29. How to find the total distance traveled by a particle moving on the x-axis.

- x(t) = position function
- x'(t) = v(t) = velocity function *|v(t)| = speed function
- x''(t) = v'(t) = a(t) = acceleration function

- The definite integral of velocity on [a, b] gives the <u>displacement</u> of a particle on [a, b].
- To find the <u>position</u> of a particle given its initial position and the velocity function, add the initial position to the displacement (integral of velocity).
- To find the <u>total distance traveled</u> on [a, b] by a particle given the velocity function…
  - **WITH A CALCULATOR** integrate |v(t)| on [a, b]
  - **WITHOUT A CALCULATOR** Split up the velocity function into the positive and negative intervals:
    - Set v(t) = 0, and split the integral at these points
    - Take the absolute value of each integral
    - The sum of these absolute values is the total distance traveled

<u>Example</u>: The velocity of a particle is given by $v(t) = 2t^2 - 2$. Find the total distance traveled when $0<t<2$.

$V(t) = 2t^2 - 2 = 0$

$t^2 = 1$

$t = \pm 1$

$$TDT = \left|\int_0^1 (2t^2 - 2)\, dt\right| + \left|\int_1^2 (2t^2 - 2) dt\right|$$
$$= \left|\left[\frac{2}{3}t^3 - 2t\right]_0^1\right| + \left|\left[\frac{2}{3}t^3 - 2t\right]_1^2\right|$$
$$= \left|\frac{2}{3} - 2\right| + \left|\left(\frac{16}{3} - 4\right) - \left(\frac{2}{3} - 2\right)\right|$$
$$= \frac{4}{3} + \frac{8}{3} = \frac{12}{3} = 4$$

30. How to use the integral to find net change (accumulation) given the rate of change as a graph or as an equation.

- When given a rate of change and asked to find the amount of something accumulated over time, integrate the rate. Make sure to add any given initial condition.

Example: The rate at which water enters a tank is given by $f(t) = 100t^2 \sin\sqrt{t}$ in gallons per hour for $0 \le t \le 7$. At time t = 0, there were 5000 gallons of water in the tank. How many gallons were there in the tank at t = 7?

# of gallons = $5000 + \int_0^7 100t^2 \sin\sqrt{t}\, dt = 13263.807\ gallons$

- If given a graph of the rate, the integral would be the area under the curve.

31. How to solve an exponential growth/decay problem

Example: Bacteria growth is modeled by the differential equation, $\frac{dy}{dx} = ky$. A bacteria culture starts with 500 bacteria and after 12 hours, there are 6000 bacteria. Find an expression for the number of bacteria after t hours, and find the number of bacteria after 2 days.

$$\frac{dy}{dx} = ky \rightarrow y = y_0 e^{kt}$$

<u>Solve for k:</u>

$6000 = 500e^{k(12)}$

$12 = e^{k(12)}$

$\ln 12 = 12k$

$\frac{\ln 12}{12} = k$

$k = .207$

$y = 500e^{.207t}$

At t = 48, $y = 500e^{.207(48)}$

$y = 10368000$

32. How to interpret differential equations geometrically using slope fields and their relationships to derivatives of implicitly defined functions.
33. How to solve a separable differential equation with given boundaries.

****See AP 2008 #5 scoring guidelines**

http://apcentral.collegeboard.com/apc/public/repository/ap08_calculus_ab_sgs.pdf

34. How to compute the area between two curves.

- Find the points of intersection between the two curves by setting them equal to each other.
- Determine which curve is the top (right) and which is the bottom (left).
- Evaluate the integral of the top curve minus the bottom curve (or right curve minus left curve if using y's)

Example:
Find the area of the region bounded by $y = 3x$ and $y = x^3 + 2x^2$.

Points of intersection:
$x^3 + 2x^2 = 3x$
$x^3 + 2x^2 - 3x = 0$
$x(x^2 + 2x - 3) = 0$
$x(x+3)(x-1) = 0$
$x = 0,1,-3$

$$Area = \int_{-3}^{0} \left((x^3 + 2x^2) - 3x\right) dx + \int_{0}^{1} \left(3x - (x^3 + 2x^2)\right) dx$$
$$= \left[x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2\right]_{-3}^{0} + \left[-x^4 - \frac{2}{3}x^3 + \frac{3}{2}x^2\right]_{0}^{1}$$
$$= \frac{71}{6}$$

35. How to compute the volume of a solid of revolution:

a. About the x-axis using the cylindrical shell method:

$Volume = \int Area.\ Area = \pi r^2$, where r is the function (distance from the curve to the axis).

Example:
Find the volume of a solid generated by revolving y = sinx around the x=axis from x = 0 to x = π.

$$Volume = \pi \int_0^{\pi} (\sin x)^2 dx = 4.934$$

b. About the y-axis

Example:
Find the volume of a solid generated by revolving the region enclosed by the y-axis, y = 2, and $x = y^2$ around the y-axis.

$$Volume = \pi \int_0^{2} (y^2)^2 dy = \pi \left[\frac{y^5}{5}\right]_0^2 = \pi \left(\frac{32}{5}\right)$$

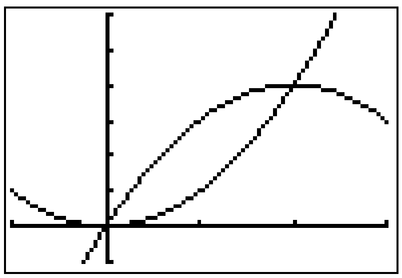
36. How to compute the volume of a solid of revolution:

a. About the x-axis with a hole using the washer method

When there is a hole in a solid of revolution, the cross section will look like a washer. Take the integral of the outer circle minus the inner circle.

Example: Find the volume of the solid generated by revolving the region bounded by the graph of $y = x^2$ and $y = 4x - x^2$ about the x-axis.

Points of intersection: x = 0, 2



$$Volume = \pi \int_0^2 ((4x - x^2)^2 - (x^2)^2)dx$$
$$= \pi \int_0^2 (16x^2 - 8x^3 + x^4 - x^4)dx$$
$$= \pi \left[\frac{16}{3}x^3 - 2x^4\right]_0^2$$
$$= \pi \left(\frac{128}{3} - 32\right) = \frac{32\pi}{3}$$

b. About some line y = a for some a.

Example: Find the volume of the solid generated by revolving the region from above about the line y = 6.

- The outer radius will now go from 6 to x^2, and the inner radius will go from 6 to $4x - x^2$.

$$Volume = \pi \int_0^2 \left((6 - x^2)^2 - \left(6 - (4x - x^2)\right)^2\right) dx$$
$$= \pi \int_0^2 (36 - 12x^2 + x^4 - 36 + 48x - 28x^2 + 8x^3 - x^4)dx$$
$$= \pi \int_0^2 (8x^3 - 40x^2 + 48x)dx$$
$$= \pi \left[2x^4 - \frac{40}{3}x^3 + 24x^2\right]_0^2$$
$$= \pi \left(32 - \frac{320}{3} + 96\right) = \pi \left(128 - \frac{320}{3}\right) = \pi \left(\frac{384}{3} - \frac{320}{3}\right) = \frac{64}{3}\pi$$

37. How to find the volume with a given cross-section.

*Remember, Volume is always equal to the integral of the function of areas of cross sections.

Example: The region bound by the curves $y = 2\sqrt{x}$ and $y = 6$ is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in the region. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Function in terms of y: $x = \left(\frac{y}{2}\right)^2 = \frac{y^2}{4}$

$$Area = (base)(3 \cdot base) = 3(base)^2$$

$$Base = \frac{y^2}{4} - 0 = \frac{y^2}{4}$$

$$Area = 3\left(\frac{y^2}{4}\right)^2$$

$$Volume = 3\int_0^6 \left(\frac{y^2}{4}\right)^2 dy$$