I LEARNING CURVE ANALYSIS

LEARNING GOALS

After reading this supplement, you should be able to:

1. Explain the concept of a learning curve and how volume is related to unit costs.

2. Develop a learning curve, using the logarithmic model.

3. Demonstrate the use of learning curves for managerial decision making.

In today's dynamic workplace, change occurs rapidly. Where there is change, there also is learning. With instruction and repetition, workers learn to perform jobs more efficiently and thereby reduce the number of direct labor hours per unit. Like workers, organizations learn.

Organizational learning involves gaining experience with products and processes, achieving greater efficiency through automation and other capital investments, and making other improvements in administrative methods or personnel. Productivity improvements may be gained from better work methods, tools, product design, or supervision, as well as from individual worker learning. These improvements mean that existing standards must be continually evaluated and new ones set.

organizational learning

The process of gaining experience with products and processes, achieving greater efficiency through automation and other capital investments, and making other improvements in administrative methods or personnel.

learning curve

A line that displays the relationship between the total direct labor per unit and the cumulative quantity of a product or service produced.

THE LEARNING EFFECT

The learning effect can be represented by a line called a **learning curve**, which displays the relationship between the total direct labor per unit and the cumulative quantity of a product or service produced. The learning curve relates to a repetitive job or task and represents the relationship between experience and productivity: The time required to produce a unit decreases as the operator or firm produces more units. The curve in Figure I.1 is a learning curve for one process. It shows that the process time per unit continually decreases until the 140th unit is produced. At that point learning is negligible and a standard time for the operation can be developed. The terms *manufacturing progress function* and *experience curve* also have been used to describe this relationship, although the experience curve typically refers to total value-added costs per unit rather than labor hours. The principles underlying these curves are identical to those of the learning curve, however. Here we use the term *learning curve* to depict reductions in either total direct labor per unit or total value-added costs per unit.

Background

The learning curve was first developed in the aircraft industry prior to World War II, when analysts discovered that the direct labor input per airplane declined with considerable regularity as the cumulative number of planes produced increased. A survey of major airplane manufacturers revealed that a series of learning curves could be developed to represent the average experience for various categories of airframes (fighters, bombers, and so on), despite the different amounts of time required to produce the first unit of each type of airframe. Once production started, the direct labor for the eighth unit was only 80 percent of that for the fourth unit, the direct labor for the twelfth was only 80 percent of that for the sixth, and so on. In each case, each doubling of the quantity reduced production time by 20 percent. Because of the consistency in the rate of improvement, the analysts concluded that the aircraft industry's rate of learning was 80 percent between doubled quantities of airframes. Of course, for any given product and company, the rate of learning may be different.

Learning Curves and Competitive Strategy

Learning curves enable managers to project the manufacturing cost per unit for any cumulative production quantity. Firms that choose to emphasize low price as a competitive strategy rely on high volumes to maintain profit margins. These firms strive to move down the learning curve (lower labor hours per unit or lower costs per unit) by increasing volume. This tactic makes entry into a market by competitors difficult. For example, in the electronics component industry, the cost of developing an integrated circuit is so large that the first units produced must be priced high. As cumulative production increases, costs (and prices) fall. The first companies in the market have a big advantage because newcomers must start selling at lower prices and suffer large initial losses.
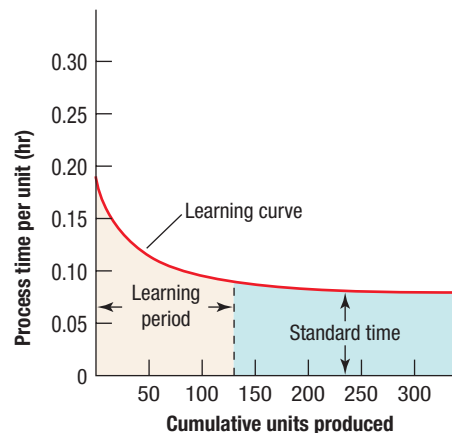

FIGURE I.1 ▲

Learning Curve, Showing the Learning Period and the Time When Standards Are Calculated

However, market or product changes can disrupt the expected benefits of increased production. For example, Douglas Aircraft management assumed that it could reduce the costs of its new jet aircraft by following a learning curve formula and committing to fixed delivery dates and prices. Continued engineering modification of its planes disrupted the learning curve, and the cost reductions were not realized. The resulting financial problems were so severe that Douglas Aircraft was forced to merge with McDonnell Company.

DEVELOPING LEARNING CURVES

In the following discussion and applications, we focus on direct labor hours per unit, although we could as easily have used costs. When we develop a learning curve, we make the following assumptions:

- The direct labor required to produce the $n + 1$st unit will always be less than the direct labor required for the nth unit.
- Direct labor requirements will decrease at a declining rate as cumulative production increases.
- The reduction in time will follow an exponential curve.

In other words, the production time per unit is reduced by a fixed percentage each time production is doubled. We can use a logarithmic model to draw a learning curve. The direct labor required for the nth unit, k_n, is

$$k_n = k_1 n^b$$

TABLE I.1 | CONVERSION FACTORS FOR THE CUMULATIVE AVERAGE NUMBER OF DIRECT LABOR HOURS PER UNIT

80% Learning Rate (n = cumulative production)						90% Learning Rate (n = cumulative production)					
n		n		n		n		n		n	
1	1.00000	19	0.53178	37	0.43976	1	1.00000	19	0.73545	37	0.67091
2	0.90000	20	0.52425	38	0.43634	2	0.95000	20	0.73039	38	0.66839
3	0.83403	21	0.51715	39	0.43304	3	0.91540	21	0.72559	39	0.66595
4	0.78553	22	0.51045	40	0.42984	4	0.88905	22	0.72102	40	0.66357
5	0.74755	23	0.50410	64	0.37382	5	0.86784	23	0.71666	64	0.62043
6	0.71657	24	0.49808	128	0.30269	6	0.85013	24	0.71251	128	0.56069
7	0.69056	25	0.49234	256	0.24405	7	0.83496	25	0.70853	256	0.50586
8	0.66824	26	0.48688	512	0.19622	8	0.82172	26	0.70472	512	0.45594
9	0.64876	27	0.48167	600	0.18661	9	0.80998	27	0.70106	600	0.44519
10	0.63154	28	0.47668	700	0.17771	10	0.79945	28	0.69754	700	0.43496
11	0.61613	29	0.47191	800	0.17034	11	0.78991	29	0.69416	800	0.42629
12	0.60224	30	0.46733	900	0.16408	12	0.78120	30	0.69090	900	0.41878
13	0.58960	31	0.46293	1,000	0.15867	13	0.77320	31	0.68775	1,000	0.41217
14	0.57802	32	0.45871	1,200	0.14972	14	0.76580	32	0.68471	1,200	0.40097
15	0.56737	33	0.45464	1,400	0.14254	15	0.75891	33	0.68177	1,400	0.39173
16	0.55751	34	0.45072	1,600	0.13660	16	0.75249	34	0.67893	1,600	0.38390
17	0.54834	35	0.44694	1,800	0.13155	17	0.74646	35	0.67617	1,800	0.37711
18	0.53979	36	0.44329	2,000	0.12720	18	0.74080	36	0.67350	2,000	0.37114

where

$$k_1 = \text{direct labor hours for the first unit}$$
$$n = \text{cumulative numbers of units produced}$$
$$b = \frac{\log r}{\log 2}$$
$$r = \text{learning rate (as decimal)}$$

We can also calculate the cumulative average number of hours per unit for the first n units with the help of Table I.1 on page I.3. It contains conversion factors that, when multiplied by the direct labor hours for the first unit, yield the average time per unit for selected cumulative production quantities.

EXAMPLE I.1	Using Learning Curves to Estimate Direct Labor Requirements

PEARSON
myomlab

Tutor I.1 in my**om**lab provides a new example for learning curve analysis.

A manufacturer of diesel locomotives needs 50,000 hours to produce the first unit. Based on past experience with similar products, you know that the rate of learning is 80 percent.

a. Use the logarithmic model to estimate the direct labor required for the 40th diesel locomotive and the cumulative average number of labor hours per unit for the first 40 units.

b. Draw a learning curve for this situation.

SOLUTION

a. The estimated number of direct labor hours required to produce the 40th unit is

$$k_{40} = 50,000(40)^{(\log 0.8)/(\log 2)} = 50,000(40)^{-0.322} = 50,000(0.30488)$$
$$= 15,248 \text{ hours}$$

We calculate the cumulative average number of direct labor hours per unit for the first 40 units with the help of Table I.1. For a cumulative production of 40 units and an 80 percent learning rate, the factor is 0.42984. The cumulative average direct labor hours per unit is 50,000(0.42984) = 21,492 hours.

b. Plot the first point at (1, 50,000). The second unit's labor time is 80 percent of the first, so multiply 50,000(0.80) = 40,000 hours. Plot the second point at (2, 40,000). The fourth is 80 percent of the second, so multiply 40,000(0.80) = 32,000 hours. Plot the point (4, 32,000). The result is shown in Figure I.2.
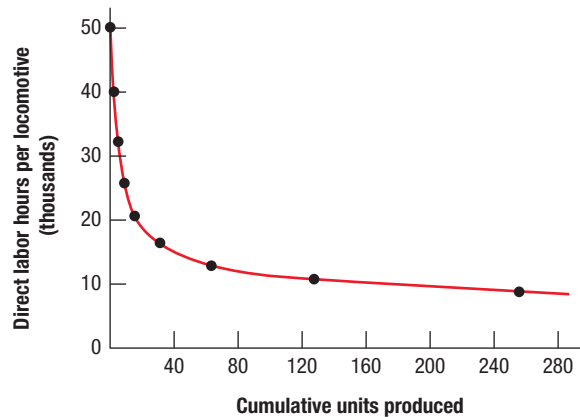
FIGURE I.2 ▶
The 80 Percent Learning Curve

DECISION POINT

Management can now use these estimates to determine the labor requirements to build the locomotives. Future hires may be necessary.

USING LEARNING CURVES

Learning curves can be used in a variety of ways. Let us look briefly at their use in bid preparation, financial planning, and labor requirement estimation.

Bid Preparation

Estimating labor costs is an important part of preparing bids for large jobs. Knowing the learning rate, the number of units to be produced, and wage rates, the estimator can arrive at the cost of labor by using a learning curve. After calculating expected labor and materials costs, the estimator adds the desired profit to obtain the total bid amount.

Financial Planning

Learning curves can be used in financial planning to help the financial planner determine the amount of cash needed to finance operations. Learning curves provide a basis for comparing prices and costs. They can be used to project periods of financial drain, when expenditures exceed receipts. They can also be used to determine a contract price by identifying the average direct labor costs per unit for the number of contracted units. In the early stages of production the direct labor costs will exceed that average, whereas in the later stages of production the reverse will be true. This information enables the financial planner to arrange financing for certain phases of operations.

Labor Requirement Estimation

For a given production schedule, the analyst can use learning curves to project direct labor requirements. This information can be used to estimate training requirements and develop production and staffing plans.

EXAMPLE I.2	Using Learning Curves to Estimate Labor Requirements

The manager of a custom manufacturer has just received a production schedule for an order for 30 large turbines. Over the next 5 months, the company is to produce 2, 3, 5, 8, and 12 turbines, respectively. The first unit took 30,000 direct labor hours, and experience on past projects indicates that a 90 percent learning curve is appropriate; therefore, the second unit will require only 27,000 hours. Each employee works an average of 150 hours per month. Estimate the total number of full-time employees needed each month for the next 5 months.

SOLUTION

The following table shows the production schedule and cumulative number of units scheduled for production through each month:

Month	Units per Month	Cumulative Units
1	2	2
2	3	5
3	5	10
4	8	18
5	12	30

   We first need to find the cumulative average time per unit using Table I.1 and the cumulative total hours through each month. We then can determine the number of labor hours needed each month. The calculations for months 1–5 follow at the top of page I.6.

Month	Cumulative Average Time per Unit	Cumulative Total Hours for All Units
1	30,000(0.95000) = 28,500.0	(2)28,500.0 = 57,000
2	30,000(0.86784) = 26,035.2	(5)26,035.2 = 130,176
3	30,000(0.79945) = 23,983.5	(10)23,983.5 = 239,835
4	30,000(0.74080) = 22,224.0	(18)22,224.0 = 400,032
5	30,000(0.69090) = 20,727.0	(30)20,727.0 = 621,810

Calculate the number of hours needed for a particular month by subtracting its cumulative total hours from that of the previous month.

$$\text{Month 1:} \quad 57,000 - 0 = 57,000 \text{ hours}$$
$$\text{Month 2:} \quad 130,176 - 57,000 = 73,176 \text{ hours}$$
$$\text{Month 3:} \quad 239,835 - 130,176 = 109,659 \text{ hours}$$
$$\text{Month 4:} \quad 400,032 - 239,835 = 160,197 \text{ hours}$$
$$\text{Month 5:} \quad 621,810 - 400,032 = 221,778 \text{ hours}$$

The required number of employees equals the number of hours needed each month divided by 150, the number of hours each employee can work.

$$\text{Month 1:} \quad 57,000/150 = 380 \text{ employees}$$
$$\text{Month 2:} \quad 73,176/150 = 488 \text{ employees}$$
$$\text{Month 3:} \quad 109,659/150 = 731 \text{ employees}$$
$$\text{Month 4:} \quad 160,197/150 = 1,068 \text{ employees}$$
$$\text{Month 5:} \quad 221,778/150 = 1,479 \text{ employees}$$

DECISION POINT

The number of employees needed increases dramatically over the 5 months. Management may have to begin hiring now so that proper training can take place.

MANAGERIAL CONSIDERATIONS IN THE USE OF LEARNING CURVES

Although learning curves can be useful tools for operations planning, managers should keep several things in mind when using them. First, an estimate of the learning rate is necessary in order to use learning curves, and it may be difficult to get. Using industry averages can be risky because the type of work and competitive niches can differ from firm to firm. The learning rate depends on factors such as process complexity and the rate of capital additions. The simpler the process, the less pronounced the learning rate. A complex process offers more opportunity than does a simple process to improve work methods and material flows. Replacing direct labor hours with automation alters the learning rate, giving less opportunity to make reductions in the required hours per unit. Typically, the effect of each capital addition on the learning curve is significant.

Another important estimate, if the first unit has yet to be produced, is that of the time required to produce it. The entire learning curve is based on it. The estimate may have to be developed by management using past experiences with similar products.

Learning curves provide their greatest advantage in the early stages of new service or product production. As the cumulative number of units produced becomes large, the learning effect is less noticeable.

Learning curves are dynamic because they are affected by various factors. For example, a short service or product life cycle means that firms may not enjoy the flat portion of the learning curve for very long before the service or product is changed or a new one is introduced. In addition, organizations utilizing team approaches will have different learning rates than they had before they introduced teams. Total quality management and continual improvement programs also will affect learning curves.

Finally, managers should always keep in mind that learning curves are only approximations of actual experience.

KEY EQUATION

Equation for the labor time for the nth unit: $k_n = k_1 n^b$

SOLVED PROBLEM

The Minnesota Coach Company has just been given the following production schedule for ski-lift gondola cars. This product is considerably different from any others the company has produced. Historically, the company's learning rate has been 80 percent on large projects. The first unit took 1,000 hours to produce.

Month	Units	Cumulative Units
1	3	3
2	7	10
3	10	20
4	12	32
5	4	36
6	2	38

a. Estimate how many hours would be required to complete the 38th unit.

b. If the budget only provides for a maximum of 30 direct labor employees in any month and a total of 15,000 direct labor hours for the entire schedule, will the budget be adequate? Assume that each direct labor employee is productive for 150 work hours each month.

SOLUTION

a. We use the learning curve formulas to calculate the time required for the 38th unit:

$$b = \frac{\log r}{\log 2} = \frac{\log 0.8}{\log 2} = \frac{-0.09691}{0.30103} = -0.322$$

$$k_n = k_1 n^b = (1,000 \text{ hours})(38)^{-0.322}$$

$$= (1,000 \text{ hours})(0.3099) = 310 \text{ hours}$$

b. Table I.1 gives the data needed to calculate the cumulative number of hours through each month of the schedule. Table I.2 shows these calculations.

The cumulative amount of time needed to produce the entire schedule of 38 units is 16,580.9 hours, which exceeds the 15,000 hours budgeted. By finding how much the cumulative total hours increased each month, we can break the total hours into monthly requirements. Finally, the number of employees required is simply the monthly hours divided by 150 hours per employee per month. The calculations are shown in Table I.3.

TABLE I.2	**CUMULATIVE TOTAL HOURS**

Month	Cumulative Units	Cumulative Average Time per Unit	Cumulative Total Hours Month for All Units
1	3	1,000(0.83403) = 834.03 hr/u	(834.03 hr/u)(3 u) = 2,502.1 hr
2	10	1,000(0.63154) = 631.54 hr/u	(631.54 hr/u)(10 u) = 6,315.4 hr
3	20	1,000(0.52425) = 524.25 hr/u	(524.25 hr/u)(20 u) = 10,485.0 hr
4	32	1,000(0.45871) = 458.71 hr/u	(458.71 hr/u)(32 u) = 14,678.7 hr
5	36	1,000(0.44329) = 443.29 hr/u	(443.29 hr/u)(36 u) = 15,958.4 hr
6	38	1,000(0.43634) = 436.34 hr/u	(436.34 hr/u)(38 u) = 16,580.9 hr

TABLE I.3 | DIRECT LABOR EMPLOYEES

Month	Cumulative Total Hours for Month	Direct Labor Workers by Month
1	2,502.1 – 0 = 2,502.1 hr	(2,502.1 hr)/(150 hr) = 16.7, or 17
2	6,315.4 – 2,502.1 = 3,813.3 hr	(3,813.3 hr)/(150 hr) = 25.4, or 26
3	10,485.0 – 6,315.4 = 4,169.6 hr	(4,169.6 hr)/(150 hr) = 27.8, or 28
4	14,678.7 – 10,485.0 = 4,193.7 hr	(4,193.7 hr)/(150 hr) = 27.9, or 28
5	15,958.4 – 14,678.7 = 1,279.7 hr	(1,279.7 hr)/(150 hr) = 8.5, or 9
6	16,580.9 – 15,958.4 = 622.5 hr	(622.5 hr)/(150 hr) = 4.2, or 5

The schedule is feasible in terms of the maximum direct labor required in any month because it never exceeds 28 employees. However, the total cumulative hours are 16,581, which exceeds the budgeted amount by 1,581 hours. Therefore, the budget will not be adequate.

PROBLEMS

1. Mass Balance Company is manufacturing a new digital scale for use by a large chemical company. The order is for 40 units. The first scale took 60 hours of direct labor. The second unit took 48 hours to complete.

   a. What is the learning rate?

   b. What is the estimated time for the 40th unit?

   c. What is the estimated total time for producing all 40 units?

   d. What is the average time per unit for producing the last 10 units (#31–#40)?

2. Cambridge Instruments is an aircraft instrumentation manufacturer. It has received a contract from the U.S. Department of Defense to produce 30 radar units for a military fighter plane. The first unit took 85 hours to produce. Based on past experience with manufacturing similar units, Cambridge estimates that the learning rate is 93 percent. How long will it take to produce the 5th unit? The 10th? The 15th? The final unit?

3. A large grocery corporation has developed the following schedule for converting frozen food display cases to use CFC-free refrigerant:

Week	Units
1	20
2	65
3	100
4	140
5	120

Historically, the learning rate has been 90 percent on such projects. The budget allows for a maximum of 40 direct labor employees per week and a total of 8,000 direct labor hours for the entire schedule. Assume 40 work hours per week. If the first unit took 30 hours to convert, is this schedule feasible? If not, how can it be altered? Are additional costs involved in altering it?

4. Freddie and Jason have just opened the Texas Toothpick, a chain-saw sharpening and repair service located on Elm Street. The Texas Toothpick promises same-week repair service. Freddie and Jason are concerned that a projected dramatic increase in demand as the end of October nears will cause service to deteriorate. Freddie and Jason have had difficulty attracting employees, so they are the only workers available to complete the work. Safety considerations require that they each work no more than 40 hours per week. The first chain-saw sharpening and repair required 7 hours of work, and an 80 percent learning curve is anticipated.

Week	Units	Cumulative Units
October 2–6	8	8
October 9–13	19	27
October 16–20	10	37
October 23–27	27	64

   a. How many total hours are required to complete 64 chain saws?

   b. How many hours of work are required for the week ending on Friday the 13th?

   c. Will Freddie and Jason be able to keep their same-week service promise during their busiest week just before Halloween?

5. The Bovine Products Company recently introduced a new automatic milking system for cows. The company just completed an order for 16 units. The last unit required 15 hours of labor, and the learning rate is estimated to be 90 percent on such systems. Another customer has just placed an order for the same system. This

customer, which owns many farms in the Midwest, wants 48 units. How many total labor hours will be needed to satisfy this order?

6. You are in charge of a large assembly shop that specializes in contract assignments. One of your customers has promised to award you a large contract for the assembly of 1,000 units of a new product. The suggested bid price in the contract is based on an average of 20 hours of direct labor per unit. You conduct a couple of test assemblies and find that, although the first unit took 50 hours, the second unit could be completed in just 40 hours.

   a. How many hours do you expect the assembly of the third unit to take?

   b. How many hours do you expect the assembly of the 100th unit to take?

   c. Is the contract's assumption about the average labor hours per unit valid or should the price be revised?

7. The personnel manager at Powerwest Inc. wants to estimate the direct and cumulative average direct labor hours for producing 30 locomotive train units during the next year. He estimates from past experience that the learning rate is 90 percent. The production department estimates that manufacturing the first unit will take 30,000 hours. Each employee averages 200 hours per month. The production rate forecast is as follows:

Month	Production Rate (units)	Month	Production Rate (units)
January	2	July	2
February	3	August	4
March	2	September	3
April	4	October	3
May	3	November	1
June	2	December	1

   a. How many direct hours are required to produce the 30th unit?

   b. How many total hours are needed to produce all 30 units?

   c. What is the maximum number of employees required next year?

   d. If the learning rate were changed to 0.85, what would be the impact on the total hours and the number of employees needed to produce 30 units?

8. The Really Big Six Corporation will hire 1,000 new accountants this year. Managers are considering whether to make or to buy office furniture for the new hires. Big can purchase office furniture for $2,000 per accountant, or it can make the desks itself. Equipment to assemble furniture can be scrounged from the company's carpentry shop. That old equipment has already been fully depreciated. Materials cost $500 per desk, and labor (and benefits) costs $30 per hour. Big hired a local shop to build a prototype desk. That desk required 100 hours of labor. If the learning curve is 90 percent, should Big make or buy the desks?

9. Although the learning curve never completely levels off to a horizontal line, if work standards are to be developed, there must be a point at which, for all practical purposes, learning is said to have stopped. If we have an 80 percent learning curve and say that the learning effect will be masked by other variables when the improvement in successive units is less than 0.5 percent, at about what unit number can the standard be set?

10. The Compton Company is manufacturing a solar grain dryer that requires methods and materials never before used by the company. The order is for 80 units. The first unit took 46 direct labor hours, whereas the 10th unit took only 24 direct labor hours.

   a. Estimate the rate of learning that occurred for this product.

   b. Use the learning rate in part (a) to estimate direct labor hours for the 80th unit.

11. The Hand-To-Mouth Company (HTM) has $200,000 in cash, no inventory, and a 90 percent learning curve. To reduce the complexity of this problem, ignore the hiring and training costs associated with dramatically increased production. Employees are paid $20 per hour every Friday for that week's work. HTM has received an order to build 1,000 oak desks over the next 15 weeks. Materials cost $400 per desk. Suppliers make deliveries each Monday and insist on cash upon delivery. The first desk takes 100 hours of direct labor to build. HTM will be paid $1,500 per desk two weeks after the desks are delivered. Should HTM take this order?

Week	Units	Week	Units	Week	Units
1	2	6	24	11	88
2	4	7	64	12	100
3	8	8	128	13	100
4	12	9	128	14	100
5	14	10	128	15	100
				Total	1,000

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