

CHAPTER 5

RATIO ANALYSIS

5.1 Meaning of Financial Statement Analysis:

The term 'Analysis' refers to rearrangement and simplification of data given in the financial statement. The analysis is done by establishing the relationship between the items of the Balance sheet and Profit and Loss Account. Financial analysis refers to an assessment of the viability, stability and profitability of a business, or Company. It is a process of examining and comparing financial data. Analysis refers to the proper arrangement of financial data. Analysis of financial statements means an attempt to determine the significance and meaning of data presented in financial statements. Such an analysis makes use of various analytical tools and techniques to data of financial statements so as to derive from them certain relationships that are significant and useful for decision making. It is performed by professionals who prepare reports using ratios that make use of information taken from financial statements and other reports. These reports are usually presented to top management as one of their basis in making business decisions. Based on these reports, management may:

1. Continue or discontinue its main operation or part of its business.
2. Make or purchase certain materials in the manufacture of its product.
3. Acquire or rent/lease certain machinery and equipment in the production of its goods.
4. Issue stocks or negotiate for a bank loan to increase its working capital.
5. Other decisions that allow management to make an informed selection on various alternatives in the conduct of its business.

Moore and Jaedicke have defined financial analysis as process of synthesis and summarization of financial operative data with a view to getting an insight in to the operative of a business enterprise.

Metcalt and Titard have defined financial analysis as process of evaluating the relationship between component parts of financial statement to obtain a better understanding of a firm's position and performance.

5.2 Advantages of Ratio Analysis

Financial statements i.e., Profit and Loss account and Balance Sheet prepared at the end of the year do not always convey to the reader the real profitability and financial health of the business. They contain various facts and figures and it is for the reader to conclude, whether these facts indicate a good or bad managerial performance. Ratio analysis is the most important tool of analysing these financial statements. It helps the reader in giving tongue to the mute heaps of figures given in financial statements. The figures then speak of liquidity, solvency, profitability etc. of the business enterprise. Some important objects and advantages derived by a firm by the use of accounting ratios are: -

5.2.1 Helpful in Analysis of Financial Statements:- Ratio analysis is an extremely device for analyzing the financial statements. It helps the bankers, creditors, investors, shareholders etc. in acquiring enough knowledge about the profitability and financial health of the business. In the light of the knowledge so acquired by them, they can take necessary decisions about their relationships with the concern.

5.2.2 Simplification of Accounting Data:- Accounting ratio simplifies and summarises a long array of accounting data and makes them understandable. It discloses the relationship between two such figures, which have a cause and effect relationship with each other.

5.2.3 Helpful in comparative study:- With the help of ratio analysis comparison of profitability and financial soundness can be made between one firm and another in the same industry. Similarly, comparison of current year figures can also be made with those of previous years with the help of ratio analysis.

5.2.4 Helpful in locating the weak spots of the business:- Current year's ratios are compared with those of the previous years and if some weak spots are thus located, remedial measures are taken to correct them.

5.2.5 Helpful in forecasting:- Accounting ratios are very helpful in forecasting and the plans for the future.

5.2.6 Estimate about the trend of the business:- If accounting ratios are prepared for a number of years, they will reveal the trend of costs, sales, profits and other important facts.

5.2.7 Fixation of Ideal Standards:- Ratios helps us in establishing ideal standards of the different item of the business. By comparing the actual ratios calculated at the end of the year with the ideal ratios, the efficiency of the business can be easily measured.

5.2.8 Effective Control:- Ratio analysis discloses the liquidity, solvency and profitability of the business enterprise. Such information enables management to assess the changes that have taken place over a period of time in the financial activities of the business. It helps them in discharging their managerial functions e.g., planning, organizing, directing, communicating and controlling more effectively.

5.3 Limitations of Ratio Analysis

Ratio analysis is a very important tool of financial analysis. But despite it's being indispensable, the ratio analysis suffers from a number of limitations. These limitations should be kept in mind while making use of the ratio analysis:-

5.3.1 False accounting data gives false ratios:- Accounting ratios are calculated on the basis of given data given in profit and loss account and balance sheet. Therefore, they will be only as correct as the accounting data on which they are based. For example, if the closing stock is over-valued, not only the profitability will be overstated but also the financial position will appear to be better. Therefore, unless the profit and loss account and balance sheet are reliable, the ratios based on them will not be reliable. There are certain limitations of financial statements as such, the ratios calculated on the basis of such financial statements will also have the same limitations.

5.3.2 Comparison not possible if different firms adopt different accounting policies:- There may be different accounting policies adopted by different firms with regard to providing depreciation, creation of provision for doubtful debts, method of valuation of closing stock etc. For instance, one firm may adopt the policy of charging depreciation on straight-Line basis, while other may charge on written-down value method. Such differences make the accounting ratios incomparable.

5.3.3 Ratio analysis becomes less effective due to price level changes:- Price level over the year goes on changing, therefore, the ratios of various years can not be compared. For e.g., one firm sells 1,000 machines for Rs 10 lacs during 1992, it again sells 1,000 machines of the same type in year 1993 but owing to rising prices the sale price was Rs 15 lacs. On the basis of ratios it will be concluded that the sales have increased by 50 % whereas in actual, sales have not increased at all. Hence, the figures of the past years must be adjusted in the light of price level changes before the ratios for the years are compared.

5.3.4 Ratios may be misleading in the absence of absolute data:- For example, X company produces 10 Lakh metres of cloth in 1992 and 15 Lakh metre in 1993, the progress is 50%. Y Company raises its production from 10 thousand metres in 1992 to 20 thousand metres in 1993, the progress is 100%, and comparison of these two firms made on the basis of ratio will disclose that the second firm is more active than the first firm. Such conclusion is quite misleading because of the difference in the size of the two firms. It is, therefore, essential to study the ratios along with the absolute data on which they are based.

5.3.5 Limited use of a Single Ratio:- The analyst should not merely rely on a single ratio. He should study several connected ratios before reaching a conclusion. For example, the Current Ratio of a firm may be quite satisfactory, whereas the Quick Ratio may be unsatisfactory.

5.3.6 Window Dressing:- Some companies in order to cover up their bad financial position resort to window dressing *i.e.*, showing a better position than the one, which really exists. They change their balance sheet in such away that the important facts and truth may be concealed.

5.3.7 Lack of proper standards:- Circumstances differ from firm to firm hence no single standard ratio can be fixed for all the firms against which the actual ratio may be compared.

5.3.8 Ratios alone are not adequate for proper conclusions:- Ratios derived from analysis of statements are not sure indicators of good or bad financial position and profitability of a firm. They merely indicate the probability of favorable or unfavorable position. The analyst has to carry out further investigations and exercise his judgment in arriving at a correct diagnosis.

5.3.9 Effect of personal ability and bias of analyst:- Another important point to keep in mind is that different persons draw different meaning of different terms. One analyst may calculate ratios on the basis of profit after interest and tax, whereas another analyst may consider profits before interest and tax; a third may consider profits after interest but before tax. Therefore, before making comparisons, one must be sure that the ratios have been calculated on the same basis.

Although ratio analysis suffers from a number of limitations as enumerated above, yet it is a very useful and widely used tool of analyzing the financial statements. Useful conclusions may be arrived at by ratio analysis provided the above-mentioned limitations are kept in mind while using the results obtained from ratio analysis.

5.4 Classification of Ratios:-

In ratio analysis the ratios may be classified into the four categories as follows;

(I) Liquidity Ratios

(II) Profitability Ratios

(III) Activity Ratios

(IV) Solvency Ratios

5.4.1 Liquidity Ratios:-

"Liquidity" refers to the ability of the firm to meet its current liabilities. The liquidity ratios, therefore, are also called 'Short-term Solvency Ratios.' These ratios are used to assess the short-term financial position of the concern. They indicate the firm's ability to meet its current obligations out of current resources.

In the words of Salomon J. Flink, "Liquidity is the ability of the firm to meet its current obligations as they fall due.

In the words of Herbert B. Mayo, "Liquidity is the ease with which assets may be converted into cash without loss."

Short-term creditors of the firm are primarily interested in the liquidity ratios of the firm as they want to know how promptly or readily the firm can meet its current liabilities. If the firm wants to take a short-term loan from the bank, the bankers also study the liquidity ratios of the firm in order to assess the margin between current assets and current liabilities.

Liquidity ratios include two ratios: -

1. Current Ratio
2. Quick Ratio

5.4.2 Profitability Ratios: -

The main object of all the business concerns is to earn profit. Profit is the measurement of the efficiency of the business. Equity shareholders of the company are mainly interested in the profitability of the company.

Profitability ratios include the following: -

1. Gross Profit Margin Ratio
2. Operating Profit Margin Ratio
3. Net Profit Margin Ratio
4. Return on Capital Employed Ratio
5. Return on Net worth Ratio
6. Earning per Share Ratio

5.4.3 Activity Ratios: -

These ratios are calculated on the basis of 'cost of sales' or 'sales'; therefore, these ratios are also called as 'Turnover Ratios'. Turnover indicates the speed or number of times the capital employed has been rotated in the process of doing business. In other words, these ratios indicate how efficiently the capital is being used to obtain sales; how efficiently the fixed assets are being used to obtain sales; and how efficiently the working capital and stock is being used to obtain sales. Higher turnover ratios indicate the better use of capital or resources and in turn lead to higher profitability. Turnover ratios include the following

1. Inventory Turnover Ratio
2. Debtors Turnover Ratio
3. Fixed Assets Turnover Ratio
4. Investment Turnover Ratio

5.4.4 Solvency Ratios: -

These ratios are calculated to assess the ability of the firms to meet its long-term liabilities as and when they become due. Long term creditors including debenture holders are primarily interested to know whether the company has ability to pay

regularly interest due to them and to repay the principal amount when it becomes due. Solvency ratios disclose the firm's ability to meet the interest costs regularly and long-term indebtedness at maturity. Solvency ratios include the following ratios; -

1. Debt-Equity Ratio
2. Interest Coverage Ratio

5.4.1 Liquidity Ratios: -

5.4.1.1 Current Ratio

The ratio is used to assess the firm's ability to meet its short-term liabilities on time. It is generally believe that 2:1 ratio shows a comfortable working capital position. However this rule should not be taken as a hard and fast rule, because ratio that is satisfactory for one company may not be satisfactory for other. It means that current assets of a business should, at least be twice of its current liabilities. The reason of assuming 2: 1 as the ideal ratio is that the current assets includes such assets as stock, debtors etc, from which full amount cannot be realized in case of need. Hence, even if half the amount is realized from the current assets on time, the firm can still meet its current liabilities in full.

$$\text{Current Ratio} = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

Current Assets = Cash & Bank Balance + Stock + Debtors + Bills Receivable + Prepaid Expenses + Investments readily convertible into cash + Loans and Advances

Current Liabilities = Creditors + Bills Payable + Bank Overdraft + Unclaimed dividend + Provision for Taxation + Proposed Dividend.

5.4.1.2 Quick Ratio

Quick or Acid Test indicates whether the firm is in a position to pay its current liabilities within a month or immediately.

An ideal acid test ratio is said to be 1:1. The idea is that for every rupee or current liabilities, there should at least be one rupee of liquid assets. This ratio is better test for short-term financial position of the company than the current ratio. Liquid assets are obtained by deducting stock-in-trade and prepaid expenses from current assets. Stock is not treated as a liquid asset because it cannot be readily converted into cash

as and when required. The current ratio of a business does not reflect the true liquid position, if its current assets consist largely of stock-in-trade.

The liquid liabilities are obtained by deducting bank overdraft from current liabilities. Bank overdraft is not included in liquid liabilities because bank overdraft is not likely to be called on demand and is treated as a sort of permanent mode of financing. Hence, it is not treated as a quick liability. If the liquid assets are equal to or more than liquid liabilities, the condition may be considered as satisfactory. Liquid ratio can be calculated as follows

$$\text{Quick Ratio} = \frac{\text{Liquid Assets}}{\text{Liquid Liabilities}}$$

5.4.2 Profitability Ratios

The main object of every business concern is to earn profits. A business must be able to earn adequate profit in relation to the capital invested in it. The following are the important profitability ratios:

5.4.2.1 Gross Profit Margin Ratio: - This ratio measures the margin of profit available on sales. The higher the gross profit ratio, the better it is. No ideal standard is fixed for this ratio; but the gross profit ratio should be adequate enough not only to cover the operating expenses but also to provide for depreciation, interest on loans, dividends and creation of reserves.

$$\text{Gross Profit Ratio} = \frac{\text{Gross Profit}}{\text{Net Sales}} \times 100$$

The ratio is compared with earlier years' ratio and important conclusions are drawn from such comparison.

5.4.2.2 Operating Profit Margin Ratio: - This ratio measures the proportion of an enterprise's cost of sales and operating expenses in comparison to its sales"

$$\text{Operating Profit Margin Ratio} = \frac{\text{EBIT}}{\text{Net Sales}}$$

EBIT = Earning Before Interest and Taxes.

Operating Ratio is a measurement of the efficiency and profitability of the business enterprise. The ratio indicates the extent of sales that is absorbed by the cost of goods

sold and operating expenses. Lower the operating ratio, the better it is, because it will leave higher margin of profit on sales.

5.4.2.3 Net Profit Margin Ratio: - This ratio measures the rate of net profit earned on sales. It helps in determining the overall efficiency of the business operation. An increase in the ratio over the previous year shows improvement in the overall efficiency of the business.

$$\text{Net Profit Ratio} = \frac{\text{Net Profit}}{\text{Net Sales}} \times 100$$

5.4.2.4 Return on Capital Employed Ratio: - This ratio reflects the overall profitability of the business. It is calculated by comparing the profit earned and the capital employed to earn it. This ratio is usually in percentage. And is also known as “Rate of Return” or “Rate on capital Employed”.

$$\text{Return on Capital employed} = \frac{\text{Profit before Interest, Tax \& Dividends}}{\text{Capital Employed}} \times 100$$

Since the capital employed includes shareholders' funds and long-term loans, interest paid on long-term loans will not be deducted from profits while calculating this ratio.

Capital Employed = Equity Share Capital + Preference Share Capital + All Reserves + P & L A/C Balance + Long term Loans - Fictitious Assets.

Or

Capital Employed = Fixed Assets + Working Capital. OR [FA + (C.A-C.L)]

This ratio measures how efficiently the capital employed in the business is being used.

5.4.2.5 Return on Net Worth Ratio: - While there is no doubt that the reference shareholders are also owners of a firm. The real owners are the ordinary shareholders who bear all the risk, participate in management and are entitled to all the profit remaining after all outside claims including preference dividends are met in full. The profitability of a firm from the owners point of view should therefore in the fitness of things be assessed in terms of the return to the ordinary shareholders. The ratio under reference serves this purpose. It is calculated by dividing the profits after taxes and preference dividends by the average equity of the ordinary shareholders thus,

$$\text{Return on Net Worth} = \frac{\text{Net Profit after Tax} - \text{Preference dividend}}{\text{Average ordinary shareholders' equity or net worth}} \times 100$$

5.4.2.6 Earning per Share Ratio: - It measures the profit available to the equity share holders on a per share basis, i.e. the amount that they can get on every share held. It is calculated by dividing the profits available to the equity shareholders by the number of the outstanding shares. The profits available to the ordinary shareholders are represented by net profits after taxes and preference dividend. Thus,

$$\text{EPS} = \frac{\text{Net profit available to equity holder}}{\text{Number of ordinary share outstanding}}$$

As a profitability ratio, the EPS can be used to draw inferences on the basis of

i) Its trend over a period of time, ii) comparison of the EPS of the other firms, iii) comparison with the industry average.

5.4.3 Activity Ratios: -

These ratios measure how well the facilities at the disposal of the concern are being utilized. These ratios are known as turnover ratios as they indicate the rapidity with which the resources available to the concern are being used to produce sales. These ratios are generally calculated on the basis of sales or cost of sales. Some of the important activity ratios are discussed below:

5.4.3.1 Inventory Turnover Ratio: - It is computed by dividing the cost of goods sold by average inventory. Thus,

$$\text{Inventory Turnover Ratio} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

The cost of goods sold means sales minus gross profit. The average inventory refers to the simple average of the opening and closing inventory. The ratio indicates how fast inventory is sold. A high ratio is good from the view point of liquidity and viceversa. A low ratio would signify that inventory does not sell fast and stays on the shelf or in the warehouse for a long time. This ratio indicates the number of times inventories replaced during the year. It measures the relationship between the cost of goods sold and the inventory level.

5.4.3.2 Debtors Turnover Ratio: - This ratio indicates the relationship between credit sales and average debtors during the year.

$$\text{Debtors Turnover Ratio} = \frac{\text{Net Credit Sales}}{\text{Average Debtors} + \text{Average Bills Receivable}}$$

Bill receivable is added in debtors for the purpose of calculation of this ratio. This ratio indicates the speed with which the amount is collected from debtors. The higher the ratio, the better it is, since it indicates that amount from debtors is being collected more quickly. The more quickly the debtors pay, the less the risk from bad debts, and so the lower the expenses of collection and increase in the liquidity of the firm. A lower debtor turnover ratio will indicate the inefficient credit sales policy of the management.

$$\text{Average Collection Period} = \frac{\text{Receivables (Debtors} + \text{Bills Receivable)}}{\text{Net Credit Sales Per day}}$$

This ratio shows the time in which the customer is paying for credit sales. Increase in this ratio indicates the excessive blockage of funds with debtors, which increases the chances of bad debts.

5.4.3.3 Fixed Asset Turnover Ratio: - This ratio is also known as the investment turnover ratio. It is based on the relationship between the cost of goods sold and assets of a firm. A reference to this was made while working out the overall profitability of a firm as reflected in its earning power.

$$\text{Fixed Asset Turnover Ratio} = \frac{\text{Cost of goods}}{\text{Average fixed assets}}$$

5.4.3.4 Investment Turnover Ratio: - It is based on relationship between the cost of goods sold and investments of firm. A reference to this was made while working out the overall profitability of the firm as reflected in its earning power. Depending upon the different concepts of assets employed, there are many variance of this ratio. Thus,

$$\text{I) Total asset turnover} = \frac{\text{Cost of goods sold}}{\text{Average total assets}}$$

$$\text{II) Fixed assets turnover} = \frac{\text{Cost of goods sold}}{\text{Average fixed assets}}$$

$$\text{III) Capital turnover} = \frac{\text{Cost of goods sold}}{\text{Average capital employed}}$$

Here, the total assets and fixed assets are net of depreciation and the assets are exclusive of fictitious assets like debit balance of profit and loss account and deferred expenditure and so on. The assets turnover ratio, however defined, measure the efficiency of a firm in managing and utilizing its assets. The higher the turnover ratio, the more efficient is the management and utilization of the assets while low turnover ratios are indicative of under utilization of available resources and presence of idle capacity.

5.4.4 Solvency Ratios: -

These ratios are calculated to assess the ability of the firms to meet its long-term liabilities as and when they become due. Long term creditors including debenture holders are primarily interested to know whether the company has ability to pay regularly interest due to them and to repay the principal amount when it becomes due. Solvency ratios disclose the firm's ability to meet the interest costs regularly and long-term indebtedness at maturity. Solvency ratios include the following ratios: -

5.4.4.1 Debt- Equity Ratio: - This, ratio establishes relationship between the outside long-term liabilities and owners' funds. It shows the proportion of long-term External Equities and Internal Equities i.e. proportion of funds provided by long-term creditors and that provided by shareholders or proprietors. A higher ratio means that outside creditors has a larger claim than the owners of the business. The company with high-debt position will have to accept stricter conditions from the lenders while borrowing money. If this ratio is lower, it is not profitable from the viewpoint of equity shareholders, as benefit of trading on equity is not availed of and the rate of equity dividend will be comparatively lower.

$$\text{Debt Equity Ratio} = \frac{\text{Long Term Liabilities} + \text{Current Liability}}{\text{Shareholders' Funds (Net Worth)}}$$

External Equities = All Long term liabilities+ Current Liabilities

Internal Liabilities= Equity share+ Preference share + Reserves & Surplus + P & L
A/c- Intangible or Fictitious Assets

5.4.4.2 Interest Coverage Ratio: - It is also known as ‘time-interest-earned ratio’. This ratio measures the debt servicing capacity of a firm in so far as fixed interest on long-term loan is concerned. It is determined by dividing the operating profits or earning before interest and taxes (EBIT) by the fixed interest charges on loans. Thus,

$$\text{Interest coverage} = \frac{\text{EBIT}}{\text{Interest}}$$

It should be noted that this ratio uses the concept of net profits before taxes because interest is tax-deductible so that tax is calculated after paying interest on long-term loan. This ratio, as the name suggests, indicates the extent to which a fall in EBIT is tolerable in that the ability of the firm to service its interest payments would not be adversely affected.

5.5 Composite Ratio obtained from Weighted Mean: -

The weighted mean is similar to an arithmetic mean (the most common type of average), where instead of each of the data points contributing equally to the final average, some data points contribute more than others. The notion of weighted mean plays a role in descriptive statistics and also occurs in a more general form in several other areas of mathematics.

If all the weights are equal, then the weighted mean is the same as the arithmetic mean. While weighted means generally behave in a similar fashion to arithmetic means, they do have a few counterintuitive properties, as captured for instance in Simpson's paradox.

The term weighted average usually refers to a weighted arithmetic mean, but weighted versions of other means can also be calculated, such as the weighted geometric mean and the weighted harmonic mean.

5.5.1 Definition of Weighted Mean

Formally, the weighted mean of a non-empty set of data $\{x_1, x_2, \dots, x_n\}$ with non-negative weights $\{w_1, w_2, \dots, w_n\}$ is the quantity $\bar{x} = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$, which means:

$$\bar{x} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n}.$$

Therefore data elements with a high weight contribute more to the weighted mean than do elements with a low weight. The weights cannot be negative. Some may be zero, but not all of them (since division by zero is not allowed).

5.5.2 Weighted Mean Ratio

When a representative ratio is to be worked out for certain purposes on the basis of more than one such ratios and the weights associated with such ratios are different the representative ratio can be the mean of such ratios. However, when ratios to be combined have different weights, the representative ratio should be weighted mean of such ratios and such a weighted mean of the ratios may be called Weighted Mean Ratio.

Suppose R_i ($i = 1, 2, \dots, n$) is ratio for the i^{th} company and w_i ($i = 1, 2, \dots, n$) is the weight (paid-up capital) for the i^{th} company. Then the composite ratio R_c can be obtained as follows

$$R_c = \frac{\sum w_i R_i}{\sum w_i}$$

where w_i ($i=1, 2, \dots, n$) are paid-up capital as weight, R_i are ratios, ($i= 1, 2, \dots, n$).

5.6 Linear Regression

In linear regression, the model specification is that the dependent variable, is a linear combination of the parameters (but need not be linear in the independent variables). For example, in simple linear regression for modeling data points there is one independent variable and two parameters and straight line:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \quad i = 1, \dots, n.$$

(In multiple linear regressions, there are several independent variables or functions of independent variables.)

Adding a term in x_i^2 to the preceding regression gives:

$$\text{Parabola: } y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i, \quad i = 1, \dots, n.$$

This is still linear regression; although the expression on the right hand side is quadratic in the independent variable, it is linear in the parameters, and in both cases, is an error term and the subscript indexes a particular observation. Given a random sample from the population, we estimate the population parameters and obtain the sample linear regression model:

The residual, is the difference between the value of the dependent variable predicted by the model, and the true value of the dependent variable. One method of estimation is ordinary least squares. This method obtains parameter estimates that minimize the sum of squared residuals (estimated errors) known as Sum of Squares of Errors (SSE) or Residuals Sum of Squares (RSS).

Minimization of this function results in a set of normal equations, a set of simultaneous linear equations in the parameters, which are solved to yield the parameter estimators,

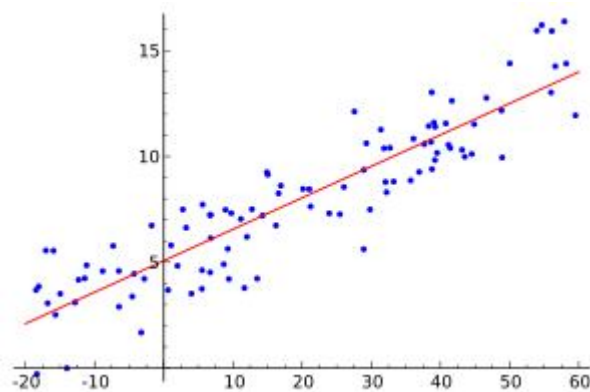


Illustration of simple linear regression on a data set is given above where a point say i^{th} point on the line is the estimate $y_i = \bar{\beta}_0 + \bar{\beta}_1 x_i$ corresponding to observed value y_i of dependent variable y .

In the case of simple regression $y = \beta_0 + \beta_1 x$ the formulas for the least squares estimates are

$$\hat{\beta}_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} \text{ and } \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

where \bar{x} is the mean (average) of the values and \bar{y} is the mean of the values. See simple linear regression for a derivation of these formulas and a numerical example.

5.7 Coefficient of correlation

The statistical tool with the help of which the closeness or strength of linear relationship between two or more than two variables is studied is called correlation. The measure of correlation called the coefficient of correlation summarizes in one figure the direction and degree (strength or closeness) of correlation. Thus correlation analysis refers to the techniques used in measuring the closeness of the relationship between the variables. An analysis of the co-variation of two or more variables is usually called correlation. A mathematical method for measuring the intensity or the magnitude of the linear relationship between two variable series was suggested by Karl Pearson (1867-1936) is by far the most widely used method in practice.

If $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ are n pairs of observations of the variables X and Y in a bivariate distribution, then

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \sum(y - \bar{y})^2}}$$

5.7.1 Coefficient of determination

In statistics, the coefficient of determination R^2 is the correlation coefficient between the dependent variable y and its linear estimator of \hat{y} and therefore is used in the context of statistical models whose main purpose is the prediction of future outcomes on the basis of other related information. R^2 is most often seen as a number between 0 and 1.0, used to describe how well a regression line fits a set of data. An R^2 near 1.0 indicates that a regression line fits the data well, while an R^2 closer to 0 indicates a regression line does not fit the data very well. It is the proportion of variability in a data set that is accounted for by the statistical model. It provides a measure of how well future outcomes are likely to be predicted by the model.

5.8 Analysis of Variance (ANOVA)

The analysis of variance frequently refer to by the contraction ANOVA is a statistical technique specially designed to test whether the means of more than two quantitative populations are equal.

The analysis of variance technique developed by R.A. Fisher in 1920's, is capable of fruitful application to a diversity of practical problems. Basically it consists of classifying and cross-classifying statistical results and testing whether the means of a specified classification differ significantly. In this way it is determined whether the given classification is important in affecting the results. From the cross-classification it could be determined whether the mean qualities of the outputs of the various machines differed significantly. Such a study would determine, for example whether uniformity in quality of outputs would be increased by standardizing the procedures of the operators and similarly whether it could be increased by standardizing the machines. Analysis of variance thus enables us to analyze the total variation of data into components which may be attributed to various "sources" or "causes" of variation.

The analysis of variance originated the agrarian research and its language is thus loaded with such agricultural terms as block and treatments which are differentiated in term of varieties of seeds, fertilizers or cultivation methods. The word treatment in analysis of variance is used to refer to any factor in the experiment that is controlled at different levels or values.

5.8.1 One-way classification

The steps in carrying out the analysis are:

1. Calculate variance between the samples. The variance between samples (group) measures the difference between the sample mean of each group and the overall mean weighted by the number of observations in each group. The variance between samples takes into account the random variations from observation to observation. It also measures difference from one group to another. The sum of squares between samples is denoted by SSC. For calculating variance between the samples we take the total of the square of the deviations of the means of various samples from the grand average and divide this total by the degree of freedom. Thus the steps in calculating variance between samples will be:

(a) Calculate the mean of each sample i.e. \bar{X}_1, \bar{X}_2 , etc.

(b) Calculate the grand average \bar{X} . Its value is obtained as follows:

$$\bar{X} = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \dots}{N_1 + N_2 + N_3 + \dots}$$

(c) Take the difference between the means of the various samples and the grand average.

(d) Square these deviations and obtain the total which will give sum of squares between the samples; and

(e) Divide the total obtained in step (d) by the degrees of freedom.

2. Calculate variance within the samples. The variance (or sum of squares) within samples measures those inter-samples differences due to change only. It is denoted by SSE. The variance within samples (groups) measures variability around the mean of each group. Since the variability is not affected by group differences it can be considered a measure of the random variations of values within a group. For calculating variance within the samples we take the total of the sums of square of the deviation of various items from the mean of values of the respective samples and divided this total by the degree of freedom.

Thus, the steps in calculating variance within the samples will be:

(a) Calculate the mean value of each sample. i.e \bar{X}_1, \bar{X}_2 , etc

(b) Take the deviations of the various items in a sample from the mean value of the respective samples:

(c) Square these deviations and obtain the total which gives the sum of square within the samples and

(d) Divide the total obtain in step (c) by the degree of freedom. The degree of freedom is obtained by deduction from the total number of items the number of samples.

3. Calculate the ratio F as follows:

$$F = \frac{\text{Between - column variance}}{\text{Within - column variance}}$$

$$F = \frac{S_1^2}{S_2^2}$$

4. Compare the calculated value of F with the table value of F for the degree of freedom at a certain critical level generally we take 5% level of significance. If the calculated value of F is greater than the table value, it is concluded that the difference in sample means is significant.

Analysis of variance (ANOVA) Table: One way classification

| Source of variation | Sum of Squares | Degree of Freedom | Mean Square MS | Variance Ratio of F |
|---------------------|--------------------------|---------------------------|-----------------------|---------------------|
| Between Samples | SSC | $V_1 = c - 1$ | $MSC = SSC / (c - 1)$ | |
| Within Samples | SSE | $V_2 = n - c$ | $MSE = SSE / (n - c)$ | MSC / MSE |
| Total | SS_T | $n - 1$ | | |

SST = Total sum of squares of variations

SSC = Sum of squares between samples (columns)

SSE = Sum of squares within samples (rows)

MSC = Mean sum of squares between samples

MSE = Mean sum of squares within samples

Compare the calculated value of F with the table value of F for the degrees of freedom at certain critical level. If the calculated value of F is greater than the value, it is concluded that the difference in sample means is significant; on the other hand, if calculated value of F is less than the table value, the difference is not significant and has arisen due to fluctuations of simple sampling.

5.9 Trend

A time series may show fluctuations in the upward and downward directions but there is a distinct tendency for it either to increase or decrease in the long run. For example, if we observe wholesale price level in India from 1951 to 1980, we shall find that despite short- run fluctuations there is a marked tendency of the prices to increase. If we observe the series of the death- rates in India over the last 50-60 years, we observe a distinct tendency for it to fall continuously because of improved health- facilities, prevention of epidemics, medical research, etc. Such long- run tendency of a time series to increase or decrease over a period of time is known as trend or secular trend. Simpson and Kafka define trend as follows:” Trend, also called secular or long- term

trend, is the basic tendency of production, sales income, employment, or the like to grow or decline over a period of time. The concept of trend does not include short-range oscillation but rather steady movement over a long time.”

5.9.1 Straight line trend

Let us discuss the fitting of a straight line first. We had shown in chapter 9 that if the equation is $y=a + bx$ then the application of the method of the least squares gives the following normal equation $\sum y = na + b\sum x$ and $\sum xy = a \sum x + b\sum x^2$ (11.4) these can be solved to obtain the value of a and b. In time series analysis, x is taken as time and y denotes the value of the variable at different points in time.

Solution of (11.4) is greatly simplified if the origin is taken at the middle of the series. This is easily done in that case where n is odd (i.e., the number of year is odd 5, 7, 9,

11, 13, etc.). The new origin here is taken as $\frac{n+1}{2}$ th year. For example, if the data is for the period 1960 to 1970, i.e., 11 year the origin will be the $\frac{11+1}{2} = 6^{\text{th}}$ year or 1965.

Then $\sum x$ will be zero and from (11.4)

$$\begin{aligned} a &= \frac{\sum y}{n} \\ b &= \frac{\sum xy}{\sum x^2} \end{aligned} \quad (11.4a)$$

A problem when n is even. What should be the middle year? The problem is resolved

by taking the origin at the mid- point of $\frac{n^{\text{th}}}{2}$ and $\frac{(n+1)}{2}$ th year. For example, if data for the period 1960 to 1971, i.e., 12 year, are given the origin will be the means of 6th and 7th year, i.e the mid point of 6th and 7th year. Thus 1965.5 would be the origin. Frequently deviations from the origin are multiplied by 2 and this value is denoted by x. This producer simplifies calculations.

5.9.2 Fitting of Parabolic Trend

A parabola is given by the equation:

$$y = a + bx + cx^2$$

Method of least squares can be employed in a similar way as described by us earlier to obtain the coefficient a, b and c. The normal normal equations are

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Choice of a new origin is to be made exactly in the same manner as described above.

In such a case $\sum x$ and $\sum x^2$ are reduced to zero, then

$$\sum y = na + c\sum x^2$$

$$\sum xy = b\sum x^2$$

$$\sum x^2y = a\sum x^3 + c\sum x^4$$

Solution of these give the value of a, b and c.