% = \frac{18}{63} * 100 = 28.57\% \sim 28.6\%$

Hence, the correct answer is 28.6.

118. We know that the selling price of 9 DVDs equals the cost price of 7 DVDs.

Let us assume that the selling price of 9 DVDs = cost price of 7 DVDs

= $k*$(LCM of 7 and 9) = \$63k (where k is a constant of proportionality).

We take the LCM since we need to take a value divisible by both 7 and 9. We could have taken \$100 as well, but then the calculation would have then been difficult.

Thus, the selling price of one DVD

$= \$\frac{63k}{9} = \$7k$ and the cost price of one DVD = $\$\frac{63k}{7} = \$9k$.

Thus, the amount saved by the customer on each DVD = $\$ (9k - 7k) = \$2k$.

So, the amount saved by the customer on 10 DVDs = $\$2k * 10 = \$20k$.

Since the man saves \$30, we have: $20k = 30 => k = 1.5$.

Thus, the cost price of one DVD = $\$9 * 1.5 = \$13.5$.

Hence, the correct answer is 13.5.

119. Say, the price of one dinner is \$1, thus for 5 dinners, it would be \$5, but the restaurant will ask you to pay for only 4 dinners or \$4. This way, they get a discount of \$5 – \$4 = \$1 on \$5 bill.

=> % discount = $\frac{1}{5} \times 100\% = 20\%$

Note that the discount would be calculated on \$5 and not on \$4, else the answer would have been 25%, which is wrong.

Hence, the correct answer is 20.

120. Since the man will get 3 shirts free, he should buy only 5 shirts.

Say, the price of one shirt is \$1, thus for 5 shirts, it would be \$5, and for 8 shirts, it would be \$8. This way, he gets a discount of \$8 – \$5 = \$3 on \$8 bill.

=> % discount = $\frac{3}{8} \times 100\% = 37.5\%$

Note that the discount would be calculated on \$8 and not on \$5, else the answer would have been 60%, which is wrong.

Hence, the correct answer is 37.5.

121. If P is the price of the boat, then $P + 7\%$ of $P = 1.07P = 6,885.45$.

Thus, $P = \frac{6,885.45}{1.07} = 6435$

Hence, the correct option is (C).

122. The question asks which of the numbers given in the options cannot be a price of the item when its original price, in whole number, is raised by 10%.

Say the original price of the item is x, a whole number, thus after the raise of 10%, it would be $x + 10\%$ of $x = 1.1x$.

So, only three option can be equated to $1.1x$, resulting x a whole number. The option that results x decimal number is the correct answer.

Let us each option one by one.

A. It implies that $1.1x = 5.50 => x = \frac{5.50}{1.1} = 5$, a whole number, thus this cannot be an answer.

B. It implies that $1.1x = 6.60 => x = \frac{6.60}{1.1} = 6$, a whole number, thus this cannot be an answer.

C. It implies that $1.1x = 11.70 => x = \frac{5.50}{1.1} = 10.64$, not a whole number, thus this is the answer.

Hence, the correct option is (C).

123. The listed price of each book =\$60.

Thus, selling price of the first book = \$60.

The customer gets 10% discount on each of the remaining three books.

Thus, discount on each book = $\$(10\% \text{ of } 60) = \$\left(\frac{10}{100} * 60\right) = \$6$.

Thus, selling price of each of the remaining three books = $\$(60 - 6) = \$54$.

Thus, total amount paid by the customer = $\$\,(60 + 54 * 3) = \$\,(60 + 162) = \$222$.

Hence, the correct answer is 222.

5.2.5 Interest & Compounding

124. Let us assume that initially there are 100 bacteria (since this is a percentage based question, the initial number of bacteria does not matter and 100 is a the most convenient number).

So, initially, after one hour, the number of bacteria present = $2 * 100 = 200$.

So, initially, after two hours, the number of bacteria present would be $2 * 200 = 400$.

With the introduction of chemical agent, after two hours the same 100 bacteria would have grown in numbers to $3 * 100 = 300$. (We need to keep the time period same in both cases in order to compare the growths).

Thus, the agent reduced the growth by $400 - 300 = 100$ bacteria.

Thus, percentage reduction in growth = $\frac{\text{Reduction}}{\text{Initial value}} * 100 = \frac{100}{400} * 100 = 25\%$.

Hence, the correct answer is 25.

125. We have to calculate the interest at the end of a year based on three terms of payment.

(A) interest calculated at the end of the year at the rate of 10% per annum

The interest at the end of the year = $\$1000 \times 10\% = \$100$

(B) interest calculated at the end of every 6 months at the rate of 9% per annum

Since the interest is calculated at the end of every 6 months, it would be calculated two times for a period of 1 year.

The interest at the end of the first 6 months = $\$1000 \times \frac{9}{2}\% = \$45$ (6 month rate of interest would be $\frac{9}{2}\%$)

Now the sum for the next 6-month would be $\$1000 + 45 = \$1045$

The interest at the end of the first 6 months = $\$1045 \times \frac{9}{2}\% = \$47$

Thus, the total interest = $\$45+\$47=\$92 < \$100$ (interest from option A)

(C) interest calculated at the end of every 3 months at the rate of 8% per annum

Since the interest is calculated at the end of every 3 months, it would be calculated four times for a period of 1 year.

Since calculating interest 4 times would be time-consuming, we apply formula of compounding.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

where A = Amount after n periods of interest, P = sum invested, and r = rate of interest

Thus, interest = $A - P$

As per the term of payment,

$$A = 1000\left(1 + \frac{2}{100}\right)^4;$$

(3 month rate of interest would be $\frac{8}{4}$ = 2%; n = 4, as the interest would be calculated 4 times in a year)

$A = 1000\left(1 + \frac{2}{100}\right)^4 = 1000 \times 1.02^4 = 1082$

Thus, interest = 1082 – 1000 = \$82 < \$100 (interest from option A)

Thus, option A yields the most interest.

Hence, the correct option is A.

126. The gain or the loss at the end of the year = 2 half-yearly payments – 1 annual payment

Annual payment = $20000 \times 6\frac{1}{4}\% = \frac{20000 \times 25}{100 \times 4}$ = \$1250

2 half-yearly payments = $2 \times \left[20000 \times (\frac{6}{2})\% = 20000 \times 3\% = \frac{20000 \times 3}{100}\right]$ = \$1200

Thus, loss = 1250 – 1200 = \$50.

Note that: The question does not state that the interest calculation as per the new term is compounded.

Hence, the correct option is (B).

127. Let the phone price be x.

First offer:

Effective price of the phone = 20% of x + 5 $*$ 20% of x

=> $0.2x + 5 * 0.2x = 0.2x + x = 1.2x$.

Effective price of the phone =$1.2x$.

Second offer:

Effective price of the phone = 10% of x + 12 $*$ 10% of x

=> $0.1x + 12 * 0.1x = 0.1x + 1.2x = 1.3x$.

Effective price of the phone =$1.3x$.

The phone is cheaper by the second offer by

$$= \left[\frac{\text{Eff. price from II offer} - \text{Eff. price from I offer}}{\text{Eff. price from I offer}}\right] \times 100\%$$

$[\frac{1.3x - 1.2x}{1.2x}] \times 100\% = \frac{0.1x}{1.2x} \times 100\% = 8.33\%$

Hence, the correct option is (B).

128. Let the initial bacteria count be 1. We want the time when the bacteria count would be more than 1.5*1 = 1.50.

Or, Bacteria count (then) > 1.5× Bacteria count (now)

Given that the count is growing at 10% per minute.

After one minute, the bacteria count would be $1 * (1 + 10\%) = 1 * 1.1 = 1.1$.

Similarly, after two minutes, the bacteria count would be $1 * (1.1)^2 = 1.21 < 1.50$, but it is less than the desired value, 1.5.

Again, after four minutes, the bacteria count would be $1 * (1.1)^4 = 1.46 < 1.50$, still less than 1.5.

Again, after five minutes, the bacteria count would be $1 * (1.1)^5 = 1.61 > 1.50$.

So, we can conclude that during the interval of 4-5 minutes, the count would be more than 1.5 times the initial count.

Hence, the correct option is (C).

5.2.6 Ratio & Proportion

129. If we want to use 13.5 liters of red paint, the amount of yellow paint required = $\frac{5}{3}\times 13.5 =$ 22.5 liters.

However, we only have 18 liters of yellow paint. So, we cannot use the entire 13.5 liters of red paint.

If we want to use 18 liters of yellow paint, the amount of red paint required = $\frac{3}{5} \times 18 =$ 10.8 liters.

Since we have 13.5 liters of red paint, we can use the entire 18 liters of yellow paint.

So, total quantity of dye = 10.8 (red) +18 (yellow) = 28.8 liters.

Hence, the correct option is (C).

130. The ratio of the ingredients, i.e. Oil : Fragrance: Water = 1 : 8 : 3.

Thus, fragrance constitutes $\frac{8}{1+8+3} = \frac{8}{12} = \frac{2}{3}^{\text{rd}}$ of the total volume of the perfume.

=> The volume of the perfume = $\frac{3}{2}*$(volume of fragrance present).

We know that the volume of fragrance used = 60 ml.

Thus, total volume of the perfume = $\frac{3}{2} * 60 = 90$ ml.

Hence, the correct option is (D).

131. We know that the lengths of three routes R, S, and P is in the ratio 3 : 4 : 3.

Let the lengths of the routes R, S, P be $3k,\ 4k,\ 3k$ respectively, where k is a constant of proportionality.

We know that the amount of fuel spent by the SUV, sedan and bike is in the ratio 6 : 2 : 3.

Let the amount of fuel spent by the SUV, sedan and bike be $6m,\ 2m,\ 3m$ respectively, where m is a constant of proportionality.

Thus, the ratio of the fuel efficiency, i.e. fuel consumed per unit distance traveled by the SUV, sedan and bike= $\frac{6m}{3k} : \frac{2m}{4k} : \frac{3m}{3k} = \frac{2m}{k} : \frac{m}{2k} : \frac{m}{k}$.

The constants $\frac{m}{k}$ can be cancelled from the ratio resulting in the required ratio to be $2 : \frac{1}{2} : 1$.

To convert the ratio to integer values, we multiply the terms of the ratio with 2 making the ratio $2 * 2 : \frac{1}{2} * 2 :\ 1 * 2 = 4 : 1 : 2$.

Hence, the correct option is (A).

Alternate solution:

Since the ratio of the lengths of R, S, P are 3 : 4 : 3, we can, without loss of generality assume the lengths to be 3, 4, 3 miles respectively.

Also, since the amount of fuel spent is in the ratio 6 : 2 : 3, we can, without loss of generality assume the amounts to be 6, 2, 3 liters respectively.

Thus, the ratio of the fuel efficiency i.e. fuel consumed per unit distance by the SUV, sedan and bike

$$= \frac{6}{3} : \frac{2}{4} : \frac{3}{3} = 2 : \frac{1}{2} : 1 = 4 : 1 : 2.$$

Hence, the correct option is (A).

132. Let x liters of the mixture be replaced with water.

Since the mixture has 80% milk, percentage of water = 100 − 80 = 20%.

Thus, quantity of water in the mixture = 20% of 40 = 8 liters.

The removed mixture of x liters also has 80% milk and 20% water.

Thus, quantity of water in x liters = 20% of $x = 0.2x$ liters.

Thus, quantity of water left when x liters of the mixture was removed = $(8 - 0.2x)$ liters.

Quantity of water added = x liters.

Thus, after addition, total volume of water = $(8 - 0.2x) + x = (8 + 0.8x)$ liters.

Also, total volume of the mixture remains at 40 liters (since we removed x liters and added back x liters).

The final volume of water now constitutes $(100 - 50) = 50\%$ of the total mixture volume.

Thus, we have: $8 + 0.8x = 50\%$ of 40 => $8 + 0.8x = 20$ => $0.8x = 12$

$$\Rightarrow x = \frac{12}{0.8} = 15.$$

Thus, the volume of mixture replaced = 15 liters.

Hence, the correct option is (B).

133. Let x liters of water be added to the existing mixture.

Since the mixture has 80% milk, percentage of water = 100 − 80 = 20%.

Thus, quantity of water in the mixture initially = 20% of 40 = 8 liters.

Quantity of water added = x liters.

Thus, after addition, total volume of water = $(8 + x)$ liters.

Also, total volume of the mixture = $(40 + x)$ liters.

The final volume of water now constitutes $(100 - 50) = 50\%$ of the total mixture volume.

Thus, we have: $8 + x = 50\%$ of $(40 + x) \Rightarrow 8 + x = 20 + 0.5x \Rightarrow 0.5x = 12$

$$\Rightarrow x = \frac{12}{0.5} = 24.$$

Thus, the volume of water added = 24 liters.

Hence, the correct option is (B).

Alternate approach:

Since the mixture has 80% milk, quantity of milk = 80% of 40 = 32 liters.

Since water was added to the mixture, the quantity of milk remained the same.

Thus, finally, this volume of milk constitutes 50% of the total mixture volume.

Hence, we can say that: 50% of (final volume of the mixture) = 32

$\Rightarrow$ Final volume of the mixture = $32 * \frac{100}{50}$ = 64 liters.

The volume has increased since we added water to the mixture.

Thus, the volume of water added = 64 − 40 = 24 liters.

134. Let us assume that the total volume of the special container be 100 ml (since this is a percentage based question, 100 is convenient number; assuming any initial value will have no effect on the answer).

Thus, initial quantity of the chemical = 80% of 100 = 80 ml.

When 40% of the mixture is removed, remaining 100 − 40 = 60% of the mixture is left.

Thus, quantity of the chemical left = 60% of 80 = 48 ml.

Since only water is added to the remaining mixture, thereby making the mixture 100 ml., the quantity of chemical does not change.

Thus, finally, there is 48 ml chemical in 100 ml of the mixture.

Hence, percentage of chemical present = $\frac{48}{100} * 100 = 48\%$.

Hence, the correct option is (D).

5.2.7 Combinatorics

135. For a 3-digit number to be even, its unit digit must be either 2 or 8.

Number of ways, unit place can be filled = 2 (either 2 or 8);

Number of ways, tenth place can be filled = 4; given that the repetition is allowed.

Number of ways, hundredth place can be filled = 4;

So, total number of ways = 2.4.4 = 32 ways.

Hence, the correct option is (C).

136. Since the letters and the digits can appear in any order, we must club them, thus we have to make a selection of out of 26 letters + 10 digits = 36 symbols.

Number of ways, each symbol can be chosen = 36;

So, total number of ways of choosing 3 symbols from 36 symbols for three places = 36^3;given that repetition is allowed.

Hence, the correct option is (D).

137. Two '3s', appearing together, can be placed in the following ways in 5-digit numbers: 33XXX, X33XX, XX33X, and XXX33. Since '3' cannot appear again, we have 9 choices for each vacant place out of digits 0-9 (excluding the digit '3').

But wait! A 5-digit number cannot begin with a digit '0', so for the 5-digit numbers that do not begin with '3' will have only 8 number of ways to its fill ten-thousandth place.

For numbers- X33XX, XX33X, and XXX33, total number of ways = $3.8.9^2 = 24.9^2$

For number- 33XXX, total number of ways = 9^3

Hence, total number of ways = $9^3 + 24.9^2$

Hence, the correct option is (D).

138. A triangle can be drawn taking any 3 vertices of a polygon.

So, the number of triangles = $C_3^7 = \frac{7.6.5}{1.2.3} = 35$

The correct answer is option C.

139. Since one specific individual must be in the team, hence the question becomes

"In how many 2-member teams can be formed out of 5 individuals provided that one specific individual must be in the team?"

Number of ways of choosing 2 members from 5 individuals = $C_2^5 = \frac{5.4}{1.2} = 10$.

The correct answer is option B.

140. Number of ways of choosing only boys out of 8 boys = $C_2^8 = \frac{8.7}{1.2} = 28$

Number of ways of choosing only girls out of 10 girls = $C_2^{10} = \frac{10.9}{1.2} = 45$

Hence, the total number of ways = 28 + 45 = 73; Since this is a case of 'OR'. we would add the number of ways.

The correct answer is option B.

141. We know that if there are total n objects, out of these there are p number of one type of identical objects, q number of another type of identical objects, and r number of another type of identical objects, then the number of ways these objects can be arranged in a row to have unique patterns = $\frac{n!}{p!.q!.r!}$

There are a total 6 letters in BANANA; so $n = 6$, and there are 3 As and 2 Ns, so $p = 2$, and $q = 2$.

Thus, the answer is $\frac{6!}{3!.2!} = 10$.

The correct answer is option B.

142. The number of ways of choosing 2 girls out of 12 students = $C_2^{12} = \frac{12.11}{1.2} = 66$.

Similarly, the number of ways of choosing 2 boys out of 10 students = $C_2^{10} = \frac{10.9}{1.2} = 45$.

Since this is a case of 'OR', we must add the number of ways, thus the answer is $66 + 45 = 111$.

The correct answer is 111.

143. We know that if there are total n objects, out of these there are p number of one type of identical objects, q number of another type of identical objects, and r number of another type of identical objects, then the number of ways these objects can be arranged in a row to have unique patterns = $\frac{n!}{p!.q!.r!}$

There are a total 6 letters in BANANA; there are 3 As and 2 Ns; since two 'Ns' have to be together, let's consider them as one letter, thus, we have so $n = 6 - 1 = 5$, and $p = 3$.

Thus, the answer is $\frac{5!}{3!} = 20$.

The correct answer is 20.

144. There are a total of 5 symbols and we have take all of them together to form codes, considering that the □ must NOT be at the beginning and the ◊ must NOT be at the end of the codes.

Number of ways 5 symbols can be arranged = $P_5^5 = 5! = 120$. We will deduct the number of codes which starts the □ with and ends with the ◊; thus this asks for the arrangement of 3 symbols.

Number of ways 3 symbols can be arranged = $P_3^3 = 3! = 6$.

Thus, the desired number of codes = $120 - 6 = 114$.

The correct answer is 114.

5.2.8 Probability

145. Probability of an event = $\frac{\text{Number of favorable events}}{\text{Number of total events}}$;

To get the number of ways, we must NOT get both the green balls, we would deduct the number of ways we get both the balls being green upon drawing two balls from the total number of ways, we can get two balls of any color.

Here, number of unfavorable events = number of ways 2 green balls can be selected from 10 balls = C_2^{10};

Number of total events = number of ways 2 balls can be selected from 30 balls = C_2^{30} ;

Hence, probability of NOT getting both the green balls

$= 1 - \frac{C_2^{10}}{C_2^{30}} = 1 - \frac{10.9/1.2}{30.29/1.2} = 1 - \frac{3}{29} = \frac{26}{29}$;

The correct answer is option D.

146. Since the coin is fair, it means that probabilities of getting a 'Head' or getting a 'Tail' upon a single toss are equal = 1/2.

Traditional approach of solving this question would be to find the probability for each time the coin turns 'Tail' and then add them, but that is a lengthy process.

An efficient approach would be to find out the probability of not getting a 'Tail' EVEN once, and then deducting it from total probability (1); this will assure that we get the 'Tails'—one time, two times, three, and all four times.

So, probability of getting at least one Head = 1 – (Probability of getting 'Heads' in all 4 tosses)

$= 1 - \left(\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}\right) = 1 - \frac{1}{16} = \frac{15}{8}$.

The correct answer is option D.

147. If we solve this question through the traditional approach, it would be time-consuming as the problem can be considered solved if all three solved it, any two of them solved it, and any one of them solved it, and then we will have to add the probabilities.

An efficient approach would be to calculate the probability of not solving the problem (it can be done in now way only when one of them can solve it), and then deduct it from the total probability (1).

So, probability of not solving the problem = P(Jack could not solve it) *P(Parker could not solve it) * P(Herman could not solve it)

=> $(1 - \frac{4}{5}).(1 - \frac{2}{3}).(1 - \frac{3}{4}) = \frac{1}{5}.\frac{1}{3}.\frac{1}{4} = 1/60.$

=> Thus, the probability of solving the problem = 1 − 1/60 = 59/60.

The correct answer is option C.

148. Total Number of actors = 5;

The condition implies that Jack and Steve must NOT be together in the selection.

Considering that Jack is chosen and Steve is left out, then the number of selections = $C_{(3-1-1)}^{(5-1-1)} = C_1^3 = 3.$

Again, considering that Steve is chosen and Jack is left out, then the number of selections = $C_{(3-1-1)}^{(5-1-1)} = C_1^3 = 3.$

Total number of favorable selections = 3 + 3 = 6.

Thus, the desired probability = $\frac{6}{C_3^5}$ = 3/5;

The correct answer is option B.

149. Number of silver coins = p; Number of gold coins = q; so total umber of coins = $p + q$;

The probability of getting 1st coin as silver coin as well as 2nd coin as silver coin = P (silver coin) * P (silver coin)

⇒ P (silver coin) = $\frac{C_1^p}{C_1^{p+q}} = \frac{p}{p+q}$

After 1st draw, there would be only $p + q - 1$ number of coins left.

⇒ So, P (silver coin) = $\frac{C_1^{p-1}}{C_1^{p+q-1}} = \frac{p-1}{p+q-1}$

⇒ Hence, the probability of getting 1st coin as silver coin as well as 2nd coin as silver coin = $\left(\frac{p}{p+q}\right)\left(\frac{p-1}{p+q-1}\right) = \frac{p(p-1)}{(p+q)(p+q-1)}$

The correct answer is option B.

You could have solved this question by picking some values for p and q. Say $p = 2$ and $q = 4$; after solving, plug in the values of p and q in the options, and cross check which option matches the derived value. The derived value is 1/15 and matches with option B.

150. Between 100 and 400, there are 400–100+1 = 301 numbers. The first few numbers divisible by '3' would be 102, 105, 108, ... and the few last ones would be .... 393, 396, 399. Similarly, the first few numbers divisible by '5' would be 100, 105, 110, ... and the few last ones would be .... 390, 395, 400. As you see that the number '105' is common, so we must take this into account.

The best way to exclude the duplicate numbers is to find the multiples of LCM of 3 & 5 = 15, thus the first few numbers divisible by '15' would be 105, 120, 135 .... and the few last ones would be .... 360, 375, 390.

Total numbers divisible by '3' between 100 & 400 = $\left[\frac{\text{Last \# – First \#}}{3} + 1\right]$ = $\frac{(399 - 102)}{3} + 1 = 100$ numbers.

Total numbers divisible by '5' between 100 & 400 = $\left[\frac{\text{Last \# – First \#}}{5} + 1\right]$ = $\frac{(400 - 100)}{5} + 1 = 61$ numbers.

Total numbers divisible by '15' between 100 & 400 = $\left[\frac{\text{Last \# – First \#}}{15} + 1\right]$ = $\frac{(390 - 105)}{15} + 1 = 20$ numbers.

Thus, total numbers divisible either by '3' or by '5' = 100 + 61 – 20 = 141.

So, the probability of winning a gift = $\frac{141}{\text{Total number of coupons}} = 141/301$

The correct answer is option B.

151. Say the number of green balls are x, so the total number of balls = $(5 + x)$;

Number of ways both the ball drawn will be green = C_2^x;

Total number of ways 2 balls can be drawn = C_2^{5+x};

Hence, the probability of getting both the balls being green =

$$\frac{C_2^x}{C_2^{(5+x)}} = \frac{x(x-1)/1.2}{(5+x).(4+x)/1.2} = \frac{1}{6} \text{ (given)};$$

This reduces to $\frac{x(x-1)}{(5+x).(4+x)} = \frac{1}{6}$;

We do not recommend you to solve the quadratic equation to get the value of x, you must plug in the values of x from the options and check if right hand side and left hand side are equal. If yes, the option is the correct answer.

Trying with $x = 4$ will satisfy the equation as $\frac{4.3}{9.8} = \frac{1}{6} \Rightarrow \frac{1}{6} = \frac{1}{6}$.

The correct answer is option B.

152. Probability of hitting the triangle = $\frac{\text{Area of the triangle}}{\text{Area of the triangle \& areas of three semi-circles}}$

Area of the equilateral triangle = $\frac{\sqrt{3}}{4}\text{side}^2 = \frac{\sqrt{3}}{4}.4^2 = 4\sqrt{3}$

Area of the three semi-circles = $3(\frac{1}{2}.\pi r^2) = \frac{3}{2}.2^2.\pi = 6\pi$; Radius = half of side = 4/2 = 2 cm.

Total area of dartboard = $4\sqrt{3} + 6\pi$

Thus, the probability of hitting the triangular part = $\frac{4\sqrt{3}}{4\sqrt{3} + 6\pi}$

The correct answer is option D.

153. Understand the following table.

Sum	**Ways**	**Number of ways**	**Frequency distribution & Probabilities**
2	{1,1}	1	$p = \frac{1}{36}$
3	{2,1}, {1,2}	2	$p = \frac{1}{18}$
4	{3,1}, {2,2}, {1,3}	3	$p = \frac{1}{12}$
5	{4,1}, {3,2}, {2,3}, {1,4}	4	$p = \frac{1}{9}$
6	{5,1}, {4,2}, {3,3}, {2,4}, {1,5}	5	$p = \frac{5}{36}$
7	{6,1}, {5,2}, {4,3}, {3,4}, {2,5}, {1,6}	6	$p = \frac{1}{6}$
8	{6,2}, {5,3}, {4,4}, {3,5}, {2,6}	5	$p = \frac{5}{36}$
9	{6,3}, {5,4}, {4,5}, {3,6}	4	$p = \frac{1}{9}$
10	{6,4}, {5,5}, {4,6}	3	$p = \frac{1}{12}$
11	{6,5}, {5,6}	2	$p = \frac{1}{18}$
12	{6,6}	1	$p = \frac{1}{36}$
	Total number of ways	**36**	

Memorise...

1. If Sum $\leq$ 7, then Number of ways = Sum – 1

2. If Sum $\geq$ 8, then Number of ways = 13 – Sum

We have to calculate the probability of getting the sum > 10. It would be

=> P(SUM > 10) = $\frac{n\{\text{\# of ways of getting a sum of 11 or 12}\}}{36} = \frac{2+1}{36} = 1/12$

The correct answer is 1/12.

154. We know that the probability of getting a ‘Head’ or a ‘Tail’ on any toss for a coin = 1/2.

We want Tail, Tail, and Head (in this order) on three tosses,

so the probability of getting TTH = (1/2)*(1/2)*(1/2) = 1/8.

The correct answer is 1/8.

This question is different from a question asking for getting two Tails and a Head. In this case, one can get two Tails in I & II, or in I & III, or in II & III positions. The answer would have been 3*1/8 = 3/8.

155. P(I > 2, II > 3, III > 4) = P(I > 2)*P(II > 3)*P(III > 4)*P(III > 5) = (4/6)*(3/6)*(2/6)*(1/6)= 1/54

The correct answer is 1/54.

156. Probability of hitting the black parts = $\frac{\text{Area of the black parts}}{\text{Area of the outermost circle}}$

Area of the outermost circle = $\pi r^2 = 4^2.\pi = 16\pi$

Area of the innermost black circle (One of the black parts) = $\pi r^2 = 1^2.\pi = \pi$

Area of the -cm band (another black part) = $3^2.\pi - 2^2.\pi = 5\pi$

Total area of black parts = $\pi + 5\pi = 6\pi$

Thus, the probability of hitting the black parts = $\frac{6\pi}{16\pi} = 3/8$

The correct answer is 3/8.

5.2.9 Statistics

Questions on Average (Arithmetic Mean)

157. We know that arithmetic mean of x and y is 80,

=> $(x + y)/2 = 80$

=> $x + y = 160 - - - - - (1)$

Also, we know that arithmetic mean of x and z is 200,

=> $(x + z)/2 = 200$

=> $x + z = 400 - - - - - (2)$

To get the value of $(z - y)/8$, we subtract equation (2) from the equation (1) and get,

=> $z - y = 400 - 160 = 240$;

=> $(z - y)/8 = 240/8 = 30$

Hence, the correct option is (C).

158. Say the three numbers are $a, 7,$ and $b; a$ being the largest and b being the smallest.

=> $(a + 7)/2 = 8$ => $a + 7 = 16$ => $a = 9$

Similarly, $(a + 7 + b)/3 = 6$

=> $9 + 7 + b = 18$

=> $b = 2$

So, the numbers are: 9, 7, & 2

So, the mean of the two smallest numbers = $(7 + 2)/2 = 4.5$

Hence, the correct option is (C).

159. Given,

Average = 7.5

=> Sum of 6 numbers = 6*average = 6*7.5 = 45

Say the number multiplied by 4 is x, so instead of x, the number becomes $4x$. Or the sum of the numbers increases by $3x$.

=> New sum = $45 + 3x$

Given that new average = $7.5 - 0.5 = 7$

=> New sum = 6* new average = 6*7 = 42

=> $45 + 3x = 42$

=> $x = -3$

Hence, the correct option is (A).

Note that the question stem stated that after multiplying a number by 4, the average decreased rather than anticipated—increased, it implies that the number must be negative, thus only options A or B could be true.

160. **Traditional approach**:

Sum of scores of 5 tests = 5 * 38 = 190;

Say the score on the 6^{th} test is x, so Sum of scores of 6 tests = 190 + x;

=> Average score = $\frac{(190 + x)}{6}$;

=> $41 = \frac{(190 + x)}{6}$; given that the average is 41.

=> $x = 56$

Hence, the correct option is (D).

Alternate approach:

You find that to increase the average score of 38 to 41, you need to increase it by 3. Since '3' is an average increase per test, so to get the total increase, you must multiply it by the total number of tests i.e. 6.

=> Total increase = 3*6 = 18;

=> Desired score in the 6^{th} test = 38 + 18 = 56

161. **Traditional approach**:

Total rainfall in first x days = $32x$;

Total rainfall in $(x + 1)$ days = $(32x + 56)$;

=> Average rainfall in $(x + 1)$ days = $\frac{(32x + 56)}{(x + 1)}$

=> $36 = \frac{(32x + 56)}{(x + 1)}$; given that 'average rainfall in $(x + 1)$ days' = 36 mm

=> $36(x + 1) = (32x + 56)$

=> $36x + 36 = 32x + 56$

=> $4x = 20$

=> $x = 5$ days

Hence, the correct option is (C).

Alternate approach:

Given that increase in average = 36 – 32 = 4;

And difference of average of x days, 32 and today's rainfall, 56 = 56 – 32 = 24;

=> $24 = 4 * (x + 1)$

=> $x = 5$ days

162. If all the elements of a set are multiplied by a constant, the mean also gets multiplied by the same constant.

However the mean remaining unchanged can happen only and only if mean = 0, so the statement I is true.

Since sum of all the elements = Mean * # of elements

Sum of all the elements = 0, as mean = 0, so the statement II is true.

Statement 3 is also not necessary because the set may contain all zeros.

Hence, the correct option is (D).

Questions on Median

163. First, arrange the set X in an ascending order, we get: $-8, -2, 4, 6, 23$

=> Median = 4 (Middle-most term)

=> Median of set Y = 4

Arranging set Y in ascending order, we get: $-8, -2, 0, 8, 12, 13$; excluding y as we do not know its value. We know that set Y has an odd number of terms, so there would be a unique term, equaling median value (here 4). Since there is no term, equaling 4, hence only y can be 4.

Hence, the correct option is (D).

164. Since there are odd number (7) of students, the median student would be $(7+1)/2 = 4^{th}$ student, and the corresponding height of the student would be the median height of seven students, when the bars are arranged in an ascending order, taking height into consideration.

It is not wise to read the value of each column and jot down on the scratch pad and then after tagging them 1, 2, 3, ..., 11 in ascending order, pick up the 5^{th} ranked value a median; it would be time-consuming.

Arranging the students as per their heights: $4'3'', 4'6'', 4'9''$, **4'9"**, $5', 5'9'', 5'9''$. Since **4'9"** is the middle-most, hence the student representing **4'9"** is the answer: Kim & Mike. Since Mike is not among options, Kim is the answer.

Hence, the correct option is (C).

165. We know that the Average (Mean) =$\dfrac{\text{Sum of heights}}{\text{Number of students}}$

Average (Mean) = $\dfrac{4'3'' + 4'6'' + 4'9'' + 4'9" + 5' + 5'9'' + 5'9''}{7}$

=>$\dfrac{31'45''}{7} = \dfrac{31' + \frac{45}{12}}{7}$; $12'' = 1'$.

=> $\dfrac{31' + 3'(\frac{9}{12})}{7} = \dfrac{34.75'}{7} = \dfrac{34.75 \times 12 \times 2.54}{7}$ centimeter; $1' = 12'' \times 2.54$ cm.

=> Average (Mean) =$\dfrac{34.75 \times 12 \times 2.54}{7} = 151.31 =\approx 151$ centimeter.

Hence, the correct answer is 151.

166. Range = Highest Value – Smallest Value

Thus, Range of heights = $5'9'' - -4'3'' = 1'6'' = 1 * 12 + 6 = 18''$.

Hence, the correct answer is 18.

Note that 18/12 = 1.5 feet is a wrong answer. The answer needed is in inches.

167. **Approach 1**:

Median of all consecutive integers being a non-integer implies that the number of consecutive integers, X is an even number.

The median, 50.5 is derived by taking the average of 50 & 51.

The list looks like this, N1, N2, N3, ..., 50, 51, ...;

Since X is even, X/2 integers will be on the left of the median, and X/2 integers will be on the right of the median, 50.5. Counting of the smaller integer must start from 51 and we must deduct X/2 to reach the smallest, thus the smallest integer = 51 – X/2.

Hence, the correct option is (A).

Approach 2:

Since we already concluded that X is an even number, we can assume X.

Say X = 4. So, the list would be:

49, 50, 51, 52; smallest being 49

Only option A, 51 – X/2 = 51 – 4/2 = 49 is applicable.

168. Since there are odd number (11) of years, the median year would be $(11 + 1)/2 = 5^{th}$ year, and the corresponding number of unemployed would be the median number of unemployed, when the columns are arranged in ascending order, taking height into consideration.

It is not wise to read the value of each column and jot down on the scratch pad and then after tagging them 1, 2, 3, ..., 11 in ascending order, pick up the 5^{th} ranked value a median; it would be time-consuming.

Better approach would be to visually pick the 5^{th} ranked bar as per the height of the bars, and then read the corresponding value—median.

Looking at the chart, we find that there are as many as 6 bars whose values are more than 17500 (Y-axis). Among them, the smallest in height (5^{th} ranked column) would be the answer. However we find that it is difficult to pin-point which of the two columns, for years 2003 or for 2011 is the smallest. Well there is no need to be very particular about it as the questions asks for the closest value of median, and does not ask for the median year.

Read the data of year 2003 and year 2011.

The least count of Y-axis is 2500; we can approximate the values of year 2003 and year 2011 to be 18000. So the median number of unemployed equals 18000.

Hence, the correct option is (D).

Questions on Average and Median

169. Given that,

Median defect per batch = Average defect per batch; for all the nine batches;

Let us first deduce the median.

By arranging the eight values of defects in ascending order, we get;

$2, 3, 3, 4, 5, 6, 7, 8$

From the options, if the # of defects in the ninth batch is 2, 3, or 4, the median would be 4—Middle-most value.

However if it is either 5, 6, 7 or 8, the median would be 5—Middle-most value, so we are not sure about the value of median, but it would be either 4 or 5.

Let us calculate the average now. It must be either 4 or 5 as it is given that average = median.

Say the # of defects in the ninth batch is x,
=> Average = $(2 + 3 + 3 + 4 + 5 + 6 + 7 + 8 + x)/9 = (38 + x)/9$;
=> $(36 + 2 + x)/9$
=> $4 + \frac{(2+x)}{9}$

$\frac{(2+x)}{9}$ must be 0 or 1, thus x must be – 2 or 7.

Since # of defects cannot be negative, so $x = 7$.

Hence, the correct option is (D).

170. Mode of any data series is the value of the characteristic that occurs most often among all the values. Here, the maximum number of students for a particular score is 12, thus the modal score representing those 12 students = 5.

Hence, the correct option is (B).

171. We know that the Average (Mean) = $\frac{\text{Sum of heights}}{\text{Number of students}}$

=> $\frac{\text{Score 0 * \# of students scoring 0 + Score 1 * \# of students scoring 1 .... + Score 10 * \# of students scoring 10}}{71}$

=> $\frac{(0*3+1*4+2*6+3*7+4*8+5*12+6*11+7*9+8*6+9*3+10*2)}{71}$

$= \frac{353}{71} = 4.97.$

Hence, the correct answer is 4.97.

Questions on Range

172. We do not know the values of a, b, & c. So,

Minimum value of range = $76 - (-43) = 119$; if any of a, b, & c is greater than 76 and/or any of a, b, & c is smaller than – 43, the range would be greater than 119.

Since range is always +ive, so option I is ruled out.

Option II is ruled out as its value is less than 119, we know that the minimum value of range should be 119.

Hence, the correct option is (C).

Questions on Standard deviation

173. Adding or subtracting a constant number from all the elements of a set does not affect the standard deviation (SD) of the set. We see that all the set—P, Q and R can be obtained by adding or subtracting a constant from the elements of the given set X, hence all the sets have equal standard deviations (SD).

Hence, the correct option is (D).

Note that, if you multiply each element of a set by a constant ‘m’, the SD would be m times the original SD. Say, each element of the set is multiplied by 3, then the SD would be 3 times the original SD = 3SD.

Similarly, if you divide each element of the set by a constant ‘m’, the SD would be 1/m times the original SD = SD/3.

Multiplying with or dividing by a negative number does not affect SD’s sign. SD is always a positive number.

174. Adding or subtracting a constant from each element in the set has no effect on standard deviation; however multiplication and division does change SD.

Approach 1:

Looking at options A, B, & C, we find that a constant term is NOT added to or NOT subtracted from each element, this implies that SD of any of the set would not be the same as the SD of set X. So only option left is: D—correct answer.

Though at first sight, it does not look that option D satisfies the criterion, expanding each element will do the trick.

Option D. $\{p + qr, q(1 + r), r(q + 1)\}$

I element: $p + qr$
II element: $q(1 + r) = q + qr$
III element: $r(q + 1) = r + qr$

You must have observed that each element is derived by adding a constant ‘qr‘ to each element of set X. This implies that SD of the set would be the same as that of set X.

Hence, the correct option is (D).

Approach 2:

Say, $p = 1, q = 2$ & $r = 3$. No need to calculate SD.

Let us analyze the options by plugging in the values of p, q & r. The values would be as follows.

A. 2, 2, 3
B. 1, 3, 3
C. 1, 2, 4
D. **7, 8, 9**: Constant ‘6’ is added to $\{p = 1, q = 2, r = 3\}$, implying same SD.
E. 0, 2, 3

175. Since SD is always +ive, so option I & II are ruled out.

SD for the given set CANNOT be ‘0’ as there are deviations among elements. For SD to be ‘0’, all the elements of a set must be equal.

Hence, the correct option is (D).

176. We know that average number of stickers $= \frac{\text{total number of stickers}}{\text{number of friends}}$

Thus, total number of stickers = (number of friends) $*$ (average number of stickers)

Since the average number of stickers with the five friends was 15, the total number of stickers with them = $5 * 15 = 75$.

The maximum number of stickers with any friend = 18.

In order that one friend should have the minimum number of stickers, we need to maximize the number of stickers with the remaining three friends (since one of them already has 18 stickers).

Thus, we assume that the number of stickers with three of the friends is 17, 16 and 15 (since the number of stickers with any friend needs to be different, we cannot assume that the three friends have 17 stickers each).

Thus, the number of stickers with the last friend = $75 - (18 + 17 + 16 + 15) = 75 - 66 = 9$.

Hence, the minimum number of stickers with any friend = 9.

Hence, the correct option is (C).

5.2.10 Data Analysis

177. From the bar chart, we know that the total government spending is $200+500+100+200 = 1000$ million dollars.

From the pie chart, total government spending was 20% of GDP. So, the GDP was $\frac{1000}{20\%} = 5000$ million dollars. Since the government's cost of operation was 200 million, it was $\frac{200}{5{,}000} \times 100\% = 4\%$ of the total GDP.

Hence, the correct option is (A).

178. From the question above, total GDP is 5000 Million dollars, so the dollar value of Management Consulting Services is 3% of $5,000 = 3 \times \frac{5{,}000}{100} = 150$ Million dollars, and Services account for the 25% or one-forth of the total GDP, or $\frac{5{,}000}{4} = 1,250$ Million dollars. So, the fraction of Management Consulting Services in the Services is $\frac{150}{1{,}250} = 0.12$, or 12%.

Hence, the correct option is (C).

179. The total advertising cost for 4 weeks = 10% of (Average length of a data request per week* Total number of data request for 4 weeks) $= 10\% \times 4 \times (200000 + 20000 + 15000 + 5000) = 10\% \times 4 \times 60000 = 24,000$ dollars.

Hence, the correct option is (D).

180. From the chart we know that Michelle's spending on travel is 19%, thus Michelle's total annual dollar spending $= \frac{100}{19} \times 38,000 = 200,000$. We know that Michelle's spending on clothing is 15%, thus her spending on clothes was 15% of $200,000 = \$30,000$.

Hence, the correct option is (D).

The following table presents quarter-wise sales and inventory for Apex Corporation for 10 steel products, in metric tons (MT). The following two questions are based on it.

S - Sales (MT); I - Inventory (MT)

Products	Quarter I		Quarter II		Quarter III		Quarter IV		Yearly Total	
	S	I	S	I	S	I	S	I	S	I
CR pipe	24	12	15	8	4	8	13	4	56	32
CR sheet	16	8	12	12	16	4	15	21	59	45
CR tube	15	15	16	8	6	15	8	4	45	42
CR wire	15	0	24	8	18	13	13	12	70	33
HR pipe	24	12	21	8	5	21	12	15	62	56
HR sheet	18	15	15	16	0	0	14	3	47	34
HR tube	8	9	8	21	8	0	9	2	33	32
HR wire	24	26	12	16	15	15	8	4	59	61
HS Billet	21	24	21	0	18	5	15	12	75	41
MS Billet	10	8	15	0	18	2	15	16	58	26
Total	**175**	**129**	**159**	**97**	**108**	**83**	**122**	**93**	**564**	**402**

181. The average sale will be calculated by dividing the total quarter-wise sale by the # of products (10), and then we can decide which quarter is the correct answer.

$$\text{Average} = \frac{\text{Total sale in a quarter}}{10}$$

Since the denominator for each quarter is a constant (10), only the numerator will determine the average; the quarter, which has the highest 'total sale' would be the answer. Here, quarter I's total sale = 175 MT (see the last row of the table) is highest among all quarters.

Hence, the correct option is (A).

182. We have to select only one statement which is correct.

Let us analyze each statement one by one.

(A) **The statement:** In quarter I, there is only one product that has the highest sale.

This is **incorrect** because in quarter I, there are as many as 3 products, and not only one, that have the highest sale (24 MT): CR pipe, HR pipe, & HR wire.

(B) **The statement:** In quarter II, for 'HS billet' and 'MS billet', the ratio of 'sale to inventory' is 0.

This is **incorrect** because in quarter II, for 'HS billet' and 'MS billet', the ratio of 'sale to inventory' is NOT 0, it is indeterminable or undefined. (We cannot determine the value of a number divided by 0)

In fact, for 'HS billet' and 'MS billet', the ratio of 'inventory to sale' (inverse) is 0, so read attentively.

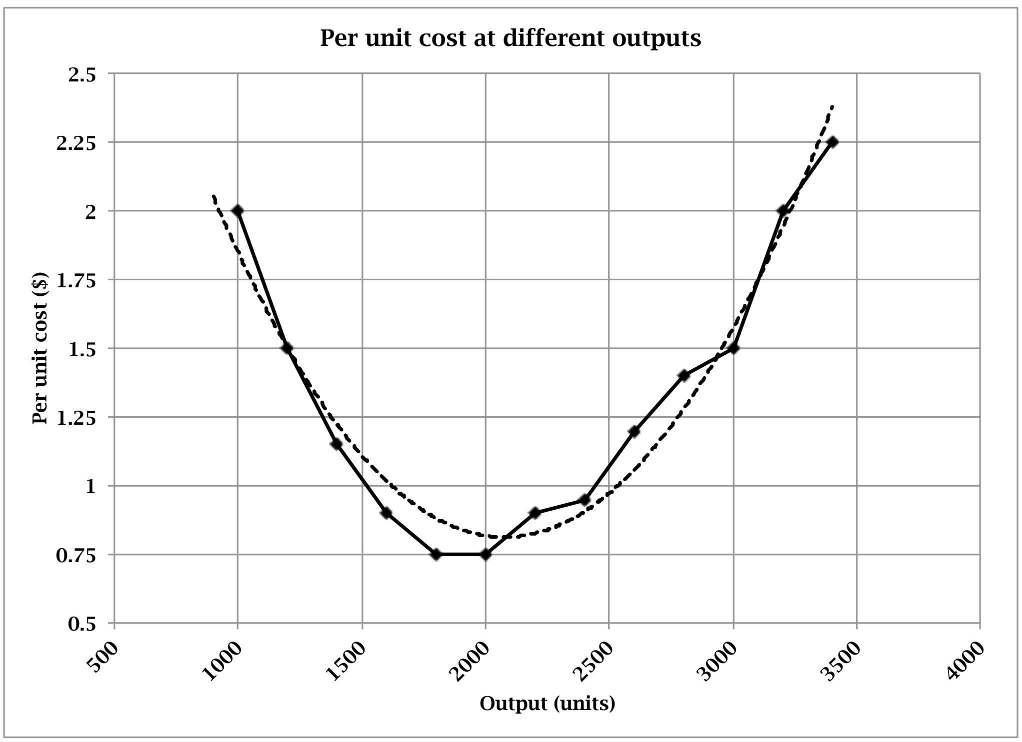
(C) **The statement:** In quarter III, the product that has the highest inventory also has the least sale.

This is **incorrect** because in quarter III, the product HR pipe has the highest inventory (21 MT), but the product HR sheet has the least sale (0).

(D) **The statement:** In quarter IV, the product that has the highest inventory also has the highest sale.

This is **correct** because in quarter IV, the product CR sheet has the highest inventory (21 MT) as well as the highest sale (15 MT); though the highest sale is shared by two more products, unlike option A statement, the option D statement does imply that CR sheet is the only product to have the highest sale.

Hence, the correct option is (D).



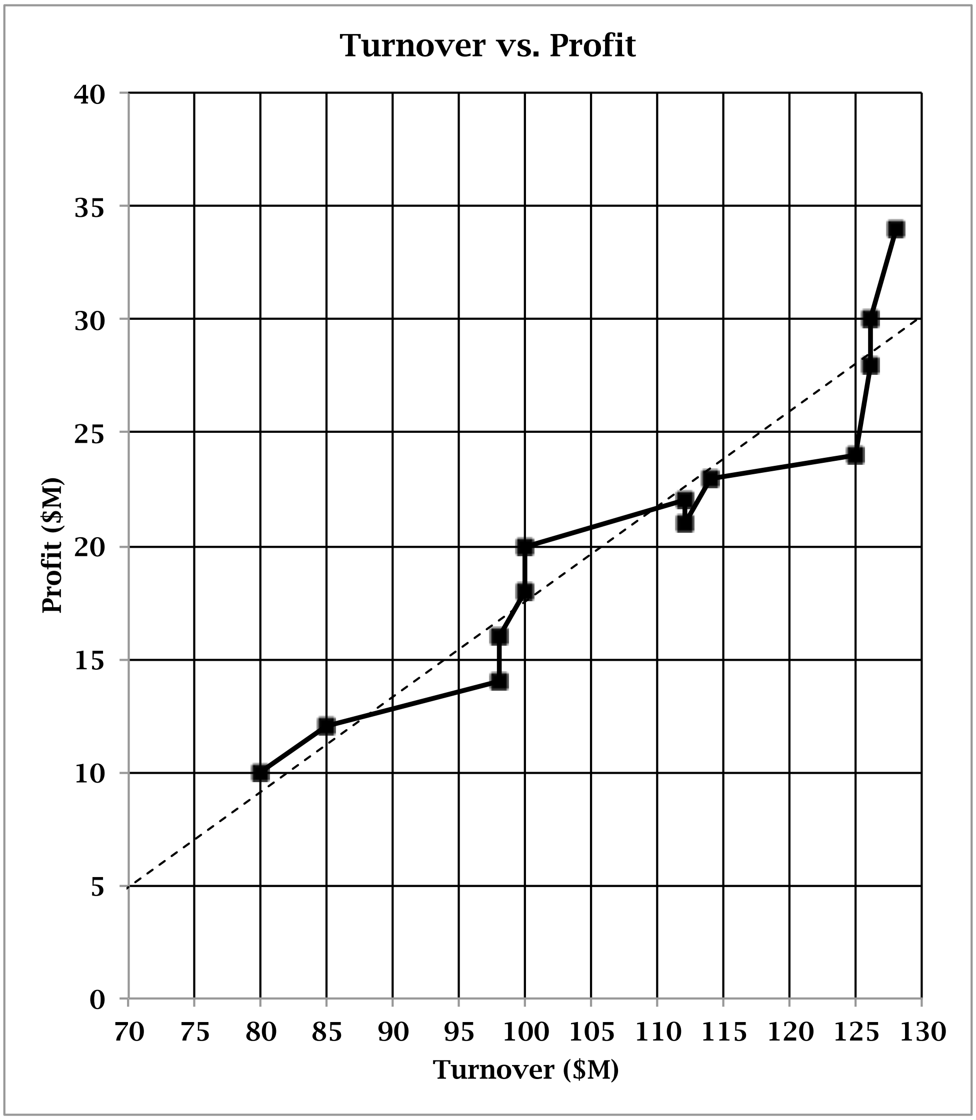
Above graph shows distribution of per unit unit cost ($) of an automotive component at different outputs of a machine. The best estimation curve is also shown with dashed line. The following two questions are based on it.

183. We want to keep the per unit cost of the item least. The curve shows that the least per unit cost is approximately \$0.80 (bottom of the curve). The corresponding output for the same would be any thing between 2000 to 2100.

Hence, the correct option is (C).

184. The per unit cost at 1200 units of output is approximately \$1.50. If we extend the horizontal line from 1.50, we meet the curve at a point which has the corresponding output equal to approximately 3000.

Hence, the correct option is (C).



Above graph shows a scatterplot for the distribution of turnovers (\$M) and their corresponding profits (\$M). The data is modeled as a linear relationship (a dashed trend-line) where p is profit and t is turnover. The following two questions are based on it.

185. First, locate \$125M in the X-axis, and then extent a line vertically to meet the trend-line; from that intersecting point, read the value at the Y-axis, which is approximately \$28M.

Hence, the correct option is (B).

186. Let us assume that the linear equation is $p = mt + c$.

Since there are two variables m & c, we must have two values of two corresponding points from the graph to get the values of m & c.

Let us choose I point @ \$70M (t); the corresponding profit = \$5M (p); thus,

=> $5 = 70m + c$.....(1);

Let us choose II point @ \$130M (t); the corresponding profit = \$30M (p); we have chosen this point as the values are easier to read for \$130M. thus,

=> $30 = 130m + c$.....(2);

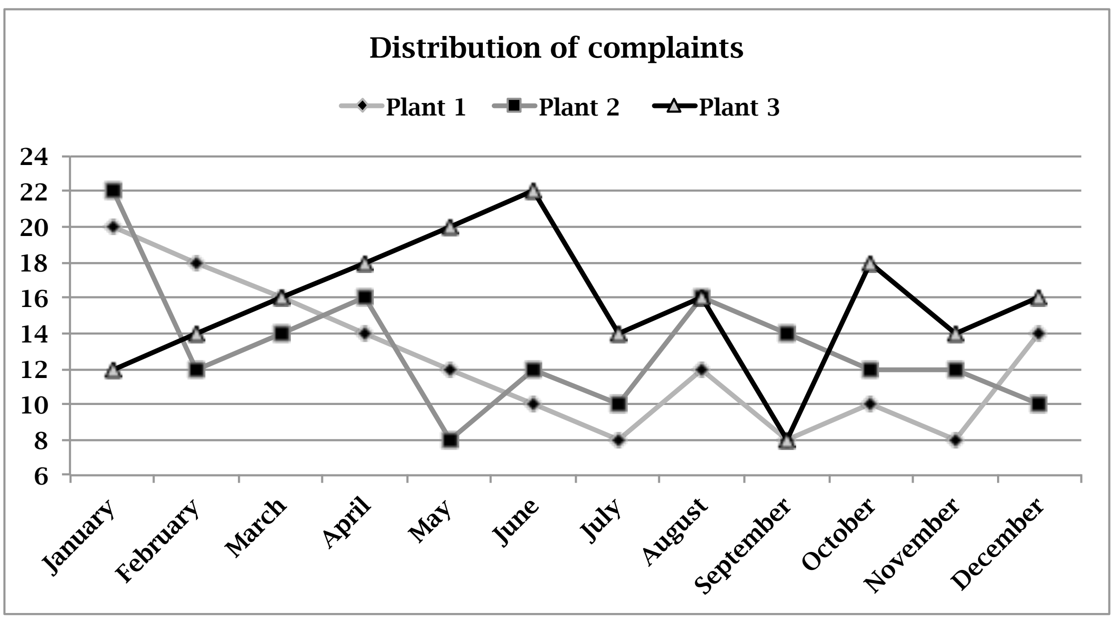
By subtracting equation (2) from (1), we get,

=> $60m = 25$ => $m = 25/60 = 0.42$.

By plugging in the value of $m = 0.42$ in equation (1), we get $c = -29.17$.

Thus, the equation is $p = 0.42t - 29.17$.

Hence, the correct option is (B).



Above line graph shows numbers of complaints received for a product over the year for three plants. The following two questions are based on it.

187. Total number of complaints in the month of January

= # of complaints in plant 1 + # of complaints in plant 2 + # of complaints in plant 3

= 20 + 22 + 12 = 54.

Similarly, total number of complaints in the month of June

= # of complaints in plant 1 + # of complaints in plant 2 + # of complaints in plant 3

= 10 + 12 + 22 = 44.

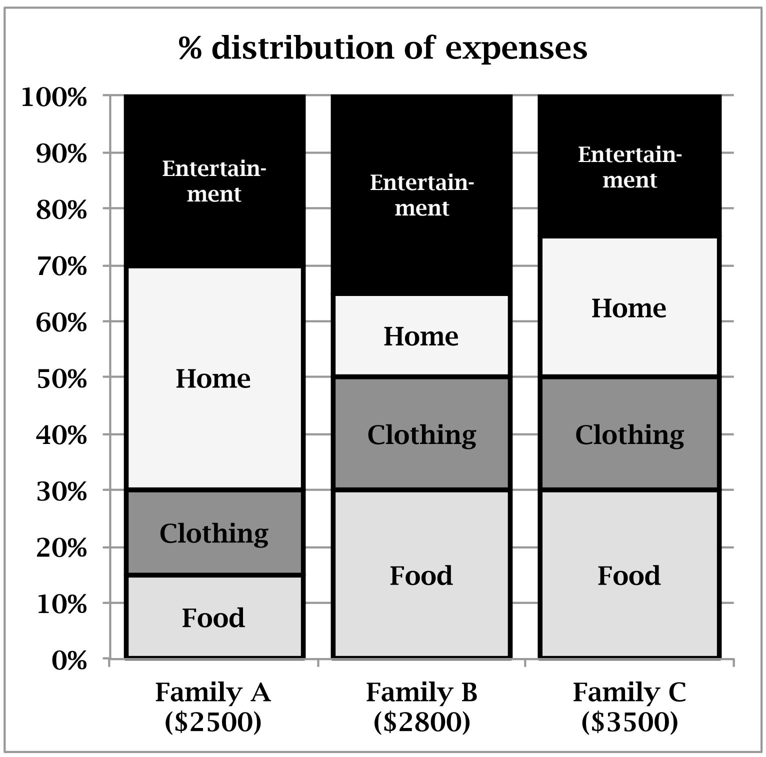
Thus, the total number of complaints in three plants in the month of January is more than that in the month of June by 54 – 44 = 10.

Hence, the correct option is (A).

188. The legend for plant 3 is given by a triangle in the graph. The best approach to figure out the answer is by observing that for how many months the triangles are placed above the legends for plants 1 (Rhombuses) and for plant 2 (Squares).

We see that for the months—April, May, June, July, October, November, and December—the triangles are placed above the rhombuses and above the squares, thus there are 7 months,

Hence, the correct answer is 7.



Above bar chart shows the percentage distribution of expenses for following heads: Food, Clothing, Home, and Entertainment for three families. Total expenses incurred by each family are also mentioned in the chart. The following two questions are based on it.

189. 'Entertainment' expenses for a family can be calculated as "% of expenses X Total expenses by the family".

The % expenses on 'Entertainment' head by the families are:

1. Family A: (100% – 70%) = 30%;
2. Family B: (100% – 65%) = 35% (Highest among the families);
3. Family C: (100% – 85%) = 15%

The total expenses by the families are:

1. Family A: 2500;
2. Family B: 2800;
3. Family C: 3500 (Highest among the families);

The total expenses on ‘Entertainment’ head by each family can be calculated as:

1. Family A: $30\% \times 2500 = \$750/-$;
2. Family B: $35\% \times 2800 = \$980/-$ (Highest among the families)—correct answer;
3. Family C: $15\% \times 3500 = \$875/-$

Hence, the correct option is (B).

190. ‘Clothing’ expenses for a family can be calculated as “% of expenses X Total expenses by the family”.

The % expenses on ‘Clothing’ head by the families are:

1. Family A: (30% – 15%) = 15% (Least among the families);
2. Family B: (50% – 30%) = 20%;
3. Family C: (50% – 30%) = 20%

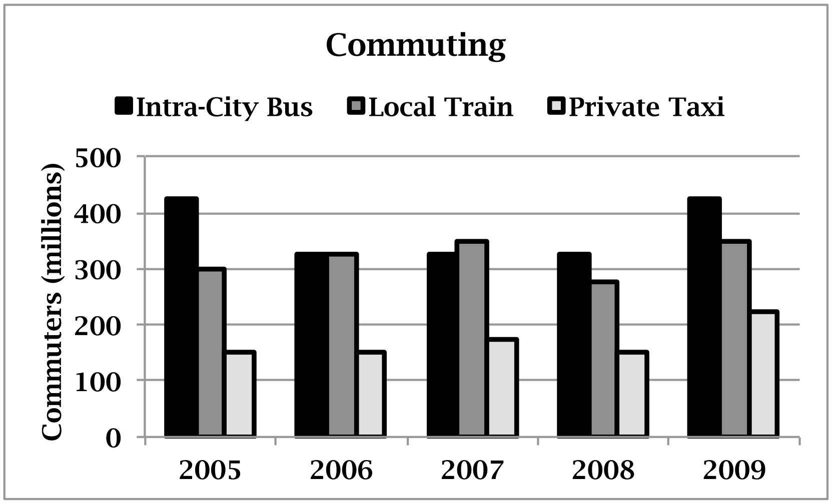
The total expenses by the families are:

1. Family A: 2500 (Least among the families);
2. Family B: 2800;
3. Family C: 3500

Since family A is least for both the parameters: % expenses on ‘Clothing’ & total expenses, its expenses on clothing would be the correct answer.

Expenses on clothing by family A: $15\% \times 2500 = \$375/-$;

Hence, the correct answer is 375.



Above multiple bar chart shows distribution of commuters for 5 years on the basis of mode of transport used by them, namely, Intra-City Bus, Local Train, and Private taxi.

191. The question asks us for which year, the

'Sum of # of commuters for Intra-City Bus + Local Train + Private taxi'

is the highest.

Looking at the graph and the options, it seems that we may have to deal with 12 data points (4 options * 3 bars per year), but it is not so, mere observation can get us the answer.

We see that the black color bar is highest for years 2005 & 2009.
Similarly, the dark grey color bar is highest for years 2007 & 2009.
Similarly, the light grey color bar is highest for year 2009.

We see that year 2009 is common in all three parameters, thus, 2009 must be the correct answer. Since the question wants us to find out the year and not the value, we must not waste our time to calculate it.

Hence, the correct option is (D).

192. The question asks us to calculate the

'Sum of # of commuters for Intra-City Bus + Local Train + Private taxi'

for all the years and answer with the least value.

Like the question above, we can solve this one with mere observation.

We see that the black color bar is lowest for years 2006, 2007, & 2009.
Similarly, the dark grey color bar is lowest for year 2008.
Similarly, the light grey color bar is lowest for years 2005, 2006, & 2008.

We see that year 2008 is common in all three parameters, thus,

‘Sum of # of commuters for Intra-City Bus + Local Train + Private taxi’

for year 2008 must be the correct answer.

‘sum of # of commuters for Intra-City Bus + Local Train + Private taxi’ for year 2008 = 325 + 275 + 150 = 750M.

Hence, the correct answer is 750.

5.3 Passport to Advanced Math

5.3.1 Polynomials

193. Given that

$(2x^5 - 3x^4 - 4x^2 + 3) - (2x^4 - 3x^2 + 4x + 13) - (2x^3 - 3x^2 - 4x)$

After opening bracket, we get,

$2x^5 - 3x^4 - 4x^2 + 3 - 2x^4 + 3x^2 - 4x - 13 - 2x^3 + 3x^2 + 4x$

$2x^5 - x^4(3 + 2) - x^2(4 - 3 - 3) - x(4 - 4) + 3 - 13$

$2x^5 - 5x^4 - x^2 - 10$

Hence, the correct option is (B).

194. Given that

$13x^5y^{-3}z^2 \times 4x^{-2}y^3z^{-3}$

We know that when we multiply or divide algebraic terms, their exponents are added or subtracted.

Thus, $13x^5y^{-3}z^2 \times 4x^{-2}y^3z^{-3} = 52x^{5-2}y^{-3+3}z^{2-3}$

$52x^3y^0z^{-1} = 52x^3z^{-1}$; as we know that $a^0 = 1$.

Hence, the correct option is (B).

195. Given that

$(-3x^4 + 3) \times (2x^4 + 4x)$

When we multiply two expressions, each term of an expression gets multiplies with each term of another expression.

Thus, $(-3 \times 2 \times x^{4+4}) + (3 \times 2x^4) - (3 \times x^{4+1}) + 3 \times 4x$

$-6x^8 + 6x^4 - 3x^5 + 12x$

Hence, the correct option is (D).

196. Given that

$\frac{6x^5 - 33x^4 + 21x^3 - 6x^2 - 30x}{3x}$

=> $\frac{\cancel{3x}(2x^4 - 11x^3 + 7x^2 - 2x - 10)}{\cancel{3x}}$; taking $3x$ common.

=> $2x^4 - 11x^3 + 7x^2 - 2x - 10$

Hence, the correct option is (B).

197. Given that

$$\frac{3x^3 - 24x^2 - 4x + 32}{x - 8}$$

=> $\dfrac{(3x^3 - 24x^2) - (4x - 32)}{x - 8}$; making pairs of terms

=> $\dfrac{3x^2(x - 8) - 4(x - 8)}{x - 8}$; anticipate that the denominator may have factor $(x - 8)$

=> $\dfrac{(3x^2 - 4)\cancel{(x - 8)}}{\cancel{(x - 8)}}$

=> $3x^2 - 4$

Hence, the correct option is (C).

198. Given that

$$\frac{8x^3 + 125}{x + 5}$$

=> $\dfrac{(2x)^3 + (5)^3}{x + 5}$

We know that $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Thus, $\dfrac{(2x)^3 + (5)^3}{x + 5} = \dfrac{\cancel{(2x + 5)}(4x^2 - 10x + 25)}{\cancel{(2x + 5)}}$

$4x^2 - 10x + 25$

Hence, the correct option is (C).

199. Given that

$$\frac{8 - 243x^6}{3x - \sqrt{2}}$$

=> $\dfrac{(2)^3 - (9x^2)^3}{3x - \sqrt{2}}$

We know that $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Thus, $\dfrac{(2)^3 - (9x^2)^3}{3x - \sqrt{2}} = \dfrac{(2 - 9x^2)(4 + 18x^2 + 81x^4)}{3x - \sqrt{2}}$

=> $\dfrac{[(\sqrt{2})^2 - (3x)^2][4 + 18x^2 + 81x^4]}{3x - \sqrt{2}}$

We know that $a^2 - b^2 = (a + b)(a - b)$

Thus by taking "−" common from the denominator and canceling the term, we get

$$\frac{[\cancel{(\sqrt{2} - 3x)}(\sqrt{2} + 3x)][4 + 18x^2 + 81x^4]}{-\cancel{(\sqrt{2} - 3x)}}$$

=> $(\sqrt{2} + 3x)(4 + 18x^2 + 81x^4)$

Hence, the correct option is (D).

5.3.2 Quadratic equations

200. Simplifying the given equation, we have:

$$\frac{3}{x-1} - \frac{2}{2x-3} = 1$$

$$=> \frac{3(2x-3) - 2(x-1)}{(x-1)(2x-3)} = 1$$

$$=> \frac{6x - 9 - 2x + 2}{2x^2 - 3x - 2x + 3} = 1$$

$$=> \frac{4x - 7}{2x^2 - 5x + 3} = 1$$

$$=> 4x - 7 = 2x^2 - 5x + 3$$

$$=> 2x^2 - 9x + 10 = 0$$

$$=> 2x^2 - 4x - 5x + 10 = 0$$

$$=> 2x(x-2) - 5(x-2) = 0$$

$$=> (2x-5)(x-2) = 0$$

$=> x = \frac{5}{2}$ or 2.

Thus, the integer value of x that satisfies the given equation is $x = 2$.

Hence, the correct answer is 2.

201.

$$\frac{3}{x} + \frac{x}{6} = \frac{3}{2} => \frac{3 * 6 + x * x}{6x} = \frac{3}{2} => \frac{(18 + x^2)}{6x} = \frac{3}{2} => 18 + x^2 = \frac{3}{2} * 6x$$

$$=> 18 + x^2 = 9x$$

$$=> x^2 - 9x + 18 = 0$$

$$=> x^2 - 6x - 3x + 18 = 0$$

$$=> x(x-6) - 3(x-6) = 0$$

$$=> (x-3)(x-6) = 0$$

$=> x = 3$ or 6.

Thus, sum of the values of x that satisfy the given equation is $3 + 6 = 9$.

Hence, the correct option is (D).

Alternate Approach:

We know that for a quadratic equation $ax^2 + bx + c = 0$, the 'Sum of roots $= -\frac{b}{a}$', thus for the equation, $x^2 - 9x + 18 = 0,\ a = 1,\ \&\ b = -9$.

=> Sum of roots $= -\frac{b}{a} = -(\frac{-9}{1}) = 9$.

202. Simplifying the equation, we have:

$$\frac{12}{x+4} + \frac{6}{x-6} = 4$$

$$=> \frac{12\,(x-6) + 6\,(x+4)}{(x+4)\,(x-6)} = 4$$

$$=> \frac{12x - 72 + 6x + 24}{x^2 + 4x - 6x - 24} = 4$$

$$=> \frac{18x - 48}{x^2 - 2x - 24} = 4$$

$$=> \frac{2\,(9x - 24)}{x^2 - 2x - 24} = 4$$

$$=> \frac{9x - 24}{x^2 - 2x - 24} = 2$$

$$=> 9x - 24 = 2(x^2 - 2x - 24)$$

$$=> 9x - 24 = 2x^2 - 4x - 48$$

$$=> 2x^2 - 13x - 24 = 0$$

$$=> 2x^2 - 16x + 3x - 24 = 0$$

$$=> 2x\,(x-8) + 3\,(x-8) = 0$$

$$=> (2x+3)\,(x-8) = 0$$

=> $x = -\frac{3}{2}$ or 8.

Thus, the integer value of x is 8.

The above is a lengthy and time-consuming process; a better approach would be to plug-in option values in the given equation. We can see that if $x = 8$, the equation is satisfied and hence it is the solution.

Hence, the correct option is (C).

Graphic quadratic equations

203. The given graph is $f(x) = (x-1)^2 + 3$.

We need to find the graph of $g(x) = (x+1)^2 + 1$.

Here, we see that $(x-1)$ has been replaced with $(x+1)$ i.e. x has been replaced with $(x+2)$.

Also, the constant 3 has been replaced by 1 i.e. the y values of the graph have been reduced by $3 - 1 = 2$ units.

Thus, the graph would shift by 2 units left (since x has been increased by two) and would shift by 2 units down (since the constant has been reduced by two).

To find the correct graph, let us pick one reference point on the original graph.

Let the point be the one where $f(x)$ has the lowermost point i.e. $(1,\ 3)$.

Thus, on shifting, this point would move to $[(1-2),\ (3-2)] => (-1, 1)$.

Thus, the graph of $g(x)$ should have the minimum position at $(-1, 1)$.

This is satisfied by the graph shown in option (D).

Hence, the correct option is (D).

204. For comparing the graphs of $f\ (x)$ and $g\ (x)$, we need to pick some reference point.

The graph of $f(x)$ has a small peak on the left and a small valley on the right. We can pick either of the two points as our reference point.

Let us work with the peak on the left.

In the graph of $f(x)$, the point of the peak is $(-2, 6)$ (we should note that each gap along the X or Y axis is of 2 units).

The same point in the graph of $g(x)$ is $(-1, 2)$.

Thus, we can say that the point has shifted to the right by 1 unit and down by 4 units.

Thus, in order to shift the graph right by one unit, we should replace x with $(x-1)$ and in order to shift the graph down by four units, we should subtract 4 overall.

Thus, option (D) satisfies the above conditions.

Hence, the correct option is (D).

205. If we compare the graphs of $f(x)$ and $g(x)$, it seems that $g(x)$ is the inverted form of $f(x)$ i.e. reflected about the X-axis.

To verify this, we need to pick some reference points.

The graph of $f(x)$ has a peak on the Y-axis and two valleys, one of the left and the other on the right.

The coordinates of the above points are $(0,6)$, $(-2,-1)$, and $(2,-1)$ respectively.

In the graph of $g(x)$, we see that peak has become the valley and the valleys have become peaks. The coordinates of the above points are $(0,-6)$, $(-2,1)$, and $(2,1)$ respectively.

Thus, we find that for each of these points, the value of x remained unchanged while the value of y has been negated.

Thus, we can say that $g(x)$ is the negation of $f(x)$ i.e. $g(x) = -f(x)$.

Thus, option (C) satisfies the above condition.

Hence, the correct option is (C).

206. If we compare the graphs of $f(x)$ and $g(x)$, it seems that $g(x)$ has been obtained by reflecting $f(x)$ about the Y-axis.

To verify this, we need to pick some reference points.

The graph of $f(x)$ has a peak on the Y-axis and intersects the X-axis at two points on the right.

The coordinates of the peak and the first point of intersection with the X-axis on the right are $(0,2)$ and $(2,0)$ respectively.

In the graph of $g(x)$, we see that peak has remained the same and the point of intersection has now come to the left at $(-2,0)$.

Thus, we find that the value of y remained unchanged while the value of x has been negated.

Thus, we can say that $g(x) = f(-x)$.

Thus, option (B) satisfies the above conditions.

Hence, the correct option is (B).

207. If we compare the graphs of $f(x)$ and $g(x)$, it seems that $g(x)$ is obtained by reflecting $f(x)$ about the X-axis i.e. x in $f(x)$ has been replaced by $(-x)$.

Also, we can observe that after reflection, the graph has been shifted up.

To verify this, let us select a point on $f(x)$ where it intersects the Y-axis.

The coordinates of the above point is $(0,-2)$.

The coordinates of the point where $g(x)$ intersects the Y-axis is $(0,-1)$.

Thus, it is clear that the graph of $f(x)$ has been shifted up by 1 unit after reflecting about the Y-axis.

Hence, we can say that $g\left(x\right) = f\left(-x\right) + 1$.

Thus, option (B) satisfies the above condition.

Hence, the correct option is (B).

208. $g\left(x\right) = \frac{x^2}{3} + 3 = \frac{1}{3}\left(x^2 + 3\right) + 2$

$=> g\left(x\right) = \frac{1}{3} * f\left(x\right) + 2$

Thus, we see that $f(x)$ has been multiplied with $\frac{1}{3}$ and then 2 has been added.

Thus, the values of y corresponding to different x values in $\frac{1}{3} * f(x)$ will be $\frac{1}{3}^{rd}$ of the corresponding values of y in $f(x)$.

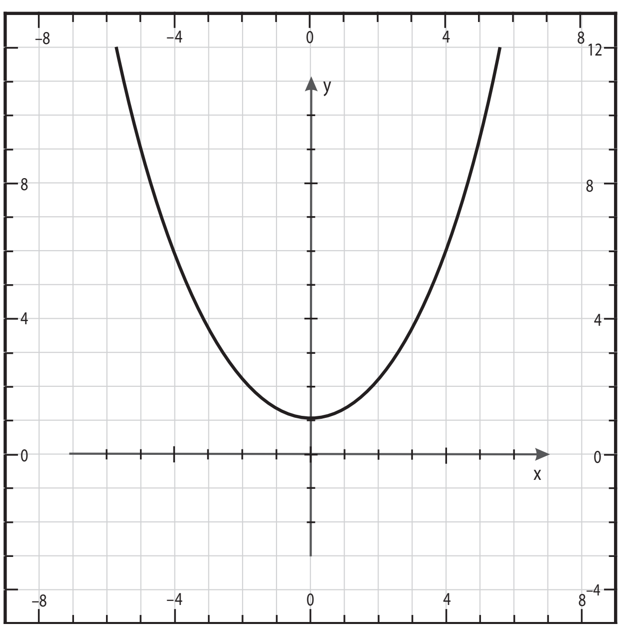
Hence, the nature of the curve in will be more flat as compared to $f(x)$ and the y values corresponding to same x values would be lower than in $f(x)$.

For example, in $f(x)$, the point where it intersects the Y-axis, the coordinates of the point is $(0, 3)$.

The corresponding value of the coordinates where $\frac{1}{3} * f(x)$ intersects the Y-axis are $\left(0, \frac{1}{3} * 3\right) = (0, 1)$ as shown below.

Also, to explain the flatness of the curve, let us observe the value of y in the graph of $f(x)$ when $x = 3$ and the corresponding value of y in the graph of $\frac{1}{3} * f(x)$ at the same value of x.

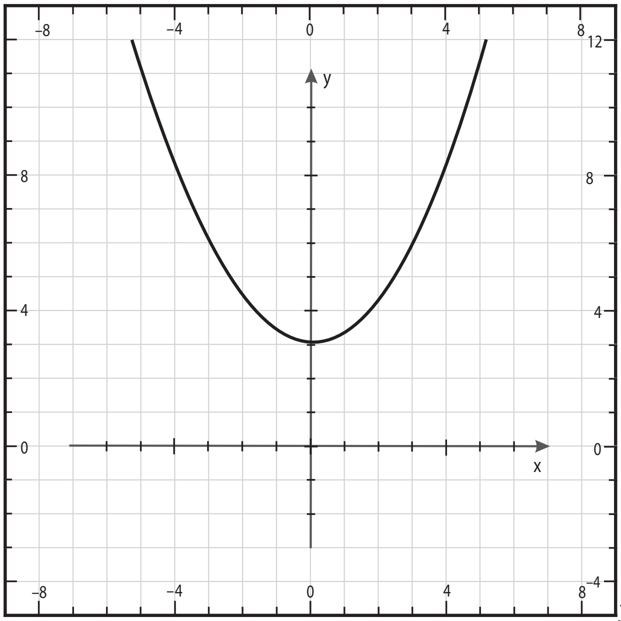
We can see that in $f(x)$, the value of y is 12 while in $\frac{1}{3} * f(x)$, the value is 4, i.e. the value has become $\frac{1}{3}^{rd}$ of the initial value.



$y = \frac{1}{3} * f(x)$

Now, since $g\left(x\right) = \frac{1}{3} * f\left(x\right) + 2$, the above graph will shift up by 2 units as shown below.

Thus, the coordinates of the point of intersection of $f(x)$ and $\frac{1}{3} * f(x) + 2$ will be the same i.e. $(0, 3)$.



$y = \frac{1}{3} * f(x) + 2$

Thus, the overall effect is that of making the graph more flat, i.e. wider without shifting the graph of $f(x)$.

Hence, the correct option is (D).

209. From the graph above, when $x = 0$, the value of y i.e. $f(0)$ is negative since the graph intersects the Y-axis below the origin.

Substituting $x = 0$ in $f(x)$, we get: $f(0) = 0 + 0 + c = c$. Thus, the value of c must be negative.

Again, the graph intersects the X-axis at two points (the roots), both of which are on the positive X-axis. Thus, both the roots of the quadratic are positive. Hence, product of the roots is also positive.

We know that the product of the roots of the quadratic function: $f(x) = ax^2 + bx + c$ is given by $\frac{c}{a}$.

Thus, we can say that $\frac{c}{a}$ is positive.

Since we know that c is negative, a must be negative as well.

Again, since both roots of the above quadratic are positive, the sum of the roots will also be positive.

We know that the sum of the roots of the quadratic function: $f(x) = ax^2 + bx + c$ is given by $-\frac{b}{a}$.

Thus, we can say that $-\frac{b}{a}$ is positive, hence, $\frac{b}{a}$ is negative.

Since we know that a is negative, b must be positive.

Thus, we have $a < 0, b > 0, \& \ c < 0$.

Hence, the correct option is (D).

210. From the graph above, when $x = 0$, the value of y i.e. $f(0)$ is negative since the graph intersects the Y-axis below the origin.

Substituting $x = 0$ in $f(x)$, we get: $f(0) = 0 + 0 + c = c$. Thus, the value of c must be negative.

Again, the graph intersects the X-axis at two points (the roots), one on the positive X-axis and the other on the negative X-axis. Hence, product of the roots is negative.

We know that the product of the roots of the quadratic function: $f(x) = ax^2 + bx + c$ is given by $\frac{c}{a}$.

Thus, we can say that $\frac{c}{a}$ is negative.

Since we know that c is negative, a must be positive.

Now, one root is negative and the other is positive. However, the negative root is bigger in magnitude than the positive root. Thus, the sum of the roots must be negative.

We know that the sum of the roots of the quadratic function: $f(x) = ax^2 + bx + c$ is given by $-\frac{b}{a}$.

Thus, we can say that $-\frac{b}{a}$ is negative, hence, $\frac{b}{a}$ is positive.

Since we know that a is positive, b must be positive as well.

Thus, we have $a > 0, b > 0, \ c < 0$.

Hence, the correct option is (C).

211. From the graph above, the intercept on the y axis is the value of $f(x)$ at $x = 0$ i.e. $f(0)$.

We can see that $f(0) = a(0)^2 + b(0) + c = c$.

Thus, the intercept on the y axis is equal to c (Note: we can see that the graph intersects the y axis below the origin. This implies that the value of c is negative or y-intercept = $-|c|$.

Hence, the correct option is (D).

212. We have $f(x) = x^2 - 12x + 20$.

Since the coefficient of x^2 is positive, the graph must be open upwards. Hence, option (B) is not possible.

Again, since the value of $f(0) = 20$ (a positive number), it implies that the graph would intersect the Y-axis above the origin.

Hence, option (D) is not possible.

In a quadratic $ax^2 + bx + c$, the sum of roots is given by $-\frac{b}{a}$.

In the above quadratic, the sum of roots would be $\left[-(\frac{-12}{1})\right] = 12$.

Thus, the sum of roots would be positive.

In option (C), both roots are negative (since the graph intersects the X-axis to the left of the origin) implying that the sum of roots should also be negative.

Hence, option (C) is not possible.

Thus, option (A), which satisfies all of the above conditions, must be the answer.

Hence, the correct option is (A).

Alternate Approach:

We have $f(x) = x^2 - 12x + 20$.

Since the value of $f(0) = +20$, hence, option (D) is not possible.

At $y = f(x) = 0$, $x^2 - 12x + 20 = 0 \Rightarrow x^2 - 2x - 10x + 20 = 0$

$\Rightarrow x(x-2) - 10(x-2) = 0 \Rightarrow (x-2)(x-10) = 0$

$\Rightarrow x = +2$ or $+10$ (Both roots are positive)

Only the graph in option (A) shows that both roots of the quadratic $f(x) = x^2 - 12x + 20$ are positive.

Hence, the correct option is (A).

213. We have $f(x) = x^2 + 6x + 9$.

Since the coefficient of x^2 is positive, the graph must be open upwards. Hence, option (C) is not possible.

Again, since the value of $f(0) = 9$, it implies that the graph would intersect the Y-axis above the origin.

Hence, options (A) and (B) are not possible.

Thus, option (D), which satisfies all of the above conditions, must be the answer.

Hence, the correct option is (D).

214. We see that the graph of $f(x)$ is open upwards.

Thus, the coefficient of x^2 must be positive.

Thus, option (D) cannot be the solution.

Again, we can observe that the graph of $f(x)$ intersects the Y–axis above the origin at the point between $(0, 4)$ and $(0, 6)$, i.e. the value of $f(0)$ is positive.

In option (A), $f(0) = 8$. Thus, though $f(0)$ is positive, the value of $f(0)$ does not fall in the above range.

Hence, option (A) cannot be the solution.

In option (B), $f(0) = 6$. Again, though $f(0)$ is positive, the value of $f(0)$ does not match the above result (i.e. between 4 and 6).

Thus, option (C) must be the answer. Let us verify.

$$f(0) = (0-3)^2 - 4 = 9 - 4 = 5.$$

Thus, $f(0)$ is positive and also lies between the values 4 and 6.

Hence, the correct option is (C).

215. We have: $f(x) = x^2 + (a^2 - b)x + b^2 - a^2$.

Since $a > 1 \Rightarrow a^2 > 1$, while $0 < b < 1$, we can say that $a^2 - b > 0$.

Also, since $0 < b < 1 \Rightarrow 0 < b^2 < 1$, we can say that $b^2 - a^2 < 0$.

Thus, we can say that the coefficient of x i.e. $(a^2 - b)$ is positive while the constant term i.e. $(b^2 - a^2)$ is negative.

So, we have $f(0) = b^2 - a^2 < 0$ i.e. the graph of $f(x)$ intersects the Y-axis below the origin.

Also, the sum of the roots is given by: $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\left(\frac{a^2-b}{1}\right) = -(a^2 - b) < 0$ [since $(a^2 - b)$ is positive].

Thus, for $f(x)$, the sum of the roots will be negative.

Thus, we can say with certainty that both roots of $f(x)$ cannot be positive (note that we cannot conclude that both roots are negative as there may be a case where one root is positive while the other is negative).

From the above conditions, we can say that:

Option (A) is not possible since both roots are shown positive, also $f(0)$ is shown positive.

Option (B) is not possible since the negative root is smaller in magnitude than the positive root implying that the sum of roots will be positive.

Option (C) is not possible since the value of $f(0)$ is shown as positive.

Thus, option (D) must be correct.

In option (D), the value of $f(0)$ is negative. Also, the magnitude of the negative root is greater than that of the positive root implying that the sum of roots will be negative.

Hence, the correct option is (D).

216. We have: $f(x) = -x^2 + (a - b)x + b^2$

Since the coefficient of x^2 is negative, the graph of $f(x)$ must be open downwards.

Also, since $0 < a < 1$ while $3 < b < 5$, we can say that $a - b < 0$ (in fact, $(a - b)$ lies between $1 - 3 = -2$ and $0 - 5 = -5$. Thus, we have: $-5 < a - b < -2$).

Also, b^2 is positive and the value of b^2 must lie between 9 and 25 i.e. $9 < b^2 < 25$ (since $3 < b < 5$).

Thus, we can say that the coefficient of x i.e. $(a - b)$ is negative while the constant term i.e. b^2 is positive.

So, we have $f(0) = b^2 > 0$ i.e. the graph of $f(x)$ intersects the Y-axis above the origin at a value between 9 and 25.

Also, the sum of the roots is given by: $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\left(\frac{a-b}{-1}\right) = a - b < 0$ [since $(a - b)$ is negative].

Thus, for $f(x)$, the sum of the roots will be negative.

Thus, we can say with certainty that both roots of $f(x)$ cannot be positive (note that we cannot conclude that both roots are negative as there may be a case where one root is positive while the other is negative).

From the above conditions, we can say that:

Option (A) is not possible since the graph of $f(x)$ is open upwards.

Option (B) is not possible since the value of $f(0)$ is shown as negative.

Option (C) is not possible though $f(0)$ is positive, since the value of $f(0) = 4$ (we had concluded that $f(0) = b^2$ and $9 < b^2 < 25$).

Thus, option (D) must be correct.

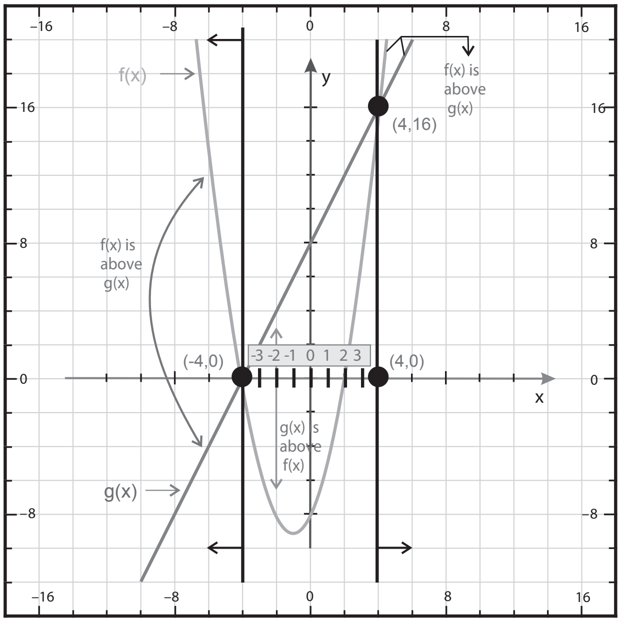
In option (D), the value of $f(0)$ is positive and lies between 9 and 25, in fact $f(0) = 18$. Also, the magnitude of the negative root is greater than that of the positive root, implying that the sum of roots will be negative (in fact, the roots are 3 and -6 implying that the sum of the roots is -3; which satisfies the condition that the sum of the roots, i.e. $(a-b)$ must lie between -2 and -5).

Hence, the correct option is (D).

217. From the graph, it is quite clear that $f(x) = x^2 + 2x - 8$ refers to the parabola and $g(x) = 2x + 8$ refers to the straight line.

From the above graph, it can be seen that $f(x)$ and $g(x)$ intersect at two points whose coordinates are given by $(-4, 0)$ and $(4, 16)$.

Let us consider the region to the left of $(-4, 0)$ and the region to the right of $(4, 16)$:



In the above diagram, we see that in both regions, i.e. to the left of $(-4, 0)$ and to the right of $(4, 16)$, the graph of $f(x)$ (i.e. the curve) is placed above the graph of $g(x)$ (i.e. the straight line).

Thus, $f(x) > g(x)$.

However, in the region in between the points of intersection of the graphs, i.e. between $(-4, 0)$ and $(4, 16)$, the graph of $g(x)$ is placed above the graph of $f(x)$.

Thus, $g(x) > f(x)$.

Thus, the integer values of x where $g(x) > f(x)$ is given by: $-3, -2, -1,\ 0,\ 1,\ 2$ and 3. The values of $x = -4\ \&+4$ are excluded as at these values of x, $g(x) = f(x)$ (as shown by the points encircled in black in the above image).

Thus, there are 7 integer values of x satisfying $g(x) > f(x)$.

Hence, the correct answer is 7.

218. The graph represents $f(x) = x^2 + ax + b$.

We can see that $f(0)$ is given in the graph as 12 (i.e. the point where $f(x)$ intersects the Y-axis).

Thus, the length of OB is 12 units.

Thus, we have $b = 12$.

In a quadratic, the product of the roots is given as $\frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{b}{1} = b = 12$.

We can see that one root is given by the point A i.e. $(2, 0)$, implying that one root of the quadratic is 2.

Since the product of the roots is 12, we can say that the other root must be $\frac{12}{2} = 6$.

Thus, coordinates of point C are $(6, 0)$.

Thus, the length of OC is 6 units.

Hence, area of $\triangle$BOC = $\frac{1}{2} *$ base $*$ height $= \frac{1}{2} * (OC) * (OB) = \frac{1}{2} * 6 * 12 = 36$ square units.

Hence, the correct answer is 36.

5.3.3 Advanced equations

219. Squaring both sides of the given equation, we have:

$$\left(\sqrt{2x-8}\right)^2 = \left(\sqrt{x+5}-3\right)^2$$

$$=> 2x-8 = \left(\sqrt{x+5}\right)^2 - 2\left(\sqrt{x+5}\right) * 3 + 3^2$$

$$=> 2x-8 = x+5+9-6\sqrt{x+5}$$

$$=> 6\sqrt{x+5} = 14+x-2x+8$$

$$=> 6\sqrt{x+5} = 22-x$$

Squaring both sides of the above equation:

$$\left(6\sqrt{x+5}\right)^2 = (22-x)^2$$

$$=> 36\,(x+5) = 484+x^2-44x$$

$$=> x^2-44x-36x+484-180 = 0$$

$$=> x^2-80x+304 = 0$$

$$=> x^2-4x-76x+304 = 0$$

$$=> x\,(x-4)-76\,(x-4) = 0$$

$$=> (x-4)\,(x-76) = 0$$

$=> x = 4$ or 76.

Now, since squaring an equation can result in extraneous solutions, we need to verify if both the above values of x satisfy the original equation.

For $x = 4$:

LHS = $\sqrt{2x-8} = \sqrt{2*4-8} = 0$.

RHS = $\sqrt{x+5}-3 = \sqrt{4+5}-3 = 3-3 = 0$.

Hence, LHS = RHS.

Thus, $x = 4$ is a correct solution.

For $x = 76$:

LHS = $\sqrt{2x-8} = \sqrt{2*76-8} = \sqrt{144} = 12$.

RHS = $\sqrt{x+5}-3 = \sqrt{76+5}-3 = 9-3 = 6$.

Hence, LHS > RHS.

Thus, $x = 76$ is not a correct solution.

Hence, we have $x = 4$.

Hence, the correct option is (A).

The above is a lengthy and time-consuming process; a better approach would be to plug-in option values in the given equation. We can see that if $x = 4$, the equation is satisfied and hence, is the solution.

220. In order to solve this equation, we need to equate the exponents on either side after having reduced either base to the same value.

Thus: $(2^{x+2})^{x+6} = 4^6$

$$=> 2^{(x+2)(x+6)} = \left(2^2\right)^6$$

$$=> 2^{(x+2)(x+6)} = 2^{12}.$$

Equating the exponents on either side:

$$(x + 2)(x + 6) = 12$$

$$=> x^2 + 8x + 12 = 12$$

$$=> x^2 + 8x = 0$$

$$=> x(x + 8) = 0$$

$=> x = 0$ or -8.

Since x is non-zero, we have $x = -8$.

Hence, the correct option is (A).

The above is a lengthy and time-consuming process; a better approach would be to plug-in option values in the given equation. We can see that if $x = -8$, the equation is satisfied and hence, is the solution.

221. In order to solve this equation, we need to equate the exponents on either side after having reduced either base to the same value.

Thus: $3^{2x-1} = 3^5$

Equating the exponents on either side:

$$2x - 1 = 5$$

$$=> 2x = 6$$

$$=> x = 3.$$

Hence, the correct answer is 3.

222. Keeping the radical term on the left and transferring the other two terms to the right, we have:

$$\sqrt{3x-8} = x - 2$$

Squaring both sides of the above equation:

$$\left(\sqrt{3x-8}\right)^2 = (x-2)^2$$

$$=> 3x - 8 = x^2 - 4x + 4$$

$$=> x^2 - 4x - 3x + 4 + 8 = 0$$

$$=> x^2 - 4x - 3x + 12 = 0$$

$$=> x\,(x-4) - 3\,(x-4) = 0$$

$$=> (x-4)\,(x-3) = 0$$

=> $x = 4$ or 3.

Now, since squaring an equation can result in extraneous solutions, we need to verify if both the above values of x satisfy the original equation.

At $x = 4$:

LHS = $\sqrt{3x-8} - x + 2 = \sqrt{3*4-8} - 4 + 2 = 2 - 4 + 2 = 0$ = RHS.

Hence, LHS = RHS.

Thus, $x = 4$ is a correct solution.

At $x = 3$:

LHS = $\sqrt{3x-8} - x + 2 = \sqrt{3*3-8} - 3 + 2 = 1 - 3 + 2 = 0$ = RHS.

Hence, LHS = RHS.

Thus, $x = 3$ is a correct solution.

Hence, we have both $x = 4$ as well as $x = 3$ as possible solutions.

Thus, absolute difference between the values of x is given by: (the larger value) – (the smaller value)

$$= 4 - 3 = 1.$$

Hence, the correct option is (B).

Note: You cannot solve this question by plugging-in values from the options as the question does not ask for the roots of the equation.

223. In order to solve this equation, take the square root on both sides.

Thus, $\sqrt{(x^2-17)^2} = \pm\sqrt{64}$

$$=> x^2 - 17 = \pm 8$$

$=> x^2 - 17 = 8$ or $x^2 - 17 = -8$

From the first equation: $x^2 - 17 = 8 => x^2 = 17 + 8 => x^2 = 25 => x = \pm 5$.

From the second equation: $x^2 - 17 = -8 => x^2 = 17 - 8 => x^2 = 9 => x = \pm 3$.

Thus, the values of x are $5,\ -5,\ 3,\ -3$.

Thus, there are four values of x.

Hence, the correct option is (D).

Note: You cannot solve this question by plugging-in values from the options as the question does not ask for the roots of the equation.

224. Squaring both sides of the given equation, we have:

$$\left(\sqrt{x-8}\right)^2 = \left(\sqrt{x+8} - 2\right)^2$$

$$=> x - 8 = (x+8) - 2 * \left(\sqrt{x+8}\right) * 2 + 4$$

$$=> x - 8 = x + 8 - 4\sqrt{x+8} + 4$$

Keeping the radical term on the left and transferring the other two terms to the right, we have:

$$4\sqrt{x+8} = x + 8 + 4 - x + 8$$

$$=> 4\sqrt{x+8} = 20$$

$$=> \sqrt{x+8} = 5$$

Squaring both sides of the above equation, we have:

$$x + 8 = 25$$

$$=> x = 17.$$

Hence, the correct answer is 17.

225. Squaring both sides of the given equation:

$$\left\{\sqrt{(-2)^{x-5}}\right\}^2 = \left\{(2)^{(x-6)}\right\}^2$$

$$=> (-2)^{x-5} = (2)^{2(x-6)}$$

$$=> (-2)^{x-5} = (2)^{2x-12}$$

$$=> \{(-1) * (2)\}^{x-5} = 2^{2x-12}$$

$$=> (-1)^{x-5} * 2^{x-5} = 2^{2x-12}$$

$$=> (-1)^{x-5} = \frac{2^{2x-12}}{2^{x-5}}$$

$$=> (-1)^{x-5} = 2^{(2x-12)-(x-5)}$$

$=> (-1)^{x-5} = 2^{x-7} \ldots$(i)

In the above equation, the LHS will take the value of $+1$ or -1 depending on whether the exponent $(x - 5)$ is even or odd respectively. However, the RHS is 2 raised to an exponent, making it positive. Hence, the value of must be $+1$ and hence, the value of $(x - 5)$ must be even.

Thus, since $(x - 5)$ is even, we have from (i): $1 = 2^{x-7} => 2^0 = 2^{x-7} => x - 7 = 0 => x = 7$.

In such a case, the value of $x - 5 = 7 - 5 = 2$, which is even, as was required.

Hence, the correct option is (B).

The above is a lengthy and time-consuming process; a better approach would be to plug-in option values in the given equation.

So, if we plug-in option values in the given equation using $x = 4,\ 7$, 8, ..., we find that at $x = 7$, the expression holds true.

At $x = 7$,

LHS = $\sqrt{(-2)^{x-5}} = \sqrt{(-2)^{7-5}} = \sqrt{(-2)^2} = \sqrt{4} = 2$ (since "$\sqrt{}$" returns only the positive square root).

RHS = $2^{(x-6)} = 2^{7-6} = 2$.

Thus, the equation is satisfied since LHS = RHS.

5.3.4 Application of functions

Following three questions are based on the following scenario:

Pete fires a pistol pointing it upwards. The height attained by the bullet above the ground t seconds after the pistol was fired is given by $h(t) = -t^2 + 12t + 30$.

226. Let us analyze the function $h(t)$.

$h(t) = -t^2 + 12t + 30 = -(t^2 - 12t) + 30 = -\left\{(t-6)^2 - 36\right\} + 30 = -(t-6)^2 + 66 \ldots$ (i)

Since $-(t-6)^2$ is the negative of a perfect square, it is always non-positive.

Hence, for $h(t)$ to have the maximum value, the value of $-(t-6)^2$ should be maximum. Thus, it should be zero.

Thus: $-(t-6)^2 = 0 => t = 6$.

Thus, the bullet would reach the maximum height above the ground 6 seconds after the pistol was fired.

The maximum height attained can be obtained by substituting $t = 6$ in (i) giving the maximum height as 66 ft.

Hence, the correct option is (D).

227.

228. The height of the bullet above the ground when it was fired is given by substituting $t = 0$ in $h(t)$.

Thus, $h(0) = -0 + 0 + 30 = 30$.

We need to find the time when the height above the ground will again be 30 feet.

Thus, $h(t) = 30$

$$=> -t^2 + 12t + 30 = 30$$

$$=> -t^2 + 12t = 0$$

$=> -t(t-12) = 0 => t = 0$ or 12.

Here, $t = 0$ refers to the time when the bullet was fired. Thus, we need the other value of t.

Hence, $t = 12$ i.e. the bullet will again reach a height of 30 feet after 12 seconds.

Hence, the correct option is (C).

Following two questions are based on the following scenario:

The total cost of manufacturing flower vases by the Potter House follows the function $c(x) = x^2 - 17x + 80$, where x denotes the number of vases manufactured. The vases are sold at a constant price of \$25 per vase.

229. Given that the total cost of manufacturing x vases = \$ $(x^2 - 17x + 80)$.

Total sales proceeds from the vases = \$$25x$.

Hence, profit = total sales – total cost = \$ $(25x - (x^2 - 17x + 80))$ = \$ $(-x^2 + 42x - 80)$.

Since there is neither profit nor loss, we have: $-x^2 + 42x - 80 = 0$

$$=> x^2 - 42x + 80 = 0$$

$$=> x^2 - 40x - 2x + 80 = 0$$

$$=> x(x - 40) - 2(x - 40) = 0$$

$$=> (x - 40)(x - 2) = 0$$

$=> x = 40$ or 2.

Hence, for neither profit nor loss, the company needs to manufacture either 2 or 40 vases.

Hence, the correct option is (B).

230. We know that the profit made by selling x vases is given by: \$ $(-x^2 + 42x - 80)$.

We need to find for what value of x the above profit will become the maximum.

Thus, we have:

$$-x^2 + 42x - 80 = -\left(x^2 - 42x\right) - 80 = -\left\{\left(x^2 - 2 * x * 21 + 21^2\right) - 21^2\right\} - 80$$

$$= -(x - 21)^2 + 21^2 - 80$$

$$= -(x - 21)^2 + 361.$$

Thus, the profit will be maximum when the term $\left\{-(x - 21)^2\right\}$, which is the negative of a perfect square, becomes zero.

This happens at $x = 21$ and the maximum value of the profit becomes \$361.

Thus, the company needs to manufacture 21 vases in order to maximize the profit.

Hence, the correct option is (D).

Following three questions are based on the following scenario:

In a laboratory experiment, it was found that the number of bacteria triples every hour. The number of bacteria at the end of six hours was found to be 72900.

231. The number of bacteria at the start of the experiment = N.

Since the number of bacteria trebles every hour, the number of bacteria at the end of every hour is shown in the table below:

At the end of	Number of bacteria
1 hour	$3 * N$
2 hours	$3 * 3N = 3^2 * N$
3 hours	$3 * 3^2 * N = 3^3 * N$
4 hours	$3 * 3^3 * N = 3^4 * N$
. . .	. . .
t hours	$3^t * N$

Thus, we see that at the end of t hours, the number of bacteria = $3^t * N$.

Hence, the correct option is (D).

232. Since the bacteria count trebles every hour, an hour before, the count would have been 1/3 of what is now i.e. 72900.

Since the bacteria count trebles every hour, an hour before, the count would have been $1/3^{rd}$ of what is now i.e. 72900.

Since 72900 is the count after 6 hours, the count would have been $\frac{1}{3}^{\text{rd}}$ after $(6 - 1) = 5$ hours.

Thus, the number of bacteria was one-third of the final number five hours after the start of the experiment.

Hence, the correct option is (D).

233. We know that the number of bacteria at the end of t hours = $3^t * N$.

We know that at the end of six hours, the number of bacteria present = 72900.

Thus, we have: $3^6 * N = 72900 => N = \frac{72900}{3^6} = \frac{72900}{729} = 1000$.

Hence, the correct option is (C).

Following two questions are based on the following scenario:

The number of vehicles plying on Waterloo Street at any hour of a day is given by the function $v\,(t) = -t^2 + 20t + 100$, with $0 \leq t \leq 24$; where t denotes the number of whole hours past 12:00 midnight.

234. We have: $v\ (t) = -t^2 + 20t + 100 = -\left(t^2 - 20t\right) + 100$

$$= -\left\{(t^2 - 2 * t * 10 + 10^2\right) - 10^2\} + 100 = -(t-10)^2 + 200.$$

The number of vehicles on the street is the maximum if the value of the term $\left\{-(t-10)^2\right\}$ is zero (since the term is the negative of a perfect square, it tends to reduce the value of the function).

This is possible at $t = 10$ i.e. 10 hours past midnight (the maximum no. of vehicles is 200).

Hence, the correct answer is 10.

235. We have: $v\ (t) = -t^2 + 20t + 100$.

The number of cars at midnight is given by: $v\ (0) = -0 + 0 + 100 = 100$.

Thus, we need to find at what other time the number of cars would also be 100.

Thus, we have: $-t^2 + 20t + 100 = 100$

$=> -t^2 + 20t = 0 => -t\ (t - 20) = 0$

$=> t = 0$ or 20.

Here, $t = 0$ refers to midnight.

Hence, the other time is at $t = 20$ i.e. 20 hours past midnight.

Hence, the correct answer is 20.

5.4 Advanced Topics in Math

5.4.1 Geometry

236. We have: $\angle$BOE =$\angle$AOD (vertically opposite angles).

Since $\angle$AOD = $2 * \angle$COE = > $\angle$BOE = $2 * \angle$COE.

Let $\angle$COE =x^0.

Thus, $\angle$BOE = $2x^0$.

Since OC is perpendicular to AB, we have $\angle$COB =$\angle$COA = 90^0.

Thus, $\angle$COB =$\angle$COE+$\angle$BOE =90^0= > $x + 2x = 90^0$= > $x = 30^0$.

Thus, $\angle$COE =30^0.

Hence, $\angle$EOA =$\angle$COE+$\angle$COA =30^0+90^0=120^0.

Hence, the correct option is (C).

237. $\angle$MPD =$\angle$BPO and $\angle$ABC =$\angle$OBP (vertically opposite angles).

Since $\angle$MPD =$\angle$ABC+20^0 => $\angle$BPO =$\angle$OBP+20^0.

Let $\angle$OBP =x^0= > $\angle$BPO =x^0+20^0.

We know that in a triangle, the sum of angles is 90^0.

In triangle BOP, $\angle$BOP =90^0 (since BO is perpendicular to OP).

Thus, $\angle$BPO+$\angle$OBP =180^0−90^0=90^0= > $x + 20^0$+x =90^0= > $x = 35^0$.

Thus, $\angle$BPO =35^0+20^0=55^0.

$\angle$CBE =$\angle$FBP (vertically opposite angles).

Also, $\angle$FBP =$\angle$BPO (alternate angles).

Thus, $\angle$CBE =$\angle$BPO =55^0.

Hence, the correct option is (C).

238. Area of the square ABCD = $(\text{side})^2$=AB^2=2^2= 4 square units.

Area of the triangle DEC = $\frac{1}{2} *$ base $*$ height =$\frac{1}{2} *$DC $*$ height =$\frac{1}{2} * 2 * 1 = 1$ square units.

Hence, the area of the shaded region

= area of square ABCD − area of triangle DEC = 4 − 1 = 3 square units.

Hence, the correct answer is 3.

239. In a parallelogram, the opposite sides are equal.

Thus, AB = CD and BC = AD.

Thus, perimeter of ABCD = AB + BC + CD + DA = 2*(AB + BC) = $2\left((3x-4)+(x+4)\right) = 8x$.

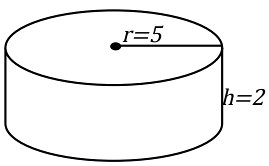
Thus, we have: $8x = 24 \Rightarrow x = 3$ units.

Hence AB = CD = $3x - 4 = 3 * 3 - 4 = 5$ units.

Thus, the area of the parallelogram ABCD = base $*$ height = CD $*$ AX = $5 * 4 = 20$ square units.

Hence, the correct answer is 20.

240. When the rectangle is folded such that AD and BC overlap, we form a cylinder whose base perimeter (perimeter of base circle) is AB and height is BC.



Let the radius of the cylinder be r.

Thus, we have: $2\pi r = 10\pi \Rightarrow r = 5$ units.

Also, height of the cylinder = $h = 2$ units.

Hence, volume of the cylinder = $\pi r^2 h = \pi * 5^2 * 2 = 50\pi$ cubic units.

Hence, the correct option is (C).

241. The approach to solve this question is by equating the volume of both the solids i.e. 'Volume of the prism = Volume of all cubes'

The volume of the prism = area of the base $*$ length = (area of $\triangle$ABC) $*$AF

$= \left(\frac{1}{2} * BC * AE\right) * AF = \frac{1}{2} * 6 * 8 * 9 = 216$ cubic units.

Volume of one cube = $(\text{side})^3 = 2^3 = 8$ cubic units.

Hence, number of cubes possible = $\frac{\text{Volume of the prism}}{\text{Volume of one cube}} = \frac{216}{8} = 27$.

Hence, the correct option is (B).

242. The area of the shaded region = (area of the square) − (area of the circle)

Area of the square = $(\text{side})^2 = 6^2 = 36$ square units.

The diameter of the circle is the same as one side of the square.

Hence, diameter of the circle = 6 units.

Hence, radius of the circle = 3 units.

Hence, area of the circle = $\pi * (\text{radius})^2 = 9\pi$ square units.

Hence, the required area = $(36 - 9\pi)$ square units = $9(4 - \pi)$ square units.

Hence, the correct option is (D).

243. If we choose a sphere of radius equal to that of the cylinder i.e. 2 units, then the diameter of the sphere would be 2*2 = 4 units.

However, since the height of the cylinder is only 3 units, a part of the sphere would protrude outside the cylinder and hence, would not completely fit inside the cylinder.

Thus, in order that the sphere completely fits inside the cylinder, we need to choose a sphere of diameter 3 units, i.e. radius 1.5 units (the sphere placed inside the cylinder is shown in the diagram above).

Hence, the volume of the sphere having radius 1.5 units = $\frac{4}{3} * \pi * (\text{radius})^3 = \frac{4}{3} * \pi * (1.5)^3$ $= \frac{4}{3} * \pi * \left(\frac{3}{2}\right)^3 = \frac{4}{3} * \pi * \frac{3}{2} * \frac{3}{2} * \frac{3}{2} = \frac{9\pi}{2}$ cubic units.

Hence, the correct option is (B).

244. The total surface area of a cube is equivalent to area of 6 identical squares.

Hence, the total surface area = $6 * (\text{side})^2 = 96k$ => $(\text{side})^2 = \frac{96\text{k}}{6} = 16\text{k}$

=> side $= \sqrt{16\text{k}} = 4\sqrt{\text{k}}$

Thus, volume of the cube = $(\text{side})^3 = \left(4\sqrt{\text{k}}\right)^3 = 4^3 * \left(\text{k}^{\left(\frac{1}{2}\right)}\right)^3 = 64k^{\left(\frac{3}{2}\right)} = 64k\sqrt{k}$.

Hence, the correct option is (B).

245. Let the radius of the sphere be r.

Thus, radius of the cylinder = $3r$.

Let the height of the cylinder be h.

Volume of the sphere = $\frac{4}{3} * \pi * r^3$.

Volume of the cylinder = $\pi * (3r)^2 * h = 9\pi r^2 h$.

Since the volume of the sphere is double that of the cylinder, we have:

$$\frac{4}{3}\pi r^3 = 2 * 9\pi r^2 h \Rightarrow \frac{r}{h} = 18 * \frac{3}{4} = \frac{27}{2} \Rightarrow \frac{h}{r} = \frac{2}{27}.$$

However, we need to find the ratio of the height of the cylinder to the radius of the cylinder, i.e. $\frac{h}{3r}$.

Thus, we have: $\frac{h}{3r} = \frac{1}{3} * \frac{h}{r} = \frac{1}{3} * \frac{2}{27} = \frac{2}{81}$.

Hence, the required ratio is 2 : 81.

Hence, the correct option is (A).

246. The area, A, of any triangle, is given by the formula

$$A = \frac{1}{2}bh$$

where h is the length of an altitude corresponding to a side with the length b.

Since $A = 35$, $bh = 70$.

The figure supporting the question shows that $b = 5k$, $h = 3k$, so $bh = 15k^2$.

Substituting $bh = 70$, we have that $15k^2 = 70$. Simplifying,

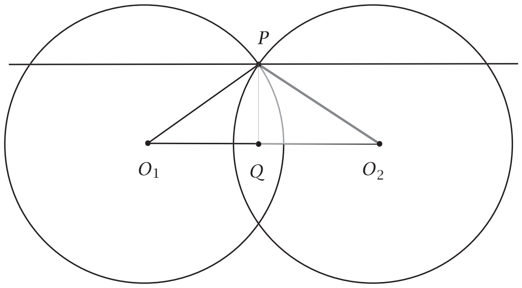
$$k^2 = \frac{70}{15} = 4\frac{2}{3} = \frac{14}{3}.$$

Thus,

$$k = \sqrt{\frac{14}{3}} = \sqrt{\frac{14 * 3}{3 * 3}} = \frac{\sqrt{42}}{3}.$$

Hence, the correct option is (D).

247. Let us understand what the question asks. It states: *The distance between the line joining the centers of the circles and the line drawn through point P parallel to the line connecting the centers* means that we have to find out the value of PQ; see the diagrams below.



The answer comes from an application of the Pythagorean theorem and right triangles.

First, notice that $\triangle PO_1O_2$ is isosceles, and PQ is the altitude of $\triangle PO_1O_2$ by construction. Therefore, $\triangle PQO_2$ is a right triangle. In an isosceles triangle, an altitude dropped from the vertex divides the base in two equal halves: O_1Q and QO_2. Each half is of length 8 since the halves make up $O_1O_2 = 16$. Now, we can get the length of PQ with the help of the Pythagorean theorem:

$$\begin{aligned} PQ &= \sqrt{(PO_2)^2 - (QO_2)^2} \\ &= \sqrt{10^2 - 8^2} = 6. \end{aligned}$$

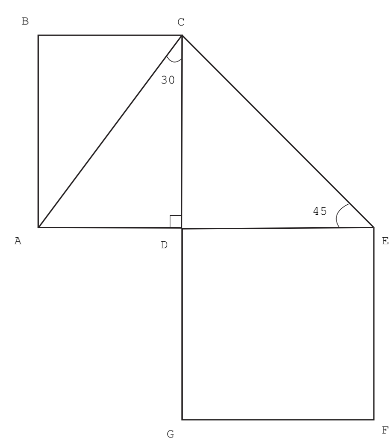
Hence, the correct option is (C).

248. The key is to use the fact that the sum of the angles in a triangle is 180°. Each of the three triangles shown in the diagram is a right triangle. It follows that $x = 180 - 45 - 90 = 45°$.

For the scalene outmost triangle, we know that $180 = 90 + 30 + (x + y)$, thus $x + y = 60°$. So, $y = 15°$, and $2x - y = 90° - 15° = 75°$.

Hence, the correct option is (D).

249. The key to this problem is to remember that the sides of a $30° - 60° - 90°$ triangle are related as $1 : \sqrt{3} : 2$, and the sides of a $45° - 45° - 90°$ triangle are related as $1 : 1 : \sqrt{2}$.



We know that area of $\triangle ACD$ is half of the area of rectangle $ABCD$ thus, area of $\triangle ACD = 2\sqrt{3}$.

Again, $\triangle ACD$ is a $30° - 60° - 90°$ triangle, so its sides are related as $AD : CD : AC :: 1 : \sqrt{3} : 2$. Hence,

Area of $\triangle ACD = 2\sqrt{3} = \frac{1}{2}AD \cdot CD = \frac{1}{2}AD \cdot (\sqrt{3}AD) = \frac{\sqrt{3}}{2}AD^2 \Rightarrow 2\sqrt{3} = \frac{\sqrt{3}}{2}AD^2 \Rightarrow \sqrt{3}AD^2 = 4\sqrt{3}$

So, $CD = \sqrt{3}.AD = 2\sqrt{3}$. So, the area of the square $DEFG = CD^2 = (2\sqrt{3})^2 = 12$.

Hence, the correct option is (D).

250. Let the distance between A and B be a and the distance between B and C be b. Then, $a + b = 150$, and $a = 5b$. To find out b, substitute the latter into the former to obtain that $6b = 150$. So, $b = 25, BC$.

Hence, the correct option is (D).

251. The key is to remember that ray R has completed one revolution around the origin. One revolution with a radius of 1 results in a circle with the circumference of 2π. Then, it is best to proceed by elimination. I is not correct because it states that $D < \frac{3\pi}{2}$, and but we know that $D > 2\pi$. II can be rewritten as $\pi > D > \frac{\pi}{2}$; like I, II is not correct because it states that $D < \pi$, and we know that $D > 2\pi$. Now, it is clear that III is the only answer. We can rewrite III as $3\pi < D < 3\frac{\pi}{2}$, thus is is clearly more that 2π.

Hence, the correct option is (C).

Alternatively, the from the diagram, we know that the ray completed one full revolution and after the completion of half revolution but before the three quarter revolution, it stops; thus we can conclude that

$[(2\pi + \frac{1}{2}.2\pi) < D < (2\pi + \frac{3}{4}.2\pi)$ or $3\pi < D < \frac{7\pi}{2}]$.

252. The key is to use the properties of two parallel lines with a transversal. To find the measure of y, we need to relate AB and L_2. Since $AB \| L_1$ & $L_1 \| L_2$, then $AB \| L_1 \| L_2$. Thus, $y = 120°$ from the properties of two parallels and a transversal.

Hence, the correct option is (D).

253. Let the side of the smallest equilateral triangle be s. We know that the area of an equilateral triangle is $\frac{\sqrt{3}}{4}(\text{side})^2$.

Then, its area is

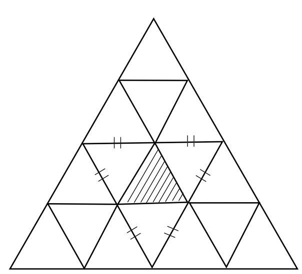
$$= \frac{\sqrt{3}}{4}s^2.$$

Since all the triangles are equilateral, the side of the largest equilateral triangle is $4s$. Its area is

$$= \frac{\sqrt{3}}{4}(4s)^2 = 4\sqrt{3}s^2.$$

Dividing the area of the smallest triangle by the largest, we get $\frac{1}{16}$.

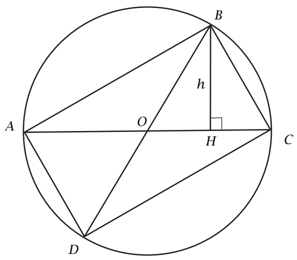
For a simple counting argument, see the figure below.



16 equilateral triangles are obtained by connecting the midpoints of the sides of the outer triangles and, then, repeating the same procedures once again for each of the four thus constructed triangles. It follows from this procedure that all 16 triangles are equal by equality of their sides.

Hence, the correct option is (A).

254. See the figure below.



First, note that $\triangle ABC = \triangle CDA$. This follows from the fact that points B, O, and D lie on the same line, which allows to split the above-mentioned triangles into pairs of triangles ABO, BOC and AOD, DOC and see that the first triangle in each pair is equal to the second triangle in the other pair, and vice versa. So, it is sufficient to focus on $\triangle ABC$.

The length of the altitude h of $\triangle ABC$ is obtained from the expression $A = \frac{1}{2}bh$ for the area A of $\triangle ABC$. We know that $b = 12$, since b being the base of the triangle and is equal to the diameter of the circle = 6.2 = 12. Thus $A = 18\sqrt{3} = \frac{1}{2}.6h$

$$h = (2 \cdot 18\sqrt{3})/12 = 3\sqrt{3}.$$

Then, the area of $\triangle BOC$ is $\frac{1}{2} \cdot OC \cdot h = \frac{1}{2} \cdot 6 \cdot h = 3 \cdot 3\sqrt{3} = 9\sqrt{3}$, and

$$\begin{aligned}
\mathbf{A}(\triangle ODC) &= \mathbf{A}(\triangle OBA) \qquad \mathbf{A} \text{ denotes area} \\
&= \mathbf{A}(\triangle ABC) - \mathbf{A}(\triangle OBC) \\
&= 18\sqrt{3} - 9\sqrt{3} \\
&= 9\sqrt{3}.
\end{aligned}$$

Hence, the correct option is (B).

255. The area of the figure is the area of three semi-circles plus the area of a triangle.

$$\text{Area} = \left[3 \cdot \left(\frac{1}{2} \cdot \pi \cdot \left(\frac{40}{2}\right)^2\right) + \frac{\sqrt{3}}{4} \cdot 40^2\right].$$

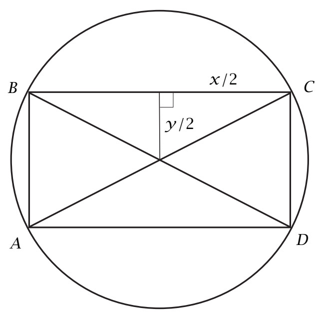
So, the volume of the figure = Height (5) * Area

$$\text{Volume} = 5 \times \left(\frac{3}{2} \cdot \pi \cdot 20^2 + \frac{\sqrt{3}}{4} \cdot 40^2\right).$$

After doing some algebra, the expression above simplifies to $3000\pi + 2000\sqrt{3}$.

Hence, the correct option is (B).

256. Assume that the length and the width of the rectangle are x & y, respectively. See the figure below for notation.



From the figure above, the Pythagorean theorem and the perimeter of $ABCD$ provide us with a system of two equations

$$\begin{aligned}
\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 &= (7.5\sqrt{2})^2, \\
\frac{x^2}{4} + \frac{y^2}{4} &= 2(7.5)^2, \\
x^2 + y^2 &= 450 \text{ (after simplification).}
\end{aligned}$$

Fro the perimeter information, we have that

$$\begin{aligned}
&2x + 2y = 60, \\
&x + y = 30 \\
&y = 30 - x.
\end{aligned}$$

After substituting the value of $y = 30 - x$ into the $x^2 + y^2 = 450$ equation, we have that

$$x^2 + (30 - x)^2 = 450,$$

and after simplifying

$$\begin{aligned}
x^2 + 30^2 - 2.30.x + x^2 &= 450 \\
2x^2 - 60x + 450 &= 0 \\
2(x^2 - 30x + 225) &= 0 \\
x^2 - 30x + 225 &= 0 \\
x^2 - 2.15.x + (15)^2 &= 0 \\
(x - 15)^2 &= 0 \\
x &= 15
\end{aligned}$$

So, $x = 15$ and $y = 30 - x = 15$, and the area of $ABCD = 15.15 = 225$.

Hence, the correct option is (B).

257. We now that the number of diagonals in a polygon with n sides $= \frac{n(n-3)}{2}$.

According to the problem statement, we have the number of diagonals equal to double the number of sides. Hence, we have the equation:

$$\frac{n\,(n-3)}{2} = 2n$$

$$=> n\,(n-3) = 4n => n\,(n-3) - 4n = 0$$

$$=> n\,\{(n-3) - 4\} = 0 => n\,(n-7) = 0$$

$=> n = 0$ or 7.

Clearly, $n \neq 0$.

Hence, $n = 7$.

Hence, the correct option is (C).

258. Let the breadth of the rectangle be s inches.

We know that the difference between the length and breadth of the rectangle is 4 inches. This means that the length must be 4 inches longer than the breadth.

Hence, the length of the rectangle is $(s + 4)$ inches.

We know that the area of a rectangle is given by (length $*$ breadth).

Thus, the initial area of the rectangle = $s(s + 4)$ square inches.

The new length of the rectangle = $(s + 4) + 6 = (s + 10)$ inches.

Thus, the new area of the rectangle = $s(s + 10)$ square inches.

Hence, increase in area = $s\,(s + 10) - s\,(s + 4) = s[(s + 10) - (s + 4)] = 6s$ square inches.

Hence, we have: $6s = 120 \Rightarrow s = 20$.

Thus, the breadth of the rectangle is 20 inches.

Hence, the correct option is (B).

259. Let the initial length of a side of the square be s inches.

We know that the area of a square is given by the square of its side.

Thus, the initial area of the square = s^2 square inches.

The new length of a side of the square becomes $2s$ inches.

Thus, the new area of the square = $(2s)^2 = 4s^2$ square inches.

Hence, increase in area = $4s^2 - s^2 = 3s^2$ square inches.

Hence, we have: $3s^2 = 300 \Rightarrow s^2 = 100 \Rightarrow s = 10$.

Thus, the initial side of the square is 10 inches.

Hence, the correct option is (A).

5.4.2 Trigonometry

260. In this question, we need to calculate the length of the hypotenuse AC where the length of one side and an angle is given.

In order to find the hypotenuse, we need to use a trigonometric ratio that involves the hypotenuse. Both Sinθ and Cosθ are two such trigonometric ratios which involve the hypotenuse.

Now, we see that the given angle is angle A while the given side BC is opposite to it. Hence, we need to use Sinθ since it deals with the opposite side and the hypotenuse.

Sin 30^0 $= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{BC}}{\text{AC}}$.

Thus, we have: $\frac{1}{2} = \frac{8}{\text{AC}}$ (since Sin $30^0 = \frac{1}{2}$ & $BC = 8$)

$=> AC = 8 * 2 = 16$.

Hence, the length of AC is 16.

Hence, the correct option is (C).

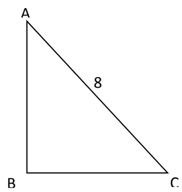
261. We know that in a right angled triangle, the square of the hypotenuse equals the sum of squares of the two legs of the triangle.

Thus, we have: $AC^2 = AB^2 + BC^2 => AC^2 = 24^2 + 7^2 = 576 + 49 = 625 = 25^2$

$=> AC = 25$.

Hence, the correct option is (C).

262. Let our triangle be ABC, right angled at B.



Since the triangle is isosceles, angles A and C must be equal.

Sum of the angles A and C = $180^0 - 90^0 = 90^0$.

Hence, each angle A = C = $\frac{90^0}{2} = 45^0$.

In this question, we need to calculate the length of the AB and BC where the length of the hypotenuse is given.

In order to find the AB and AC, we need to use a trigonometric ratio that involves the hypotenuse. Both Sinθ and Cosθ are two such trigonometric ratios which involve the hypotenuse.

Thus, we have: Sin A = Sin $45^0 = \frac{1}{\sqrt{2}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{\mathrm{BC}}{8} => \mathrm{BC} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$.

Similarly, Sin C = Sin $45^0 = \frac{1}{\sqrt{2}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{\mathrm{AB}}{8} => AB = \frac{8}{\sqrt{2}} = 4\sqrt{2}$.

Hence, sum of the other two sides = AB + BC = $4\sqrt{2} + 4\sqrt{2} = 8\sqrt{2}$.

Hence, the correct option is (D).

263. To find AB, given the length of BC and angle A, we need to use tanθ since this trigonometric ratio relates the two legs of a right angled triangle.

Thus, tan $43^0 = \frac{\mathrm{BC}}{\mathrm{AB}} => AB = \frac{\mathrm{BC}}{\tan 43^0} = \frac{8}{\tan 43^0}$.

To find AC, given the length of BC and angle A, we need to use Sinθ since this trigonometric ratio relates the opposite side and the hypotenuse of a right angled triangle.

Thus, Sin $43^0 = \frac{\mathrm{BC}}{\mathrm{AC}} => AC = \frac{\mathrm{BC}}{\sin 43^0} = \frac{8}{\sin 43^0}$.

Thus, the difference between AB ad AC = $AC - AB = \frac{8}{\sin 43^0} - \frac{8}{\tan 43^0}$.

Hence, the correct option is (A).

264. Since BE = EC, we have EC = $\frac{\mathrm{BC}}{2} = \frac{8}{2} = 4$.

Also, angle C = $180^0 -$ (B + A) $= 180^0 - \left(30^0 + 90^0\right) = 60^0$.

Now, in triangle DEC, Cos C = $\frac{\mathrm{CE}}{\mathrm{DC}} =>$ Cos $60^0 = \frac{4}{\mathrm{DC}} => \frac{1}{2} = \frac{4}{\mathrm{DC}} =>$ DC = 8.

Hence, the correct option is (B).

265. In the triangle, corresponding to angle A, we know the values of the opposite side and the hypotenuse. Hence, we work with Sin A.

Sin A $=\frac{BC}{AC}=\frac{4}{8}=\frac{1}{2}$.

We know that Sin $30^0=\frac{1}{2}$.

Thus, A $=30^0$.

Hence, the correct option is (B).

266. Let the garden be the triangle ABC as shown beside, right angled at B.

For a right angled triangles triangle, the longest side is the hypotenuse = 8 m long.

We need to use Sinθ and Cosθ to find the other two sides of the triangle since only these two ratios relate the sides to the hypotenuse.

Sin $30^0=\frac{BC}{AC}=>\frac{1}{2}=\frac{BC}{8}=>$ BC = 4 m.

Cos $30^0=\frac{AB}{AC}=>\frac{\sqrt{3}}{2}=\frac{AB}{8}=>$ AB = $4\sqrt{3}$ m.

Thus, perimeter of the triangle = AB + BC + AC = $4\sqrt{3}+4+8 = (12+4\sqrt{3})$ m.

Hence, the correct option is (D).

267. Let the triangle be ABC, right angled at B.

Let angle A be 30^0.

Thus, the other angle C must be $180^0-\left(90^0+30^0\right)=60^0$.

The smallest side must be opposite to the smallest angle.

Hence, we have BC = 4 (the smallest side).

We know that area of a triangle is gives as: Area $=\frac{1}{2}*$Base $*$ Height.

Here, we can take BC as the base and AB as the height.

Thus, we need to find the length of AB.

Since we need the relation between the two legs of the right angled triangle, we work with tanθ.

Thus, tan A = tan $30^0=\frac{BC}{AB}=>\frac{1}{\sqrt{3}}=\frac{4}{AB}=>$ AB = $4\sqrt{3}$.

Hence, area of the triangle = $\frac{1}{2}*AB*BC=\frac{1}{2}*4\sqrt{3}*4=8\sqrt{3}$.

Hence, the correct option is (B).

268. Here, we need to focus on triangle BDC in order to find the length of BD.

We know that angle B is 90^0 and angle A is 30^0.

Thus, angle C = $180^0-\left(90^0+30^0\right)=60^0$.

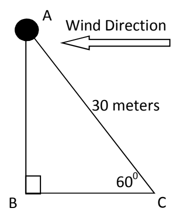
In triangle BDC, with respect to angle C, BD is the opposite side and BC is the hypotenuse.

Thus, in order to relate these two sides, we use Sinθ.

Thus, Sin C = Sin $60^0 = \frac{BD}{BC} => \frac{\sqrt{3}}{2} = \frac{BD}{4} =>$ BD = $2\sqrt{3}$.

Hence, the correct option is (C).

269. Let the balloon be at the point A as shown in the diagram below.



AC is the rope with which the balloon is tied to the ground (the ground being BC).

Due to the wind, the balloon sways and its rope makes an angle of 60^0 with the ground.

We need to calculate the current height of the balloon above the ground level i.e. AB.

Thus, in triangle ABC, with reference to angle C, we know the hypotenuse and we need to calculate the opposite side. Hence, we work with Sinθ.

Thus, we have: Sin $60^0 = \frac{AB}{AC} => \frac{\sqrt{3}}{2} = \frac{AB}{30} =>$ AB = $15\sqrt{3}$ meters.

Hence, the correct option is (D).

270. We can simplify the above as follows:

$$\frac{3\pi}{4}^c + \frac{11\pi}{5}^c - \frac{7\pi}{10}^c = \left(\frac{3\pi * 5 + 11\pi * 4 - 7\pi * 2}{20}\right)^c = \left(\frac{15\pi + 44\pi - 14\pi}{20}\right)^c = \left(\frac{45\pi}{20}\right)^c = \left(\frac{9\pi}{4}\right)^c$$

We know that π radians are equivalent to 180^0.

Thus, $\frac{9\pi}{4}$ radians are equivalent to $180^0 * \frac{9}{4} = 405^0$.

Thus, the required value is 405^0.

Hence, the correct option is (C).

271. We can simplify the above as follows: $\frac{7\pi}{4}^c + \frac{4\pi}{3}^c - \frac{5\pi}{12}^c - 180^0 = \left(\frac{7\pi*3+4\pi*4-5\pi*1}{12}\right)^c - 180^0$
$= \left(\frac{21\pi+16\pi-5\pi}{12}\right)^c - 180^0 = \left(\frac{32\pi}{12}\right)^c - 180^0 = \frac{8\pi}{3}^c - 180^0$.

We know that π radians are equivalent to 180^0.

Thus, $\frac{8\pi}{3}$ radians are equivalent to $180^0 * \frac{8}{3} = 480^0$.

Thus, $\frac{8\pi}{3}^c - 180^0 = 480^0 - 180^0 = 300^0$. (We do not find an option with 300°)

Converting back to radians: Since $180^0 = \pi$ radians, we have:

$300^0 = \left(\frac{\pi}{180} * 300\right)^c = \frac{5\pi}{3}^c$.

Thus, the required value is 300^0 or $\frac{5\pi}{3}^c$.

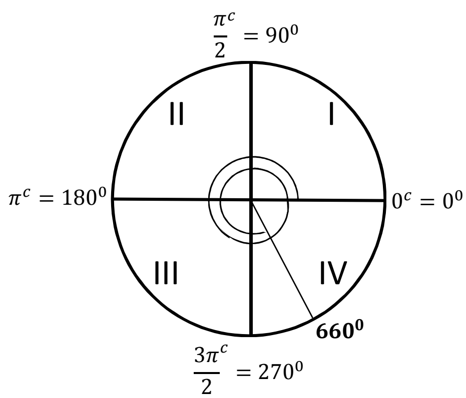
Hence, the correct option is (B).

272. We know that π radians are equivalent to 180^0.

Thus, $\frac{11\pi}{3}$ radians are equivalent to $180^0 * \frac{11}{3} = 660^0$.

Hence, the first statement is correct.

The unit circle along with the quadrants marked is shown below. The values of the angles at the four major points are also shown.



Now, $660^0 = 360^0 * 1 + 300^0$.

Thus, in order to trace the angle, we need to make one full revolution from 0^0 and then cover 300^0 extra. In the diagram, the angle corresponding to $660^0 => 300^0$ is shown.

Thus, we finally end in the fourth quadrant. Thus, the second statement is not correct.

We also know that in the fourth quadrant, the value of Cosθ is positive.

Thus, the third statement is correct.

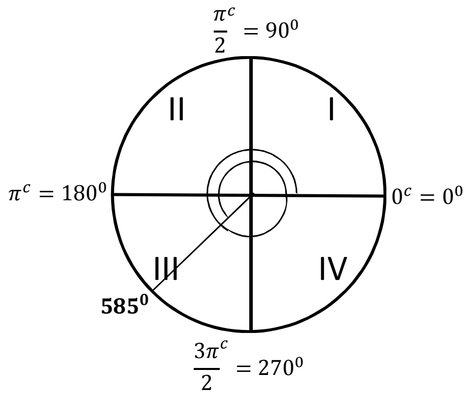
Hence, the correct option is (D).

273. We know that π radians are equivalent to 180^0.

Thus, $\frac{13\pi}{4}$ radians are equivalent to $180^0 * \frac{13}{4} = 585^0$.

Hence, the first statement is correct.

The unit circle along with the quadrants marked is shown below. The values of the angles at the four major points are also shown.



Now, $585^0 = 360^0 * 1 + 225^0$.

Thus, in order to trace the angle, we need to make one full revolution from 0^0 and then cover 225^0 extra. In the diagram, the angle corresponding to $585^0 \Rightarrow 225^0$ is shown.

Thus, we finally end in the third quadrant.

We know that in the third quadrant, the value of tanθ is positive while the value of Cosθ is negative.

Thus, the second and third statements are not correct.

Hence, the correct option is (A).

274. We can see that the value of Cosθ is positive.

We know that the value of Cosθ is positive in the first and fourth quadrants only. Thus, θ should lie in either the first or fourth quadrants.

Hence, the first statement is not correct.

Again, the value of tanθ is positive in the first quadrant while it is negative in the fourth quadrant. Since θ lies in either the first or fourth quadrant, the value of tanθ can be both positive as well as negative.

Hence, the second statement is correct.

Again, the value of Sinθ is positive in the first quadrant and negative in the fourth quadrant. Since θ lies in either the first or fourth quadrant, the value of Sinθ may be both positive as well as negative.

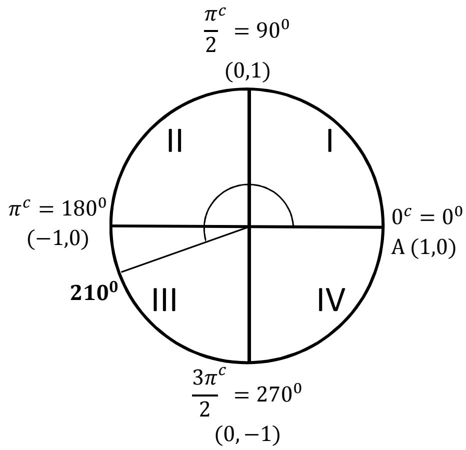
Hence, the third statement is not correct.

Hence, the correct option is (B).

275. We know that π radians are equivalent to 180^0.

Thus, $\frac{7\pi}{6}$ radians are equivalent to $180^0 * \frac{7}{6} = 210^0$.

The above unit circle along with the quadrants marked is shown below. The values of the angles at the four major points are also shown.



In the diagram, the angle corresponding to 210^0 is also shown.

Thus, we finally end in the third quadrant.

We know that in the third quadrant, the values of both $X$and Y coordinates are negative.

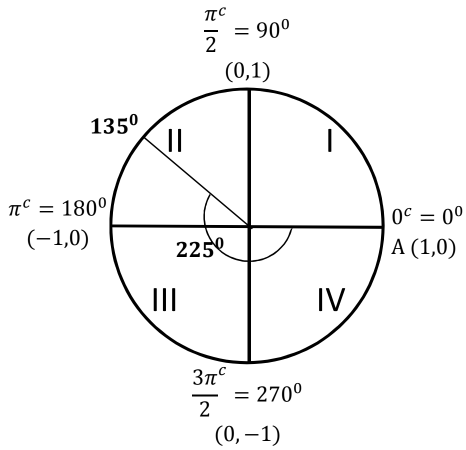
Thus, option (D), which has both coordinates as negative, must be the correct answer.

Hence, the correct option is (D).

276. We know that π radians are equivalent to 180^0.

Thus, $\frac{5\pi}{4}$ radians are equivalent to $180^0 * \frac{5}{4} = 225^0$.

The above unit circle along with the quadrants marked is shown below. The values of the angles at the four major points are also shown.



In the diagram, the angle corresponding to 225^0 in the clockwise direction is shown.

One should note that 225^0 in the clockwise direction is equivalent to $360^0 - 225^0 = 135^0$.

Thus, we finally end in the second quadrant.

We know that in the second quadrant, the value of X coordinate is negative while that of the Y coordinate is positive.

Thus, option (B), which satisfies the above conditions, must be the answer.

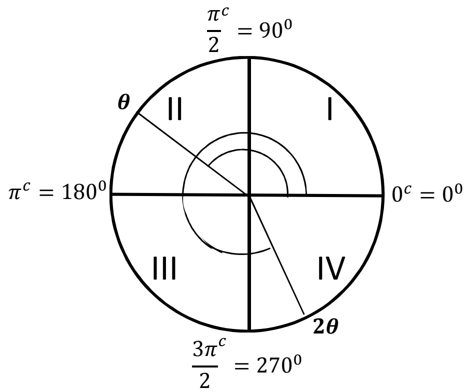
Hence, the correct option is (B).

277. We know that π radians are equivalent to 180^0.

Thus, $\frac{4\pi}{5}$ radians are equivalent to $180^0 * \frac{4}{5} = 144^0$ and $\frac{5\pi}{6}$ radians are equivalent to $180^0 * \frac{5}{6} = 150^0$.

Thus, we have $144^0 \le \theta \le 150^0$ i.e. θ lies in the second quadrant.

The angle is shown in the diagram below.



Thus, the value of Cosθ is negative.

Hence, the first statement is not correct.

Again, since $144^0 \le \theta \le 150^0$, we have $2 * 144^0 \le 2\theta \le 2 * 150^0 => 288^0 \le 2\theta \le 300^0$.

Thus, 2θ lies in the fourth quadrant. The angle is shown in the diagram below.

Thus, the value of Cos(2θ) is positive and the value of Sin(2θ) is negative.

Hence, both second and third statements are correct.

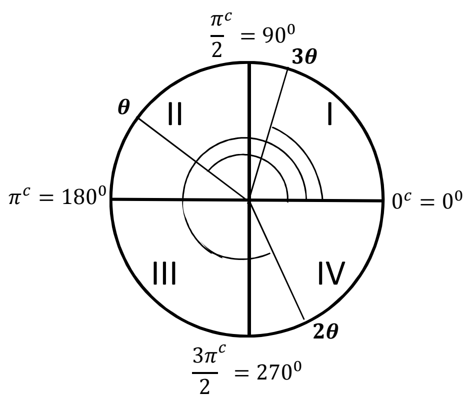
Hence, the correct option is (C).

278. We know that π radians are equivalent to 180^0.

Thus, $\frac{3\pi}{4}$ radians are equivalent to $180^0 * \frac{3}{4} = 135^0$ and $\frac{4\pi}{5}$ radians are equivalent to $180^0 * \frac{4}{5} = 144^0$.

Thus, $135^0 < \theta < 144^0$ i.e. θ lies in the second quadrant. The angle is shown in the diagram below.

Thus, the value of tanθ is negative.



Hence, the first statement is correct.

Again, since $135^0 < \theta < 144^0$, we have $2 * 135^0 < 2\theta < 2 * 144^0 => 270^0 < 2\theta < 288^0$.

Thus, 2θ lies in the fourth quadrant. The angle is shown in the diagram below.

Thus, the value of $\tan(2\theta)$ is negative.

Hence, the second statement is correct.

Again, since $135^0 < \theta < 144^0$, we have $3 * 135^0 < 3\theta < 3 * 144^0 => 405^0 < 3\theta < 432^0$.

Thus, 3θ lies in the first quadrant. The angle is shown in the diagram below.

Thus, the value of $\tan(3\theta)$ is positive.

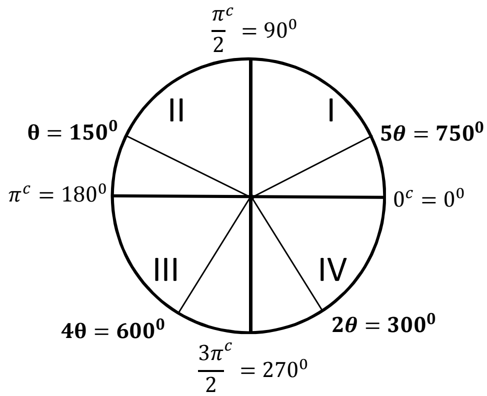
Hence, the third statement is correct.

Thus, all statements are correct.

Hence, the correct option is (D).

279. We have θ=150^0=> 2θ=300^0, 4θ=600^0 and 5θ=750^0.

Let us first plot the angles $\theta, 2\theta, 4\theta$ and 5θ on the unit circle.



Thus, we can see that θ lies in the second quadrant, 2θ lies in the fourth quadrant, 4θ lies in the third quadrant and 5θ lies in the first quadrant.

Verifying each statement:

Statement I: Sin(2θ) is negative (angle in fourth quadrant) and Cos(4θ) is negative (angle in third quadrant).

Thus, both have the same sign.

Hence, the first statement is correct.

Statement II: tan(2θ) is negative (angle in fourth quadrant) and Cos(5θ) is positive (angle in first quadrant).

Thus, they have opposite signs.

Hence, the second statement is correct.

Statement III: Sin (4θ) is negative (angle in third quadrant) and tan(5θ) is positive (angle in first quadrant).

Thus, they have opposite signs.

Hence, the third statement is not correct.

Hence, the correct answer is (C).

5.4.3 Complex Numbers

280. We can rewrite the given expression $\sqrt{-25}$ as a product of two square roots:

$\sqrt{-25} = \sqrt{25} \times \sqrt{-1} = 5 \times i$ (Since $i^2 = -1,\ i = \sqrt{-1}$)

Hence, the correct option is (A).

281. We can simplify the given expression as follows:

$$\frac{4\sqrt{-9}}{3i} = \frac{4 \times \sqrt{9} \times \sqrt{-1}}{3i} = \frac{4 \times 3 \times i}{3i} = 4$$

Hence, the correct option is (C).

282. We can simplify the given expression as follows:

$$(3 - i) \times (2 + 5i) = 3 \times 2 + 3 \times 5i - 2 \times i - 5i \times i = 6 + 15i - 2i - 5i^2$$

$$= 6 + 13i - 5 \times (-1) = 6 + 13i + 5 = 11 + 13i.$$

Hence, the correct option is (C).

283. We can simplify the given expression as follows:

$$(2 - 3i) + (1 + 8i) = (2 + 1) + (8i - 3i) = 3 + 5i$$

Hence, the correct option is (B).

284. We can simplify the given expression as follows:

$$(2 - 3i) \times (1 + 8i) + i \times (2 + 5i) = (2 \times 1 - 3i \times 1 + 2 \times 8i - 3i \times 8i) + (2i + 5i \times i)$$

$$= \left(2 - 3i + 16i - 24i^2\right) + \left(2i + 5i^2\right) = (2 + 13i - 24 \times -1) + (2i + 5 \times (-1))$$

$$= (2 + 13i + 24) + (2i - 5) = (26 + 13i) + (2i - 5) = 21 + 15i$$

Hence, the correct option is (D).

285. We know that:

$$i^2 = \sqrt{-1} \times \sqrt{-1} = -1$$

$$i^3 = i \times i^2 = i \times (-1) = -i$$

$$i^4 = i \times i^3 = i \times (-i) = -i^2 = -(-1) = 1$$

$$i^5 = i \times i^4 = i \times 1 = i$$

Hence, the required sum = $(i^2 + i^3 + i^4 + i^5) = -1 - i + 1 + i = 0$.

Hence, the correct answer is (A).

286. We can simplify the given expression as follows:

$$(\sqrt{-3}) \times (\sqrt{-6}) \times (\sqrt{-8}) \times i = \left(\sqrt{-1} \times \sqrt{3}\right) \times \left(\sqrt{-1} \times \sqrt{6}\right) \times \left(\sqrt{-1} \times \sqrt{8}\right) \times i$$

$= \sqrt{3} \times \sqrt{6} \times \sqrt{8} \times \left(\sqrt{-1}\right)^3 \times \sqrt{-1} = \sqrt{3} \times \left(\sqrt{2} \times \sqrt{3}\right) \times \sqrt{8} \times \left(\sqrt{-1}\right)^2 \times \left(\sqrt{-1}\right)^2$; Clubbing $\sqrt{-1}$ and replacing the value of i.

$$= (\sqrt{3})^2 \times \left(\sqrt{2} \times \sqrt{8}\right) \times (-1) \times (-1)$$

$$= (\sqrt{3})^2 \times \sqrt{16} \times (-1) \times (-1) = 3 \times 4 \times 1 = 12$$

Hence, the correct option is (C).

287. We can simplify the given expression as follows:

$i \times i^2 \times i^3 \times i^4 = i^{1+2+3+4} = i^{10} = (i^2)^5 = (-1)^5 = -1$. [Since $(-1)^{\text{odd}}$ always equals to -1.]

Hence, the correct answer is (A).

288. We can simplify the given expression as follows:

$$\{(1-i) + (1+i)\} \times \{(1+i) \times (1-i)\} = \{1 + i + 1 - i\} \times \{1 \times 1 - 1 \times i + i \times 1 - i \times i\}$$

$$= 2 \times \left(1 - i + i - i^2\right)$$

$$= 2 \times (1 + 1) = 2 \times 2 = 4.$$

Hence, the correct option is (D).

289. We see that the denominator of the expression contains a term $(1 + i)$. A simplified method to get rid of roots/surds, in this case it is an imaginary number, is to rationalize the denominator $(1 + i)$ of the fraction $(\frac{1-i}{1+i})$.

To rationalize a fraction, we multiply & divide the fraction with the conjugate of the fraction $\left(\frac{1-i}{1+i}\right)$. If the denominator is $(a + bi)$, then the conjugate is $(a - bi)$, and vice-versa.

In this question, the denominator is $(1 + i)$, so we multiply the given fraction with a fraction$\left(\frac{1-i}{1-i}\right)$.

$$= \left(\frac{1-i}{1+i}\right) = \left(\frac{1-i}{1+i}\right) \times \left(\frac{1-i}{1-i}\right) = \frac{(1-i)\times(1-i)}{(1+i)\times(1-i)} = \frac{1-i-i+i^2}{1+i-i-i^2} = \frac{1-2i-1}{1+1} = \frac{-2i}{2} = -i$$

Hence, the correct option is (A).

5.5 Higher Order Thinking Questions

290. Part 1

First, you must identify that Alex is the fastest worker, finishing (1/8)th part of the work in an hour, while Cherry is the slowest worker, finishing (1/48)th part of the work in an hour.

Second, note that *Anyone can take a start and others will follow*. It means that anybody: Alex, Betty, or Cherry can start or/and complete the job.

When Alex, Betty, and Cherry each work, in that order, for an hour, then in a total of three hours, they accomplish:

$\frac{1}{8} + \frac{1}{24} + \frac{1}{48} = \frac{9}{48} = \left(\frac{3}{16}\right)^{\text{th}}$ of the job.

After five rounds, in a total of $5 * 3 =$ Č15 hours, they complete $\frac{15}{16}$ part of the job So, $\frac{1}{16}$ of the job remains.

To find the **minimum hours to do the job**, after the initial five rounds, select the **fastest worker, Alex**, to take the next turn. He would need only $\frac{1}{2}$ an hour to complete the remaining $\frac{1}{16}$ part of the job [Alex can do $\frac{1}{8}$ part of job in one hour, so he will take $\frac{1}{2}$ an hour to do $\frac{1}{16}$ part of the job]. Thus, the minimum hours needed to do the job is $15 + 0.5 = 15.50$ hours.

The correct answer is option 15.5.

Part 2

To find the **maximum hours to do the job**, choose the **slowest worker (Cherry)** to take the next turn. She will complete $\frac{1}{48}$ part of the work in an hour, now choose the next slower worker, Betty, to do another $\frac{1}{24}$ part of the job in the next hour. This covers the remaining part of the job after the initial five rounds:

$\frac{1}{48} + \frac{1}{24} = \frac{3}{48} = \frac{1}{16}$ part of the job = Remaining part of the job

So, the maximum hours needed to do the job would be $15 + 2 = 17$ hours.

The correct answer is option 17.

291. Since the expression itself does not describe anything, we will have to read each option one by one.

(A) Time taken by x persons = $x * \left[\frac{h \text{ hours}}{y \text{ persons}}\right] = \frac{xh}{y}$

The result does not match with the given expression, hence option A cannot be an answer.

(B) Time taken by x persons = $x * \left[\frac{y \text{ hours}}{h \text{ persons}}\right] = \frac{xy}{h}$

The result does not match with the given expression, hence option B cannot be an answer.

(C) Time taken by $2h$ persons = $2h * \left[\frac{2y \text{ hours}}{x \text{ persons}}\right] = \frac{4hy}{x}$

The result does not match with the given expression, hence option C cannot be an answer.

(D) Time taken by y persons = $y * \left[\frac{2h \text{ hours}}{2x \text{ persons}}\right] = \frac{yh}{x}$

The result matches with the given expression, hence option D is the correct answer.

The correct answer is option (D).

292. Glancing at the options, we find that there are two situations: one, emptying a tank and two, filling a tank.

Considering that the given expression $\frac{1}{x} - \frac{1}{y} = \frac{1}{10}$ for filling a tank, we can deduce that the expression is a mathematical model, describing that "An empty tank can be filled in 10 hours with the help of two pipes: one, an inlet pipe, running alone can fill the tank in x hours and two, an outlet pipe, running alone can empty the tank in y hours."

It is important that the rate of inlet pipe, here $\frac{1}{x}$, must be greater than that of outlet pipe, here $\frac{1}{y}$; else the tank will never be filled. Thus, option B is the correct answer.

Let us mathematically represent each option.

(A) $\frac{1}{y} - \frac{1}{x} = \frac{1}{10}$

(B) $\frac{1}{x} - \frac{1}{y} = \frac{1}{10}$; correct answer

(C) $\frac{1}{y} - \frac{1}{x} = \frac{1}{10}$;

Rate of outlet pipe $\frac{1}{y}$ > Rate of inlet pipe $\frac{1}{x}$, else tank can never be empty.

(D) $\frac{1}{2x} - \frac{1}{2y} = \frac{1}{5} \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{2}{5}$

The correct answer is option (B).

293. Since the given situation is that an inlet pipe fills the tank and an outlet pipe empties it, to make sure that the tank is filled, rate of the inlet pipe must be higher than that of the outlet pipe; else the tank can never be filled (Even running for infinite hours).

$\Rightarrow \frac{1}{x} > \frac{1}{y} \Rightarrow x < y$

So option C is the correct answer. Although option A also includes the condition expressed in option C, two options cannot be correct in SAT MCQ question type.

=> The second part of option A: $y < z$ must not be true!

Let us see how.

Since we have to negate the condition $y < z$, assume some values for $x, y,$ & z, such that $y \geq z$.

Say $y = 30,$ & $z = 15$ $(y > z)$.

Thus, $\frac{1}{x} - \frac{1}{30} = \frac{1}{15} => \frac{1}{x} = \frac{1}{30} + \frac{1}{15} = \frac{1+2}{30} = \frac{3}{30} = \frac{1}{10} => x = 10$ hours.

The result $x = 10 < y = 15$ is justified as discussed above.

So, had one of the options been: $x < y$ & $y > z$, had it been correct?

No, it is not!

Let's see how.

Say $y = 20,$ & $z = 30$ $(y < z)$.

Thus, $\frac{1}{x} - \frac{1}{20} = \frac{1}{30} => \frac{1}{x} = \frac{1}{30} + \frac{1}{20} = \frac{2+3}{60} = \frac{5}{60} = \frac{1}{12} => x = 12$ hours.

The result $x = 12 < y = 20$ is justified as discussed above.

Thus we cannot conclusively conclude anything between y & z. Even there can be a situation where $y = z$. Try with $x = 10,$ $y = 20,$ & $z = 20$.

The correct answer is option (C).

294. In the expression $\frac{10}{x} + \frac{5}{y} = \frac{1}{6}, \frac{1}{x}$ represents the portion of job one slower machine does in one hour, so $\frac{10}{x}$ represents the portion of job 10 slower machines do in one hour.

Similarly, $\frac{5}{y}$ represents the portion of job 5 faster machines do in one hour. Option D is the correct answer.

It is to be noted that x & y represent time, each slower and each faster machine, respectively, takes to complete the job, working alone.

The correct answer is option (D).

295. Part I

To find the equivalent of Rupee 5500 in Dinar, first we need to convert Rs. 5500 to $ and then $ to Dinar, since Rs to Dinar conversion rate is not given in the table.

From the table, we know that Rs to $ rate on July is 56.50. So, Rs. 5500 would be $= \frac{5500}{56.50} = \$97.35$. Again, on July, $ to Dinar is 3.5. So $ 97.35 = Dinar $\frac{96.49}{3.5}$ = Dinar 27.81.

Adding 5% of bank charges, we get, total Dinar needed in the bank account = 27.81+5% = $27.81 + 27.81 * \left(\frac{5}{100}\right) = 27.81 + 1.391 =$ Dinar 29.20.

Roger must have at least Dinar 29.20 in his bank account.

The correct answer is 29.2.

Part II

Roger can buy the Mac if he has the money in a particular currency greater than or equal to the cost of MaC Now calculate the cost of Mac in different currencies.

In Sept, Real to \$ rate is 2.4. So, the equivalent \$ for Real 2400 =\$ (2400/2.4) = \$1000. Roger has \$1100, so he can buy the Mac spending only dollars.

Similarly, Rs. to \$ rate is 61.50. So, the equivalent Rs. for \$1000 (cost of Mac) = Rs. (1000*61.50) = Rs. 61,500. But Roger has only Rs. 50,000, so he cannot buy the Mac spending only rupees.

Similarly, \$ to Dinar rate is 3.6. So, the equivalent Dinar for \$1000 = (1000/3.6) Dinar = 277.8 Dinar. But Roger has only Dinar 250, so he cannot buy the Mac spending only Dinar.

Hence Roger can buy the Mac only using Dollars.

The correct answer is option (A).

296. There are 5 divisions between 1.04 & 1.05, thus each interval represents a difference of $\frac{(1.05-1.04)}{5} = \frac{0.01}{5} = 0.002$, and the arrow points to the middle of the third interval between 1.044 and 1.046, thus it points to 1.045.

Alternatively, since the arrow points to the middle of 1.04 and 1.05, the correct reading would be the average of 1.04 and 1.05 = $\frac{(1.04+1.05)}{2} = 1.045$.

Hence, the correct option is (C).

297. The standard formula for the calculation of compound Interest is given by:

$A = P\left(1 + \frac{r}{100}\right)^n$; where A = Amount P = Principal, r = rate of interest, & n = term of interest or period

Here, Compound Interest, $CI = A - P = P\left[1 - \left(1 + \frac{r}{100}\right)^n\right]$;

Since rate of interest r is given in percent, so, we can replace $\frac{r}{100}$ with $r\%$.

=> $CI = P\left[1 - (1 + r\%)^n\right]$

For, $r = 5\%$ & $n = 4$ years;

=> $CI = P\left[1 - (1 + 5\%)^4\right]$; the result matches with option A.

Let us see what other options represent.

(A) $CI = P\left[1 - (1 + 5\%)^4\right]$ –Not a desired answer

(B) $CI = P\left[1 - \left(1 + \frac{10\%}{2}\right)^{2*2}\right] = CI = P\left[1 - (1 + 5\%)^4\right]$; since the calculation of interest is half-yearly compounding, half-yearly rate = $\frac{10\%}{2} = 5\%$, & the number of time interest would be calculated (# of 6-month periods in 2 years) = 2*2 = 4. Option B is not a desired answer.

(C) $CI = P\left[1-\left(1+\frac{20\%}{4}\right)^{1*4}\right] = CI = P\left[1-(1+5\%)^{4}\right]$; since the calculation of interest is quarterly compounding, quarterly rate = $\frac{20\%}{4}$ = 5%, & the number of time interest would be calculated (# of 3-month periods in 1 years) = 1*4 = 4. Option C is not a desired answer.

(D) This option is similar to option C.
$CI = P\left[1-\left(1+\frac{1.25\%}{4}\right)^{16*4}\right] = CI = P\left[1-(1+0.3125\%)^{64}\right]$; the result does not match with the expression given in the question, so option D is the desired and the correct answer.

The correct answer is option (D).

298. Part 1:

We have to calculate:

# of Engineering undergrads − # of Management undergrads

In the pie chart, we find that the number of Engineering undergrads and Management undergrads are 18% and 12% of total undergrads, respectively. So,

# of Engineering undergrads − # of Management undergrads

= 18% – 12% = 6% of total undergrads

If we know the value of total number of undergrads, we can get the answer. However, we don't really need to know it. We already know that the number of the **Art undergrads** = **6% of the total number of undergrads = 126**, and since this is the number we are looking for, we don't need to calculate it. Simply make use of number of Art undergrads, which is 126.

The correct answer is option 126.

Part 2:

After answering part 1, you might be tempted to simply calculate:

# of Engineering undergrads − # of Science undergrads

= 18% – 12% = 6% as the answer, however it is wrong.

What we really need to calculate is:

$$\frac{\text{\# of Engineering undergrads} - \text{\# of Science undergrads}}{\text{\# of Science undergrads}} * 100\%$$

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$$= \frac{\text{18\% of total undegrads} - \text{12\% of total undegrads}}{\text{12\% of total undegrads}} * 100\%$$

$$= \frac{18\% \ - 12\%}{12\%} * 100\% = 50\%$$

We need not know the value of total number of undergrads, as it cancels out.

Note that the denominator is the number of Science undergrads (12% of total undergrads) and not the number of engineering undergrads (18% of total undergrads). Had you mistakenly put the number of engineering undergrads in the denominator, you would have wrongly answered 33.33%.

The correct answer is option 50%.

299. Since the expression itself does not tell anything, let us see each option one by one.

Since in case of team selection, order is not important, we must apply combination to calculate the number of ways.

Number of ways a team of 3 can be formed out of 5 = ${}^5_3C$.

The result does not match with the given expression, hence option A cannot be an answer.

(A) This option is illogical. We cannot select two teams of three each from 5 players.

(B) Since in case of taking unique photographs, order is important, we must apply permutation.

Number of ways unique photographs of 3 can be taken out of 5 =${}^5_3P$.

Though at first sight, the result does not seem to match with the given expression, it is the correct answer.

Let us calculate the values of ${}^5_3P$ & ${}^5_3C.3!$.

$${}^5_3P = 5.4.3 = 60 \ \&$$

$${}^5_3C.3! = \frac{5.4.3}{1.2.3} * 3.2.1 = 5.4.3 = 60.$$

Since the results match, option C is the correct answer.

Another approach to calculate the number of ways unique photographs of 3 out of 5 is to first select 3 out of 5 people and arrange 3 people.

Thus, Number of ways = (Selecting of 3 out of 5) * (Arranging 3) = ${}^5_3C.3!$.

The result matches with the given expression.

Though we got the correct answer, and there is no need to look at option D, for the sake of understating, we have a look at it.

(C) Number of ways three persons can be selected out of five players to be seated in a round table arrangement is given by:

= (Selecting of 3 out of 5) * (Seating 3 persons in a round table arrangement) = ${}^{5}_{3}C * (3-1)! = {}^{5}_{3}C * 2!$.

The result does not match with the given expression, hence option D cannot be an answer.

The correct answer is option (C).

300. This is a special kind of question. We need not calculate the area of the triangle, but we must be assured that after adding an addition piece of information, the area can be calculated.

Needless to say that with the limited information given in the question, we cannot calculate the area.

Let us analyze each statement one by one.

(A) $XZ = 24$: Since the figure is not drawn to scale, two cases are possible: Either $XZ = ZY = 24, \& \; YZ = 12$ or $XZ = 24, \& \; XY = YZ = 18$. Each case will give a different area. Note that we have to calculate the unique area of the triangle.

(B) $Y^\circ = 30^\circ$: Similar to option A, two cases are possible: either $Y^\circ = X^\circ = 30^\circ$, & $Z^\circ = 120^\circ$, implying that $YZ = XZ$ or $Y^\circ = Z^\circ = 30^\circ$, and $X^\circ = 120^\circ$, implying that $XZ = XY$. Each case will give a different area.

(C) $XY : YZ : XZ :: 1 : 1 : 2$: Option (C) is the correct answer. As per the ratio, the sides are $3a, 3a,$ & $4a$. Thus, the perimeter = $3a + 3a + 4a = 10a = 60 => a = 6$. This gives three sides as 18, 18, & 24. Once we have three sides, we can calculate the unique area of the triangle.

(D) Sum of two sides: Despite this information since we do not know which two sides are equal, so similar to options A & B, two cases are possible and we cannot get the unique area of the triangle.

Hence, the correct option is (C).

Chapter 6

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Please email your questions to info@manhattanreview.com. We will be happy to answer you. You questions can be related to a concept, an application of a concept, an explanation of a question, a suggestion for an alternate approach, or anything else you wish to ask regarding the SAT.

Please do mention the page number when quoting from the book.

Best of luck!

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Manhattan Review's origin can be traced directly back to an Ivy League MBA classroom in 1999. While teaching advanced quantitative subjects to MBAs at Columbia Business School in New York City, Professor Dr. Joern Meissner developed a reputation for explaining complicated concepts in an understandable way. Prof. Meissner's students challenged him to assist their friends, who were frustrated with conventional test preparation options. In response, Prof. Meissner created original lectures that focused on presenting standardized test content in a simplified and intelligible manner, a method vastly different from the voluminous memorization and so-called tricks commonly offered by others. The new methodology immediately proved highly popular with students, inspiring the birth of Manhattan Review.

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About the Author

Professor Dr. Joern Meissner has more than 25 years of teaching experience at the graduate and undergraduate levels. He is the founder of Manhattan Review, a worldwide leader in test prep services, and he created the original lectures for its first test preparation classes. Prof. Meissner is a graduate of Columbia Business School in New York City, where he received a PhD in Management Science. He has since served on the faculties of prestigious business schools in the United Kingdom and Germany. He is a recognized authority in the areas of supply chain management, logistics, and pricing strategy. Prof. Meissner thoroughly enjoys his research, but he believes that grasping an idea is only half of the fun. Conveying knowledge to others is even more fulfilling. This philosophy was crucial to the establishment of Manhattan Review, and remains its most cherished principle.

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